Numerical modelization of thin structures: axisymmetric thermoelastoplastic shells and 1D

Summarized:

One presents a numerical formulation for the modelization of structures to mean surface of particular geometry:

- shells with symmetry of revolution around the axis Oy ,
- invariant shells with unspecified section along the axis Oz .

One describes the isotropic thermoelastoplastic case completely, in the frame of the theories of COILS - KIRCHHOFF and of HENCKY-MINDLIN-REISSNER, as well as the various studied loadings, for the selected isoparametric finite element.

The examples of validation suggested show qualities of the finite element.

Contents

1 Introduction

One is interested in what follows to the mechanical modelization of thin structures to mean surface of particular geometry:

- shells with symmetry of revolution around the axis Oy ,
- shells with invariant unspecified sections along the axis Oz .

More particularly, one limits oneself if the mechanical parameters (materials, loadings) are independent of a direction of space (the circumference for the shells of revolution, the axis Oz shells C_PLAN and D_PLAN).

For the resolution of chained thermomechanical problems, one must use before the finite element of thermal shell describes in [R3.11.01] according to the case in his axisymmetric version, or his invariant plane version according to Oz .

One gives hereafter first of all a progress report on the description of the mechanical model: kinematics, thermoelastoplastic constitutive law. Then one presents the selected finite element, the interpolation and the integration method.

One gives finally some numerical results of application, by comparison with analytical solutions.

2 Continuous problem

the geometry is defined in a unidimensional way:

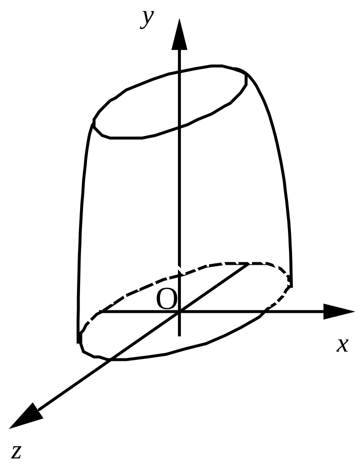
- by the generator in the plane $(O\,xy)$ for a shell of revolution,
- by the section of the shell in the plane $(O\,xy)$ for an invariant shell in z .

In this last case, by analogy with the two-dimensional problems, one considers two cases:

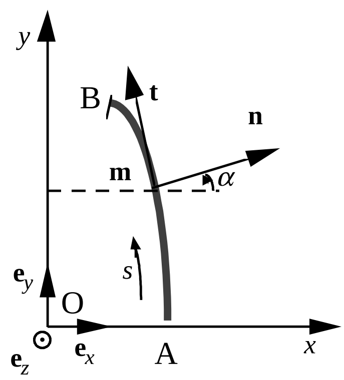
- the case “plane stresses”, i.e. that of a free shell according to the direction Oz , or that of an arc in the plane $O\,xy$,
- the case “plane strains”, i.e. when displacements according to the direction Oz are null.

2.1 Description of the geometry, of the kinematics

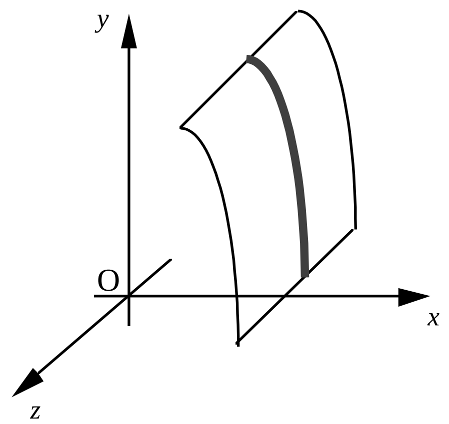
One considers a shell of revolution of axis Oy , or an invariant shell according to the axis Oz . For both, mean surface is defined by the curve $\omega = AB$ in the plane Oxy : ω is a generator for the shell of revolution, or the section for the invariant shell according to Oz .



Appear 2.1-a: Shell of revolution



Appears 2.1-b: Generator



Appears 2.1-c: Shell with invariant section according to Oz

the curve $\omega = AB$ is parameterized by the curvilinear abscisse s. One will note partial derivatives $\frac{\partial}{\partial s}$ by: $,s$.

In a point m of ω one defines the local coordinate system (n, t, e_z) by:

$$t = \frac{\mathbf{Om}_{,s}}{\|\mathbf{Om}_{,s}\|} \quad ; \quad n \wedge t = e_z \; .$$

One notes also the angle α such as:

$$n = \cos\alpha\, e_x + \sin\alpha\, e_y \; .$$

The curvature of ω is defined by:

$$\frac{1}{R} = -n . t_{,s} = \alpha_{,s}$$

In the case of the shell of revolution, the position on the parallel passing by m is noted θ. The tangent vector on this parallel is e_θ. For the generator located in the plane Oxy, $\theta = 0$ and $e_\theta = -e_z$. The radius of curvature of the parallel in m is:

$R_\theta = \frac{r}{\cos\alpha}$ where r is the X-coordinate x of the point m of ω.

On the other hand, for an invariant shell according to z this parallel is a right generator, directed according to e_z, of curvature null.

The kinematical transformations of the shell are defined by the displacement U of the point m of mean surface, like by the rotation β_s of the norm n at the point m. The vector U can be expressed in local base:

$$U_{(s)} = U_{(s)} . t_{(s)} + W_{(s)} . n_{(s)} \; .$$

Or in Cartesian base:

$$U_{(s)}=u_x(s)e_x+u_y(s)e_y \ .$$

The strains of the shell associated with this transformation $(U\ ,\ \beta_s)$ are determined by:

- a membrane strain tensor E ,
- a tensor of variation of curvature K ,
- a vector of strain of distortion tranverse γ .

This last appears in the theory of shells of HENCKY-MINDLIN-NAGHDI and not in that of COILS. According to displacement U and rotation β_s , these quantities are expressed (cf [bib1]):

Case	Shell of revolution	invariant Shell according to Oz
U expressed in local base (n,t,e_z)	$E_{ss}=U_{,s}+\frac{W}{R}$	$E_{ss}=U_{,s}+\frac{W}{R}$
	$E_{\theta\theta}=\frac{1}{r}(-U\sin\alpha+W\cos\alpha)$	
	$K_{ss}=\beta_{s,s}$	$K_{ss}=\beta_{s,s}$
	$K_{\theta\theta}=-\frac{\sin\alpha}{r}\beta_s$	
	$\gamma_s=\beta_s+W_{,s}-\frac{U}{R}$	$\gamma_s=\beta_s+W_{,s}-\frac{U}{R}$
U expressed in global database $(e_x,\ e_y,\ e_z)$	$E_{ss}=u_{y,s}\cos\alpha-u_{x,s}\sin\alpha$	$E_{ss}=u_{y,s}\cos\alpha-u_{x,s}\sin\alpha$
	$E_{\theta\theta}=\frac{u_x}{r}$	
	$K_{ss}=\beta_{s,s}$	$K_{ss}=\beta_{s,s}$
	$K_{\theta\theta}=-\frac{\sin\alpha}{r}\beta_s$	
	$\gamma_s=\beta_s+u_{x,s}\cos\alpha+u_{y,s}\sin\alpha$	$\gamma_s=\beta_s+u_{x,s}\cos\alpha+u_{y,s}\sin\alpha$

Note:

The change of meaning of the curvilinear abscisse s does not modify the values of: $\beta_s,\ E_{ss},E_{\theta\theta}$, but changes the sign of $\alpha,\ U,\ W,\ R,\ K_{ss},\ K_{\theta\theta}$ ˜

In the frame of the theory of COILS, the condition $\gamma_s=0$ (the norms with the shell remain it after strain) results in a direct relationship between rotations β_s and the slope $W_{,s}$. The components of the tensor variation of curvature are according to displacement in the local base:

$$K_{ss}=-W_{,ss}+\frac{U_{,s}}{R}-U\frac{R_{,s}}{R^2}$$

$$K_{\theta\theta}=\frac{\sin\alpha}{r}\left(W_{,s}-\frac{U}{R}\right)$$

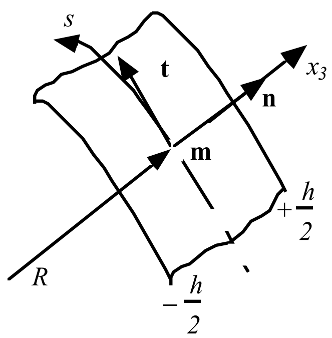
If displacement is expressed in global database:

$$K_{ss}= \frac{1}{R}\left(u_{x,s}\sin\alpha-u_{y,s}\cos\alpha\right)-u_{x,ss}\cos\alpha-u_{y,ss}\sin\alpha$$

$$K_{\theta\theta}=\frac{\sin\alpha}{r}\left(u_{x,s}\cos\alpha+u_{y,s}\sin\alpha\right)$$

One notices that the statement of the variations of curvature according to displacement in theory of COILS is rather complicated and that it utilizes second derivative. If one requires an interpolation conforms i.e. here C^1 , this requires the use of finite elements of high degree.

