Voluminal shell element SHB with 8 nodes

Summarized:

We present in this document the theoretical formulation of element SHB8 and its numerical establishment for implicit nonlinear incremental analyses (large displacements, small rotations, small strains).

It is about a three-dimensional cubic element with 8 nodes with a called privileged direction thickness. Thus, it can be used to represent thin structures while correctly taking into account the phenomena through the thickness (bending, elastoplasticity), grace a numerical integration to 5 Gauss points in this privileged direction.

In order to reduce the computing time considerably and to draw aside various blockings likely to appear, this element under-is integrated. It requires consequently a mechanism of stabilization in order to null control the modes of strain to energy (modes of Hourglass).

In addition to its cost of relatively weak computation and its good performances in elastoplasticity, this element has another advantage. Since it is based on a three-dimensional formulation and that it has only degrees of freedom of translation, it is very easy to couple it with elements 3D voluminal, which is very useful in systems where voluminal shells and elements must cohabit. Moreover, it makes it possible to easily model the plates and shells with variable thickness.

Contents

1 Introduction

Of many recent works proposed to use a voluminal formulation for thin structures. Two principal families of methods, which rest all on the introduction of a strain field applied ("assumed strain"), emerge. The methods of the first family consist in using a conventional numerical integration with an adequate control of all the modes of blocking and locking (volume, transverse shears, membrane). The methods of the second family consist under-integrating the elements to remove blockings and controlling the modes of *Hourglass* which rise from this under-integration (see [bib3] [bib4]). The two approaches were studied in details in the case of an elastic behavior. On the other hand, very few works treat elastoplastic case.

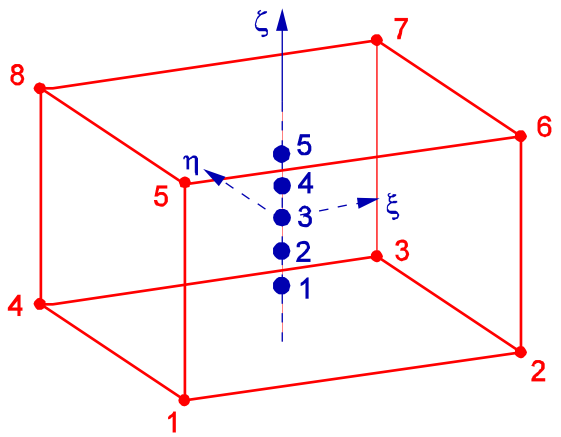
The element presented here rests on an under-integrated formulation especially developed for the elastoplastic behavior of structures in bending. The basic idea first of all consists in making sure that there is sufficiently of Gauss points in the thickness to represent the phenomenon of bending correctly, then calculating stiffness of stabilization in an adaptive way according to the plastic state of the element. That represents an unquestionable improvement compared to the classical formulations for the forces of stabilization, because these last rest on an elastic stabilization which becomes too rigid when the effects of plasticity dominate the response of structure.

Element SHB8 is a continuous three-dimensional cube with eight nodes, in which a privileged direction, called thickness, was selected. It can thus be used to model thin structures and to take into account the phenomena which develop in the thickness in the frame of the three-dimensional mechanics of the continuums. Being given that this element under-is integrated, it displays modes of *Hourglass* which must be stabilized. We chose the method of stabilization introduced by Belytschko, Bindeman and Flanagan [bib3] [bib4]. This element (entitled SHB8PS then) and this method of stabilization were initially implemented in an explicit formulation by Abed-Meraim and Combescure [bib2]. The numeric work implementation of this element in an implicit nonlinear frame was proposed by Legay and Combescure in [bib1].

This documentation describes the formulation of this element, its numeric work implementation for the prediction of elastic and elastoplastic structural instabilities, like its establishment in *the Code_Aster*. For the nonlinear problems, an implicit incremental formulation of Newton-Raphson type is used [R5.03.01]. The balance equations are solved by the method of Lagrangian the update. The control of the increments of load and displacement is based on a continuation method close to the algorithm of Riks [bib5].

2 Kinematics of the element

element SHB8 is a hexahedron with 8 nodes. The five points of integration are selected along the direction ζ in the reference of the local coordinates. The shape of the element of reference as well as the points of integration are represented on [Figure 2-a].



Appear 2-a: Geometry of the element of reference and points of integration

This element is isoparametric and has the same linear interpolation and the same kinematics as the hexaèdraux elements with 8 standard nodes.

3 Variational formulation

the formulation used for the construction of element SHB8 differs from a classical formulation simply by the choice of an applied strain $\dot{\bar{\varepsilon}}$, therefore of an operator discretized gradient, making it possible to avoid the induced parasitic modes by under integration.

Thus, the variational principle is written:

$$\delta\pi\left(v,\dot{\bar{\varepsilon}}\right)=\int_V \delta\left(\dot{\bar{\varepsilon}}\right):\sigma\, dV-\delta\dot{u}\, f^{ext}=0$$

where π represents the total virtual power, δ the variation, v the velocity field, $\dot{u}$ the nodal velocities, $\dot{\bar{\varepsilon}}$ strain rate applied (assumed strain disastrous), σ the stress of Cauchy, V volume brought up to date and the f^{ext} external forces.

The discretized equations thus require the only interpolation velocity v and strain rate applied $\dot{\bar{\varepsilon}}$ in the element. We now will build element SHB8 from this equation. The complete developments and the demonstrations concerning this element are exposed in details in [bib2].

4 Discretization

4.1 Discretization of the field of displacement

the spatial coordinates x_i of the element are connected to the nodal coordinates x_{iI} by means of the isoparametric shape functions N_i by the formulas:

$$x_i = x_{iI} N_I(\xi, \eta, \zeta) = \sum_{I=1}^{8} N_I(\xi, \eta, \zeta) x_{iI}$$

In the continuation, and except contrary mention, one will adopt summation convention for the repeated indices. The indices in small letters i vary from one to three and represent the directions of the spatial coordinates. Those in capital letters I vary from one to eight and correspond to the nodes of the element.

