Finite elements of right pipe and curve with ovalization, swelling and warping in elastoplasticity

Summarized:

This document presents the modelization of a finite element of pipe usable in computations of pipework in elasticity or plasticity. The pipes, curves or rights, can be relatively thick (thickness ratio on radius of the cross-sectional area until 0.2) and are subjected to various combined loadings - internal pressure, cross-bendings and anti-plane, torsion, extension - and can have a nonlinear behavior.

This linear element combines at the same time properties of shells and beams. The average fiber of the pipe behaves like a beam and the surface of the pipe like a shell. The element carried out is a pipe section right or curve in small rotations and strains, with an elastoplastic behavior in plane stresses.

Three modelizations, corresponding to three different element types, are available:

- `TUYAU_3M`, which takes into account 3 modes of Fourier to the maximum, and which can lean on meshes with 3 nodes or 4 nodes.
- `TUYAU_6M`, which takes as a count up to 6 modes of Fourier, and leans on meshes with 3 nodes.

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1 Introduction

There exist an important bibliography on the modelization of the pipework and of many finite elements of right and bent pipes are available in the great codes of finite elements. Syntheses were already realized [bib1], [bib5], [bib6], in the past that one supplemented by incorporating the last developments known in the field [bib11]. The effects important to take into account are swelling due to the internal pressure and the ovalization of the cross-sectional areas by combined bendings plane and anti-plane. One places oneself on the assumption of small rotations and strains in the frame of this document.

It is of a linear element with 3 or 4 nodes, about standard curved beam or right with local plasticity taking into account ovalization, warping and swelling. The kinematics of beam is enriched by a kinematics of shell for the description of the behavior of the cross-sectional areas. This kinematics is discretized in M modes of Fourier of which the number M must at the same time be sufficient to obtain good performances in plasticity and not too large to limit the computing time. The literature encourages us to use $M=6$ [bib9], [bib13] in plasticity. In elasticity, for thick pipes, one can be satisfied with $M=2$ or $M=3$.

2 The various theories of shells and beams for the finite elements of right or bent pipes

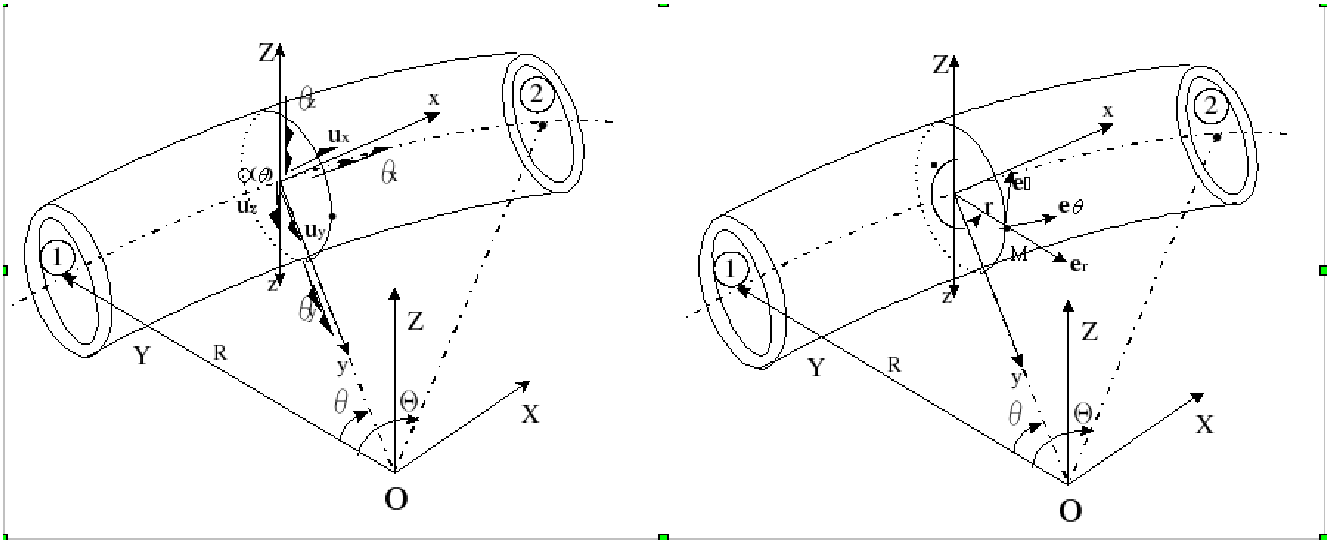
One presents in this chapter the elements of kinematics in three-dimensional curvilinear geometry, like their restrictions in the frame as of beam models and of shell. Indeed, to build the finite element of pipework enriched which answers the specifications defined in introduction, one exploits a technique of decomposition of the three-dimensional kinematics. The kinematics of shell brings there the description of ovalization, swelling and warping, while the kinematics of beam described there the general motion of line of pipework.

The various theories of shells and beams used for each element translate the assumptions chosen a priori on the type of strains and behaviors.

2.1 The pipe in beam theory

2.1.1 Case of a pipe elbow

a first approaches relatively simple come down to regard the elbow represented below as a hollow beam of circular section. The beam is obtained by rotation of angle Θ of the circular section around OZ . A point of the beam is located by its distance r compared to the axis of the beam and by the two angles θ , ϕ where θ is the longitudinal angle with OY indicated above and ϕ the trigonometrical angle with OZ measured on the circular section.



Appear 2.1.1-a: Geometry and kinematics of the elbow in beam theory

In the coordinate system curvilinear ($r \quad \theta$, ϕ), the relations between displacements $\boldsymbol{u}$ of the points of the elbow of position $r_0=\mathbf{OM}=-\mathrm{R}\,\boldsymbol{e}_y(\theta)+r\,\boldsymbol{e}_r(\theta,\phi)$ and the strains of Green - Lagrange are given by the following tensor in the natural base ($r \quad \theta$, ϕ):

$$2\,f_{\alpha\beta}=\frac{\partial\left(\boldsymbol{r}_0+\boldsymbol{u}\right)}{\partial\alpha}\cdot\frac{\partial\left(\boldsymbol{r}_0+\boldsymbol{u}\right)}{\partial\beta}-\frac{\partial\boldsymbol{r}_0}{\partial\alpha}\cdot\frac{\partial\boldsymbol{r}_0}{\partial\beta}\quad(\alpha,\beta)\in\{r,\theta,\phi\}\ .$$

The vectors units in the directions ($r \quad \theta$, ϕ) are:

$$\boldsymbol{e}_r=\frac{\partial\boldsymbol{r}_0}{\partial r},\boldsymbol{e}_\theta=\frac{1}{A}\frac{\partial\boldsymbol{r}_0}{\partial\theta},\boldsymbol{e}_\phi=\frac{1}{B}\frac{\partial\boldsymbol{r}_0}{\partial\phi}\ \text{ where }\ A=\sqrt{\frac{\partial\boldsymbol{r}_0}{\partial\theta}\cdot\frac{\partial\boldsymbol{r}_0}{\partial\theta}}\ \text{et}\ B=\sqrt{\frac{\partial\boldsymbol{r}_0}{\partial\phi}\cdot\frac{\partial\boldsymbol{r}_0}{\partial\phi}}\ .$$

If one expresses the position of a point of the elbow in the local toric orthonormal base ($\boldsymbol{e}_r,\boldsymbol{e}_\theta,\boldsymbol{e}_\phi$) by (y_r,y_θ,y_ϕ) there are the following relations:

$$\boldsymbol{e}_r=\frac{\partial\boldsymbol{r}_0}{\partial y_r},\boldsymbol{e}_\theta=\frac{\partial\boldsymbol{r}_0}{\partial y_\theta},\boldsymbol{e}_\phi=\frac{\partial\boldsymbol{r}_0}{\partial y_\phi}\ .$$

The form of the tensor of the strains of Green-Lagrange in this base is then:

$$2\,\varepsilon_{\alpha\beta}=\frac{\partial\left(\boldsymbol{r}_0+\boldsymbol{u}\right)}{\partial y_\alpha}\cdot\frac{\partial\left(\boldsymbol{r}_0+\boldsymbol{u}\right)}{\partial y_\beta}-\frac{\partial\boldsymbol{r}_0}{\partial y_\alpha}\cdot\frac{\partial\boldsymbol{r}_0}{\partial y_\beta}\ .$$

The relations of transition between the statement of the strains of Green-Lagrange in the curvilinear coordinate system and the local toric base previously definite are:

$$\varepsilon_{rr}=f_{rr}$$
$$\varepsilon_{\theta\theta}=\frac{f_{\theta\theta}}{A^2}$$
$$\varepsilon_{\phi\phi}=\frac{f_{\phi\phi}}{B^2}$$
$$\varepsilon_{\theta\phi}=\frac{f_{\theta\phi}}{AB}$$
$$\varepsilon_{\theta r}=\frac{f_{\theta r}}{A}$$
$$\varepsilon_{\phi r}=\frac{f_{\phi r}}{B}$$

The use of this base is particularly interesting because the behavior models in the orthonormal toric base are simple of use. For elbow Ci above, if it is considered that the strains remain small, it obtains then [bib4] after linearization of the strains of Green - Lagrange:

$$\varepsilon_{rr}=\frac{\partial u_r}{\partial r}$$
$$\varepsilon_{\theta\theta}=\frac{1}{A}\frac{\partial u_\theta}{\partial\theta}+\frac{u_\phi}{AB}\frac{\partial A}{\partial\phi}+\frac{u_r}{R_\theta}$$
$$\varepsilon_{\phi\phi}=\frac{1}{B}\frac{\partial u_\phi}{\partial\phi}+\frac{u_r}{R_\phi}$$
$$2\varepsilon_{\theta\phi}=\gamma_{\theta\phi}=\frac{1}{B}\frac{\partial u_\theta}{\partial\phi}+\frac{1}{A}\frac{\partial u_\phi}{\partial\theta}-\frac{u_\theta}{AB}\frac{\partial A}{\partial\phi}$$
$$2\varepsilon_{\theta r}=\gamma_{\theta r}=\frac{1}{A}\frac{\partial u_r}{\partial\theta}-\frac{u_\theta}{R_\theta}+\frac{\partial u_\theta}{\partial r}$$
$$2\varepsilon_{\phi r}=\gamma_{\phi r}=\frac{1}{B}\frac{\partial u_r}{\partial\phi}-\frac{u_\phi}{R_\phi}+\frac{\partial u_\phi}{\partial r}$$

with:

$$A=R+r\sin\phi\ ,B=r\ ,R_\theta=\frac{R+r\sin\phi}{\sin\phi}\ ,R_\phi=r$$

The statements of the strains established above are written then:

$$\varepsilon_{rr}=\frac{\partial u_r}{\partial r}$$
$$\varepsilon_{\theta\theta}=\frac{1}{R+r\sin\phi}\left(\frac{\partial u_\theta}{\partial\theta}+u_\phi\cos\phi+u_r\sin\phi\right)$$
$$\varepsilon_{\phi\phi}=\frac{1}{r}\left(\frac{\partial u_\phi}{\partial\phi}+u_r\right)$$
$$2\varepsilon_{\theta\phi}=\gamma_{\theta\phi}=\frac{1}{R+r\sin\phi}\left(\frac{\partial u_\phi}{\partial\theta}-u_\theta\cos\phi\right)+\frac{1}{r}\frac{\partial u_\theta}{\partial\phi}$$
$$2\varepsilon_{r\theta}=\gamma_{r\theta}=\frac{1}{R+r\sin\phi}\left(\frac{\partial u_r}{\partial\theta}-u_\theta\sin\phi\right)+\frac{\partial u_\theta}{\partial r}$$
$$2\varepsilon_{r\phi}=\gamma_{r\phi}=\frac{1}{r}\left(\frac{\partial u_r}{\partial\phi}-u_\phi\right)+\frac{\partial u_\phi}{\partial r}$$

Displacement u_r, u_θ, u_ϕ, of a point of the elbow in the toric base associated with the cross-sectional area of observation can be easily expressed according to displacements and of rotations associated with the center with the cross-sectional area. Indeed, if one notes u_1, u_2, u_3 displacement in the local curvilinear base $(o(\theta), \boldsymbol{x}(\theta), \boldsymbol{y}(\theta), \boldsymbol{z}(\theta))$ associated with the cross-sectional area as indicated on [Figure 2.1.1-a] one has the following relations, valid in the frame of the kinematics of the beams of Timoshenko [R3.08.01]:

$$u_1(r,\theta,\phi)=u_x(\theta)+\theta_z(\theta)r\sin\phi-\theta_y(\theta)r\cos\phi$$
$$u_2(r,\theta,\phi)=u_y(\theta)+\theta_x(\theta)r\cos\phi$$
$$u_3(r,\theta,\phi)=u_z(\theta)-\theta_x(\theta)r\sin\phi$$

where u_x, u_y, u_z is the displacement of translation of the section and θ_x θ_y, θ_z the rotation of its center o. The statement of the components of displacement in the local toric orthonormal base ($\boldsymbol{e}_r, \boldsymbol{e}_\theta, \boldsymbol{e}_\phi$) is obtained by change of reference:

$$u_\theta(r,\theta,\phi)=u_1(r,\theta,\phi)=u_x(\theta)+\theta_z(\theta)r\sin\phi-\theta_y(\theta)r\cos\phi$$
$$u_\phi(r,\theta,\phi)=u_3(r,\theta,\phi)\sin\phi-u_2(r,\theta,\phi)\cos\phi=u_z(\theta)\sin\phi-u_y(\theta)\cos\phi-\theta_x(\theta)r$$
$$u_r(r,\theta,\phi)=-[u_3(r,\theta,\phi)\cos\phi+u_2(r,\theta,\phi)\sin\phi]=-[u_z(\theta)\cos\phi+u_y(\theta)\sin\phi]$$