The tensors E, K, γ make it possible to express the three-dimensional strain ε in the thickness. On [Figure 2.1-d], one compared to the indicates x_3 by the position in $\left]-\frac{h}{2},\frac{h}{2}\right[$ the thickness average fiber, at the point m , of curvilinear abscisse s on ω .



Appear 2.1-d

In a point of the thickness, displacement expresses itself in total reference:

$$U(s,x_3)=\left(u_x(s)-\beta_s(s).x_3\ \sin\alpha(s)\right).e_x+\left(u_y(s)+\beta_s(s).x_3\cos\alpha(s)\right).e_y$$

In order to take account of the variation of metric in the thickness (due to the curvature of mean surface), one defines the functions:

$$\rho_s(x_3)=1+\frac{x_3}{R} \;\; ; \;\; \rho_\theta(x_3)=1+\frac{x_3}{r}.\cos\alpha$$

For a sufficiently thin shell, this correction is negligible:

$$\rho_s\approx 1 \, ; \, \rho_\theta \approx 1$$

In practice this correction carried out if `MODI_METRIQUE=' OUI'` in `AFFE_CARA_ELEM` [U4.42.01] is useless if the ratios h/R and h/R_θ , when they exist, are lower than $1/15$.

In theory of HENCKY-MINDLIN-NAGHDI, the components of the strain tensor ε are:

$$\begin{cases} \varepsilon_{ss}(s, x_3)=\dfrac{1}{\rho_s}(E_{ss}+x_3 K_{ss}) \\ \varepsilon_{\theta\theta}(s, x_3)=\dfrac{1}{\rho_\theta}(E_{\theta\theta}+x_3 K_{\theta\theta}) \quad \text{(only in the case shell of revolution)} \\ \varepsilon_{sx_3}(s, x_3)=\dfrac{1}{2\rho_s}\gamma_s \end{cases}$$

2.2 thermoelastoplastic Equilibrium

One considers that the material constitutive of the shell is thermoelastoplastic isotropic. The usually allowed assumption is made that the transverse normal stress is null: $\sigma_{x_3 x_3}\equiv 0$. The most general constitutive law is written then:

$$\begin{pmatrix} s_{11} \\ s_{22} \\ s_{1x_3} \end{pmatrix}=\begin{pmatrix} C_{1111} & C_{1122} & 0 \\ C_{2211} & C_{2222} & 0 \\ 0 & 0 & C_{11x_3x_3} \end{pmatrix}\begin{pmatrix} \varepsilon_{11}-\varepsilon_{11}^{th} \\ \varepsilon_{22}-\varepsilon_{22}^{th} \\ \varepsilon_{1x_3} \end{pmatrix}$$

where $C(\varepsilon, \mu)$ components C_{ijkl} is the local matrix of behavior in plane stresses and μ represents all the local variables when the behavior is nonlinear. In the continuation index 1 refers to the curvilinear abscisse and 2 to θ or z . With the above definite three-dimensional strains, one associates the components of the tensor then forced σ :

- in the case of a shell of revolution:

$$\begin{cases} \sigma_{ss}=C_{ssss}(\varepsilon_{ss}-\varepsilon_{ss}^{th})+C_{ss\theta\theta}(\varepsilon_{\theta\theta}-\varepsilon_{\theta\theta}^{th}) \\ \sigma_{\theta\theta}=C_{\theta\theta ss}(\varepsilon_{ss}-\varepsilon_{ss}^{th})+C_{\theta\theta\theta\theta}(\varepsilon_{\theta\theta}-\varepsilon_{\theta\theta}^{th}) \\ \sigma_{sx_3}= \;\; C_{ssx_3x_3}\varepsilon_{sx_3} \end{cases}$$

- in the case shell invariant according to the direction z and free in z (“plane stresses”):

$$\begin{cases} \sigma_{ss} = (C_{ssss} - \frac{C_{sszz} C_{zzss}}{C_{zzzz}})(\varepsilon_{ss} - \varepsilon_{ss}^{th}) \\ \sigma_{zz} = 0 \\ \sigma_{sx_3} = C_{ssx_3x_3} \varepsilon_{sx_3} \end{cases}$$

- in the case shell invariant according to the direction z and blocked in z ("plane strains"):

$$\begin{cases} \sigma_{ss} = C_{ssss}(\varepsilon_{ss} - \varepsilon_{ss}^{th}) \\ \sigma_{zz} = C_{zzss}(\varepsilon_{ss} - \varepsilon_{ss}^{th}) \\ \sigma_{sx_3} = C_{ssx_3x_3} \varepsilon_{sx_3} \end{cases}$$

One draws the statement from it from the elastic strain energy of strain, which one will deduce the stiffness matrix according to the kinematics from shell seen in the paragraph [§2.1]:

- in the case shell of revolution:

$$W^{él} = \frac{1}{2} \int_{\omega} \int_0^{2\pi} \int_{-h/2}^{h/2} \left[C_{ssss} \varepsilon_{ss}^2 + C_{\theta\theta\theta\theta} \varepsilon_{\theta\theta}^2 + (C_{ss\theta\theta} + C_{\theta\theta ss}) \varepsilon_{ss} \varepsilon_{\theta\theta} + 2C_{ssx_3x_3} \varepsilon_{sx_3}^2 \right] \left(\rho_s + \rho_\theta - 1 \right) . r\, ds\, d\theta\, dx_3$$

- in the case invariant shell according to z, in "plane stresses":

$$W^{él} = \frac{1}{2} \int_{\omega} \int_{-h/2}^{h/2} \left[(C_{ssss} - \frac{C_{sszz} C_{zzss}}{C_{zzzz}}) \varepsilon_{ss}^2 + 2C_{ssx_3x_3} \varepsilon_{sx_3}^2 \right] \rho_s . ds dx_3$$

- in the case invariant shell according to z, in "plane strains":

$$W^{él} = \frac{1}{2} \int_{\omega} \int_{-h/2}^{h/2} \left[C_{ssss} \varepsilon_{ss}^2 + 2C_{ssx_3x_3} \varepsilon_{sx_3}^2 \right] \rho_s . ds\, dx_3$$

Note:

In thermoelasticity, if one notes E the modulus of YOUNG and ν the Poisson's ratio, one a:

$$C_{iiii} = \frac{E}{1-\nu^2} ; C_{iijj} = \frac{E\nu}{1-\nu^2} \forall (i,j) \in \{1,2\} ; C_{11x_3x_3} = \frac{E}{1+\nu}$$

One defines the following quantities:

- the membrane stiffness of a shell of revolution:

$$[C_{ij}]=\int_{-h/2}^{h/2}\frac{\rho_s+\rho_\theta-1}{\rho_i\rho_j}.\begin{bmatrix}C_{ssss} & C_{ss\theta\theta}\\ C_{\theta\theta ss} & C_{\theta\theta\theta\theta}\end{bmatrix}dx_3$$; who is worth:

$$\frac{Eh}{1-\nu^2}\begin{bmatrix}1 & \nu\\ \nu & 1\end{bmatrix}$$ in elasticity and absence of correction of metric in the thickness;

- the stiffness of membrane-flexure coupling of a shell of revolution:

$$[B_{ij}]=\int_{-\frac{h}{2}}^{\frac{h}{2}}x_3.\frac{\rho_s+\rho_q-1}{\rho_i\rho_j}.\begin{bmatrix}C_{ssss} & C_{ss\theta\theta}\\ C_{\theta\theta ss} & C_{\theta\theta\theta\theta}\end{bmatrix}dx_3$$, which is null in elasticity and absence of correction of metric in the thickness;

- the flexural rigidity of a shell of revolution:

$$[D_{ij}]=\int_{-\frac{h}{2}}^{\frac{h}{2}}x_3^2.\frac{\rho_s+\rho_\theta-1}{\rho_i\rho_j}.\begin{bmatrix}C_{ssss} & C_{ss\theta\theta}\\ C_{\theta\theta ss} & C_{\theta\theta\theta\theta}\end{bmatrix}dx_3$$, which is worth:

$$\frac{Eh^3}{12(1-\nu^2)}\begin{bmatrix}1 & \nu\\ \nu & 1\end{bmatrix}$$ in elasticity and absence of correction of metric in the thickness;

- the transverse stiffness of distortion of a shell of revolution:

$$G_{sx_3}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{\rho_s+\rho_\theta-1}{\rho_s^2}.C_{ssx_3x_3}dx_3$$, which is worth:

$$\frac{Eh}{1+\nu}$$ in elasticity and absence of correction of metric in the thickness.

For an invariant shell according to the direction z , one considers in these statements only the terms $ij=ss$; moreover one must replace there $(\rho_s+\rho_\theta-1)$ by ρ_s : one thus defines the coefficients $C_{ss}^D,\ B_{ss}^D,\ D_{ss}^D$ and $C_{ss}^C,\ B_{ss}^C,\ D_{ss}^C$ for the case, respectively, of plane strains or plane stresses. In elasticity, the coefficients $C_{ss}^C,\ B_{ss}^C,\ D_{ss}^C$, are the products of the coefficients $C_{ss}^D,\ B_{ss}^D,\ D_{ss}^D$ par. $1-\nu^2$ Lastly, the coefficient of transverse stiffness of distortion G_{sx_3} is identical for the three modelizations to the correction of metric near.