The same shape functions are used to define the field of displacement of the element u_i according to nodal displacements u_{iI} :

$$u_i = u_{iI} N_i(\xi, \eta, \zeta)$$

Trilinear isoparametric shape functions are chosen:

$$\begin{cases} N_I(\xi, \eta, \zeta) = \frac{1}{8}(1+\xi_I \xi)(1+\eta_I \eta)(1+\zeta_I \zeta) \\ \xi, \eta, \zeta \in [-1,1], \quad I=1,\ldots,8 \end{cases}$$

These shape functions transform a unit cube in space (ξ, η, ζ) into an unspecified hexahedron in space (x_1, x_2, x_3) .

4.2 Operator gradient discretized

the gradient $u_{i,j}$ of the field of displacement is a function of the displacement $\mathbf{U}_{iI}$ of the node I in the direction i :

$$u_{i,j} = \mathbf{U}_{iI} N_{I,j}$$

The linear strain tensor is given by the symmetric part of the displacement gradient:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

Let us introduce the three vectors $\mathbf{b}_i$, derived from the shape functions to Gauss points P_3 :

$$\mathbf{b}_i^T(P_3) = \left.\frac{\partial N}{\partial x_i}\right|_{\xi=0,\ \eta=0,\ \zeta=0}$$

Also let us introduce the following vectors:

$$
\begin{aligned}
\mathbf{s}^T &= (\ 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1\ )\\
\mathbf{h}_1^T &= (\ 1 \quad 1 \quad -1 \quad -1 \quad -1 \quad -1 \quad 1 \quad 1\ )\\
\mathbf{h}_2^T &= (\ 1 \quad -1 \quad -1 \quad 1 \quad -1 \quad 1 \quad 1 \quad -1\ )\\
\mathbf{h}_3^T &= (\ 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1\ )\\
\mathbf{h}_4^T &= (\ -1 \quad 1 \quad -1 \quad 1 \quad 1 \quad -1 \quad 1 \quad -1\ )\\
\mathbf{X}_1^T &= (\ -1 \quad 1 \quad 1 \quad -1 \quad -1 \quad 1 \quad 1 \quad -1\ )\\
\mathbf{X}_2^T &= (\ -1 \quad -1 \quad 1 \quad 1 \quad -1 \quad -1 \quad 1 \quad 1\ )\\
\mathbf{X}_3^T &= (\ -1 \quad -1 \quad -1 \quad -1 \quad 1 \quad 1 \quad 1 \quad 1\ )
\end{aligned}
$$

The three vectors $\mathbf{X}_i^T$ represent the nodal coordinates of the eight nodes. The four vectors $\mathbf{h}_\alpha^T$ respectively represent the functions h_1 h_2 , h_3 and h_4 for each of the eight nodes, which are defined by:

$$h_1 = \eta\zeta \qquad h_2 = \zeta\xi \qquad h_3 = \xi\eta \qquad h_4 = \xi\eta\zeta$$

Let us introduce finally the four following vectors:

$$\boldsymbol{\gamma}_\alpha = \frac{1}{8}\left[\mathbf{h}_\alpha - \sum_{j=1}^{3}\left(\mathbf{h}_\alpha^T.\mathbf{X}_j\right)\mathbf{b}_j\right]$$

The gradient of the field of displacement can be now written in the form (without any approximation [bib3]):

$$u_{i,j} = \left(\mathbf{b}_j^T + \sum_{\alpha=1}^{4} \mathbf{h}_{\alpha,j}\gamma_\alpha^T\right)\cdot\mathbf{U}_i = \left(\mathbf{b}_j^T + \mathbf{h}_{\alpha,j}\gamma_\alpha^T\right)\cdot\mathbf{U}_i$$

Or, in the form of vector:

$$\nabla_S \mathbf{u} = \begin{bmatrix} u_{x,x} \\ u_{y,y} \\ u_{z,z} \\ u_{x,y}+u_{y,x} \\ u_{x,z}+u_{z,x} \\ u_{y,z}+u_{z,y} \end{bmatrix}$$

with $\mathbf{U}_i$ nodal displacement in the direction i. The symmetric operator gradient (noted ∇_S) discretized connecting the strain tensor to the vector of nodal displacements

$$\nabla_s \mathbf{u} = \mathbf{B}\cdot\mathbf{u}$$

takes the matric shape then:

$$\mathbf{B}=\begin{bmatrix} \mathbf{b}_x^T+h_{\alpha,x}\gamma_\alpha^T & 0 & 0 \\ 0 & \mathbf{b}_y^T+h_{\alpha,y}\gamma_\alpha^T & 0 \\ 0 & 0 & \mathbf{b}_z^T+h_{\alpha,z}\gamma_\alpha^T \\ \mathbf{b}_y^T+h_{\alpha,y}\gamma_\alpha^T & \mathbf{b}_x^T+h_{\alpha,x}\gamma_\alpha^T & 0 \\ \mathbf{b}_z^T+h_{\alpha,z}\gamma_\alpha^T & 0 & \mathbf{b}_x^T+h_{\alpha,x}\gamma_\alpha^T \\ 0 & \mathbf{b}_z^T+h_{\alpha,z}\gamma_\alpha^T & \mathbf{b}_y^T+h_{\alpha,y}\gamma_\alpha^T \end{bmatrix}$$

The detailed formulation was presented by Belytschko in [bib3].