The introduction of this field of displacement into the statement of the linearized strains enables us to obtain the statement of the three-dimensional strains associated with the kinematics of beam:

$$\varepsilon_{rr}=0$$
$$\varepsilon_{\theta\theta}=\frac{1}{R+r\sin\phi}(u_{x,\theta}-u_y-r\theta_x\cos\phi+\theta_{z,\theta}r\sin\phi-\theta_{y,\theta}r\cos\phi)$$
$$\varepsilon_{\phi\phi}=0$$
$$2\varepsilon_{\theta\phi}=\frac{1}{R+r\sin\phi}(-u_x\cos\phi-u_{y,\theta}\cos\phi+u_{z,\theta}\sin\phi-r\theta_{x,\theta}+\theta_y r\cos^2\phi-\theta_z r\sin\phi\cos\phi)$$
$$+(\theta_z\cos\phi+\theta_y\sin\phi)$$
$$2\varepsilon_{r\theta}=\frac{1}{R+r\sin\phi}(-u_x\sin\phi-u_{y,\theta}\sin\phi-u_{z,\theta}\cos\phi+\theta_y r\sin\phi\cos\phi-\theta_z r\sin^2\phi)$$
$$+(\theta_z\sin\phi-\theta_y\cos\phi)$$
$$2\varepsilon_{r\phi}=0$$

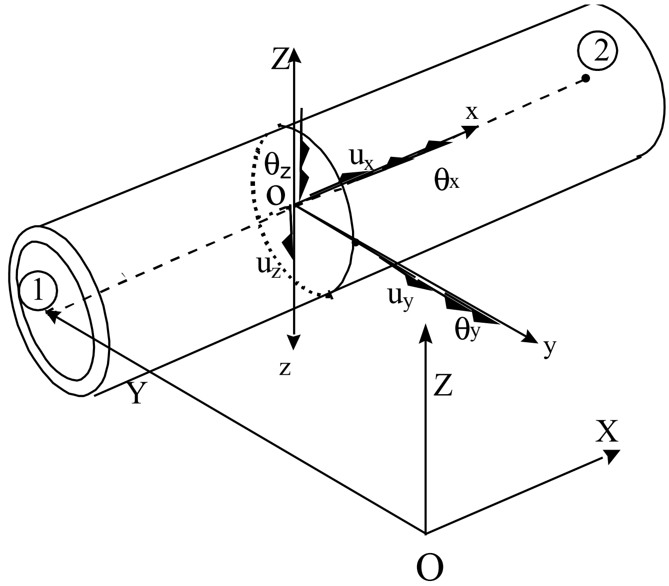
2.1.2 Cases of the right pipe

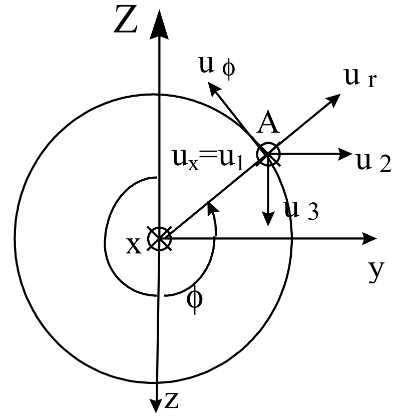
the statements of the strains established above also apply to the case of the right pipe, where one replaces θ by s where s is the curvilinear abscisse along average fiber of the pipe, with :

$$A=1\ ,B=r\ ,1/R_\theta=0\ ,R_\phi=r\ .$$

The statements given for the elbow are written then for the right pipe:

$$\varepsilon_{rr}=\frac{\partial u_r}{\partial r}$$
$$\varepsilon_{xx}=\frac{\partial u_x}{\partial x}$$
$$\varepsilon_{\phi\phi}=\frac{1}{r}(\frac{\partial u_\phi}{\partial\phi}+u_r)$$
$$2\varepsilon_{x\phi}=\gamma_{x\phi}=\frac{\partial u_\phi}{\partial x}+\frac{1}{r}\frac{\partial u_x}{\partial\phi}$$
$$2\varepsilon_{rx}=\gamma_{rx}=\frac{\partial u_r}{\partial x}+\frac{\partial u_x}{\partial r}$$
$$2\varepsilon_{r\phi}=\gamma_{r\phi}=\frac{1}{r}(\frac{\partial u_r}{\partial\phi}-u_\phi)+\frac{\partial u_\phi}{\partial r}$$





Section transverse : vue de 1 vers 2

Appear 2.1.2-a: Geometry and kinematics of a right pipe in beam theory

Like previously, displacement, u_r, u_x, u_ϕ of a point of the pipe in the toric base associated with the cross-sectional area of observation can be easily expressed according to displacements and of rotations associated with the center with the cross-sectional area. Indeed, if one notes u_1, u_2, u_3 displacement in the local curvilinear base (o, x, y, z) associated with the cross-sectional area as indicated on figure Ci - below one has the following relations:

$$u_1(r,x,\phi)=u_x(x)+\theta_z(x)r\sin\phi-\theta_y(x)r\cos\phi$$
$$u_2(r,x,\phi)=u_y(x)+\theta_x(x)r\cos\phi$$
$$u_3(r,x,\phi)=u_z(x)-\theta_x(x)r\sin\phi$$

and:

$$u_x(r,x,\phi)=u_1(r,x,\phi)=u_x(x)+\theta_z(x)r\sin\phi-\theta_y(x)r\cos\phi$$
$$u_\phi(r,x,\phi)=u_3(r,x,\phi)\sin\phi-u_2(r,x,\phi)\cos\phi=u_z(x)\sin\phi-u_y(x)\cos\phi-\theta_x(x)r$$
$$u_r(r,x,\phi)=-[u_3(r,x,\phi)\cos\phi+u_2(r,x,\phi)\sin\phi]=-[u_z(x)\cos\phi+u_y(x)\sin\phi]$$

The introduction of this field of displacement into the statement of the strains given below enables us to obtain the statement of the strains associated with the kinematics with beam:

$$\varepsilon_{rr}=0$$
$$\varepsilon_{xx}=u_{x,x}+\theta_{z,x}r\sin\phi-\theta_{y,x}r\cos\phi$$
$$\varepsilon_{\phi\phi}=0$$
$$2\varepsilon_{x\phi}=-r\theta_{x,x}+(\theta_y+u_{z,x})\sin\phi+(\theta_z-u_{y,x})\cos\phi$$
$$2\varepsilon_{rx}=(\theta_z-u_{y,x})\sin\phi-(\theta_y+u_{z,x})\cos\phi$$
$$2\varepsilon_{r\phi}=0$$

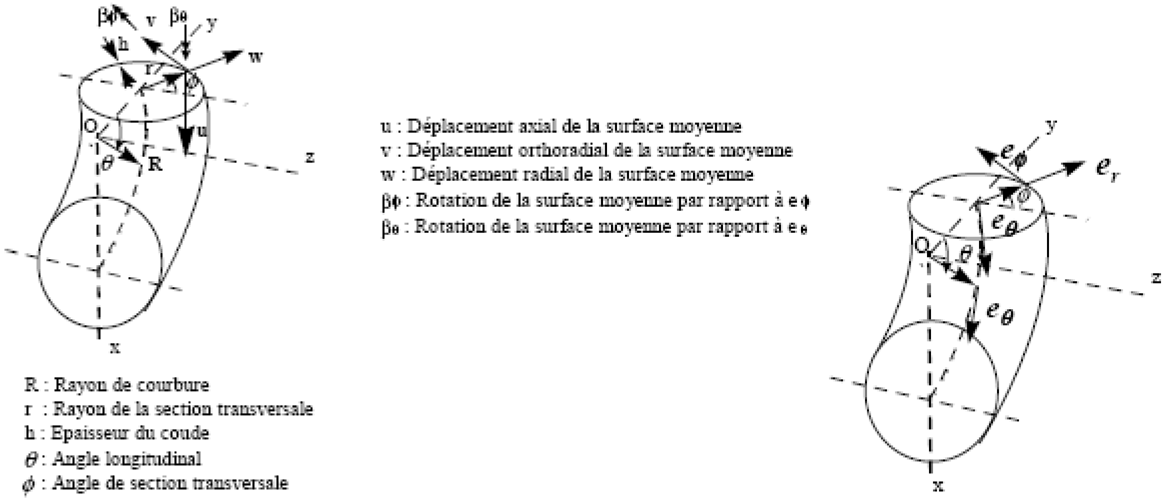
2.1.3 Remarks

the fact that $\varepsilon_{rr}, \varepsilon_{\phi\phi}$ et $\varepsilon_{r\phi}$ are simultaneously null watch which kinematics of beam cannot represent the strains of the cross-sectional areas to average fiber of the pipe. Indeed, the cross-sectional areas are actuated by a rigid body motion, which prohibits to model warping, swelling and ovalization.

2.2 The pipe in linearized theory of shell

2.2.1 general Case

the pipe elbow is regarded as a thin shell of revolution (portion of torus). Mean surface is obtained by rotation of angle Θ of a circle of radius a whose center is at a distance R from the axis of revolution Oz. One indicates by H the thickness of the elbow. One forces this thickness to remain constant like with the section of the elbow being perfectly circular. A point on mean surface is characterized by the two angles θ,ϕ and its position $-h/2\leq\zeta\leq+h/2$ compared to the mean surface, where θ is the longitudinal angle, variable enters 0 and Θ, and ϕ the angle measured on the cross-sectional area.



Appear 2.2.1-a: Geometry and kinematics of the elbow in theory of shell

One places oneself first of all in the frame of the linearized theory of the shells with transverse shears such as it was described for example in Washizu [bib14]. It limits our study to the frame of the small strains. Moreover, large rotations of mean surface are not taken into account. Displacements and rotations are thus defined compared to the initial geometry of the elbow. **If displacements of the points of mean surface in the three directions θ axial, ϕ orthoradiale and ζ radial are noted u,v and w those of any point of the elbow are written in the following way** :

$$u_\theta=u(\theta,\phi)+\zeta\beta_\phi(\theta,\phi)$$
$$u_\phi=v(\theta,\phi)-\zeta\beta_\theta(\theta,\phi)$$
$$u_\zeta=w(\theta,\phi)$$

where β_θ and β_ϕ are rotations compared to the vectors e_θ and e_ϕ respectively.
The strains in any point are thus given by [bib14]:

$$\varepsilon_{\theta\theta}=\frac{E_{\theta\theta}+\zeta\kappa_{\theta\theta}}{1+\zeta/R_\theta}$$
$$\varepsilon_{\phi\phi}=\frac{E_{\phi\phi}+\zeta\kappa_{\phi\phi}}{1+\zeta/R_\phi}$$
$$2\varepsilon_{\theta\phi}=\gamma_{\theta\phi}=\frac{2E_{\theta\phi}+2\zeta\kappa_{\theta\phi}}{\left(1+\zeta/R_\theta\right)\left(1+\zeta/R_\phi\right)}$$
$$2\varepsilon_{\theta\zeta}=\gamma_{\theta\zeta}=\frac{2E_{\theta\zeta}}{\left(1+\zeta/R_\theta\right)}$$
$$2\varepsilon_{\phi\zeta}=\gamma_{\phi\zeta}=\frac{2E_{\phi\zeta}}{\left(1+\zeta/R_\phi\right)}$$

with:

$$A=R+a\sin\phi,\ B=a,\ R_\theta=\frac{R+a\sin\phi}{\sin\phi},\ R_\phi=a.$$

where $E_{\theta\theta}, E_{\phi\phi}$ and $E_{\theta\phi}$ are the membrane strains of mean surface, $\kappa_{\theta\theta},\kappa_{\phi\phi},\kappa_{\theta\phi}$ the strains of bending of mean surface and the $E_{\theta\zeta}, E_{\phi\zeta}$ transverse distortions. The strains of mean surface are connected to displacements of mean surface by replacing the field of displacement of the preceding paragraph by that given above. One finds then:

$$E_{\theta\theta}=\frac{1}{A}\frac{\partial u}{\partial\theta}+\frac{v}{AB}\frac{\partial A}{\partial\phi}+\frac{w}{R_\theta}$$
$$E_{\phi\phi}=\frac{1}{B}\frac{\partial v}{\partial\phi}+\frac{w}{R_\phi}$$
$$2E_{\theta\phi}=\frac{1}{B}\frac{\partial u}{\partial\phi}+\frac{1}{A}\frac{\partial v}{\partial\theta}-\frac{u}{AB}\frac{\partial A}{\partial\phi}$$
$$\kappa_{\theta\theta}=\frac{1}{A}\frac{\partial\beta_\phi}{\partial\theta}-\frac{\beta_\theta}{AB}\frac{\partial A}{\partial\phi}$$
$$\kappa_{\phi\phi}=-\frac{1}{B}\frac{\partial\beta_\theta}{\partial\phi}$$
$$2\kappa_{\theta\phi}=\frac{1}{B}\frac{\partial\beta_\phi}{\partial\phi}-\frac{\beta_\phi}{AB}\frac{\partial A}{\partial\phi}-\frac{1}{A}\frac{\partial\beta_\theta}{\partial\theta}+\left[\frac{1}{R_\theta}\frac{1}{B}\frac{\partial u}{\partial\phi}+\frac{1}{R_\phi}\left(\frac{1}{A}\frac{\partial v}{\partial\theta}-\frac{u}{AB}\frac{\partial A}{\partial\phi}\right)\right]$$
$$2E_{\theta\zeta}=\beta_\phi+\frac{1}{A}\frac{\partial w}{\partial\theta}-\frac{u}{R_\theta}$$
$$2E_{\phi\zeta}=-\beta_\theta+\frac{1}{B}\frac{\partial w}{\partial\phi}-\frac{v}{R_\phi}$$