One can thus express elastic strain energy according to the strain tensors of shell: E, K, γ by:

- for a shell of revolution:

$$W^{él}=\frac{1}{2}\int_{\omega}\int_{0}^{2\pi}[C_{ss}E_{ss}^{2}+2\mathrm{B}_{ss}E_{ss}K_{ss}+D_{ss}K_{ss}^{2}+C_{\theta\theta}E_{\theta\theta}^{2}+2\mathrm{B}_{\theta\theta}E_{\theta\theta}K_{\theta\theta}+D_{qq}K_{\theta\theta}^{2}$$
$$+2\left(C_{s\theta}E_{ss}E_{\theta\theta}+B_{s\theta}(E_{ss}K_{\theta\theta}+E_{\theta\theta}K_{ss})+D_{s\theta}K_{ss}.K_{\theta\theta}\right)$$
$$+\frac{G_{sx_3}}{2}\gamma_s^2]r.ds.d\theta$$

- for an invariant shell according to z in “plane stresses”:

$$W^{él}=\frac{1}{2}\int_{\omega}\left[C_{ss}^{C}E_{ss}^{2}+2\mathrm{B}_{ss}^{C}E_{ss}.K_{ss}+D_{ss}^{C}K_{ss}^{2}+\frac{G_{sx_3}}{2}\gamma_s^2\right].ds$$

- for an invariant shell according to z in “plane strains”:

$$W^{él}=\frac{1}{2}\int_{w}\left[C_{ss}^{D}E_{ss}^{2}+2\mathrm{B}_{ss}^{D}E_{ss}.K_{ss}+D_{ss}^{D}K_{ss}^{2}+\frac{G_{sx_3}}{2}\gamma_s^2\right].ds$$

formulate these statements, it is necessary to add the potential associated with the thermal stresses, which will be a contribution to the second member (that one will express below in total reference):

- in the case shell of revolution:

$$L_{(V)}^{th}=\int_{\omega}\int_{0}^{2\pi}\int_{-h/2}^{h/2}\left[\alpha\left(T-T^{réf}\right)\left(\left(C_{ssss}+C_{ss\theta\theta}\right)\varepsilon_{ss}+\left(C_{\theta\theta ss}+C_{\theta\theta\theta\theta}\right)\varepsilon_{\theta\theta}\right)\right]r\,d\theta\,dx_3\,ds$$

statement which for an isotropic elastic behavior becomes:

$$L_{(V)}^{th}=\int_{\omega}\int_{0}^{2\pi}\int_{-h/2}^{h/2}\left[\frac{E\alpha}{1-\nu}\left(T-T^{réf}\right)\left(\frac{v_x}{r}-v_{x,s}\sin\alpha+v_{y,s}\cos\alpha+x_3\left(\beta_{s,s}-\frac{\sin\alpha}{r}\beta_s\right)\right)\right]r\,d\theta\,dx_3\,ds$$

- in the case invariant according to z in “plane stresses”:

$$L_{(V)}^{th}=\int_{\omega}\int_{-h/2}^{h/2}\left[\alpha\left(T-T^{réf}\right)\left(C_{ssss}-\frac{C_{sszz}C_{zzss}}{C_{zzzz}}\right)\varepsilon_{ss}\right]dx_3\,ds$$

statement which for an isotropic elastic behavior becomes:

$$L_{(V)}^{th}=\int_{\omega}\int_{-h/2}^{h/2}\left[E\alpha\left(T-T^{réf}\right)\left(-v_{x,s}\sin\alpha+v_{y,s}\cos\alpha+x_3\beta_{s,s}\right)\right]dx_3\,ds$$

- in the case invariant according to z in "plane strains":

$$L_{(V)}^{th}=\int_{\omega}\int_{-h/2}^{h/2}\left[\alpha\left(T-T^{réf}\right)C_{ssss}\,\varepsilon_{ss}\right]dx_3\,ds$$

  statement which for an isotropic elastic behavior becomes:

$$L_{(V)}^{th}=\int_{\omega}\int_{-h/2}^{h/2}\left[\frac{E\alpha}{1-\nu}\left(T-T^{réf}\right)\left(-v_{x,s}\sin\alpha+v_{y,s}\cos\alpha+x_3\beta_{s,s}\right)\right]dx_3\,ds$$

In these three statements, one deliberately neglected the correction of metric in the thickness (terms in ρ_s, ρ_θ seen for the stiffness). Moreover the temperature T which appears is defined by the model thermal shell in three fields (cf [R3.11.01]):

$$T(s,\ x_3)=T^m(s).\left(1-\left(\frac{x_3}{h}\right)^2\right)+T^s(s)\frac{x_3}{2\mathrm{h}}\left(1+\frac{x_3}{h}\right)+T^i(s)\frac{x_3}{2\mathrm{h}}\left(-1+\frac{x_3}{h}\right)$$

From the set of these statements, one deduces the tensors from forces generalized N and M (normal force and bending moments) associated with the generalized strains E and K by the principle of the virtual wors. They are related to the tensor of the three-dimensional $\tau_{\alpha\beta}$ stresses by:

$$N_{\alpha\beta}=\int_{-h/2}^{h/2}\tau_{\alpha\beta}\,dx_3$$
$$M_{\alpha\beta}=\int_{-h/2}^{h/2}x_3\,.\,\tau_{\alpha\beta}\,dx_3$$

(where one neglected the variations of metric in the thickness).

Note:

Transverse energy of shears

The model of shell presented above, said HENCKY-MINDLIN-NAGHDI, rests on a kinematical assumption: the parameters W and β_s indicate the normal displacement of the point m of mean surface ω and the rotation of the normal vector n .

One also frequently the model finds known as REISSNER which rests on a static assumption of the transverse distribution of the shearing stresses. The kinematical parameters deduced W and β_S in this model are weighted averages in the thickness of normal displacement and local rotations. If one wishes to place oneself in this frame, it is enough to affect the coefficient $\kappa=5/6$ at the end of transverse energy of shears (in γ_s^2). (cf [bib7], [bib9]).

Lastly, if one wants, for a thin shell, to be located in the frame of the model of - KIRCHHOFF COILS, one can neutralize the energy of shears with a great value of κ (who penalizes the condition $\gamma_s=0$), for example $10^6 h/R$, where h is the thickness and R a characteristic radius of curvature or a distance characteristic of the loadings: (cf [feeding-bottle 2]). In practice the user can inform the value of κ under key word `A_CIS` of the command `AFFE_CARA_ELEM` [U4.42.01].

3 Formulation of the finite element. Discretization

3.1 Description of the selected finite element

3.1.1 Motivations

the choice of frame HENCKY-MINDLIN-NAGHDI to describe the kinematics of shell, presented to the paragraph [§2], led to statements of the strains where the derivatives are limited to order 1, contrary to the model of LOVE-KIRCHHOFF. This offers the advantage of being able to use a finite element of a restricted nature while ensuring conformity. The natural choice is the element of LAGRANGE $P2$, isoparametric, which makes it possible to have a fine representation of a curved geometry and good estimates of the stresses.

The degrees of freedom are of course displacements and the rotations.

As it is known as previously, the model of LOVE-KIRCHHOFF can be recovered by penalization for a very κ large parameter affecting the transverse energy of shears.

This formulation joined the category of shells of the finite elements known as “degenerated”, i.e. built by injecting the kinematics of shell in elements of three-dimensional continuums: cf [bib10].

As for all the finite elements of shells, of the particular aspects must be analyzed: the taking into account of the rigid modes and the risks of blocking of membrane or shears.

In the case of the axisymmetric shell of revolution, there is only one rigid mode: translation according to the axis of symmetry Oy .

On the other hand, in the case of the invariant shell according to the direction Oz , one has three rigid modes: two translations in the plane (xOy) and rotation around Oz .

So that the finite element is powerful, it is necessary that the approximations retained for the description of displacement ensure an exact representation of the strain state null (rigid mode). In practice, as the notion of rigid mode is defined compared to the total reference one thus decides to describe displacements in global database (e_x, e_y) , in which the rigid modes (functions closely connected) are represented by the selected interpolation.

As for the risks of blocking out of membrane and transverse shears, the usual processing consists in a selective numerical integration (cf [bib2]), but the practice reveals that these phenomena seldom appear for the shells of revolution.

3.1.2 General presentation of the element

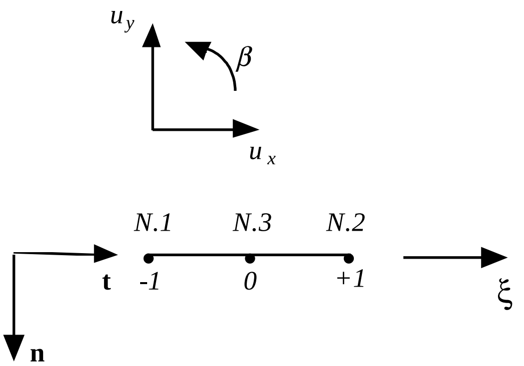
the selected element of reference is quadratic, isoparametric with three nodes and three degrees of freedom per node. These degrees of freedom are:

u_x, u_y : components of displacement U in total reference

β_s : rotation around e_z norm n .