4.3 Stiffness matrix

the stiffness matrix of the element is given by:

$$\mathbf{K}_e=\int_{\Omega_e}\mathbf{B}^T\cdot\mathbf{C}\cdot\mathbf{B}\,d\,\Omega$$

The five points of integration considered are on the same vertical line. Their coordinates are (ξ,η,ζ) and their weights of integration are the roots of the polynomial of Gauss-Legendre:

	ξ	η	ζ	ω
P (1)	0	0	$\zeta_1=0.91$	$\omega_1=0.24$
P (2)	0	0	$\zeta_2=0.54$	$\omega_2=0.48$
P (3)	0	0	0	0.57
P (4)	0	0	$-\zeta_2$	ω_2
P (5)	0	0	$-\zeta_1$	ω_1

Thus, the statement of the stiffness $\mathbf{K}_e$ are:

$$\mathbf{K}_e=\sum_{j=1}^{5}\omega(\zeta_j)J(\zeta_j)\mathbf{B}^T(\zeta_j)\cdot\mathbf{C}\cdot\mathbf{B}(\zeta_j)$$

where $J(\zeta_j)$ is the Jacobian, calculated at the Gauss point j, of the transformation between the unit configuration of reference and an arbitrary hexahedron. The matrix of elastic behavior $\mathbf{C}$ selected has the following form:

$$\mathbf{C}=\begin{bmatrix} \bar\lambda+2\mu & \bar\lambda & 0 & 0 & 0 & 0 \\ \bar\lambda & \bar\lambda+2\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & E & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}$$

where E is the Young modulus, ν the Poisson's ratio, $\mu=\frac{E}{2(1+\nu)}$ the shear modulus and $\bar\lambda=\frac{E\nu}{1-\nu^2}$ the coefficient of modified Lamé. This model is specific with element SHB8. It resembles that which one would have in the case of the assumption of the plane stresses, put except for the term (3,3). One can note that this choice involves an artificial anisotropic behavior.
This choice makes it possible to satisfy all the tests without introducing blocking.

4.4 Geometrical stiffness matrix K σ

By introducing the quadratic strain $\mathbf{e}^Q$:

$$\mathbf{e}^Q=\frac{1}{2}\left(\sum_{k=1}^{3} u_{k,i} u_{k,j}\right)$$

one can define this geometrical stiffness matrix by:

$$\mathbf{u}^T\cdot\mathbf{K}_\sigma\cdot\mathbf{u}=\int_{\Omega_0}\sigma:\mathbf{e}^Q(\mathbf{u},\mathbf{u})\,d\Omega=\int_{\Omega_0}\sigma:(\nabla\mathbf{u}^T\cdot\nabla\mathbf{u})\,d\Omega$$

In order to express this matrix in discretized space, let us introduce the operators quadratic gradient discretized $\mathbf{B}^\mathbf{Q}$ such as:

$$\mathbf{e}^Q(\mathbf{u}(\zeta_j),\mathbf{u}(\zeta_j))=\begin{bmatrix} \mathbf{e}^Q_{\mathbf{xx}} & \mathbf{e}^Q_{\mathbf{yy}} & \mathbf{e}^Q_{\mathbf{zz}} \\ \mathbf{e}^Q_{\mathbf{yy}} & \mathbf{e}^Q_{\mathbf{zz}} & \mathbf{e}^Q_{\mathbf{xy}}+\mathbf{e}^Q_{\mathbf{yx}} \\ \mathbf{e}^Q_{\mathbf{zz}} & \mathbf{e}^Q_{\mathbf{xy}}+\mathbf{e}^Q_{\mathbf{yx}} & \mathbf{e}^Q_{\mathbf{xz}}+\mathbf{e}^Q_{\mathbf{zx}} \\ \mathbf{e}^Q_{\mathbf{xy}}+\mathbf{e}^Q_{\mathbf{yx}} & \mathbf{e}^Q_{\mathbf{xz}}+\mathbf{e}^Q_{\mathbf{zx}} & \mathbf{e}^Q_{\mathbf{yz}}+\mathbf{e}^Q_{\mathbf{zy}} \\ \mathbf{e}^Q_{\mathbf{xz}}+\mathbf{e}^Q_{\mathbf{zx}} & \mathbf{e}^Q_{\mathbf{yz}}+\mathbf{e}^Q_{\mathbf{zy}} & \\ \mathbf{e}^Q_{\mathbf{yz}}+\mathbf{e}^Q_{\mathbf{zy}} & & \end{bmatrix}=\begin{bmatrix} \mathbf{U}^T\cdot\mathbf{B}^\mathbf{Q}_{\mathbf{xx}}(\zeta_j)\cdot\mathbf{U} & \mathbf{U}^T\cdot\mathbf{B}^\mathbf{Q}_{\mathbf{yy}}(\zeta_j)\cdot\mathbf{U} & \mathbf{U}^T\cdot\mathbf{B}^\mathbf{Q}_{\mathbf{zz}}(\zeta_j)\cdot\mathbf{U} \\ \mathbf{U}^T\cdot\mathbf{B}^\mathbf{Q}_{\mathbf{yy}}(\zeta_j)\cdot\mathbf{U} & \mathbf{U}^T\cdot\mathbf{B}^\mathbf{Q}_{\mathbf{zz}}(\zeta_j)\cdot\mathbf{U} & \mathbf{U}^T\cdot\mathbf{B}^\mathbf{Q}_{\mathbf{xy}}(\zeta_j)\cdot\mathbf{U} \\ \mathbf{U}^T\cdot\mathbf{B}^\mathbf{Q}_{\mathbf{zz}}(\zeta_j)\cdot\mathbf{U} & \mathbf{U}^T\cdot\mathbf{B}^\mathbf{Q}_{\mathbf{xy}}(\zeta_j)\cdot\mathbf{U} & \mathbf{U}^T\cdot\mathbf{B}^\mathbf{Q}_{\mathbf{xz}}(\zeta_j)\cdot\mathbf{U} \\ \mathbf{U}^T\cdot\mathbf{B}^\mathbf{Q}_{\mathbf{xy}}(\zeta_j)\cdot\mathbf{U} & \mathbf{U}^T\cdot\mathbf{B}^\mathbf{Q}_{\mathbf{xz}}(\zeta_j)\cdot\mathbf{U} & \mathbf{U}^T\cdot\mathbf{B}^\mathbf{Q}_{\mathbf{yz}}(\zeta_j)\cdot\mathbf{U} \\ \mathbf{U}^T\cdot\mathbf{B}^\mathbf{Q}_{\mathbf{xz}}(\zeta_j)\cdot\mathbf{U} & \mathbf{U}^T\cdot\mathbf{B}^\mathbf{Q}_{\mathbf{yz}}(\zeta_j)\cdot\mathbf{U} & \\ \mathbf{U}^T\cdot\mathbf{B}^\mathbf{Q}_{\mathbf{yz}}(\zeta_j)\cdot\mathbf{U} & & \end{bmatrix}$$