That is to say still:

$$E_{\theta\theta}=\frac{1}{R+a\sin\phi}\left(\frac{\partial u}{\partial\theta}+v\cos\phi+w\sin\phi\right)$$
$$E_{\phi\phi}=\frac{1}{a}\left(\frac{\partial v}{\partial\phi}+w\right)$$
$$\gamma_{\theta\phi}=\frac{1}{R+a\sin\phi}\left(\frac{\partial v}{\partial\theta}-u\cos\phi\right)+\frac{1}{a}\frac{\partial u}{\partial\phi}$$
$$\kappa_{\theta\theta}=\frac{1}{R+a\sin\phi}\left(\frac{\partial\beta_\phi}{\partial\theta}-\beta_\theta\cos\phi\right)$$
$$\kappa_{\phi\phi}=-\frac{1}{a}\frac{\partial\beta_\theta}{\partial\phi}$$
$$\kappa_{\theta\phi}=\frac{1}{a}\frac{\partial\beta_\phi}{\partial\phi}-\frac{1}{R+a\sin\phi}\left(\frac{\partial\beta_\theta}{\partial\theta}+\beta_\phi\cos\phi\right)+\left[\frac{\sin\phi}{R+a\sin\phi}\frac{1}{a}\frac{\partial u}{\partial\phi}+\frac{1}{R+a\sin\phi}\frac{1}{a}\left(\frac{\partial v}{\partial\theta}-u\cos\phi\right)\right]$$
$$\gamma_{\theta\zeta}=\beta_\phi+\frac{1}{R+a\sin\phi}\left(\frac{\partial w}{\partial\theta}-u\sin\phi\right)$$
$$\gamma_{\phi\zeta}=-\beta_\theta+\frac{1}{a}\left(\frac{\partial w}{\partial\phi}-v\right)$$

In this theory there are thus five unknowns; 3 displacements u, v and w like two rotations β_θ, β_ϕ. If the assumption of Coils-Kirchhoff is applied (thin tube) the transverse shears are null and there are nothing any more but 3 displacements u, v and w since:

$$\beta_\phi=-\frac{1}{R+a\sin\phi}\left(\frac{\partial w}{\partial\theta}-u\sin\phi\right)$$
$$\beta_\theta=\frac{1}{a}\left(\frac{\partial w}{\partial\phi}-v\right)$$

2.2.2 Case of the right pipe

If one applies these equations to the case of the right pipe with:

$$A=1\ , B=a\ , 1/R_\theta=0\ , R_\phi=a .$$

One finds the more usual statement for this kind of geometry:

$$E_{xx}=\frac{\partial u}{\partial x}$$
$$E_{\phi\phi}=\frac{1}{a}(\frac{\partial v}{\partial \phi}+w)$$
$$2E_{x\phi}=\frac{\partial v}{\partial x}+\frac{1}{a}\frac{\partial u}{\partial \phi}$$
$$\kappa_{xx}=\frac{\partial \beta_\phi}{\partial x}$$
$$\kappa_{\phi\phi}=-\frac{1}{a}\frac{\partial \beta_x}{\partial \phi}$$
$$2\kappa_{x\phi}=\frac{1}{a}\frac{\partial \beta_\phi}{\partial \phi}-\frac{\partial \beta_x}{\partial x}+[\frac{1}{a}\frac{\partial v}{\partial x}]$$
$$2E_{x\zeta}=\beta_\phi+\frac{\partial w}{\partial x}$$
$$2E_{\phi\zeta}=-\beta_x+\frac{1}{a}(\frac{\partial w}{\partial \phi}-v)$$

In this theory there are thus five unknowns; 3 displacements u, v and w like two rotations β_x, β_ϕ. If the assumption of Coils-Kirchhoff is applied (thin tube) the transverse shears are null and there are nothing any more but 3 displacements u, v and w since:

$$\beta_\phi=-\frac{\partial w}{\partial x}$$
$$\beta_x=\frac{1}{a}(\frac{\partial w}{\partial \phi}-v)$$

2.2.3 Notice

One can 3D introduce directly the kinematics of shell into the strain field. In this case one a:

$$\varepsilon_{\theta\theta}=E_{\theta\theta}+\zeta\kappa_{\theta\theta}$$
$$\varepsilon_{\phi\phi}=E_{\phi\phi}+\zeta\kappa_{\phi\phi}$$
$$2\varepsilon_{\theta\phi}=\gamma_{\theta\phi}=2E_{\theta\phi}+2\zeta\kappa_{\theta\phi}$$
$$2\varepsilon_{\theta\zeta}=\gamma_{\theta\zeta}=2E_{\theta\zeta}$$
$$2\varepsilon_{\phi\zeta}=\gamma_{\phi\zeta}=2E_{\phi\zeta}$$

where the statements $E_{\theta\theta}, E_{\phi\phi}$ and $E_{\theta\phi}$ for the membrane strains, $\kappa_{\theta\theta}, \kappa_{\phi\phi}, \kappa_{\theta\phi}$ the strains of bending and $E_{\theta\zeta}, E_{\phi\zeta}$ the transverse distortions are given by the following statement in the general case:

$$E_{\theta\theta}=\frac{1}{R+r\sin\phi}\left(\frac{\partial u}{\partial\theta}+v\cos\phi+w\sin\phi\right)$$
$$E_{\phi\phi}=\frac{1}{r}\left(\frac{\partial v}{\partial\phi}+w\right)$$
$$2E_{\theta\phi}=\frac{1}{r}\frac{\partial u}{\partial\phi}+\frac{1}{R+r\sin\phi}\left(\frac{\partial v}{\partial\theta}-u\cos\phi\right)$$
$$\kappa_{\theta\theta}=\frac{1}{R+r\sin\phi}\left(\frac{\partial\beta_\phi}{\partial\theta}-\beta_\theta\cos\phi\right)$$
$$\kappa_{\phi\phi}=-\frac{1}{r}\frac{\partial\beta_\theta}{\partial\phi}$$
$$2\kappa_{\theta\phi}=\frac{1}{r}\frac{\partial\beta_\phi}{\partial\phi}-\frac{1}{R+r\sin\phi}\left(\frac{\partial\beta_\theta}{\partial\theta}+\beta_\phi\cos\phi\right)$$
$$2E_{\theta\zeta}=\beta_\phi\frac{R+a\sin\phi}{R+r\sin\phi}+\frac{1}{R+r\sin\phi}\left(\frac{\partial w}{\partial\theta}-u\sin\phi\right)$$
$$2E_{\phi\zeta}=-\beta_\theta\frac{a}{r}+\frac{1}{r}\left(\frac{\partial w}{\partial\phi}-v\right)$$

It is noticed that to order 1 in ζ the two ways of proceeding give identical results. It is the definition of the strain of membrane or bending which changes. In the first case it is independent of the position in the thickness and is calculated for the average radius of the cross-sectional area of the pipe, whereas it depends on it in the case on the approach 3D. The term between hook in the statement of [§2.2.1]. represent a coupling between bending and the membrane which appears when one expresses $R+r\sin\phi$ and r according to $R+a\sin\phi$ and a. In the continuation of our analysis we will use this statement 3D degenerated of the kinematics of shell.
If moreover we use the assumption of Coils-Kirchhoff for the transverse shears, $E_{\theta\zeta}=E_{\phi\zeta}=0$ one finds well the following statements of rotations:

$$\beta_\phi=-\frac{1}{R+a\sin\phi}\left(\frac{\partial w}{\partial\theta}-u\sin\phi\right)$$
$$\beta_\theta=\frac{1}{a}\left(\frac{\partial w}{\partial\phi}-v\right)$$

and:

$$\kappa_{\theta\theta}=\frac{1}{R+r\sin\phi}\left[-\frac{1}{R+a\sin\phi}\left(\frac{\partial^2 w}{\partial\theta^2}-\frac{\partial u}{\partial\theta}\sin\phi\right)-\frac{\cos\phi}{a}\left(\frac{\partial w}{\partial\phi}-v\right)\right]$$
$$\kappa_{\phi\phi}=-\frac{1}{ar}\left(\frac{\partial^2 w}{\partial\phi^2}-\frac{\partial v}{\partial\phi}\right)$$
$$2\kappa_{\theta\phi}=\left(\frac{\partial w}{\partial\theta}-u\sin\phi\right)\left[\frac{\cos\phi}{(R+r\sin\phi)(R+a\sin\phi)}+\frac{a\cos\phi}{r(R+a\sin\phi)^2}\right]$$
$$-\frac{\partial^2 w}{\partial\theta\partial\phi}\left[\frac{1}{a(R+r\sin\phi)}+\frac{1}{r(R+a\sin\phi)}\right]$$
$$+\frac{\partial v}{\partial\theta}\frac{1}{a(R+r\sin\phi)}+\left(\frac{\partial u}{\partial\phi}\sin\phi+u\cos\phi\right)\frac{1}{r(R+a\sin\phi)}$$

the statements for $E_{\theta\theta}, E_{\phi\phi}$ and $E_{\theta\phi}$ remainder unchanged.

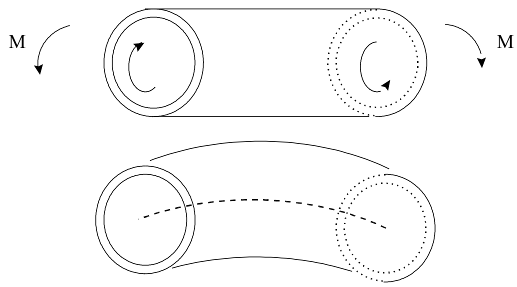
One can easily extend this remark to the case of the right pipe.

2.3 Analyzes pipes right and bent

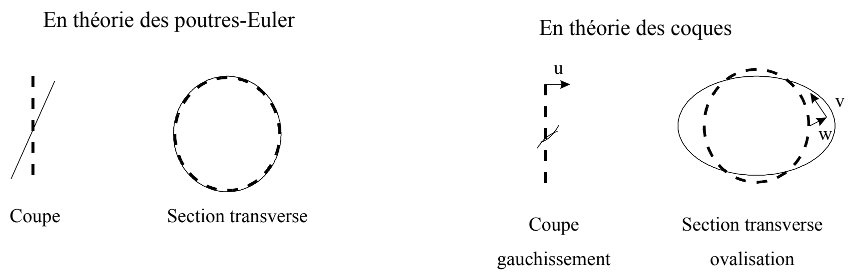
In conclusion of the two preceding analyses one can model the pipe as a beam element whose section is a thin shell. This interpretation is made in most codes ([bib2], [bib8], [bib9], [bib10], [bib12], etc…). In the absence of warping of the cross-sectional areas (i.e the cross-sectional areas remain plane) the axial displacement of beam gives the new position of the cross-sectional area and the displacements of ovalization (it is enough to then take u=0 in the mean equations of shells) make it possible to know how this one becomes deformed. The total deflection is obtained like superposition of the strains of beam and the strains of ovalization. The field of displacement which one represents on the figure below writes: .

$$U=U^p+U^s$$ In

the first field of displacement the image of the cross-sectional area is an identical cross-sectional area obtained by translation and rotation of the first. In the second field of displacement, the cross-sectional area is deformed. Appear



flexion-torsion d'une poutre droite



2.3-a: Decomposition of displacement in fields of beam and shell The modelization

finite element must thus give an account of two different mechanical responses: that of the beam and that of the shell for ovalization, swelling and warping. These three last modelizations utilize degrees of freedom which are not nodal (decomposition in Fourier series for example). Mixed elements

3 shell-beam for the right and curved pipes Kinematics

3.1 One

breaks up the field of displacement into a macroscopic part of "beam" and a local additional part of "shell". is V the useful space of the fields of three-dimensional displacements definite on an unspecified section of pipe. For

the beam part, as in [R3.03.03], one introduces the space of $\mathbf{T}$ the fields associated with a torsor (defined by two vectors): For

$$\mathbf{T}=\{v\in V/\exists(T,\Omega) \text{ tel que } \mathbf{v}(M)=T+\Omega\wedge\mathbf{GM}\}$$

the fields of displacement of, $\mathbf{T}$ is T the translation of the section (or the point), G Ω infinitesimal rotation and the fields are $\boldsymbol{v}$ displacements preserving the plane section S and not deformed there (One uses still the assumptions of NAVIER-BERNOULLI). is

$\mathbf{T}$ a vectorial subspace of finished size equalizes to 6. It has additional orthogonal for the scalar product on: V .

Any $$\mathbf{T}^{\perp}=\left\{\mathbf{v}\in V/\int_S \mathbf{v}.\mathbf{w}=0\ \forall\ \mathbf{w}\in\mathbf{T}\right\}$$

field of $\boldsymbol{u}$ V breaks up then in a single way all in all of an element of and $\mathbf{T}$ an element of: $\mathbf{T}^{\perp}$.

$$\boldsymbol{u}=\boldsymbol{u}^p+\boldsymbol{u}^s\ \boldsymbol{u}^p\in\mathbf{T}\ ,\ \boldsymbol{u}^s\in\mathbf{T}^{\perp}$$ One

applies then for displacements of surface of the pipe defined in [§2.2] decomposition in following Fourier series who check the preceding principle of orthogonality with displacements of beam until order 3 in the thickness of the pipe: (uniform

$$u(x,\phi)=\sum_{m=2}^{M} u_m^i(x)\cos m\phi$$
$$+\sum_{m=2}^{M} u_m^o(x)\sin m\phi$$
$$v(x,\phi)=w_{n1}^i(x)\sin\phi+\sum_{m=2}^{M} v_{nm}^i(x)\sin m\phi$$
$$\cdots-w_{n1}^o(x)\cos\phi+\sum_{m=2}^{M} v_{nm}^o(x)\cos m\phi$$
$$w(x,\phi)=w_n^o(x)$$ radial expansion) where
$$\cdots+\sum_{m=1}^{M} w_{nm}^i(x)\cos m\phi$$
$$\cdots+\sum_{m=1}^{M} w_{nm}^o(x)\sin m\phi$$

is x the curvilinear abscisse along the elbow or of the right pipe, indifferently, and M the number of modes of Fourier. Rotations and $\beta_x(x,\phi)$ $\beta_\phi(x,\phi)$ result from $u(x,\phi), v(x,\phi)$ et $w(x,\phi)$ the relations from Coils-Kirchhoff [§2.2.1]. Note:

One

can note that in the decomposition of $v(x,\phi)$ et $w(x,\phi)$ the terms in $\cos\phi$ et $\sin\phi$ are not completely independent because of orthogonality with displacements of beam. This makes it possible moreover to avoid rigid body motions, because if are v_{n1}^i, v_{n1}^o et w_{n1}^i, w_{n1}^o independent, one can find a solution non-zero giving of the strains null. In addition in the statement of one $u(x,\phi)$ notes the absence of the terms in and $\cos\phi$ formulates $\sin\phi$ *present in the beam part. If*
one neglects the variation of metric with the thickness of the pipe the conditions of rigorous orthogonality between displacements of beam and those of the surface of the pipe are satisfied. In the contrary case, to satisfy this condition rigorously one would need a development in Fourier series of rotations and $\beta_x(x,\phi)$ starting $\beta_\phi(x,\phi)$ for order 2. This is incompatible with the assumptions of Love_Kirchhoff for these rotations. Constitutive law

3.2

the behavior of the new element is a behavior 3D in plane stresses, because the total behavior of structure is that of a thin shell. It results from it that and $\sigma_{\zeta\zeta}=0$ the constitutive law is written in a general way in the following way: In

$$\begin{pmatrix}\sigma_{xx}\\ \sigma_{\phi\phi}\\ \sigma_{x\phi}\\ \sigma_{\zeta\phi}\\ \sigma_{x\zeta}\end{pmatrix}=C\begin{pmatrix}\varepsilon^p_{xx}+\varepsilon^s_{xx}\\ \varepsilon^p_{\phi\phi}+\varepsilon^s_{\phi\phi}\\ \gamma^p_{x\phi}+\gamma^s_{x\phi}\\ \gamma^p_{r\phi}+\gamma^s_{\zeta\phi}\\ \gamma^p_{xr}+\gamma^s_{x\zeta}\end{pmatrix}=C\begin{pmatrix}\varepsilon_{xx}\\ \varepsilon_{\phi\phi}\\ \gamma_{x\phi}\\ \gamma_{\zeta\phi}\\ \gamma_{x\zeta}\end{pmatrix}$$

our case one will neglect the transverse shears for the shell part of our field of displacement. It thus results from it that. $\gamma^s_{\zeta x}=\gamma^s_{\zeta\phi}=0$ As in addition [§2.1.2] it was shown that it $\gamma^p_{r\phi}=0$ results from it that. $\sigma_{\zeta\phi}=0$ For an elastic behavior one has as follows: and

$$\begin{pmatrix}\sigma_{xx}\\ \sigma_{\phi\phi}\\ \sigma_{x\phi}\\ \sigma_{x\zeta}\end{pmatrix}=\frac{E}{1-\nu^2}\begin{pmatrix}1 & \nu & 0 & 0\\ \nu & 1 & 0 & 0\\ 0 & 0 & \frac{1-\nu}{2} & 0\\ 0 & 0 & 0 & \frac{1-\nu}{2}\end{pmatrix}\begin{pmatrix}\varepsilon_{xx}\\ \varepsilon_{\phi\phi}\\ \gamma_{x\phi}\\ \gamma_{x\zeta}\end{pmatrix} \quad . \quad C=\frac{E}{1-\nu^2}\begin{pmatrix}1 & \nu & 0 & 0\\ \nu & 1 & 0 & 0\\ 0 & 0 & \frac{1-\nu}{2} & 0\\ 0 & 0 & 0 & \frac{1-\nu}{2}\end{pmatrix}$$ Work

3.3 of strain

the general statement of the work of strain 3D for the element of elbow with the type of above mentioned behavior is worth: where

$$W_{def}=\int_0^l\int_0^{2\pi}\int_{-h/2}^{h/2}(\varepsilon_{xx}\sigma_{xx}+\varepsilon_{\phi\phi}\sigma_{\phi\phi}+\gamma_{x\phi}\sigma_{x\phi}+\gamma_{x\zeta}\sigma_{x\zeta})dV$$

is l the curvilinear abscisse which applies to $l=(R+r\sin\phi)\theta$ an elbow where is θ the angle traversed to describe the elbow. In the case of an elbow, one has thus and $dV=(R+r\sin\phi)d\theta r d\phi d\zeta$ for a right pipe where $dV=dxrd\phi d\zeta$. ζ is the position in the thickness of the elbow which varies between - h/2 and +h/2. In the continuation, in order to reduce the notations, the second statement will be employed. Elastic

3.4 internal energy of the elbow In the case of

an elastic behavior, the elastic internal energy of the elbow is expressed in the following way: This

$$\Phi_{\text{int}}=\frac{1}{2}\int_0^l\int_0^{2\pi}\int_{-h/2}^{h/2}\left(\frac{E}{1-\nu^2}(\varepsilon^2_{xx}+\varepsilon^2_{\phi\phi}+2\nu\varepsilon_{xx}\varepsilon_{\phi\phi})+G(\gamma^2_{x\phi}+\gamma^2_{x\zeta})\right)dV$$

energy can be broken up into part of energy of beam, part of energy for the surface of the pipe and the terms of coupling of the type. $\int_{-h/2}^{h/2}\int_0^l\int_0^{2\pi}\varepsilon^p_{xx}.\varepsilon^s_{xx}dV$ Work

3.5 of the forces and couples external With

decomposition of displacements stated at the top of paragraph, the work of the forces being exerted on the pipe is expressed in the following way: by

$$W_{ext}=\int_0^l\int_{-h/2}^{+h/2}\int_0^{2\pi}\boldsymbol{F}_v.(\boldsymbol{U}^P+\boldsymbol{U}^S)dV+\int_0^l\int_0^{2\pi}\boldsymbol{F}_s.(\boldsymbol{U}^P+\boldsymbol{U}^S)(a\pm h/2)d\phi dx+\int_{-h/2}^{+h/2}\int_0^{2\pi}\boldsymbol{F}_c.(\boldsymbol{U}^P+U^S)r\,d\phi\,d\zeta=$$
$$\int_0^l\int_{-h/2}^{+h/2}\int_0^{2\pi}\boldsymbol{F}_v.\boldsymbol{U}^p dV+\int_0^l\int_0^{2\pi}\boldsymbol{F}_s.\boldsymbol{U}^p(a\pm h/2)d\phi dx+\int_{-h/2}^{+h/2}\int_0^{2\pi}\boldsymbol{F}_c.\boldsymbol{U}^p r\,d\phi\,d\zeta+$$
$$\int_0^l\int_{-h/2}^{+h/2}\int_0^{2\pi}\boldsymbol{F}_v.\boldsymbol{U}^s dV+\int_0^l\int_0^{2\pi}\boldsymbol{F}_s.\boldsymbol{U}^s(a\pm h/2)d\phi\,dx+\int_{-h/2}^{+h/2}\int_0^{2\pi}\boldsymbol{F}_c.\boldsymbol{U}^s r\,d\phi\,d\zeta=W_{ext}^p+W_{ext}^s$$

simple linear decomposition, where are $\boldsymbol{F_v},\boldsymbol{F_s},\boldsymbol{F_c}$ the voluminal forces, surface and of contour being exerted on the pipe, respectively. One

determines as follows: and

$$W_{ext}^p=\int_0^l\int_{-h/2}^{+h/2}\int_0^{2\pi}\boldsymbol{F}_v.\boldsymbol{U}^p dV+\int_0^l\int_0^{2\pi}\boldsymbol{F}_s.\boldsymbol{U}^p(a\pm h/2)d\phi dx+\int_{-h/2}^{+h/2}\int_0^{2\pi}\boldsymbol{F}_c.\boldsymbol{U}^p r\,d\phi\,d\zeta$$

:

$$W_{ext}^s=\int_0^l\int_{-h/2}^{+h/2}\int_0^{2\pi}\boldsymbol{F}_v.\boldsymbol{U}^s dV+\int_0^l\int_0^{2\pi}\boldsymbol{F}_s.\boldsymbol{U}^s(a\pm h/2)d\phi\,dx+\int_{-h/2}^{+h/2}\int_0^{2\pi}\boldsymbol{F}_c.\boldsymbol{U}^s r\,d\phi\,d\zeta$$

The work of the external forces can thus be separate in two contributions distinct from the same forces, on the kinematics of beam and its additional. Principle of virtual work

3.6 It

is written in the following way: with $\delta W_{ext}=\delta W_{ext}^p+\delta W_{ext}^s=\delta W_{int}$: External

$$\delta W_{int}=\int_0^l\int_0^{2\pi}\int_{-h/2}^{h/2}(\sigma_{xx}\delta\varepsilon_{xx}+\sigma_{\phi\phi}\delta\varepsilon_{\phi\phi}+\sigma_{x\phi}\delta\gamma_{x\phi}+\sigma_{x\zeta}\delta\gamma_{x\zeta})dV$$

3.6.1 forces part and couples for the beam part

the discretization of the principle of virtual work for the external forces gives: :

$$\delta W_{ext}^p=\int_0^l(f_x\delta u_x+f_y=\delta u_y+f_z\delta u_z+m_x\delta\theta_x+m_y\delta\theta_y+m_z\delta\theta_z)dx+$$
$$[\phi_x\delta u_x+\phi_y\delta u_y+\phi_z\delta u_z+\mu_x\delta\theta_x+\mu_y\delta\theta_y+\mu_z\delta\theta_z]_{0,l}$$

f_x,f_y,f_z linear forces acting according to, x and y passing z by the center of gravity of the cross-sectional areas: where

$f_i=\int_{-h/2}^{+h/2}\int_0^{2\pi}\boldsymbol{F}_v.\boldsymbol{e}_i r\,d\phi\,d\zeta+\int_0^{2\pi}\boldsymbol{F}_s.\boldsymbol{e}_i(a\pm h/2)d\phi$ are $\boldsymbol{e}_x,\boldsymbol{e}_y,\boldsymbol{e}_z$ the vectors of the local curvilinear base. :

m_x,m_y,m_z linear couples acting around the axes, x and y : z where

$m_i=\int_{-h/2}^{+h/2}\int_0^{2\pi}(\boldsymbol{r}\times\boldsymbol{F}_v).\boldsymbol{e}_i r\,d\phi\,d\zeta+\int_0^{2\pi}(\boldsymbol{r}\times\boldsymbol{F}_v).\boldsymbol{e}_i(a\pm h/2)d\phi$ are $\boldsymbol{e}_x,\boldsymbol{e}_y,\boldsymbol{e}_z$ the vectors of the local curvilinear base. :

ϕ_x, ϕ_y, ϕ_z concentrated forces acting according to, x and y passing z by the center of gravity of the cross-sectional areas: where

$\phi_i = \int_{-h/2}^{+h/2} \int_0^{2\pi} \boldsymbol{F}_c . \boldsymbol{e}_i \, r \, d\phi \, d\zeta$ are $\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z$ the vectors of the local curvilinear base. :

μ_x, μ_y, μ_z moments concentrated around the axes, x and y : z where

$\mu_i = \int_{-h/2}^{+h/2} \int_0^{2\pi} (\boldsymbol{r} \times \boldsymbol{F}_c) . \boldsymbol{e}_i \, r \, d\phi \, d\zeta$ are $\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z$ the vectors of the local curvilinear base. External

3.6.2 forces part and couples for the shell part It

is supposed that the external forces applied to the elbow are independent of the thickness of the elbow where

$$\delta W_{ext}^s = \int_0^l \int_0^{2\pi} (F_x \delta u + F_\phi \delta v + F_r \delta w + M_x \delta \beta_x + M_\phi \delta \beta_\phi) d\theta d\phi$$
$$+ \int_0^{2\pi} [\Phi_x \delta u + \Phi_\phi \delta v + \Phi_r \delta w + M_\theta \delta \beta_\theta + M_\phi \delta \beta_\phi]_{0,l} d\phi$$

: :

F_x, F_ϕ, F_r surface forces acting according to, x and ϕ : r where

$F_i = \int_{-h/2}^{+h/2} \boldsymbol{F}_v . \boldsymbol{e}_i \, r \, d\zeta + \boldsymbol{F}_s . \boldsymbol{e}_i (a \pm h/2)$ are $\boldsymbol{e}_x, \boldsymbol{e}_\phi, \boldsymbol{e}_r$ the vectors of the local toric base. :

M_x, M_ϕ surface couples acting around and x : ϕ where

$M_i = \int_{-h/2}^{+h/2} (\zeta \mathbf{e_r} \times \mathbf{F_v}) . \mathbf{e_i} \, r d\zeta + (\pm h/2 \mathbf{e_r} \times \mathbf{F_s}) . \mathbf{e_i} (a \pm h/2)$ are $\boldsymbol{e}_x, \boldsymbol{e}_\phi, \boldsymbol{e}_r$ the vectors of the local toric base. :

$\Phi_x, \Phi_\phi, \Phi_r$ linear forces acting according to, x and ϕ : r where

$\Phi_i = \int_{-h/2}^{+h/2} \boldsymbol{F}_c . \boldsymbol{e}_i \, r \, d\zeta$ are $\boldsymbol{e}_x, \boldsymbol{e}_\phi, \boldsymbol{e}_r$ the vectors of the local toric base. :

M_x, M_ϕ linear couples acting around and x : ϕ where

$M_i = \int_{-h/2}^{+h/2} (\zeta \boldsymbol{e}_r \times \boldsymbol{F}_c) . \boldsymbol{e}_i \, r \, d\zeta$ are $\boldsymbol{e}_x, \boldsymbol{e}_\phi, \boldsymbol{e}_r$ the vectors of the local toric base. Note:

When

the external forces applied are independent of ϕ work external on the kinematics of shell is null except that of the compressive forces corresponding to the forces according to. $\boldsymbol{e}_r$ It is also noticed that the statements of the moments linear and concentrated compared to are r null. One finds although exerted moment ago perpendicular to the plane of the shell. Generalized

3.7 forces If

is S the area of the cross-sectional area of S the pipe, one poses: :

$N = \int_S \sigma_{xx} dS$ normal force at the center of gravity of the cross-sectional area. and the

$T_y=\int_S \sigma_{xy}\,dS=-\int_S (\sin\phi\,\sigma_{x\zeta}+\cos\phi\,\sigma_{x\phi})\,dS$