See [Figure 3.1.2-a].

This element is a generalization of the curved beam element plane. It is well adapted to the discretization of the shells with variable meridian R curvature, cf [bib2].

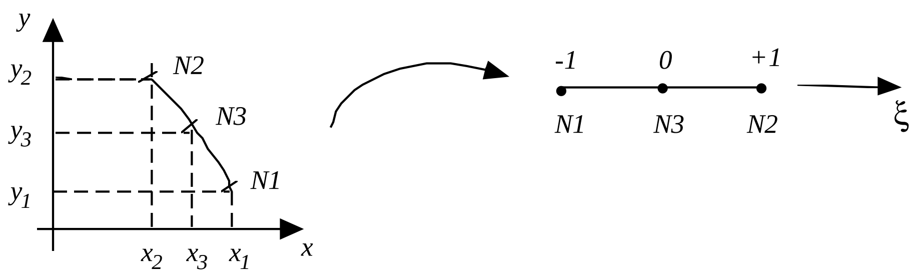


Appear 3.1.2-a: Element of reference

the shape functions (basic) are the polynomials of LAGRANGE:

$$\hat{N}_1(\xi)=\xi\frac{-1+\xi}{2};\hat{N}_2(\xi)=\xi\frac{1+\xi}{2};\hat{N}_3(\xi)=1-\xi^2$$

3.1.3 Transformations finite element/finite element of reference



the geometry is interpolated using the coordinates $(x_i,\ y_i)$ of the three nodes $N1, N3, N2$:

$$x(\xi)=\sum_{i=1}^{3} x_i\hat{N}_i(\xi)\ ;\ y(\xi)=\sum_{i=1}^{3} y_i\hat{N}_i(\xi)$$

In the same way using the degrees of freedom $\left(u_{x_i},\, u_{y_i},\, \beta_{s_i}\right)$ on the nodes, one a:

$$u_x(\xi)=\sum_{i=1}^{3} u_{x_i}\hat{N}_i(\xi)\;;\; u_y(\xi)=\sum_{i=1}^{3} u_{y_i}\hat{N}_i(\xi)$$
$$\beta_s(\xi)=\sum_{i=1}^{3} \beta_{s_i}\hat{N}_i(\xi)$$

One also needs the jacobian of the transformation:

$$m(\xi)=\frac{ds}{d\xi}(\xi)=\sqrt{\left(x_{,\xi}\right)^2+\left(y_{,\xi}\right)^2}$$

And vectors of the local base:

$$t(\xi)=\frac{1}{m(\xi)}\left(x_{,\xi}e_x+y_{,\xi}e_z\right)$$
$$n(\xi)=\frac{1}{m(\xi)}\left(y_{,\xi}e_x-x_{,\xi}e_z\right)$$

Finally:

$$\cos\,\alpha=\frac{y_{,\xi}}{m(\xi)}\;;\sin\,\alpha=\frac{-x_{,\xi}}{m(\xi)}$$

the meridian curvature is obtained by:

$$\frac{1}{R}=-\left(n\,.\,t_{,\xi}\right).\frac{d\xi}{ds}=\frac{1}{m^3(\xi)}\left(x_{,\xi}.\,y_{,\xi\xi}-y_{,\xi}.\,x_{,\xi\xi}\right)$$

Because of the interpolation $P2$, the second derivative which appears below are expressed using the coordinates of the three nodes by:

$$x_{,\xi\xi}=x_1+x_2-2\,.\,x_3 \qquad y_{,\xi\xi}=y_1+y_2-2\,.\,y_3$$

3.1.4 surface Numerical integration

For numerical integrations along the element one uses a formula of numerical integration to four Gauss points, single for all the terms to be integrated. This formula reveals the blockings mentioned in the paragraph [§3.1.1] in the event of extremely localised plasticization. One thus advises to avoid the use of these elements in plasticity for the moment. The formula of numerical integration to four Gauss points will be replaced later on by a formula with two Gauss points supposed avoiding these nuisances.

3.1.5 Numerical integration in the thickness

For an elastic behavior, insofar as it is admitted that one limits oneself to uniform elastic characteristics in the thickness, the stiffness $[C_{ij}]$, $[B_{ij}]$, $[D_{ij}]$ and G_{sx_3} defined in the paragraph [§2.2] are calculated exactly.

For a nonlinear behavior, one subdivides the initial thickness out of N layers of thickness identical numbered in the meaning of the norm to the mean surface of the element. For each layer one uses three points of integration. The points of integration are located in higher skin of layer, in the middle of the layer and in lower skin of layer. For N layers, the number of points of integration is of $2N+1$. One advises to use from 3 to 5 layers in the thickness for a number of points of integration being worth 7,9 and 11 respectively.
For each layer, one calculates the state of the stresses $(\sigma_{11,}\sigma_{22,}\sigma_{12})$ and all the local variables, in the middle of the layer and in skins higher and lower of layer, starting from the local plastic behavior and of the local strain field $(\varepsilon_{11,}\varepsilon_{22,}\varepsilon_{12})$. The positioning of the points of integration enables us to have the rightest estimates, because not extrapolated, in skins lower and higher of layer, where it is known that the stresses are likely to be maximum. The plastic behavior does not understand for the moment the terms of transverse shears which are treated in an elastic way, because the transverse shears are uncoupled from the membrane behavior in plane stresses.

Cordonnées of the points	Weight ω_l
$\xi_1=-1$	$1/3$
$\xi_2=0$	$4/3$
$\xi_3=+1$	$1/3$
$\int_{-1}^{1} y(\xi)d\xi=$	$\sum_{i=1}^{n}\omega_i y(\xi_i)$

Formulates numerical integration for a layer in the thickness in plasticity

For a thermo-elastic behavior, one uses integration, by layer in the thickness $]-\frac{h}{2},+\frac{h}{2}[$ described previously in the nonlinear field, of the thermomechanical terms seen in the paragraph [§2.2]. It is then necessary to use `STAT_NON_LINE` with an elastic behavior.

Note:

One already mentioned with [§2.2]. and in [R3.07.04] that the value of the coefficient of correction in transverse shears for the shell elements was obtained by identification of elastic complementary energies after resolution of the equilibrium 3D. This method is not usable any more in elastoplasticity and the choice of the coefficient of correction in transverse shears is posed then. The transverse terms of shears are thus not affected by plasticity and are treated elastically, for want of anything better. If one places oneself in theory of Coils-Kirchhoff for a value of this coefficient of $10^6 h/R$ (h being the thickness of the shell and R its average radius of curvature) the transverse terms of shears become negligible and the approach is more rigorous.

3.2 Formulation of the elementary terms

3.2.1 Masses, center of gravity, matrix of inertia

In the case of the shells of revolution, the mass is worth:

$$\int_{\omega}\int_{0}^{2\pi}\int_{-h/2}^{h/2}\rho(\rho_s+\rho_\theta-1)\,dx_3\,r\,d\theta\,ds=\int_{\omega}\int_{0}^{2\pi}\rho\,h\,r\,d\theta\,ds=2\pi\rho h\int_{\omega}r\,ds$$

where ρ is the presumedly constant density of the element.

The position of the center of inertia is given in the reference $Oxyz$ of [§2.1] by:

$$x_G=0$$

$$y_G=\frac{\int_{\omega}\mathrm{yr}\,ds+\frac{h^2}{12}\int_{\omega}\sin\alpha\left(\frac{1}{R}+\frac{\cos\alpha}{r}\right)r ds}{\int_{\omega}r\,ds}$$

$$z_G=0$$

the terms of the matrix of inertia compared to O in the reference $Oxyz$ of [§2.1] have then as a statement:

$$I_{xx/O}=2\pi\rho\int_{\omega}\left[h\left(\frac{x^2}{2}+y^2\right)+\frac{h^3}{12}\left(\sin^2\alpha+\frac{\cos^2\alpha}{2}+\delta x\cos\alpha+2\delta y\sin\alpha\right)\right]r ds$$

$$I_{yy/O}=2\pi\rho\int_{\omega}\left[hx^2+\frac{h^3}{12}\left(\cos^2\alpha+2\delta x\cos\alpha\right)\right]r ds$$

$$I_{zz/O}=2\pi\rho\int_{\omega}\left[h\left(\frac{x^2}{2}+y^2\right)+\frac{h^3}{12}\left(\sin^2\alpha+\frac{\cos^2\alpha}{2}+\delta x\cos\alpha+2\delta y\sin\alpha\right)\right]r ds$$

where

$$\delta=\frac{1}{R}+\frac{\cos\alpha}{r}\ .$$

In the case of the invariant shells according to Oz, the mass is worth:

$$\int_{\omega}\int_{-h/2}^{h/2}\rho\,\rho_s\,dx_3\,ds=\rho h\int_{\omega}ds\ .$$