The various terms $\mathbf{B}^\mathbf{Q}_\mathbf{ij}$ are given by the following equations:

$$\mathbf{B}^\mathbf{Q}_{\mathbf{xx}}(\zeta_j)=\begin{bmatrix} \mathbf{B_x}\cdot\mathbf{B_x^T} & 0 & 0 \\ 0 & \mathbf{B_x}\cdot\mathbf{B_x^T} & 0 \\ 0 & 0 & \mathbf{B_x}\cdot\mathbf{B_x^T} \end{bmatrix}$$

$$\mathbf{B}^\mathbf{Q}_{\mathbf{yy}}(\zeta_j)=\begin{bmatrix} \mathbf{B_y}\cdot\mathbf{B_y^T} & 0 & 0 \\ 0 & \mathbf{B_y}\cdot\mathbf{B_y^T} & 0 \\ 0 & 0 & \mathbf{B_y}\cdot\mathbf{B_y^T} \end{bmatrix}$$

$$\mathbf{B}^\mathbf{Q}_{\mathbf{zz}}(\zeta_j)=\begin{bmatrix} \mathbf{B_z}\cdot\mathbf{B_z^T} & 0 & 0 \\ 0 & \mathbf{B_z}\cdot\mathbf{B_z^T} & 0 \\ 0 & 0 & \mathbf{B_z}\cdot\mathbf{B_z^T} \end{bmatrix}$$

$$\mathbf{B}^\mathbf{Q}_{\mathbf{xy}}(\zeta_j)=\begin{bmatrix} \mathbf{B_x}\cdot\mathbf{B_y^T}+\mathbf{B_y}\cdot\mathbf{B_x^T} & 0 & 0 \\ 0 & \mathbf{B_x}\cdot\mathbf{B_y^T}+\mathbf{B_y}\cdot\mathbf{B_x^T} & 0 \\ 0 & 0 & \mathbf{B_x}\cdot\mathbf{B_y^T}+\mathbf{B_y}\cdot\mathbf{B_x^T} \end{bmatrix}$$

$$\mathbf{B}^{\mathbf{Q}}_{\mathbf{xz}}(\zeta_j)=\begin{bmatrix}\mathbf{B_x}\cdot\mathbf{B_z^T}+\mathbf{B_z}\cdot\mathbf{B_x^T} & 0 & 0\\ 0 & \mathbf{B_x}\cdot\mathbf{B_z^T}+\mathbf{B_z}\cdot\mathbf{B_x^T} & 0\\ 0 & 0 & \mathbf{B_x}\cdot\mathbf{B_z^T}+\mathbf{B_z}\cdot\mathbf{B_x^T}\end{bmatrix}$$

$$\mathbf{B}^{\mathbf{Q}}_{\mathbf{yz}}(\zeta_j)=\begin{bmatrix}\mathbf{B_y}\cdot\mathbf{B_z^T}+\mathbf{B_z}\cdot\mathbf{B_y^T} & 0 & 0\\ 0 & \mathbf{B_y}\cdot\mathbf{B_z^T}+\mathbf{B_z}\cdot\mathbf{B_y^T} & 0\\ 0 & 0 & \mathbf{B_y}\cdot\mathbf{B_z^T}+\mathbf{B_z}\cdot\mathbf{B_y^T}\end{bmatrix}$$

where $\mathbf{B}_i = \mathbf{b}_i^T + h_{\alpha,i}\gamma_\alpha^T$ avec $i = x, y, z$

With these notations, the geometrical stiffness matrix $\mathbf{k}_\sigma$ at the Gauss point ζ_j east given by:

$$\mathbf{k}_\sigma(\zeta_j)=\sigma_{xx}(\zeta_j).\mathbf{B}^{\mathbf{Q}}_{\mathbf{xx}}(\zeta_j)+\sigma_{yy}(\zeta_j).\mathbf{B}^{\mathbf{Q}}_{\mathbf{yy}}(\zeta_j)+\sigma_{zz}(\zeta_j).\mathbf{B}^{\mathbf{Q}}_{\mathbf{zz}}(\zeta_j)$$
$$+\sigma_{xy}(\zeta_j).\mathbf{B}^{\mathbf{Q}}_{\mathbf{xy}}(\zeta_j)+\sigma_{xz}(\zeta_j).\mathbf{B}^{\mathbf{Q}}_{\mathbf{xz}}(\zeta_j)+\sigma_{yz}(\zeta_j).\mathbf{B}^{\mathbf{Q}}_{\mathbf{yz}}(\zeta_j)$$

and the geometrical stiffness matrix of the element is given by:

$$\mathbf{K}_\sigma=\sum_{j=1}^{5}\omega(\zeta_j)J(\zeta_j)\mathbf{k}_\sigma(\zeta_j)$$

4.5 Stamp pressure Kp

the following compressive forces are present in the tangent matrix via the matrix $\mathbf{K_P}$, because the following external forces depend on displacement. The following compressive forces are written:

$$\int_{\partial\Omega} p\,\mathbf{n}\cdot\mathbf{u}\,dS=\int_{\partial\Omega 0} p\,\mathbf{det}[F(\mathbf{u})]\mathbf{n}_0^T\cdot F(\mathbf{u})^{-1T}\,dS_0=p\,\mathbf{F}_0-p\,\mathbf{K_p}\cdot\mathbf{U}$$
$$F(\mathbf{u})=1+\nabla\mathbf{u}$$

by means of notations:

- $\mathbf{n}_0{}^T=(n_x, n_y, n_z)$, norm on the surface external of the element in the reference configuration
- $\tilde{\mathbf{b}}_\mathbf{i}$, vector of dimension 4, drift of the shape functions to the 4 nodes of the face of the element charged in pressure
- S_0 area with the face charged in pressure

the preceding formulation leads to a nonsymmetrical matrix. It is known that one can nevertheless use a symmetric formulation if the external forces due to the pressure derive from a potential. It is the case if the compressive forces do not work on the border of the modelled field. It is thus considered that the symmetric part of the matrix is enough. The symmetrized matrix takes the following shape:

$$\mathbf{K_p}=S_0\begin{bmatrix} 0 & \tilde{\mathbf{b}}_{\mathbf{y}}^T n_x-\tilde{\mathbf{b}}_{\mathbf{x}}^T n_y & \tilde{\mathbf{b}}_{\mathbf{z}}^T n_x-\tilde{\mathbf{b}}_{\mathbf{x}}^T n_z \\ 0 & \tilde{\mathbf{b}}_{\mathbf{y}}^T n_x-\tilde{\mathbf{b}}_{\mathbf{x}}^T n_y & \tilde{\mathbf{b}}_{\mathbf{z}}^T n_x-\tilde{\mathbf{b}}_{\mathbf{x}}^T n_z \\ 0 & \tilde{\mathbf{b}}_{\mathbf{y}}^T n_x-\tilde{\mathbf{b}}_{\mathbf{x}}^T n_y & \tilde{\mathbf{b}}_{\mathbf{z}}^T n_x-\tilde{\mathbf{b}}_{\mathbf{x}}^T n_z \\ 0 & \tilde{\mathbf{b}}_{\mathbf{y}}^T n_x-\tilde{\mathbf{b}}_{\mathbf{x}}^T n_y & \tilde{\mathbf{b}}_{\mathbf{z}}^T n_x-\tilde{\mathbf{b}}_{\mathbf{x}}^T n_z \\ \tilde{\mathbf{b}}_{\mathbf{x}}^T n_y-\tilde{\mathbf{b}}_{\mathbf{y}}^T n_x & 0 & \tilde{\mathbf{b}}_{\mathbf{z}}^T n_y-\tilde{\mathbf{b}}_{\mathbf{y}}^T n_z \\ \tilde{\mathbf{b}}_{\mathbf{x}}^T n_y-\tilde{\mathbf{b}}_{\mathbf{y}}^T n_x & 0 & \tilde{\mathbf{b}}_{\mathbf{z}}^T n_y-\tilde{\mathbf{b}}_{\mathbf{y}}^T n_z \\ \tilde{\mathbf{b}}_{\mathbf{x}}^T n_y-\tilde{\mathbf{b}}_{\mathbf{y}}^T n_x & 0 & \tilde{\mathbf{b}}_{\mathbf{z}}^T n_y-\tilde{\mathbf{b}}_{\mathbf{y}}^T n_z \\ \tilde{\mathbf{b}}_{\mathbf{x}}^T n_y-\tilde{\mathbf{b}}_{\mathbf{y}}^T n_x & 0 & \tilde{\mathbf{b}}_{\mathbf{z}}^T n_y-\tilde{\mathbf{b}}_{\mathbf{y}}^T n_z \\ \tilde{\mathbf{b}}_{\mathbf{x}}^T n_z-\tilde{\mathbf{b}}_{\mathbf{z}}^T n_x & \tilde{\mathbf{b}}_{\mathbf{y}}^T n_z-\tilde{\mathbf{b}}_{\mathbf{z}}^T n_y & 0 \\ \tilde{\mathbf{b}}_{\mathbf{x}}^T n_z-\tilde{\mathbf{b}}_{\mathbf{z}}^T n_x & \tilde{\mathbf{b}}_{\mathbf{y}}^T n_z-\tilde{\mathbf{b}}_{\mathbf{z}}^T n_y & 0 \\ \tilde{\mathbf{b}}_{\mathbf{x}}^T n_z-\tilde{\mathbf{b}}_{\mathbf{z}}^T n_x & \tilde{\mathbf{b}}_{\mathbf{y}}^T n_z-\tilde{\mathbf{b}}_{\mathbf{z}}^T n_y & 0 \\ \tilde{\mathbf{b}}_{\mathbf{x}}^T n_z-\tilde{\mathbf{b}}_{\mathbf{z}}^T n_x & \tilde{\mathbf{b}}_{\mathbf{y}}^T n_z-\tilde{\mathbf{b}}_{\mathbf{z}}^T n_y & 0 \end{bmatrix}$$

It is a matrix $(12 \text{x} 12)$, which it is necessary to multiply by displacements of the 4 nodes of the face to which one applies a pressure.

The formulation is similar to that used in 3D, described in [R3.03.04].

5 Stabilization of the element

5.1 Motivations

the under-integration of element SHB8 (5 Gauss points only) aims at reducing the computing time considerably (gradient displacement, constitutive law,…). It also makes it possible of the finite elements to draw aside the various blockings met in the numeric work implementation.

However, this under-integration does not have only advantages: it unfortunately introduces parasitic modes associated with an energy null (mode of *Hourglass* or sand glass). In static, that can lead to a singularity of the total stiffness matrix for certain boundary conditions. In transient dynamics, on the other hand, that led to modes of sand glass which will deform the mesh in an unrealistic way and which end up exploding the solution. This deficiency of the stiffness matrix, due to under-integration, must thus be compensated by adding to the elementary stiffness a matrix of stabilization. The core of the new stiffness, thus obtained, must be reduced to the only modes corresponding to rigid solid motions.