$T_z=\int_S \sigma_{xz}\,dS=\int_S (\sin\phi\,\sigma_{x\phi}-\cos\phi\,\sigma_{x\zeta})\,dS$ shears according to and y . z :

$M_x=\int_S (y\,\sigma_{xz}-z\,\sigma_{xy})\,dS=-\int_S \sigma_{x\phi}\,dS$ twisting moment around. x :

$M_y=\int_S z\,\sigma_{xx}\,dS=-\int_S r\cos\phi\,\sigma_{xx}\,dS$ bending moment around. y :

$M_z=-\int_S y\,\sigma_{xx}\,dS=\int_S r\sin\phi\,\sigma_{xx}\,dS$ bending moment around. z Numerical

4 discretization of the variational formulations Discretization

4.1 of the fields of displacement and strain for the beam part In

a point of average fiber, the field of displacement of beam is in the local curvilinear reference defined in [§2.1]: This $\mathbf{U}^p=\begin{pmatrix} u_x \\ u_y \\ u_z \\ \theta_x \\ \theta_y \\ \theta_z \end{pmatrix}$

field can be discretized in the following way: and

$\boldsymbol{u}=\sum_{k=1}^{N} H_k(\theta)[u_x^k \boldsymbol{x}_k+u_y^k \boldsymbol{y}_k+u_z^k \boldsymbol{z}_k]$ It should be noted $\theta=\sum_{k=1}^{N} \overline{H_k}(\theta)[\theta_x^k \boldsymbol{x}_k+\theta_y^k \boldsymbol{y}_k+\theta_z^k \boldsymbol{z}_k]$

that the nodal values are given in the local coordinate systems attached to the nodes and that and $\boldsymbol{u}$ must θ be expressed in the local coordinate system associated with the current point. Curved beam

4.1.1 One

obtains then: and

$\begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}=\sum_{k=1}^{N} H_k(\theta)\begin{pmatrix} u_x^k(\boldsymbol{x}_k.\boldsymbol{x})+u_y^k(\boldsymbol{y}_k.\boldsymbol{x}) \\ u_x^k(\boldsymbol{x}_k.\boldsymbol{y})+u_y^k(\boldsymbol{y}_k.\boldsymbol{y}) \\ u_z^k \boldsymbol{z}_k \end{pmatrix}$ According to $\begin{pmatrix} \theta_x \\ \theta_y \\ \theta_z \end{pmatrix}=\sum_{k=1}^{N} \bar{H}_k(\theta)\begin{pmatrix} \theta_x^k(\boldsymbol{x}_k.\boldsymbol{x})+\theta_y^k(\boldsymbol{y}_k.\boldsymbol{x}) \\ \theta_x^k(\boldsymbol{x}_k.\boldsymbol{y})+\theta_y^k(\boldsymbol{y}_k.\boldsymbol{y}) \\ \theta_z^k \boldsymbol{z}_k \end{pmatrix}$

the kinematics of beam presented higher to [§2.1]: Knowing

$$\begin{aligned}
&\varepsilon_{rr}=0 \\
&\varepsilon_{\theta\theta}=\frac{1}{R+r\sin\phi}(u_{x,\theta}-u_y-r\theta_x\cos\phi+\theta_{z,\theta}r\sin\varphi-\theta_{y,\theta}r\cos\phi) \\
&\varepsilon_{\phi\phi}=0 \\
&2\varepsilon_{\theta\phi}=\frac{1}{R+r\sin\phi}(-u_x\cos\phi-u_{y,\theta}\cos\varphi+u_{z,\theta}\sin\phi-r\theta_{x,\theta}+\theta_y r\cos^2\phi-\theta_z r\sin\phi\cos\phi) \\
&\quad+(\theta_z\cos\phi+\theta_y\sin\phi) \\
&2\varepsilon_{r\theta}=\frac{1}{R+r\sin\phi}(-u_x\sin\phi-u_{y,\theta}\sin\phi-u_{z,\theta}\cos\phi+\theta_y r\sin\phi\cos\phi-\theta_z r\sin^2\phi) \\
&\quad+(\theta_z\sin\phi-\theta_y\cos\phi) \\
&2\varepsilon_{r\phi}=0
\end{aligned}$$

that and $\mathbf{x}_{,\theta}=-\mathbf{y}$ with $\mathbf{y}_{,\theta}=\mathbf{x}$ moreover.

$\boldsymbol{x}.\boldsymbol{x}_k=\boldsymbol{y}.\boldsymbol{y}_k=\cos(\theta-\theta_k)=C_k \text{ et } \boldsymbol{y}.\boldsymbol{x}_k=-\boldsymbol{x}.\boldsymbol{y}_k=\sin(\theta-\theta_k)=S_k$ That implies strain for the field: Maybe

$$\varepsilon_{\theta\theta}=\frac{1}{R+r\sin\phi}\sum_{k=1}^{N}[H'_k(u_x^k\cos(\theta-\theta_k)-u_y^k\sin(\theta-\theta_k))+H_k(-u_x^k\sin(\theta-\theta_k)-u_y^k\cos(\theta-\theta_k))$$
$$-H_k(u_x^k\sin(\theta-\theta_k)+u_y^k\cos(\theta-\theta_k))-r\bar{H}_k\cos\phi(\theta_x^k\cos(\theta-\theta_k)-\theta_y^k\sin(\theta-\theta_k))$$
$$-r\bar{H}'_k\cos\phi(\theta_x^k\sin(\theta-\theta_k)+\theta_y^k\cos(\theta-\theta_k))-r\bar{H}_k\cos\phi(\theta_x^k\cos(\theta-\theta_k)-\theta_y^k\cos(\theta-\theta_k))$$
$$+r\bar{H}'_k\sin\phi\,\theta_z^k]$$
$$\varepsilon_{\phi\phi}=0$$
$$\gamma_{\theta\phi}=\sum_{k=1}^{N}\frac{1}{R+r\sin\phi}[-H_k\cos\phi(u_x^k\cos(\theta-\theta_k)-u_y^k\sin(\theta-\theta_k))$$
$$-H'_k\cos\phi(u_x^k\sin(\theta-\theta_k)+u_y^k\cos(\theta-\theta_k))-H_k\cos\phi(u_x^k\cos(\theta-\theta_k)-u_y^k\sin(\theta-\theta_k))$$
$$+H'_k u_z^k\sin\phi-r\bar{H}'_k(\theta_x^k\cos(\theta-\theta_k)-\theta_y^k\sin(\theta-\theta_k))-r\bar{H}_k(-\theta_x^k\sin(\theta-\theta_k)-\theta_y^k\cos(\theta-\theta_k))$$
$$+r\bar{H}_k\cos^2\phi(\theta_x^k\sin(\theta-\theta_k)+\theta_y^k\cos(\theta-\theta_k))-r\bar{H}_k\theta_z^k\sin\phi\cos\phi]$$
$$+\bar{H}_k\sin\phi(\theta_x^k\sin(\theta-\theta_k)+\theta_y^k\cos(\theta-\theta_k))+\bar{H}_k\theta_z^k\cos\phi$$
$$\gamma_{r\theta}=\sum_{k=1}^{N}\frac{1}{R+r\sin\phi}[-H_k\sin\phi(u_x^k\cos(\theta-\theta_k)-u_y^k\sin(\theta-\theta_k))$$
$$-H'_k\sin\phi(u_x^k\sin(\theta-\theta_k)+u_y^k\cos(\theta-\theta_k))-H_k\sin\phi(u_x^k\cos(\theta-\theta_k)-u_y^k\sin(\theta-\theta_k))$$
$$-H'_k u_z^k\cos\phi+r\bar{H}_k\sin\phi\cos\phi(\theta_x^k\sin(\theta-\theta_k)+\theta_y^k\cos(\theta-\theta_k))-r\bar{H}_k\theta_z^k\sin^2\phi]$$
$$-\bar{H}_k\cos\phi(\theta_x^k\sin(\theta-\theta_k)+\theta_y^k\cos(\theta-\theta_k))+\bar{H}_k\theta_z^k\sin\phi$$

in matric form: where

$$\begin{pmatrix}\varepsilon_{\theta\theta}^P\\ \gamma_{\theta\phi}^P\\ \gamma_{\theta\zeta}^P\end{pmatrix}=\sum_{k=1}^{N}\boldsymbol{B}_k^P\boldsymbol{U}_k^P \text{ is } \mathbf{U}_k^p=\begin{pmatrix}u_x^k\\ u_y^k\\ u_z^k\\ \theta_x^k\\ \theta_y^k\\ \theta_z^k\end{pmatrix}$$
the field of displacement to the node and k

$$\mathbf{B}_k^P=\begin{pmatrix}
\frac{H'_kC_k}{R+r\sin\phi}-\frac{2H_kS_k}{R+r\sin\phi} & \frac{-H'_kS_k}{R+r\sin\phi}-\frac{2H_kC_k}{R+r\sin\phi} & 0 & \frac{-r\overline{H'_k}S_k\cos\phi}{R+r\sin\phi}-\frac{2r\overline{H_k}C_k\cos\phi}{R+r\sin\phi} & \frac{-r\overline{H'_k}C_k\cos\phi}{R+r\sin\phi}+\frac{2r\overline{H_k}S_k\cos\phi}{R+r\sin\phi} & \frac{r\overline{H'_k}\sin\phi}{R+r\sin\phi}\\
0 & 0 & 0 & 0 & 0 & 0\\
\frac{-H'_kS_k\cos\phi}{R+r\sin\phi}-\frac{2H_kC_k\cos\phi}{R+r\sin\phi} & \frac{-H'_kC_k\cos\phi}{R+r\sin\phi}+\frac{2H_kS_k\cos\phi}{R+r\sin\phi} & \frac{H'_k\sin\phi}{R+r\sin\phi} & \frac{\overline{H_k}S_k(R\sin\phi+2r)}{R+r\sin\phi}-\frac{r\overline{H'_k}C_k}{R+r\sin\phi} & \frac{\overline{H_k}C_k(R\sin\phi+2r)}{R+r\sin\phi}+\frac{r\overline{H'_k}S_k}{R+r\sin\phi} & \frac{R\overline{H_k}\cos\phi}{R+r\sin\phi}\\
\frac{-H'_kS_k\sin\phi}{R+r\sin\phi}-\frac{2H_kC_k\sin\phi}{R+r\sin\phi} & \frac{-H'_kC_k\sin\phi}{R+r\sin\phi}+\frac{2H_kS_k\sin\phi}{R+r\sin\phi} & \frac{-H'_k\cos\phi}{R+r\sin\phi} & \frac{-R\overline{H_k}S_k\cos\phi}{R+r\sin\phi} & \frac{-R\overline{H_k}C_k\cos\phi}{R+r\sin\phi} & \frac{R\overline{H_k}\sin\phi}{R+r\sin\phi}
\end{pmatrix}$$

the transition matrix of the strains at the field of displacement is written as follows: Straight beam $\boldsymbol{B}^P = \left(\boldsymbol{B}_1^P \cdots \boldsymbol{B}_N^P \right)$

4.1.2 and

$$\begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} = \sum_{k=1}^{N} H_k(x) \begin{pmatrix} u_x^k \\ u_y^k \\ u_z^k \end{pmatrix} \text{ According to } \begin{pmatrix} \theta_x \\ \theta_y \\ \theta_z \end{pmatrix} = \sum_{k=1}^{N} \bar{H}_k(x) \begin{pmatrix} \theta_x^k \\ \theta_y^k \\ \theta_z^k \end{pmatrix}$$

the kinematics of beam presented higher [§2.1]: that

$$\begin{aligned}
&\varepsilon_{rr}=0\\
&\varepsilon_{xx}=u_{x,x}+\theta_{z,x} r\sin\phi-\theta_{y,x} r\cos\phi\\
&\varepsilon_{\phi\phi}=0\\
&2\varepsilon_{x\phi}=-r\theta_{x,x}+(\theta_y+u_{z,x})\sin\phi+(\theta_z-u_{y,x})\cos\phi\\
&2\varepsilon_{rx}=(\theta_z-u_{y,x})\sin\phi-(\theta_y+u_{z,x})\cos\phi\\
&2\varepsilon_{r\phi}=0
\end{aligned}$$

implies strain for the field: Maybe

$$\begin{aligned}
&\varepsilon_{xx}=\sum_{k=1}^{N}\left(H_k^{'} u_x^k-\overline{H^{'}}_k r\cos(\phi)\theta_y^k+\overline{H^{'}}_k r\sin(\phi)\theta_z^k\right)\\
&\varepsilon_{\phi\phi}=0\\
&\gamma_{x\varphi}=\sum_{k=1}^{N}\left(-H_k^{'}\cos(\phi)u_y^k+H_k^{'}\sin(\phi)u_z^k-\overline{H^{'}}_k r\theta_x^k+\overline{H}_k\sin(\phi)\theta_y^k+\overline{H}_k\cos(\phi)\theta_z^k\right)\\
&\gamma_{rx}=\sum_{k=1}^{N}\left(-H_k^{'}\sin(\phi)u_y^k-H_k^{'}\cos(\phi)u_z^k-\overline{H}_k\cos(\phi)_y^k+\overline{H}_k\sin(\phi)\theta_z^k\right)
\end{aligned}$$

in matric form: where

$$\begin{pmatrix} \varepsilon_x^P \\ \varepsilon_\phi^P \\ \gamma_{x\phi}^P \\ \gamma_{x\zeta}^P \end{pmatrix} = \sum_{k=1}^{N} \boldsymbol{B}_k^P \boldsymbol{U}_k^P \text{ is } \mathbf{U}_k^p = \begin{pmatrix} u_x^k \\ u_y^k \\ u_z^k \\ \theta_x^k \\ \theta_y^k \\ \theta_z^k \end{pmatrix} \text{ the field of displacement to the node and } k$$