The position of the center of inertia is defined in the reference Oxy of [§2.1] by:

$$x_G=\frac{\int_{\omega}x\,ds+\frac{h^2}{12}\int_{\omega}\frac{\cos\alpha}{R}ds}{\int_{\omega}ds}$$

$$y_G=\frac{\int_{\omega}y\,ds+\frac{h^2}{12}\int_{\omega}\frac{\sin\alpha}{R}ds}{\int_{\omega}ds}$$

the terms of the matrix of inertia compared to O in the reference $Oxyz$ of [§2.1] have then as a statement:

$$I_{xx/O}=\rho\int_{\omega}\left[hy^2+\frac{h^3}{12}(\sin^2\alpha+2\,\delta\,y\sin\alpha)\right]ds$$
$$I_{xy/O}=I_{yx/O}=\rho\int_{w}\left[hxy+\frac{h^3}{12}(\sin\alpha\cos\alpha+\delta\,x\sin\alpha+\delta\,y\cos\alpha)\right]ds$$
$$I_{yy/O}=\rho\int_{\omega}\left[hx^2+\frac{h^3}{12}(\cos^2\alpha+2\,\delta\,x\cos\alpha)\right]ds$$
$$I_{zz/O}=\rho\int_{\omega}\left[h(x^2+y^2)+\frac{h^3}{12}(1+2\,\delta\,x\cos\alpha+2\delta\,y\sin\alpha)\right]ds$$

where $\delta=\frac{1}{R}$.

The terms in $\frac{h^2}{12}$ for the centres of inertia and $\frac{h^3}{12}$ the matrixes of inertia are not taken into account in the programming. That amounts neglecting the variation of metric with the curvature in the computation of these terms.

3.2.2 Mass matrix

the term: $\int_{\omega}\int_0^{2\pi}\int_{-h/2}^{h/2}\rho\,v(s,x_3).\bar{v}(s,x_3)\,r\,dx_3\,d\theta\,ds$, of kinetic energy is treated by considering the constant density ρ in the thickness and the correction of metric due to the negligible curvature. The intégrande is burst in three terms:

- $\rho\,h(u_x.\bar{u}_x+u_y.\bar{u}_y)$ kinetic energy of translation
- $\rho\frac{h^3}{12}\beta_{s.}\bar{\beta}_s$ kinetic energy of rotation
- $\rho\frac{h^3}{12}\delta\left(-\sin\alpha(u_x\bar{\beta}_s+\bar{u}_x\beta_s)+\cos\alpha(u_y\bar{\beta}_s+\bar{u}_y\beta_s)\right)$ kinetic energy of coupling, with:

$\delta=\frac{1}{R}+\frac{\cos\alpha}{r}$ for the case axisymmetric shell of revolution.

$\delta=\frac{1}{R}$ for the case invariant shell according to Oz (moreover in this case the integral disappears $\int_0^{2\pi}\ r\,d\theta$).

3.2.3 Second member of centrifugal force

In the case of the shells of revolution, one considers a vector rotation: $\Omega=\omega_2 . e_y$, carried by the axis of revolution. The term of the second corresponding member is:

$$\int_\omega \int_0^{2\pi} \int_{-h/2}^{h/2} \rho\,\omega_2^2 . r\left(\bar{u}_x-\bar{\beta}_s . x_3 \sin\alpha\right) dx_3\, r\, d\theta\, ds$$
$$=\int_\omega \int_0^{2\pi} h\,\rho\,\omega_2^2 r^2 . \bar{u}_x\, d\theta\, ds$$

(one neglects the correction of metric in the thickness).

In the case of the invariant shells according to Oz , a vector rotation is considered: $\Omega=\omega_3 . e_z$, perpendicular to the plane of the section ω .

The second member is then:

$$\int_\omega h\,\rho\,\omega_3^2\left(x . \bar{u}_x+y . \bar{u}_y\right) ds$$

3.2.4 Second member of gravity

In the case of the shells of revolution, gravity is directed according to e_y .

The second member is:

$$\int_\omega \int_0^{2\pi} \rho\, g\, h\, \bar{u}_y\, r\, d\theta\, ds$$

formulate the invariant shells according to Oz , this one is directed in the plane xOy $g=g_x e_x+g_y e_y$.

The second member is:

$$\int_\omega \rho\left(g_x . e_x+g_y . e_y\right) ds$$

3.2.5 Second member of distributed loads

These distributed loads can be two forces in the plane (xOy) and the couple M_z carried by the axis Oz . Two forces, which one considers that they are applied to mean surface ω , could be provided in total or (e_x, e_y) local reference (t, n) . The second member is:

$$\int_\omega \int_0^{2\pi} \left(F_x \bar{u}_x+F_y \bar{u}_y+M_z \bar{\beta}_s\right) r\, d\theta\, ds$$

(in invariant shell according to z , the integral $\int_0^{2\pi} r\, d\theta$ disappears).

Note:

The specific actions are treated as nodal forces where they are applied, since they work in the degrees of freedom of the finite element.

3.3 Computation of the strains and the stresses

After resolution, one has the possibility with operator CALC_CHAMP [U4.81.04] of calculating with the nodes the elementary fields according to:

- generalized strains $E_{\alpha\beta}$, $K_{\alpha\beta}$: option DEGE_ELNO,
- three-dimensional strains $\varepsilon_{\alpha\beta}$ on average fiber and in skins internal and external (with or without correction of curvature): option EPSI_ELNO,
- three-dimensional stresses $\sigma_{\alpha\beta}$ on average fiber and in skins internal and external (with or without correction of curvature): option SIGM_ELNO,
- forces generalized $N_{\alpha\beta}$, $M_{\alpha\beta}$ (with or without correction of curvature): option EFGE_ELNO.

These values with the nodes are obtained by extrapolation starting from the values with Gauss points of the element, according to the exposed method in [bib4] [R3.06.03].

Lastly, one can have also the values $N_{\alpha\beta}$, $M_{\alpha\beta}$ with Gauss points of the element: option SIEF_ELGA.

No postprocessing of stresses or generalized forces is for the moment available for nonlinear behaviors materials.

4 Validation - Case test

One considers hereafter, to judge capacities of this formulation, some examples of application (cf [bib10]).

4.1 Roll under pressure interns

One studies a vertical roll subjected to a pressure interns p constant on the part $y<0$, and null on $y>0$: to see [Figure 4.1-a].

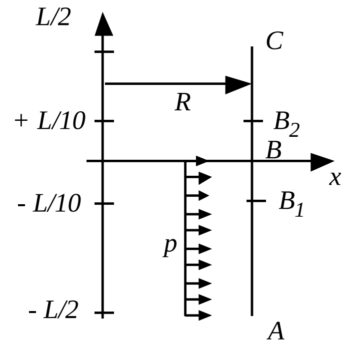


Figure 4.1 - a: Rolls under axisymmetric pressure

the radius is: $R=4m$, the thickness $t=0.25m$, the length $L=10m$. This one is selected so that the edge effects free $y=\pm L/2$ are negligible on the solution (into axisymmetric, L must check: $\frac{1}{2}L>3\sqrt{Rt}=3m$ here).

The material is elastic $(E=1Pa,\ \nu=0.3)$.

The boundary conditions are: $p=1\mathrm{N}/m^2$, vertical displacement in A no one.

One the model chooses the solution obtained by LOVE-KIRCHHOFF.

To reach it numerically, one takes as shear coefficient: $\kappa=10^6$, to inhibit the distortions γ_s . The analytical solution is:

for $y\le 0\ :\ u_x(y)\ =\ \frac{P}{8\alpha^4 D}(2-e^{\alpha y}\cos\alpha y),\ \beta_s(y)\ =\ \frac{P}{8\alpha^3 D}e^{\alpha y}(\cos\alpha y-\sin\alpha y)$

$y\ge 0\ :\ u_x(y)\ =\ \frac{P}{8\alpha^4 D}e^{-\alpha y}\cos\alpha y,\qquad \beta_s(y)\ =\ \frac{P}{8\alpha^3 D}e^{-\alpha y}(\cos\alpha y+\sin\alpha y)$

with $D=\frac{Et^3}{12(1-\nu^2)}\quad 4\alpha^4=\frac{Et}{DR^2}$.