5.2 Modes of "Hourglass"

being given that the points of integration are on the same vertical line (privileged direction), the derivatives of the functions h_3 and h_4 are cancelled in these points. The operator discretized gradient is thus reduced to:

$$\mathbf{B}=\begin{bmatrix} \mathbf{b}_x^T+\sum_{\alpha=1}^{2} h_{\alpha,x}\gamma_\alpha^T & 0 & 0 \\ 0 & \mathbf{b}_y^T+\sum_{\alpha=1}^{2} h_{\alpha,y}\gamma_\alpha^T & 0 \\ 0 & 0 & \mathbf{b}_z^T+\sum_{\alpha=1}^{2} h_{\alpha,z}\gamma_\alpha^T \\ \mathbf{b}_y^T+\sum_{\alpha=1}^{2} h_{\alpha,y}\gamma_\alpha^T & \mathbf{b}_x^T+\sum_{\alpha=1}^{2} h_{\alpha,x}\gamma_\alpha^T & 0 \\ \mathbf{b}_z^T+\sum_{\alpha=1}^{2} h_{\alpha,z}\gamma_\alpha^T & 0 & \mathbf{b}_x^T+\sum_{\alpha=1}^{2} h_{\alpha,x}\gamma_\alpha^T \\ 0 & \mathbf{b}_z^T+\sum_{\alpha=1}^{2} h_{\alpha,z}\gamma_\alpha^T & \mathbf{b}_y^T+\sum_{\alpha=1}^{2} h_{\alpha,y}\gamma_\alpha^T \end{bmatrix}$$

The modes of *Hourglass* are modes of displacement to energy null, i.e they check. $\mathbf{Bu}=0$ Six modes, others that rigid modes of solids, which check this equation are: Stabilization

$$\begin{bmatrix} \mathbf{h}_3 & 0 & 0 \\ 0 & \mathbf{h}_3 & 0 \\ 0 & 0 & \mathbf{h}_3 \end{bmatrix}\begin{bmatrix} \mathbf{h}_4 & 0 & 0 \\ 0 & \mathbf{h}_4 & 0 \\ 0 & 0 & \mathbf{h}_4 \end{bmatrix}$$

5.3 of the type "Assumed Strain Method" In

this approach, inspired of works of Belytschko, Bindeman and Flanagan [bib3] [bib4], the derivatives of $\mathbf{b_i}$ the shape functions are not calculated with but are not Gauss points realized on the element: Thus

$$\hat{\mathbf{b}}_i^T=\frac{1}{V}\int_{\Omega_e}\mathbf{N}_{,i}(\xi,\eta,\zeta)d\Omega \quad , \quad i=1,2,3$$

, the new operator discretized gradient can be written:

$$\hat{\mathbf{B}}=\mathbf{B}+\hat{\mathbf{B}}_{\mathbf{stab}}$$

The statement of $\hat{\mathbf{B}}_{\mathbf{stab}}$ is given by: and

$$\hat{\mathbf{B}}_{\mathbf{stab}}=\begin{bmatrix} \sum_{\alpha=3}^{4} h_{\alpha,x}\hat{\gamma}_\alpha^T & 0 & 0 \\ 0 & \sum_{\alpha=3}^{4} h_{\alpha,y}\hat{\gamma}_\alpha^T & 0 \\ 0 & 0 & h_{3,z}\hat{\gamma}_3^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & h_{4,y}\hat{\gamma}_4^T \end{bmatrix}$$

that of the vectors by $\hat{\gamma}_\alpha$:

$$\hat{\gamma}_\alpha=\frac{1}{8}\left[\mathbf{h}_\alpha-\sum_{j=1}^{3}\left(\mathbf{h}_\alpha^T\cdot\mathbf{X}_j\right)\hat{\mathbf{b}}_j\right]$$

The new stiffness matrix becomes:

$$\mathbf{K}=\int_{\Omega_e}\mathbf{B}^T\cdot\mathbf{C}\cdot\mathbf{B}\;d\,\Omega+\int_{\Omega_e}\mathbf{B}^T\cdot\mathbf{C}\cdot\hat{\mathbf{B}}_{\mathbf{stab}}\;d\,\Omega+\int_{\Omega_e}\hat{\mathbf{B}}_{\mathbf{stab}}{}^T\cdot\mathbf{C}\cdot\mathbf{B}\;d\,\Omega+\underbrace{\int_{\Omega_e}\hat{\mathbf{B}}_{\mathbf{stab}}{}^T\cdot\mathbf{C}\cdot\hat{\mathbf{B}}_{\mathbf{stab}}\;d\,\Omega}_{\mathbf{K}^{\mathbf{stab}}}$$

The last term of the preceding equation () is enough $\mathbf{K}^{\mathbf{stab}}$ to stabilize the element. One can thus reduce the stiffness matrix stabilized to:

$$\mathbf{K}=\mathbf{K}_e+\mathbf{K}^{\mathbf{stab}}$$

$$\mathbf{K}^{\mathbf{stab}}=\int_{\Omega_e}\hat{\mathbf{B}}_{\mathbf{stab}}{}^T\cdot\mathbf{C}\cdot\hat{\mathbf{B}}_{\mathbf{stab}}\,d\,\Omega$$

The many cases which were studied showed that it is enough to calculate the diagonal terms of the matrix of stabilization $\mathbf{K}^{\mathbf{stab}}{}_{ii}$, $i=1,2,3$ which is given by:

$$\mathbf{K}_{11}^{\mathbf{stab}}=H_{11}(\bar{\lambda}+2\mu)[\hat{\gamma}_3\hat{\gamma}_3^T+\frac{1}{3}\hat{\gamma}_4\hat{\gamma}_4^T]$$

$$\mathbf{K}_{22}^{\mathbf{stab}}=H_{22}(\bar{\lambda}+2\mu)[\hat{\gamma}_3\hat{\gamma}_3^T+\frac{1}{3}\hat{\gamma}_4\hat{\gamma}_4^T]$$

$$\mathbf{K}_{33}^{\mathbf{stab}}=\mu\frac{H_{33}}{3}E\hat{\gamma}_4\hat{\gamma}_4^T$$

The coefficients H_{ii} themselves are given by the following equation, in which there is no summation on the repeated indices: Strategy

$$H_{ii}=\frac{1}{3}\frac{\left(\mathbf{X}_j^T\cdot\mathbf{X}_j\right)\left(\mathbf{X}_k^T\cdot\mathbf{X}_k\right)}{\left(\mathbf{X}_i^T\cdot\mathbf{X}_i\right)}$$

6 for nonlinear computations Not

6.1 - geometrical linearities One

treats here the case of large displacements, but with weak rotations (see further) and small strains. One adopts for that an up to date put Lagrangian formulation. In nonlinear

we seek to write the equilibrium between external internal forces and force at the end of the increment of load (located by the index 2):

$$F_2^{\text{int}} = F_2^{extr}$$

The statement of the internal forces is written: In

$$F_2^{\text{int}} = \int_{\Omega_2} \mathbf{B}_2^T \sigma_2 \, dV$$

the preceding equation the operator is $\mathbf{B}_2$ the operator allowing to pass from the displacement to the linear strain calculated on the geometry at the end of the step, the stress is σ_2 the stress of Cauchy at the end of the step and integration is made on the volume deformed Ω_2 at the end of the step.