:

$$\boldsymbol{B}_k^P=\begin{pmatrix} H_k^{'} & 0 & 0 & 0 & -r\cos(\phi)\overline{H_k^{'}} & r\sin(\phi)\overline{H_k^{'}} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\cos(\phi)H_k^{'} & \sin(\phi)H_k^{'} & -r\overline{H_k^{'}} & \sin(\phi)\overline{H_k} & \cos(\phi)\overline{H_k} \\ 0 & -\sin(\phi)H_k^{'} & -\cos(\phi)H_k^{'} & 0 & -\cos(\phi)\overline{H_k} & \sin(\phi)\overline{H_k} \end{pmatrix}$$

The transition matrix of the strains at the field of displacement is written as follows: Discretization $\boldsymbol{B}^P=\left(\boldsymbol{B}_1^P\cdots\boldsymbol{B}_N^P\right)$

4.2 of the fields of displacement and strain for the additional part One

discretizes the field of displacement for the surface of the pipe in the form: with $\boldsymbol{U}^s=\sum_{k=1}^{N}H_k(x)\boldsymbol{U}_k^s$:

and

$$\mathbf{U}^s=\begin{bmatrix} u_m^i \\ v_m^i \\ w_m^i \\ u_m^o \\ v_m^o \\ w_m^o \\ w_1^i \\ w_1^o \\ w^o \end{bmatrix} \text{. One } \mathbf{U}_k^s=\begin{bmatrix} u_{km}^i \\ v_{km}^i \\ w_{km}^i \\ u_{km}^o \\ v_{km}^o \\ w_{km}^o \\ w_{k1}^i \\ w_{k1}^o \\ w_k^o \end{bmatrix} \Bigg\} m=2,M$$

has as follows: if

$$\begin{pmatrix} u(x,\phi) \\ v(x,\phi) \\ w(x,\phi) \end{pmatrix}=\begin{pmatrix} \cos(m\phi) & 0 & 0 & \sin(m\phi) & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & \sin(m\phi) & 0 & 0 & \cos(m\phi) & 0 & \vdots & \sin(\phi) & -\cos(\phi) & 0 \\ 0 & 0 & \cos(m\phi) & 0 & 0 & \sin(m\phi) & \vdots & \cos(\phi) & \sin(\phi) & 1 \end{pmatrix}\boldsymbol{U}^s$$

$m=2,M$

the indices of m $\boldsymbol{U}_k^s$ are ordered in the following way:

$$\boldsymbol{U}_k^s=\begin{vmatrix} u_{k\ m=2}^i \\ v_{k\ m=2}^i \\ w_{k\ m=2}^i \\ u_{k\ m=2}^o \\ v_{k\ m=2}^o \\ w_{k\ m=2}^o \\ \vdots \\ u_{k\ m=M}^i \\ v_{k\ m=M}^i \\ w_{k\ m=M}^i \\ u_{k\ m=M}^o \\ v_{k\ m=M}^o \\ w_{k\ m=M}^o \\ w_{k\ 1}^i \\ w_{k\ 1}^o \\ w_k^o \end{vmatrix}$$

The kinematics of shell presented higher to [§2.2] is: Bend

$$\begin{aligned} \varepsilon_{\theta\theta}&=E_\theta+\zeta\kappa_x \\ \varepsilon_{\phi\phi}&=E_\phi+\zeta\kappa_\phi \\ \gamma_{\theta\phi}&=2E_{\theta\phi}+2\zeta\kappa_{\theta\phi} \\ \gamma_{\theta\zeta}&=2E_{\theta\zeta}=0 \\ \gamma_{\phi\zeta}&=2E_{\phi\zeta}=0 \end{aligned}$$

4.2.1 With

: and

$$\begin{aligned} E_{\theta\theta}&=\frac{1}{R+r\sin\phi}\left(\frac{\partial u}{\partial\theta}+v\cos\phi+w\sin\phi\right) \\ E_{\phi\phi}&=\frac{1}{r}\left(\frac{\partial v}{\partial\phi}+w\right) \\ 2E_{\theta\phi}&=\frac{1}{r}\frac{\partial u}{\partial\phi}+\frac{1}{R+r\sin\phi}\left(\frac{\partial v}{\partial\theta}-u\cos\phi\right) \end{aligned}$$

: allows

$$\kappa_{\theta\theta}=\frac{1}{R+r\sin\phi}\left[-\frac{1}{R+a\sin\phi}\left(\frac{\partial^2 w}{\partial\theta^2}-\frac{\partial u}{\partial\theta}\sin\phi\right)-\frac{\cos\phi}{a}\left(\frac{\partial w}{\partial\phi}-v\right)\right]$$

$$\kappa_{\phi\phi}=-\frac{1}{ar}\left(\frac{\partial^2 w}{\partial\varphi^2}-\frac{\partial v}{\partial\phi}\right)$$

$$2\kappa_{\theta\phi}=\left(\frac{\partial w}{\partial\theta}-u\sin\phi\right)\left[\frac{\cos\phi}{(R+r\sin\phi)(R+a\sin\phi)}+\frac{a\cos\phi}{r(R+a\sin\phi)^2}\right]$$
$$-\frac{\partial^2 w}{\partial\theta\partial\phi}\left[\frac{1}{a(R+r\sin\phi)}+\frac{1}{r(R+a\sin\phi)}\right]$$
$$+\frac{\partial v}{\partial\theta}\frac{1}{a(R+r\sin\phi)}+\left(\frac{\partial u}{\partial\phi}\sin\phi+u\cos\phi\right)\frac{1}{r(R+a\sin\phi)}$$

to break up the strain field of shell on the modes of Fourier in the following way: with

$$\begin{pmatrix}\varepsilon^s_{xx}\\ \varepsilon^s_{\phi\phi}\\ \gamma^s_{x\phi}\\ \gamma^s_{x\zeta}\end{pmatrix}=\sum_{k=1}^{N}\boldsymbol{B}^s_k\,\boldsymbol{U}^s_k$$: where

$$\boldsymbol{B}^s_k=\left(\boldsymbol{B}^{si}_{k\ m=2}\quad\cdots\quad\boldsymbol{B}^{si}_{k\ m=M}\quad\boldsymbol{B}^{so}_{k\ m=2}\quad\cdots\quad\boldsymbol{B}^{so}_{k\ m=M}\quad\boldsymbol{B}^{sg}_k\right)$$

and

$$\mathbf{B}^{si}_{km}=\begin{pmatrix}
\frac{H'_k\cos m\phi}{R+r\sin\phi}\left(1+\frac{\zeta\sin\phi}{(R+a\sin\phi)}\right) & \frac{H_k\sin m\phi\cos\phi}{R+r\sin\phi}\left(1+\frac{\zeta}{a}\right) & \frac{H_k\cos m\phi\sin\phi}{R+r\sin\phi}-\frac{\zeta H''_k\cos m\phi}{(R+r\sin\phi)(R+a\sin\phi)}+\frac{\zeta H_k\, m\sin m\phi\cos\phi}{a(R+r\sin\phi)}\\
0 & \frac{m}{r}H_k\left(1+\frac{\zeta}{a}\right)\cos m\phi & \frac{1}{r}H_k\left(1+\frac{\zeta m^2}{a}\right)\cos m\phi\\
-\frac{m}{r}H_k\sin m\phi-\frac{H_k\cos m\phi\cos\phi}{R+r\sin\phi}-\zeta\frac{\cos\phi\sin\phi}{(R+a\sin\phi)}\left[\frac{H_k\cos m\phi}{(R+r\sin\phi)}+\frac{aH_k\cos m\phi}{r(R+a\sin\phi)}\right]+\zeta H_k\frac{(\cos m\phi\cos\phi-m\sin m\phi\sin\phi)}{r(R+a\sin\phi)} & \frac{H'_k\left(1+\frac{\zeta}{a}\right)\sin m\phi}{R+r\sin\phi} & \zeta\left(\frac{mH'_k\sin m\phi}{r(R+a\sin\phi)}+\frac{mH'_k\sin m\phi}{a(R+r\sin\phi)}\right)+\zeta\frac{\cos\phi}{R+a\sin\phi}\left[\frac{\cos m\phi\, H'_k}{(R+r\sin\phi)}+\frac{a\cos m\phi\, H'_k}{r(R+a\sin\phi)}\right]\\
0 & 0 & 0
\end{pmatrix}$$

$$\mathbf{B}^{so}_{km}=\begin{pmatrix}
\frac{H'_k\sin m\phi}{R+r\sin\phi}[1+\frac{\zeta\sin\phi}{(R+a\sin\phi)}] & \frac{H_k\cos m\phi}{R+r\sin\phi}\left(1+\frac{\zeta}{a}\right)\cos\phi & \frac{H_k\sin m\phi\sin\phi}{R+r\sin\phi}-\frac{\zeta H''_k\sin m\phi}{(R+r\sin\phi)(R+a\sin\phi)}-\frac{\zeta H_k\, m\cos m\phi\cos\phi}{a(R+r\sin\phi)}\\
0 & -\frac{m}{r}H_k\left(1+\frac{\zeta}{a}\right)\sin m\phi & \frac{1}{r}H_k\left(1+\frac{\zeta m^2}{a}\right)\sin m\phi\\
\frac{m}{r}H_k\cos m\phi-\frac{H_k\sin m\phi\cos\phi}{R+r\sin\phi}-\zeta\frac{\cos\phi\sin\phi}{R+a\sin\phi}[\frac{H_k\sin m\phi}{R+r\sin\phi}+\frac{aH_k\sin m\phi}{r(R+a\sin\phi)}]+\zeta H_k\frac{(m\cos m\phi\sin\phi+\sin m\phi\cos\phi)}{r(R+a\sin\phi)} & \frac{H'_k\left(1+\frac{\zeta}{a}\right)\cos m\phi}{R+r\sin\phi} & -\left(\frac{\zeta mH'_k\cos m\phi}{r(R+a\sin\phi)}+\frac{\zeta mH'_k\cos m\phi}{a(R+r\sin\phi)}\right)+\zeta\frac{\cos\phi}{R+a\sin\phi}[\frac{H'_k\sin m\phi}{R+r\sin\phi}+\frac{aH'_k\sin m\phi}{r(R+a\sin\phi)}]\\
0 & 0 & 0
\end{pmatrix}$$

right

$$\mathbf{B}^{sg}_{k}=\begin{pmatrix}
\frac{2H_k\cos\phi\sin\phi}{R+r\sin\phi}-\frac{\zeta H''_k\cos\phi}{(R+r\sin\phi)(R+a\sin\phi)}+\frac{2\zeta H_k\cos\phi\sin\phi}{a(R+r\sin\phi)} & \frac{H_k(\sin^2\phi-\cos^2\phi)}{R+r\sin\phi}-\frac{\zeta H''_k\sin\phi}{(R+r\sin\phi)(R+a\sin\phi)}-\frac{2\zeta H_k\cos^2\phi}{a(R+r\sin\phi)} & \frac{H_k\sin\phi}{(R+r\sin\phi)}-\frac{\zeta H''_k}{(R+a\sin\phi)(R+r\sin\phi)}\\
\frac{2}{r}\left(1+\frac{\zeta}{a}\right)H_k\cos\phi & \frac{2}{r}\left(1+\frac{\zeta}{a}\right)H_k\sin\phi & \frac{H_k}{r}\\
H'_k\sin\phi[\frac{\left(1+\frac{2\zeta}{a}\right)}{R+r\sin\phi}+\frac{\zeta}{r(R+a\sin\phi)}]+\frac{\zeta H'_k\cos\phi}{R+a\sin\phi}[\frac{\cos\phi}{R+r\sin\phi}+\frac{a\cos\phi}{r(R+a\sin\phi)}] & -H'_k\cos\phi[\frac{\left(1+\frac{2\zeta}{a}\right)}{R+r\sin\phi}+\frac{\zeta}{r(R+a\sin\phi)}]+\frac{\zeta H'_k\sin\phi}{R+a\sin\phi}[\frac{\cos\phi}{R+r\sin\phi}+\frac{a\cos\phi}{r(R+a\sin\phi)}] & \frac{\zeta H'_k\cos\phi}{(R+r\sin\phi)(R+a\sin\phi)}+\frac{\zeta H'_k a\cos\phi}{r(R+a\sin\phi)^2}\\
0 & 0 & 0
\end{pmatrix}$$