The forces generalized are $(\sin\alpha=0)$:

$$N_{\theta\theta}=\frac{Et}{R}u_x(y)\ ;\ M_{ss}=Du_x^{''}(y)=\frac{p}{4\alpha^2}e^{-|y|}\ \sin\alpha\, y$$

the three-dimensional stresses are:

$$\sigma^{\theta\theta}=\frac{N_{\theta\theta}}{t}+12\frac{M_{\theta\theta}x_3}{t^3}\ ;\ \sigma^{ss}=12\frac{M_{ss}x_3}{t^3},$$ from where:

for $y\leq 0$:

$$\begin{cases}\sigma^{\theta\theta}(y,x_3)=\frac{pR}{t}\left(1-\frac{e^{\alpha y}}{2}\left(\cos\alpha y+2\nu\frac{x_3}{t}\sqrt{\frac{3}{1-\nu^2}}\ \sin\alpha\, y\right)\right)\\ \sigma^{ss}(y,x_3)=\frac{pR}{t}.\frac{x_3}{t}\sqrt{\frac{3}{1-\nu^2}}e^{\alpha y}\ \sin\alpha y\end{cases}$$

for $y\geq 0$:

$$\begin{cases}\sigma^{\theta\theta}(y,x_3)=\frac{pR}{t}\frac{e^{-\alpha y}}{2}\left(\cos\alpha\, y-2\nu\frac{x_3}{t}\sqrt{\frac{3}{1-\nu^2}}\ \sin\alpha y\right)\\ \sigma^{ss}(y,x_3)=\frac{pR}{t}.\frac{x_3}{t}\sqrt{\frac{3}{1-\nu^2}}e^{-\alpha y}\ \sin\alpha y\end{cases}$$

For a regular mesh of one hundred meshes and two hundred nodes, one finds:

	Reference	Aster	% difference
Displacement U_x			
Point A	63.9488	63.922	– 0.042
Item B	32.000	32.005	0.015
Item C	0.05120	0.08755	
Rotation β_S			
Point A	0.06583	0.04057	
Item B	41.133	41.165	0.078
normal Force $N_{\theta\theta}$			
Point B	2.0000	2.0003	0.015
Point B_1 (with $-L/10$)	3.84429	3.8442	0.002
Moment M_{ss}			
Point B_1	4.01497 10-2	4.013 10-2	0.05

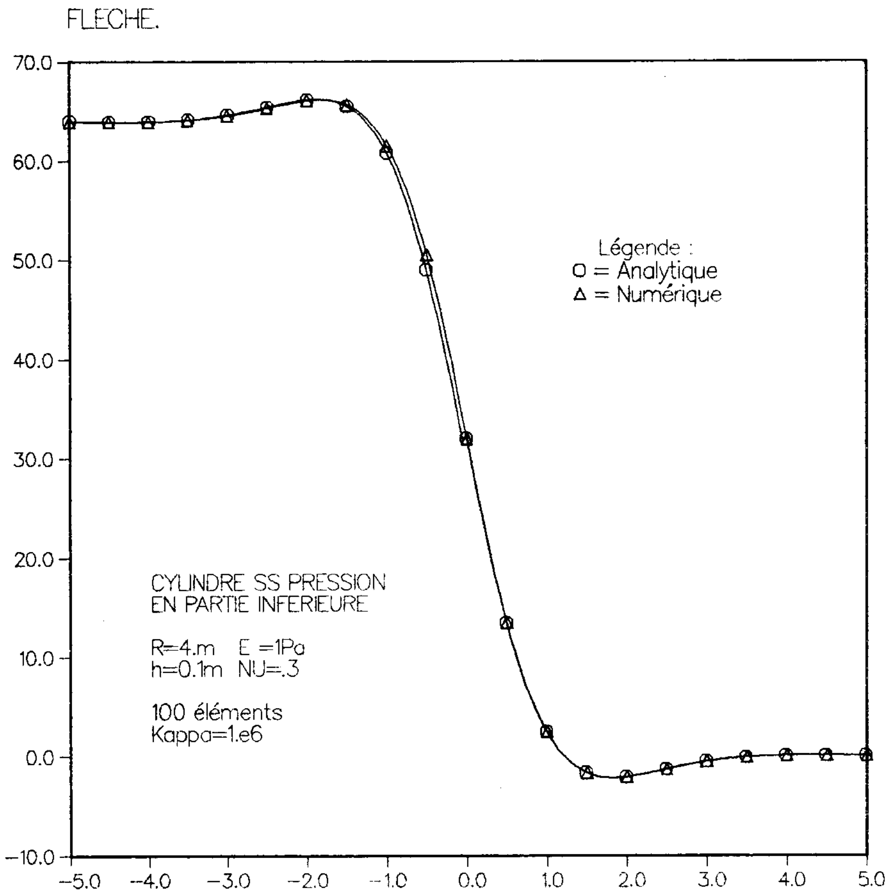
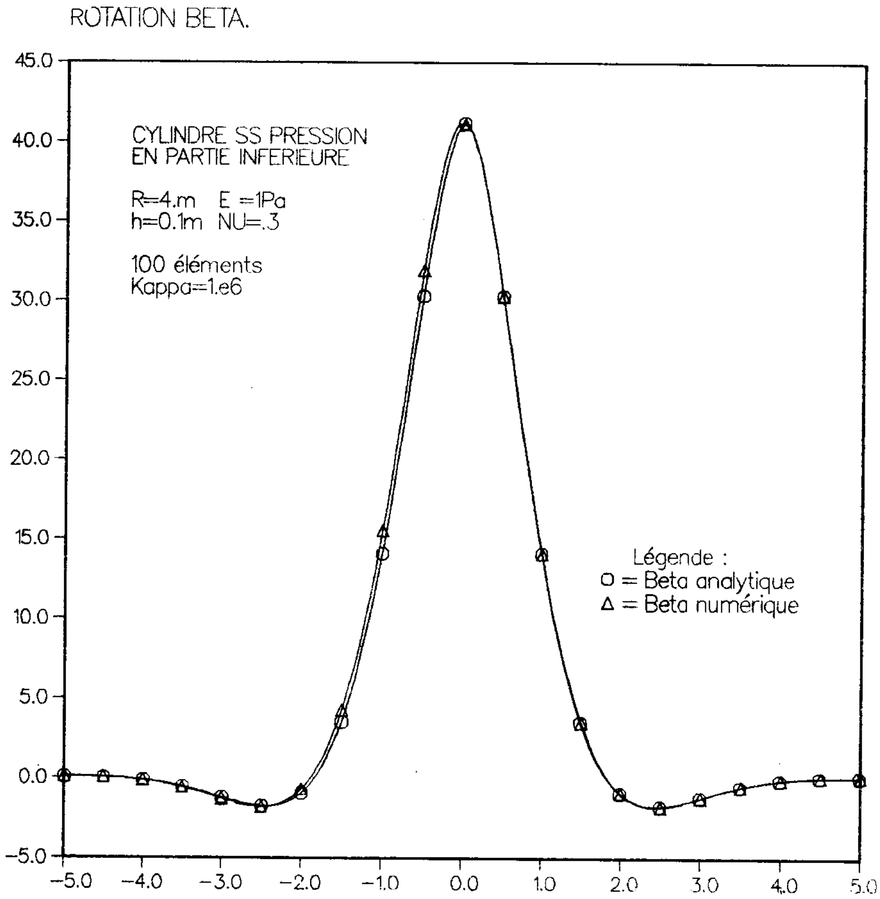
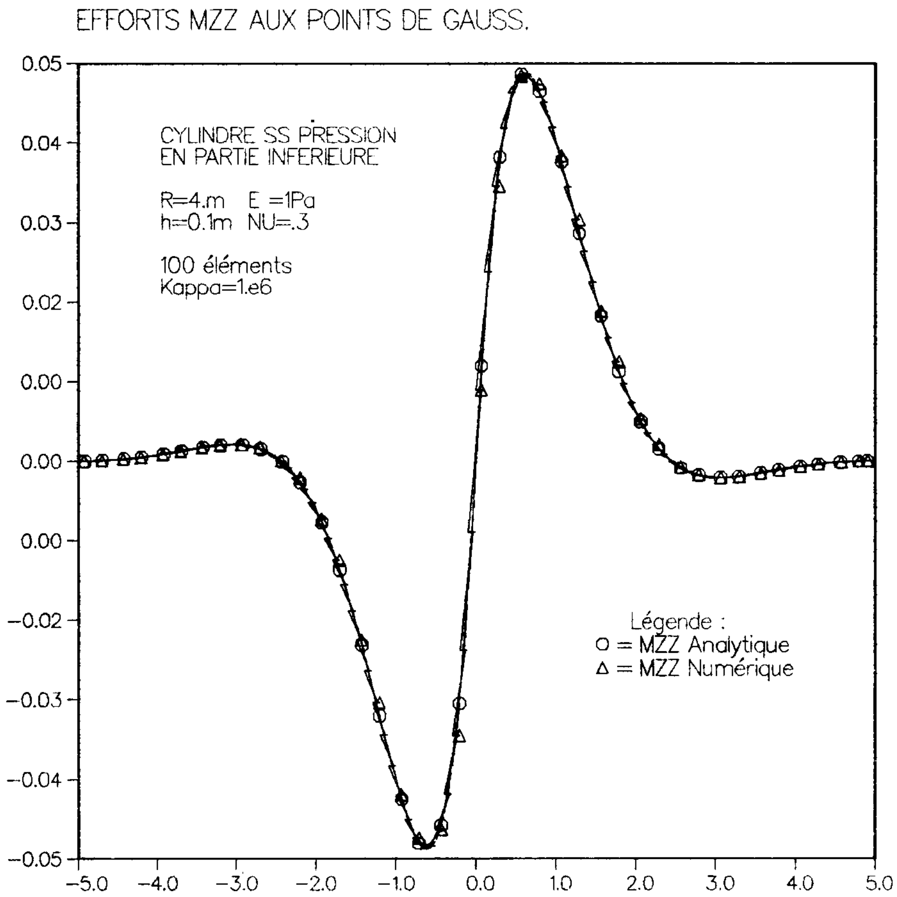


Figure 4.1-b: Cylinder under pressure marks with arrows



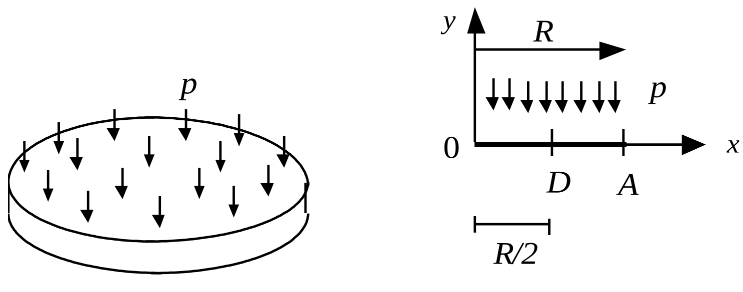
Appears 4.1-c: Rotation of the cylinder under pressure.