The element is programmed in small rotations. Indeed the increment of strain is calculated by means of only the linear strain:

$$\Delta \mathbf{E} = \frac{1}{2}\left(\underline{\underline{\nabla_1(\Delta \underline{\mathbf{u}})}} + \underline{\underline{\nabla_1^T(\Delta \underline{\mathbf{u}})}} \right)$$

The operator gradient is calculated on the geometry of beginning of step. This writing of the strain is restricted with small rotations (<5 degrees). One

can without difficulty of extending the formulation to large rotations by including in the strain the terms of second order: In

$$\Delta \underline{\underline{E}} = \frac{1}{2}\left(\underline{\underline{\nabla_1(\Delta \mathbf{u})}} + \underline{\underline{\nabla_1^T(\Delta \mathbf{u})}} + \underline{\underline{\nabla_1^T(\Delta \mathbf{u})}} \cdot \underline{\underline{\nabla_1(\Delta \mathbf{u})}} \right)$$

elasticity, the constitutive law is written: where

$$\Delta \underline{\underline{\pi}} = \underline{\underline{\mathbf{C}}}' \Delta \underline{\underline{E}}$$

is $\mathbf{C}$ the matrix of Hooke. Let us notice that for the SHB8 this matrix is a transverse orthotropic matrix which is written in the axes of the lamina:

$$[\mathrm{C}'] = \begin{bmatrix} \lambda+2\mu & \mu & 0 & 0 & 0 & 0 \\ \mu & \lambda+2\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & E & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}$$

The formula making it possible to calculate the stress of Cauchy from $\underline{\underline{\sigma_2}}$ the stress of Piola Kirchoff II is $\underline{\underline{\pi_2}}$:

$$\begin{cases} \underline{\underline{\pi_2}} = \underline{\underline{\sigma_1}} + \Delta\underline{\underline{\pi}} \\ \underline{\underline{\sigma_2}} = \dfrac{1}{\det\left(\underline{\underline{F}}\right)} \underline{\underline{F^T}}\,\underline{\underline{\pi_2}}\,\underline{\underline{F}} \\ \underline{\underline{F}} = \underline{\underline{I}} + \underline{\underline{\nabla_1 \Delta\mathbf{u}}} \end{cases}$$

The combination of the four last equations with the statement of the internal forces gives the formulation of the element in large deformations into Lagrangian updated. Let us notice

that this up to date put Lagrangian formulation is completely equivalent to the total Lagrangian formulation for which the internal forces are written: In

$$F_2^{\text{int}} = \int_{\Omega_0} \left(\mathbf{B} + \mathbf{B}^{NL}(\mathbf{u}) \right)_0^T \pi_2 \, dV$$

this case all integrations are made on the initial geometry Ω_0 the stress used π_2 is the stress of Piola Kirchoff II. This last method is probably preferable when the mesh becomes deformed significantly and thus makes it possible to deal with the problems in large deformations but requires the development of the operator. $\mathbf{B}^{NL}(\mathbf{u})$

The increment of strain in Lagrangian total is expressed on the initial geometry of structure.

$$\Delta\,\underline{\underline{E}} = \frac{1}{2}\left(\underline{\underline{\nabla_0 (\Delta\,\mathbf{u})}} + \underline{\underline{\nabla_0^T (\Delta\,\mathbf{u})}} + \underline{\underline{\nabla_0^T (\Delta\,\mathbf{u})}} \cdot \underline{\underline{\nabla_0 (\Delta\,\mathbf{u})}} \right)$$

The combination of the two preceding equations gives the formulation of the element in large deformations in linear behavior material. Small

6.2 displacements In the case of

small displacements one confuses geometry in the beginning and end of step, stress of Cauchy and Piola Kirchoff II, moreover one uses the linear statement of the strains. Forces

6.3 of stabilization

the forces of stabilization make it possible to avoid the modes of sand glass and are added in the computation of the residues to balance the contribution of the stiffness matrix of stabilization to the first member. The forces of stabilization, $\mathbf{F}^{\mathbf{stab}}$ to add to the internal forces, F_2^{int} are written: For

$$\mathbf{F}^{\mathbf{stab}} = \mathbf{K}^{\mathbf{stab}}\mathbf{U}$$

reasons of effectiveness, one chooses not to assemble again to compute: $\mathbf{K}^{\mathbf{stab}}$ at $\mathbf{F}^{\mathbf{stab}}$ the end of the step, but rather to build from $\mathbf{F}^{\mathbf{stab}}$ qu $\hat{\mathbf{B}}_{\mathbf{stab}}$ "one calculated previously. One must for that place oneself in the reference frame corotationnel of medium of step suggested in [bib3]. For this reason, one N" does not obtain an exact statement of, $\mathbf{F}^{\mathbf{stab}}$ and some additional iterations are generally necessary to converge. These some iterations are however unimportant compared to the cost of computation saved while not assembling. $\mathbf{K}^{\mathbf{stab}}$ Plasticity