4.2.2 Pipe With

:

$$E_{xx}=\frac{\partial u}{\partial x}$$

$$E_{\varphi\varphi}=\frac{1}{r}\left(\frac{\partial v}{\partial\varphi}+w\right)$$

$$2E_{x\varphi}=\frac{\partial v}{\partial x}+\frac{1}{r}\frac{\partial u}{\partial\varphi}$$

$$\kappa_{xx}=\frac{\partial\beta_\varphi}{\partial x}$$

$$\kappa_{\varphi\varphi}=-\frac{1}{a}\frac{\partial\beta_x}{\partial\varphi}$$

$$2\kappa_{x\varphi}=\frac{1}{r}\frac{\partial\beta_\varphi}{\partial\varphi}-\frac{\partial\beta_x}{\partial x}$$

$$\beta_\phi=-\frac{\partial w}{\partial x}$$

$$\beta_x=\frac{1}{a}\left(\frac{\partial w}{\partial\phi}-v\right)$$

the strain field of shell breaks up on the modes of Fourier in the following way: That is to say

$$\varepsilon_{xx}=\frac{\partial}{\partial x}\sum_{n=1}^{N}\sum_{m=2}^{M}H_n(x)\left(\cos m\varphi u_{nm}^{i}+\sin m\varphi u_{nm}^{o}\right)$$

$$-\zeta\frac{\partial^2}{\partial x^2}\sum_{n=1}^{N}H_n(x)\left[w_n^o+\sum_{m=1}^{M}\left(\cos m\varphi w_{nm}^{i}+\sin m\varphi w_{nm}^{o}\right)\right]$$

$$\varepsilon_{\varphi\varphi}=\frac{1}{r}\frac{\partial}{\partial \varphi}\sum_{n=1}^{N}H_n(x)\left[\sin\varphi w_{n1}^{i}-\cos\varphi w_{n1}^{o}+\sum_{m=2}^{M}\left(\sin m\varphi v_{nm}^{i}+\cos m\varphi v_{nm}^{o}\right)\right]$$

$$+\frac{1}{r}\sum_{n=1}^{N}H_n(x)\left[w_n^o+\sum_{m=1}^{M}\left(\cos m\varphi w_{nm}^{i}+\sin m\varphi w_{nm}^{o}\right)\right]$$

$$+\frac{\zeta}{a\,r}\frac{\partial}{\partial \varphi}\sum_{n=1}^{N}H_n(x)\left[\sin\varphi w_{n1}^{i}-\cos\varphi w_{n1}^{o}+\sum_{m=2}^{M}\left(\sin m\varphi v_{nm}^{i}+\cos m\varphi v_{nm}^{o}\right)\right]$$

$$-\frac{\zeta}{a\,r}\frac{\partial^2}{\partial \varphi^2}\sum_{n=1}^{N}H_n(x)\left[w_n^o+\sum_{m=1}^{M}\left(\cos m\varphi w_{nm}^{i}+\sin m\varphi w_{nm}^{o}\right)\right]$$

$$\gamma_{x\varphi}=\frac{\partial}{\partial x}\sum_{n=1}^{N}H_n(x)\left[\sin\varphi w_{n1}^{i}-\cos\varphi w_{n1}^{o}+\sum_{m=2}^{M}\left(\sin m\varphi v_{nm}^{i}+\cos m\varphi v_{nm}^{o}\right)\right]$$

$$+\frac{1}{r}\frac{\partial}{\partial \varphi}\sum_{n=1}^{N}\sum_{m=2}^{M}H_n(x)\left(\cos m\varphi u_{nm}^{i}+\sin m\varphi u_{nm}^{o}\right)$$

$$+\frac{\zeta}{a}\frac{\partial}{\partial x}\sum_{n=1}^{N}H_n(x)\left[\sin\varphi w_{n1}^{i}-\cos\varphi w_{n1}^{o}+\sum_{m=2}^{M}\left(\sin m\varphi v_{nm}^{i}+\cos m\varphi v_{nm}^{o}\right)\right]$$

$$-\left(\frac{\zeta}{r}+\frac{\zeta}{r}\right)\frac{\partial^2}{\partial x\,\partial \varphi}\sum_{n=1}^{N}H_n(x)\left[w_n^o+\sum_{m=1}^{M}\left(\cos m\varphi w_{nm}^{i}+\sin m\varphi w_{nm}^{o}\right)\right]$$

$$\gamma_{x\zeta}=0$$

still: This

$$
\begin{aligned}
\varepsilon_{xx} &= \sum_{n=1}^{N} \sum_{m=2}^{M} H_n^{'}(x)\left(\cos m\phi\, u_{nm}^{i} + \sin m\phi\, u_{nm}^{o}\right)\\
&-\zeta \sum_{n=1}^{N} H_n^{''}(x)\left[w_n^{o} + \sum_{m=1}^{M}\left(\cos m\phi\, w_{nm}^{i} + \sin m\phi\, w_{nm}^{o}\right)\right]\\
\varepsilon_{\phi\phi} &= \frac{1}{r}\sum_{n=1}^{N} H_n(x)\left[\cos\phi\, w_{n1}^{i} + \sin\phi\, w_{n1}^{o} + \sum_{m=2}^{M}\left(m\cos m\phi\, v_{nm}^{i} - m\sin m\phi\, v_{nm}^{o}\right)\right]\\
&+\frac{1}{r}\sum_{n=1}^{N} H_n(x)\left[w_n^{o} + \sum_{m=1}^{M}\left(\cos m\phi\, w_{nm}^{i} + \sin mf w_{nm}^{o}\right)\right]\\
&+\frac{\zeta}{ar}\sum_{n=1}^{N} H_n(x)\left[\cos\phi\, w_{n1}^{i} + \sin\phi\, w_{n1}^{o} + \sum_{m=2}^{M}\left(m\cos m\phi\, v_{nm}^{i} - m\sin m\phi\, v_{nm}^{o}\right)\right]\\
&+\frac{\zeta}{ar}\sum_{n=1}^{N} H_n(x)\left[\sum_{m=1}^{M}\left(m^2\cos m\phi\, w_{nm}^{i} + m^2\sin m\phi\, w_{nm}^{o}\right)\right]\\
\gamma_{x\phi} &= \sum_{n=1}^{N} H_n^{'}(x)\left[\sin\phi\, w_{n1}^{i} - \cos f w_{n1}^{o} \sum_{m=2}^{M}\left(\sin m\phi\, v_{nm}^{i} + \cos m\phi\, v_{nm}^{o}\right)\right]\\
&+\frac{1}{r}\sum_{n=1}^{N}\sum_{m=2}^{M} H_n(x)\left(-m\sin m\phi\, u_{nm}^{i} + m\cos m\phi\, u_{nm}^{o}\right)\\
&+\frac{\zeta}{a}\sum_{n=1}^{N} H_n^{'}(x)\left[\sin\phi\, w_{n1}^{i} - \cos\phi\, w_{n1}^{o} \sum_{m=2}^{M}\left(\sin m\phi\, v_{nm}^{i} + \cos m\phi\, v_{nm}^{o}\right)\right]\\
&-\left(\frac{\zeta}{r}+\frac{\zeta}{a}\right)\sum_{n=1}^{N} H_n^{'}(x)\left[\sum_{m=1}^{M}\left(-m\sin m\phi\, w_{nm}^{i} + m\cos m\phi\, w_{nm}^{o}\right)\right]\\
\gamma_{x\zeta} &= 0
\end{aligned}
$$

gives in matric form: with

$$
\begin{pmatrix} e_{xx}^{s} \\ e_{ff}^{s} \\ g_{xf}^{s} \\ g_{xz}^{s} \end{pmatrix} = \sum_{k=1}^{N} B_k^{s} U_k^{s} \text{ : where } B_k^{s} = \left(B_{k\ m=2}^{si} \ \cdots \ B_{k\ m=M}^{si} \ B_{k\ m=2}^{so} \ \cdots \ B_{k\ m=M}^{so} \ B_k^{sg}\right)
$$

: ,

$$
B_{km}^{si} = \begin{pmatrix}
H_k^{'}\cos(m\varphi) & 0 & -\zeta H_k^{''}\cos(m\varphi) \\
0 & \frac{m}{r} H_k \cos(m\varphi)\left(1+\frac{\zeta}{a}\right) & \frac{1}{r} H_k \cos(m\varphi)\left(1+\frac{\zeta\ m^2}{a}\right) \\
-\frac{m}{r} H_k \sin(m\varphi) & H_k^{'}\sin(m\varphi)\left(1+\frac{\zeta}{a}\right) & \left(\frac{\zeta}{r}+\frac{\zeta}{a}\right) m H_k^{'}\sin(m\varphi) \\
0 & 0 & 0
\end{pmatrix} \text{ and}
$$

$$B_{km}^{so}=\begin{pmatrix} H_k^{'}\sin(m\varphi) & 0 & -\zeta H_k^{''}\sin(m\varphi) \\ 0 & -\frac{m}{r} H_k \sin(m\varphi)\left(1+\frac{\zeta}{a}\right) & \frac{1}{r} H_k \sin(m\varphi)\left(1+\frac{\zeta\, m^2}{a}\right) \\ \frac{m}{r} H_k \cos(m\varphi) & H_k^{'}\cos(m\varphi)\left(1+\frac{\zeta}{a}\right) & -\left(\frac{\zeta}{r}+\frac{\zeta}{a}\right) m H_k^{'}\cos(m\varphi) \\ 0 & 0 & 0 \end{pmatrix}$$

Discretization

$$B_k^{sg}=\begin{pmatrix} -\zeta\, H_k^{''}\cos(\varphi) & -\zeta\, H_k^{''}\sin(\varphi) & -\zeta\, H_k^{''} \\ \frac{2}{r}\left(1+\frac{\zeta}{a}\right) H_k \cos(\varphi) & \frac{2}{r}\left(1+\frac{\zeta}{a}\right) H_k \sin(\varphi) & \frac{H_k}{r} \\ \left(1+\frac{2\,\zeta}{a}+\frac{\zeta}{r}\right) H_k^{'}\sin(\varphi) & -\left(1+\frac{2\,\zeta}{a}+\frac{\zeta}{r}\right) H_k^{'}\cos(\varphi) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

4.3 of the strain field total with

$$\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{\varphi\varphi} \\ \gamma_{x\varphi} \\ \gamma_{x\zeta} \end{pmatrix}=\begin{pmatrix} \varepsilon_{xx}^P \\ \varepsilon_{\varphi\varphi}^P \\ \gamma_{x\varphi}^P \\ \gamma_{x\zeta}^P \end{pmatrix}+\begin{pmatrix} \varepsilon_{xx}^s \\ \varepsilon_{\varphi\varphi}^s \\ \gamma_{x\varphi}^s \\ \gamma_{x\zeta}^s \end{pmatrix}=\sum_{k=1}^{N} B_k^P\, U_k^P+\sum_{k=1}^{N} B_k^s\, U_k^s=\sum_{k=1}^{N} B_k\, U_k=BU$$ and

$B=\left(B_k^P\, B_k^s\right)_{k=1,\,N}$ Stiffness matrix $U=\begin{pmatrix} U_k^P \\ U_k^s \end{pmatrix}_{k=1,\,N}$

4.4

the variational formulation of the work of strain is: that is to say

$$\delta W_{def}=\int_0^l \int_0^{2\pi} \int_{-h/2}^{h/2} \left\{ \begin{pmatrix} \delta\varepsilon_{xx} & \delta\varepsilon_{\varphi\varphi} & \delta\gamma_{x\varphi} & \delta\gamma_{x\zeta} \end{pmatrix} \begin{pmatrix} \sigma_{xx} \\ \sigma_{\varphi\varphi} \\ \sigma_{x\varphi} \\ \sigma_{x\zeta} \end{pmatrix} \right\} rd\,\varphi\, dxd\,\zeta$$

still:

$$\delta\, W_{def}=\int_0^l \int_0^{2\pi} \int_{-h/2}^{h/2} \left\{ \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{\varphi\varphi} & \gamma_{x\varphi} & \gamma_{x\zeta} \end{pmatrix} C \begin{pmatrix} \delta\varepsilon_{xx} \\ \delta\varepsilon_{\varphi\varphi} \\ \delta\gamma_{x\varphi} \\ \delta\gamma_{x\zeta} \end{pmatrix} \right\} rd\,\varphi\, dxd\,\zeta$$

$$dW_{def}=\int_0^l\int_0^{2\pi}\int_{-h/2}^{h/2}\left\{\left(\sum_{k=1}^{N}B_k U_k\right)^T C\left(\sum_{k=1}^{N}B_k\delta U_k\right)\right\} r\,d\phi\,dx\,d\zeta$$

$$=\int_0^l\int_0^{2\pi}\int_{-h/2}^{h/2}\left\{\left(\sum_{k=1}^{N}U_{k^T}B_{k^T}\right)C\left(\sum_{k=1}^{N}B_k\,\delta U_k\right)\right\} r\,d\phi\,dx\,d\zeta$$

$$=\int_0^l\int_0^{2\pi}\int_{-h/2}^{h/2}\left\{\begin{pmatrix}U_{1^T} & \dots & U_{N^T}\end{pmatrix}B^T CB\begin{pmatrix}\delta U_1\\ \dots\\ \delta U_N\end{pmatrix}\right\} r\,d\phi\,dx\,d\zeta$$

$$=\begin{pmatrix}U_{1^T} & \dots & U_{N^T}\end{pmatrix}K\begin{pmatrix}\delta U_1\\ \dots\\ \delta U_N\end{pmatrix}$$

The principle of the virtual wors is written then where $\delta U^T\mathbf{K}\,U=F\ \ \delta U$ is $\mathbf{K}$ the stiffness matrix which is worth: Note:

$$\mathbf{K}=\int_0^l\int_0^{2\pi}\int_{-h/2}^{h/2}\left\{\mathbf{B}^T\mathbf{C}\,\mathbf{B}\right\} r\,d\varphi\,dxd\,\zeta$$

One

makes no assumption on the constitutive law. This statement is thus in particular valid in the case as of nonlinear behaviors (plasticity). Mass matrix

4.5

the terms of the mass matrix are obtained after discretization of the following variational formulation of the noncentrifugal terms of inertia: with

$$dW_{mass}^{ac}=\int_0^l\int_0^{2p}\int_{-h/2}^{+h/2}\rho\,\ddot{u}\,.\,dv\,rdxd\ fdz\quad .\quad u=\begin{pmatrix}u_1(x,\varphi,r)\\ u_2(x,\varphi,r)\\ u_3(x,\varphi,r)\\ u(x,\varphi,r)\\ v(x,\varphi,r)\\ w(x,\varphi,r)\end{pmatrix}$$

The notations used are those of [§2.1]: and u_1,u_2 are u_3 displacements of beam in a point of the section and u,v are w displacements of average fiber of this section in this same point.