Appear 4.1-d: Axial bending moments of the cylinder under pressure

4.2 Plates circular clamped under uniform pressure [V3.03.100]

One considers the plate of radius $R=1\,m$, of thickness $t=0{,}1\,m$ (see [Figure 4.2-a] below) embedded on its circumference.



Appear 4.2-the

material is elastic $(E=1.Pa\,,\nu=0.3)$. The pressure is: $p=1.N/m^2$.

The boundary conditions are in O : $\beta_s=0.$, in A : $u_x=u_y=0.$, $\beta_s=0$.

One is interested in the solutions of the models of REISSNER $\left(\kappa=\frac{5}{6}\right)$ and LOVE-KIRCHHOFF (one will take $\kappa=10^6$).

The analytical solution is for the deflection:

$$u_y(x)=-\frac{pR^4}{64\,D}\left(1-\left(\frac{x}{R}\right)^2\right)\left(1-\left(\frac{x}{R}\right)^2+\varphi\right).$$

with $D = \frac{Et^3}{12(1-\nu^2)}$; $\varphi = \frac{16}{5}\left(\frac{t}{R}\right)^2 \frac{1}{1-\nu}$ si $\kappa = \frac{5}{6}$; $\varphi = 0$ for the solution - KIRCHHOFF COILS.

The distortion is indeed: $\gamma_s(x)=-\frac{pR^2}{16\,D}\frac{x}{2}\varphi$.

Rotation β_s is: $\beta_s(x) = \frac{pR^2}{16D}\, x\left(1 - \left(\frac{x}{R}\right)^2\right)$.

The variations of curvature are $(\sin\alpha = +1)$:

$$K_{ss}(x) = -\frac{pR^2}{16D}\left(1-3\left(\frac{x}{R}\right)^2\right)$$

$$K_{\theta\theta}(x) = -\frac{pR^2}{16D}\left(1-\left(\frac{x}{R}\right)^2\right)$$

The bending moments are $(\sin\alpha=+1)$:

$$M_{ss}(x) = \frac{pR^2}{16}\left((3+\nu)\left(\frac{x}{R}\right)^2 - (1+\nu)\right)$$

$$M_{\theta\theta}(x) = \frac{pR^2}{16}\left((1+3\nu)\left(\frac{x}{R}\right)^2 - (1+\nu)\right)$$

The stresses are written:

$$\sigma_{ss}(x,x_3) = \frac{E}{1-\nu^2}x_3[K_{ss}(x)+\nu K_{\theta\theta}(x)]$$

$$\sigma_{\theta\theta}(x,x_3) = \frac{E}{1-\nu^2}x_3[K_{\theta\theta}(x)+\nu K_{ss}(x)]$$

One notices independence in κ rotation, variations of curvature and bending moments. In the center O of the plate:

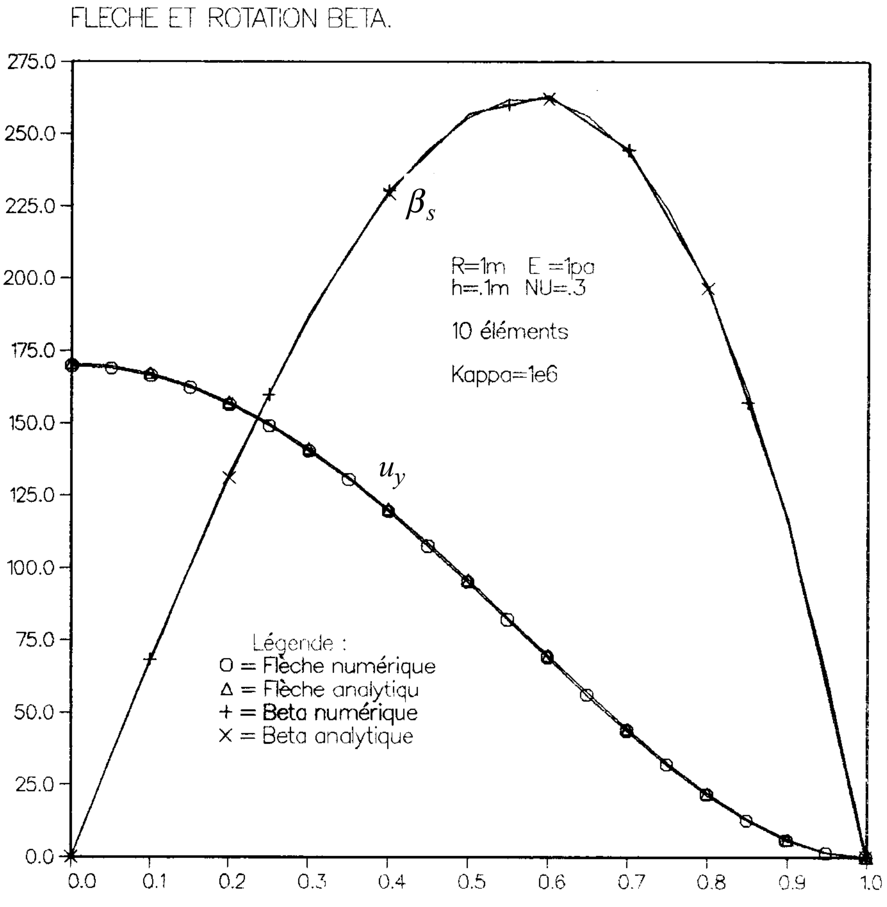
$$u_y(0)=-\frac{pR^4}{64D}(1+\varphi),\ \ M_{ss}(0)=M_{\theta\theta}(0)=-\frac{pR^2}{16}(1+\nu),$$

$$K_{ss}(0)=K_{\theta\theta}(0)=-\frac{pR^2}{16D}.$$

$$s_{ss}(0,\pm t/2)=s_{\theta\theta}(0,\pm t/2)=m\frac{E}{1-\nu}\frac{t}{2}\frac{pR^2}{16D}\ .$$

It is noticed that one is in compression in higher skin of plate.

With the fixed support A : $M_{ss}(R)=\frac{pR^2}{8}$; $M_{\theta\theta}(R)=\nu\frac{pR^2}{8}$.



Appear 4.2-b: Marks with arrows, rotation of a circular plate embedded

For a regular mesh of 10 meshes (21 nodes) one finds:

		Reference	*Aster*	% difference
Displacement u_y				
Not D $\left(\kappa=\frac{5}{6}\right)$		– 101.827	– 101.7769	0.049
	LOVE-KIRCHHOFF	– 95.9765	– 95.0395	0.978
Point O $\left(\kappa=\frac{5}{6}\right)$		– 178.424	– 178.368	0.031
	LOVE-KIRCHHOFF	– 170.625	– 169.761	0.507
Rotation β_s				
Not D $\left(\kappa=\frac{5}{6}\right)$			256.001	0.024
	LOVE-KIRCHHOFF	255.94	257.123	0.462
Variation of curvature K_{ss}				
Not D $\left(\kappa=\frac{5}{6}\right)$			173.406	1.60
	LOVE-KIRCHHOFF	170.625	162.765	4.61
Variation of curvature $K_{\theta\theta}$				
D $\left(\kappa=\frac{5}{6}\right)$			514.001	0.024
	LOVE-KIRCHHOFF	511.875	512.242	0.46
Moment M_{ss}				
Not O $\left(\kappa=\frac{5}{6}\right)$		– 0.08125	– 0.081751	+0.617
	LOVE-KIRCHHOFF		– 0.081394	– 0.18
Point A $\left(\kappa=\frac{5}{6}\right)$		formulates	0.12373	– 1.02
	LOVE-KIRCHHOFF		0.10717	– 14.3
Moment $M_{\theta\theta}$				
O $\left(\kappa=\frac{5}{6}\right)$		– 0.08125	– 0.081751	0.617
	LOVE-KIRCHHOFF		– 0.081394	– 0.18
Point A $\left(\kappa=\frac{5}{6}\right)$		0.03750	0.037121	– 1.01
	LOVE-KIRCHHOFF		0.032146	– 14.3

One notices that the solution LOVE-KIRCHHOFF ($k=10^6$) is less quite approximate than that by REISSNER $\left(k=\frac{5}{6}\right)$ on the variations of curvature and the bending moments. On the other hand, displacements and rotations are well calculated.