6.4

a first version of the element treated only the elastoplastic behavior of Von Mises, with isotropic hardening. In each of the 5 points of integration, the formulas and the usual programming of plasticity 3D was used, with the matrix of linear behavior orthotropic C' . This resulted in modifying the usual three-dimensional elastoplastic flow algorithm by replacing the usual matrix of Hooke by C the matrix of transverse orthotropic behavior. C' The nonlinear problem was solved by a method of Newton. For

more generality allows, and to give access to all the constitutive laws, another strategy from now on is used, similar to that used for the COQUE_3D [R 3.07.04]. It is a question of uncoupling the behavior according to mean surface, of the transverse behavior, according to the norm with the element. The method consists in supposing that the element is in plane stress state in the local coordinate system of each point of Gauss quadrature and that the strains except plane are elastic. That involves then immediately that the total deflections except plane are equal to the elastic strain. After integration of the behavior in plane stresses, the stresses except plane are calculated in an elastic way. Let us call $\underline{\underline{C}}^{CPT}$ the tangent matrix in plane stresses. The tangent matrix of behavior for the selected behavior is written: This

$$\underline{\underline{C}}^{T}=\begin{bmatrix} C^{CPT}_{xxxx} & C^{CPT}_{xxyy} & 0 & C^{CPT}_{xxxy} & 0 & 0 \\ C^{CPT}_{xyyx} & C^{CPT}_{yyyy} & 0 & C^{CPT}_{yyxy} & 0 & 0 \\ 0 & 0 & E & 0 & 0 & 0 \\ C^{CPT}_{xyxx} & C^{CPT}_{xyyy} & 0 & C^{CPT}_{xyxy} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}$$

method thus makes it possible to connect elements SHB to all the constitutive laws available in plane stresses in the Code Aster (in an analytical way or via the method due to Borst). Establishment

7 of element SHB8 in the Code_Aster *Description*

7.1 This

element leans on meshes 3D the voluminal HEXA 8. Use

7.2 This

element is used in the following way: Mesh

7.2.1 It

is necessary by means of to check the good directional sense of the sides of indicated elements (compatibility with the privileged direction) ORIE_SHB of operator MODI _MAILLAGE. Modelization

7.2.2

meshes to assign modelization SHB 8 to the indicated HEXA 8 . Material

7.2.3

the element supposes that the coefficients E, Young modulus and NU , Poisson's ratio. are given (as well into linear as in nonlinear) for the computation of the elastic or tangent stiffness matrix (transverse terms). It is thus necessary to inform key word ELAS in DEFI _MATERIAU. All

the nonlinear behaviors (compatible with a modelization in plane stresses) are usable. It should be noted that

thermal thermal expansion is not taken into account in version 9 of Code_Aster for elements SHB. Boundary conditions

7.2.4 and loading

the usual voluminal loadings are usable: body forces, gravity.
The forces of pressure (and the other surface forces) are applied to elements of sides, as in 3D (under key word PRES_REP). One will have taken care as a preliminary to define meshes of skin QUAD4 and to suitably direct the outgoing norms with these meshes of skin using command MODI _MAILLAGE key word ORIE_PEAU_3D. No

development was necessary for the compressive forces distributed and the following compressive forces. Indeed, these loadings lean on meshes of skin identical to those of the elements 3D voluminal. Computation

7.2.5 in linear elasticity

the options of postprocessing available are SIEF_ELNO and SIEQ_ELNO . Computation

7.2.6 in linear buckling

option RIGI _MECA_GE being activated in the catalog of the element, it is possible to carry out a classical computation of buckling after assembly of the stiffness matrixes elastic and geometrical. Geometrical

7.2.7 nonlinear computation One

can carry out nonlinear computations in small strains (DEFORMATION=' PETIT') or with an approximation of the large deformations (DEFORMATION= `PETIT_REAC') or large displacements (DEFORMATION=' GREEN') in STAT_NON_LINE or DYNA_NON_LINE .
The strategy used being based on the use of a tangent stiffness matrix during iterations (reactualization at the beginning of step only), one will take care to use another option only that which is activated by default, namely REAC_ITER = 0 pennies NEWTON . Nonlinear

7.2.8 computation material All

the nonlinear behaviors of continuums are usable (key word RELATION = under COMP_INCR). If the behavior is not integrated in an analytical way in plane stresses, the method of Borst [R5.03.03] is automatically used.

The strategy used being based on the use of a tangent stiffness matrix during iterations (reactualization at the beginning of step only), one will take care to use another option only that which is activated by default, namely REAC_ITER = 0 pennies NEWTON . Characteristics

7.3 of the establishment

the forces of stabilization of the element require the storage of a vector of size 12 for each Gauss point. We chose to store these terms like additional components of the stress field. Validation

7.4

the tests validating this element are the following: SDLS

1) 109: Eigenfrequencies of a cylindrical ring thick SSLS
2) 101: Plate circular posed under pressure SSLS
3) 105: hemisphere doubly pinch [V3.03.105] classical test to check the convergence of the element, SSLS
4) 108: beam bored in bending, test allowing to check the absence of blocking [V3.03.108], SSLS
5) 123: sphere under external pressure [V3.03.123] to validate the loadings of pressure and the orthotropic behavior particular to this element, SSLS
6) 124: thin plate in bending with various slenderness, to delimit the field of use of the element [V3.03.124]. The results are correct (less than 1% with the analytical solution) for ratios of slenderness (thickness/width) going from 1 to 5 10-3, SSLS
7) 125: buckling (modes of Eulerian) of a free cylinder under external pressure [V3.03.125] this test makes it possible to validate the geometrical nature of stiffness, SSLS
8) 129: Modelization D a shell doubly sinusoidal SSNS
9) 101: breakdown of a cylindrical roof [V6.03.101]. This test makes it possible to validate geometrical nonlinear computation and elastoplasticity, SSNS
10) 102: buckling of a shell with stiffeners in large displacements and following pressure [V6.03.102]. Bibliography

8 LEGAY

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9 of the versions Index

document Version	**Aster Author**	**(S) Organization (S) Description**	**of the modifications A**
7.2	S.	BAGUET, A.COMBESCURE INSA Lyon J. Mr. PROIX EDF R & D AMA initial	Version B
9.4	Trinh	Vuong God, X Desroches EDF R & D AMA Modification	of Bstat §5.3, and the integration of the nonlinear behaviors §6.4