These differences are due to the relative thickness of this plate, with respect to the coarseness of the selected mesh. The figures hereafter show the comparison of the solutions analytical and numerical, in case LOVE-KIRCHHOFF, on meshes of 10 and 100 elements.

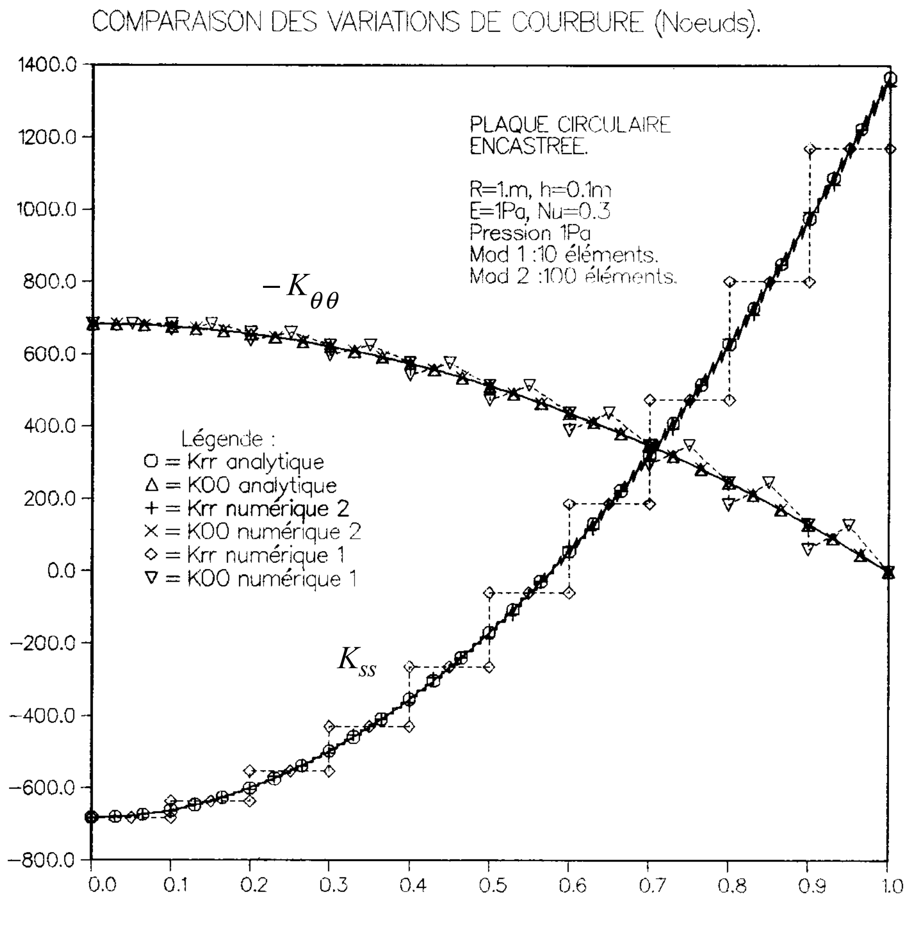


Figure 4.2 - C: Variations of curvature of a clamped circular plate

the layout of the variations of curvature K_{ss} and K_{qq} illustrates the fact that these two components are not approximate same way: first is linear since derived from a shape function $P2$, while second is constant per pieces.

4.3 Axisymmetric modal analysis of a thin spherical envelope [V2.03.007]

One considers a sphere, of average radius $R_m = 2.5\,m$, thickness $t = 0.10\,m$.
The material is elastic ($E = 200000\,MPa$, $\nu = 0{,}3$), of density $\rho = 7800\ kg/m^3$.

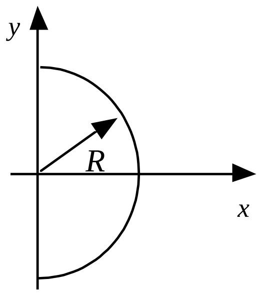


Figure 4.3-a : Sphere

One studies his axisymmetric free vibrations in frame LOVE-KIRCHHOFF $(k = 10^6)$.

One meshes uses a mesh made up of 40 and 81 nodes. One is interested in the frequencies understood enters 220 and $375\,Hz$. Compared to the reference solution [V2.03.007] one finds like the first 5 frequencies:

N°	1	2	3	4	5
Reference	237.25	282.85	305.2	324.2	346.8
Aster	237.32	282.78	304.95	323.7	346.2

Table 4.3-a: Frequencies of the axisymmetric modes

5 Conclusion

the finite elements which we propose were selected with a quite particular aim: computation of axisymmetric thin structures, or orthogonal sections of infinite shells with independence in the direction z , the concern of obtaining a good accuracy on the membrane and flexional solution while having a simple element of establishment and not too expensive.

The choice of the degrees of freedom allows a good representation of the boundary conditions. Moreover, this displacement formulation and rotation lead to elements of smaller degree: the elements are $P2$ out of membrane and $P2$ bending. It appears that they are easy to handle and that their formulation makes it possible to use a structure of pre and post simple processor, significant advantage to carry out rather fine meshes (unidimensional) and to display the results easily (on a simple curve). Selected kinematics: formulation of HENCKY-MINDLIN-NAGHDI, in displacements and rotations of mean surface makes it possible to utilize the transverse energy of shears (interesting for the shells of average thickness).

This energy can be affected of a factor of correction k : if one wants to place oneself in theory of REISSNER, it is enough to choose $k=5/6$ instead of 1 (but of course, the deflection W and the rotations β are in this theory only weighted averages in the thickness). Moreover, the formulation of shell of LOVE-KIRCHHOFF (for very mean structures) can be simulated by penalization of the condition of nullity of the transverse distortion, by choosing a factor $k=10^6\times h/L$, h being the thickness and L a characteristic distance (radius of curvature, enforcement zone of the loads…).

The nonlinear behaviors in plane stresses are available for these elements. It is announced however that the stresses generated by the transverse distortion are treated elastically, for want of anything better. Indeed, the taking into account of non-zero constant transverse shears on the thickness and the determination of the correction associated on the shear stiffness compared to a model satisfying the boundary conditions are not possible and thus return the use of these elements, when the transverse shears are non-zero, rigorously impossible in plasticity. In any rigor, for nonlinear behaviors, it would thus be necessary to use these elements in the frame of the theory of Coils-Kirchhoff.

Elements corresponding to the machine elements exist in thermal; the thermomechanical sequences are thus available with of the finite elements of thermal shells to three nodes described in [R3.11.01] according to the case in its axisymmetric version, or its invariant plane version according to Oz .

In the treated benchmarks, the phenomena of blocking did not appear. The decomposition of strain energy will make it possible, where necessary, to integrate in a selective way the terms responsible for blocking, such a modification not having to raise particular difficulties. A more detailed study must of course be carried out on this subject, as for the numerical methods to use to avoid this blocking when the thickness becomes low.

The possible developments are:

- anisotropy in order to be able to deal the multi-layer shells,
- with the problems of buckling,
- decomposition in Fourier series to study nonaxisymmetric problems of shells of revolution,
- the taking into variable account of one thickness…

6 Bibliography

1) B. ALMROTH - D. BRUSH: Buckling of bars, punts and shells. Mc Graw-Hill 1975.

2) J.L. BATOZ - G. DHATT: Modelization of structures by finite elements. Volume 3 Shells. Hermes 1992.

3) D. BUI - F. VOLDOIRE: Presentation of a finite element of cylindrical shell P2 out of membrane and Morley in bending. Note EDF-DER-MMN, HI 71/6715, of the 10.10.90.

4) X. DESROCHES: Computation of the nodal stresses by a local method of lissage by least squares. Note EDF-DER-MMN of the 20.01.92 [R3.06.03].

5) G. DHATT - G. TOUZOT: A presentation of the method of the elements finis.2ème edition. Maloine SA 1984.

6) GREEN - ZERNA: Theoretical elasticity. Univ. Oxford 1954.

7) TIMOSHENKO and WOINOWSKY-KRIEGER: Plates and shells. Béranger 1961.

8) F. VOLDOIRE: Formulation and numerical evaluating of a model of elastoplastic shell axisymmetric enriched. Note EDF-DER-MMN, HI-73/7518, of the 04.02.92.

9) D. BUI: Shears in the plates and the shells: modelization and computation. Note EDF - DER-MMN, HI-71/7784, of the 20.02.92.

10) S. ANDRIEUX - F. VOLDOIRE: Models of shells. Applications in linear static. School of Numerical Summer CEA-EDF-INRIA of Analysis 1992.

7 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
4	F.VOLDOIRE, C.SEVIN EDF-R&D/AMA	initial Text
5	P.MASSIN, up to date	Put EDF-R&D/AMA