The discretization gives then: where

$$\mathbf{u}=\sum_{k=1}^{N} H_k\ \mathbf{N}_k \begin{vmatrix} u_x^k \\ u_y^k \\ u_z^k \\ \theta_x^k \\ \theta_y^k \\ \theta_z^k \\ u_{km}^i \\ v_{km}^i \\ w_{km}^i \\ u_{km}^o \\ v_{km}^o \\ w_{km}^o \\ w_{k1}^i \\ w_{k1}^o \\ w_k^o \end{vmatrix} \quad \left.\begin{matrix} u_{km}^i \\ v_{km}^i \\ w_{km}^i \\ u_{km}^o \\ v_{km}^o \\ w_{km}^o \end{matrix}\right\} m=2,\ M$$

the matrixes have N_k as a statement:

$$\mathbf{N}_k=\begin{bmatrix} \mathbf{x}_k\cdot\mathbf{x} & \mathbf{y}_k\cdot\mathbf{x} & 0 & -r\ \cos\phi(\mathbf{x}_k\cdot\mathbf{y}) & -r\ \cos\phi(\mathbf{y}_k\cdot\mathbf{y}) & r\sin\phi & 0 & & & \dots & & & 0 & 0 & 0 \\ \mathbf{x}_k\cdot\mathbf{y} & \mathbf{y}_k\cdot\mathbf{y} & 0 & r\ \cos\phi(\mathbf{x}_k\cdot\mathbf{x}) & r\ \cos\phi(\mathbf{y}_k\cdot\mathbf{x}) & 0 & 0 & & & \dots & & & 0 & 0 & 0 \\ 0 & 0 & 1 & -r\ \sin\phi(\mathbf{x}_k\cdot\mathbf{x}) & -r\ \sin\phi(\mathbf{y}_k\cdot\mathbf{y}) & 0 & 0 & & & \dots & & & 0 & 0 & 0 \\ 0 & & \dots & & & & \cos(m\phi) & 0 & 0 & \sin(m\phi) & 0 & 0 & 0 & 0 & 0 \\ 0 & & \dots & & & & 0 & \sin(m\phi) & 0 & 0 & \cos(m\phi) & 0 & \sin(\phi) & \cos(\phi) & 0 \\ 0 & & \dots & & & & 0 & 0 & \cos(m\phi) & 0 & 0 & \sin(m\phi) & \cos(\phi) & \sin(\phi) & 1 \end{bmatrix}$$

The mass matrix has then as a statement: .

$$M=\int_0^l \int_0^{2\pi} \int_{-h/2}^{+h/2} \rho\ N^T\ N\ rdxd\ \varphi d\zeta$$ with

.
$N=\left(H_k N_k\right)_{k=1,\ N}$ Note:

In the case of

the right pipe, one has and $x_k.x=y_k.y=1$. $x_k.y=y_k.x=0$ *Shape functions*

4.6 One

chooses at least quadratic shape functions for the part beam (displacements and rotations) in order to avoid the phenomena of numerical blocking [bib3]. This choice implies the use of a finite element with three or four nodes. In the case of an element with 3 nodes, the shape functions are quadratic, and for an element with 4 nodes, the shape functions will be cubic. For the additional part, one chooses to take the same shape functions as for the beam part.

The quadratic shape functions (element with 3 nodes) are the following ones:

$$H_1(x) = \left(\frac{2x}{l}-1\right)\left(\frac{x}{l}-1\right)$$
$$H_2(x) = \frac{x}{l}\left(\frac{2x}{l}-1\right)$$
$$H_3(x) = -4\frac{x}{l}\left(\frac{x}{l}-1\right)$$

The cubic shape functions (element with 4 nodes) are the functions of Lagrange of order 3: Numerical integration

$$H_1(x) = \frac{(\xi-\xi_2)(\xi-\xi_3)(\xi-\xi_4)}{(\xi_1-\xi_2)(\xi_1-\xi_3)(\xi_1-\xi_4)}$$
$$H_2(x) = \frac{(\xi-\xi_1)(\xi-\xi_3)(\xi-\xi_4)}{(\xi_2-\xi_1)(\xi_2-\xi_3)(\xi_2-\xi_4)}$$
$$H_3(x) = \frac{(\xi-\xi_1)(\xi-\xi_2)(\xi-\xi_4)}{(\xi_3-\xi_1)(\xi_3-\xi_2)(\xi_3-\xi_4)}$$
$$H_4(x) = \frac{(\xi-\xi_1)(\xi-\xi_2)(\xi-\xi_3)}{(\xi_4-\xi_1)(\xi_4-\xi_2)(\xi_4-\xi_3)}$$
$$-1 \le \xi \le 1$$

4.7

integration is done by the method of Gauss along average fiber, the method of Simpson in the thickness and on the circumference. For the Gauss quadrature, one uses 3 points of integration for the elements with 3 nodes, as for the elements with 4 nodes (those under-are thus integrated). Integration in the thickness is an integration by layers of which the number could be built-in later on by the user. For each layer one takes 3 points of Simpson, the 2 points ends being common with the close layers. Thus for layers n one uses points $2n+1$. The number of sectors for integration on the circumference, could also be built-in later on by the user. Currently the numbers of layers and sectors are fixed at their maximum value: 3 layers (7 points) and 16 sectors (33 points), which gives 693 points of integration on the whole. The integration of Simpson comes down to calculating the sum of the values of the function at the points of integrations (ends and medium of each layer or sector) affected of the weights given by the table below. Cordonnées

of the points Weights	=
$-\sqrt{(3/5)}$ -0,77459 66692 41483 5	9=0,55555 55555 55556 0
8	9=0,88888 88888 88889 =0,77459
$\sqrt{(3/5)}$ 66692 41483 5	9=0,55555 55555 55556 Gauss quadrature

on average fiber Weight

$$\frac{1}{3}\,\frac{4}{3}\,\frac{2}{3}\,\frac{4}{3}\,\frac{2}{3}\cdots\cdots\frac{2}{3}\,\frac{4}{3}\,\frac{2}{3}\,\frac{4}{3}\,\frac{1}{3}$$

of the points of integration for the method of Simpson Thus
for a function on $f(x,\varphi,\zeta)$ a right geometry one a: being

$$\int_0^l\int_0^{2\pi}\int_{-h/2}^{h/2} f(x,\varphi,\zeta)\, r\; dx\; d\varphi\; d\zeta=\frac{l}{2}\int_{-1}^{1}\int_0^{2\pi}\int_{-h/2}^{h/2}\tilde{f}(\tilde{x},\varphi,\zeta)\; r\; d\,\tilde{x}\; d\varphi\; d\zeta$$

$$=\frac{l}{2}\,\frac{h}{2\,N_{COU}}\,\frac{2\pi}{2\,N_{SECT}}\sum_{k=1}^{NPG}\;\sum_{n=1}^{2N_{COU}+1}\;\sum_{m=1}^{2N_{SECT}+1}\left[w_k\; w_n\; w_m\; r_{N_{COU}}\;\tilde{f}(\tilde{x}_{NPG},\varphi_{N_{SECT}},\zeta_{N_{COU}})\right]$$

to them $w_k\; w_n\; w_m$ respectively weights of integration over the length, the circumference and in the thickness, ordered as the two tables show it above. Discretization

4.8 of work external

the variational formulation of external energy for the beam part is written: and

$$\delta W^p_{ext}=\int_0^l (f_x\delta u_x+f_y\delta u_y+f_z\delta u_z+m_x\delta\theta_x+m_y\delta\theta_y+m_z\delta\theta_z)\,dx+$$
$$[\varphi_x\,\delta u_x+\varphi_y\,\delta u_y+\varphi_z\,\delta u_z+\mu_x\,\delta\theta_x+\mu_y\,\delta\theta_y+\mu_z\,\delta\theta_z]_{0,l}$$

for the additional part she is written in taking into account only the loading of pressure: By

$$\delta W^s_{ext}=\int_0^l F_r\,\delta w\, dx+[\Phi_r\,\delta w]_{0,l}$$

taking account of the discretization of displacements, one can write: and

$$\delta W^{\mathbf{p}}_{ext}=\sum_{k=1}^{N}\int_0^l (f_x H_k(x)(\mathbf{x_k}.\mathbf{x}\,\delta u^k_x+\mathbf{y_k}.\mathbf{x}\,\delta u^k_y)+f_y H_k(x)(\mathbf{x_k}.\mathbf{y}\,\delta u^k_x+\mathbf{y_k}.\mathbf{y}\,\delta u^k_y)+f_z H_k(x)\delta u^k_z+$$
$$m_x\overline{H_k}(x)(\mathbf{x_k}.\mathbf{x}\,\delta\theta^k_x+\mathbf{y_k}.\mathbf{x}\,\delta\theta^k_y)+m_y\overline{H_k}(x)(\mathbf{x_k}.\mathbf{y}\,\delta\theta^k_x+\mathbf{y_k}.\mathbf{y}\,\delta\theta^k_y)+m_z\overline{H_k}(x)\delta\theta^k_z)dx+$$
$$[\phi_x H_k(x)\delta u^k_x+\phi_y H_k(x)\delta u^k_y+\phi_z H_k(x)\delta u^k_z+\mu_x\overline{H_k}(x)\delta\theta^k_x+\mu_y\overline{H_k}(x)\delta\theta^k_y+\mu_z\overline{H_k}(x)\delta\theta^k_z]_{0,l}$$

$$=\sum_{k=1}^{N}\Bigg(\int_0^l H_k(x)\big(f_x(\mathbf{x_k}.\mathbf{x})+f_y(\mathbf{x_k}.\mathbf{y})\big)dx+[\phi_x H_k(x)]_{0,l}\;\vdots\;\int_0^l H_k(x)\big(f_x(\mathbf{y_k}.\mathbf{x})+f_y(\mathbf{y_k}.\mathbf{y})\big)dx+[\phi_y H_k(x)]_{0,l}\;\vdots\;\int_0^l f_z H_k(x)dx+[\mu_x\overline{H_k}(x)]_{0,l}\;\vdots$$
$$\int_0^l\overline{H_k}(x)\big(m_x(\mathbf{x_k}.\mathbf{x})+m_y(\mathbf{x_k}.\mathbf{y})\big)dx+[\mu_x\overline{H_k}(x)]_{0,l}\;\vdots\;\int_0^l\overline{H_k}(x)\big(m_x(\mathbf{y_k}.\mathbf{x})+m_y(\mathbf{y_k}.\mathbf{y})\big)dx+[\mu_y\overline{H_k}(x)]_{0,l}\;\vdots\;\int_0^l m_z\overline{H_k}(x)dx+[\mu_z\overline{H_k}(x)]_{0,l}\Bigg)\delta\mathbf{U}^p_k$$
$$=\sum_{k=1}^{N}\mathbf{F}^p_k\,\delta\mathbf{U}^p_k=\mathbf{F}^p\,\delta\mathbf{U}^p$$

Note

$$\delta\, W^s_{ext}=\sum_{k=1}^{N}\left(\int_0^l F_r\, H_k(x)\, dw^o_k\, dx+[F_r\, H_k(x)\, dw^o_k]_{0,l}\right)$$
$$=\sum_{k=1}^{N}\left(0\quad 0\quad 0\quad 0\quad 0\quad 0\quad \int_0^l F_r H_k(x)\,dx+[F_r H_k(x)]_{0,l}\right)\delta U^s_k$$
$$=\sum_{k=1}^{N}F^s_k\,\delta U^s_k=F^s\,\delta U^s$$

: For

the extreme nodes of the elbow one has and $x_k . x = y_k . y = 1$. $x_k . y = y_k . x = 0$ *In the case of the right pipe, one has and* $x_k . x = y_k . y = 1$ *for* $x_k . y = y_k . x = 0$ *all the element. Geometrical*

5 characteristics of the pipe section One

has in this chapter some useful results to characterize the element pipe and which are calculated by computation option MASS_INER `of` the Code_Aster *. In* the continuation the index indicates d the results for the right pipe and the index for c the curved pipe. Volume

- **:** .

$$V_d = \int_0^l \int_0^{2\pi} \int_{-h/2}^{h/2} r \; dx \; d\varphi \; d\zeta = 2\pi lah$$

$$V_c = \int_0^l \int_0^{2\pi} \int_{-h/2}^{h/2} r \; dx \; d\varphi \; d\zeta = \int_0^{\Theta} \int_0^{2\pi} \int_{-h/2}^{h/2} [R + (a+\zeta)\sin\varphi](a+\zeta) d\theta \; d\varphi \; d\zeta = 2\pi R\Theta ah$$

Center of gravity

- **:** The position of this last is calculated starting from the point medium with the two extreme nodes of the pipe section, in the reference associated with the internal node with the element (cf [§2.1.1]). In this reference, the coordinates of the center of gravity are: and

$$\begin{pmatrix} x_{Gd} \\ y_{Gd} \\ z_{Gd} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} x_{Gc} \\ y_{Gc} \\ z_{Gc} \end{pmatrix} = -R \begin{pmatrix} 0 \\ \frac{2}{\Theta} \sin\frac{\Theta}{2} (1 + \frac{1}{2R^2}[a^2 + \frac{h^2}{4}]) - \cos\frac{\Theta}{2} \\ 0 \end{pmatrix}$$

- **inertia Stamps: It** is relatively easy to calculate the matrix of inertia at the center of curvature of the elbow in O the reference defined above. To have his statement the fact then is used that: where

$$I_{xx}(G) = I_{xx}(O) - mb^2$$
$$I_{yy}(G) = I_{yy}(O)$$
$$I_{zz}(G) = I_{zz}(O) - mb^2$$

B is the distance between the center of gravity and the center of curvature which is worth: .

$$b = R \frac{2}{\Theta} \sin\frac{\Theta}{2} (1 + \frac{1}{2R^2}[a^2 + \frac{h^2}{4}])$$ In the case of

a right pipe, the notion of center of curvature does not have a meaning. and O coincide G with the node interns element and the medium of the segment uniting the two top nodes. If
one notes A the area of the cross-sectional area and his I inertia compared to the center of the section one can write: and

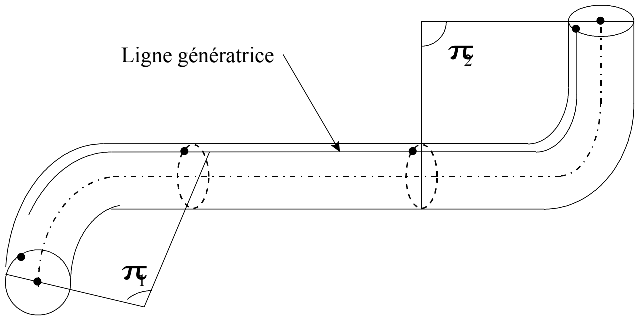
$$I^d_{xx}(O) = \rho l I$$
$$I^d_{yy}(O) = \rho l (I/2 + Al^2/12)$$
$$I^d_{zz}(O) = \rho l (I/2 + Al^2/12)$$

.

$$I^c_{xx}(O) = \rho R\Theta (\frac{I}{2} + [AR^2 + 3\frac{I}{2}][\frac{1}{2} + \frac{\sin\Theta}{4\Theta}])$$
$$I^c_{yy}(O) = \rho R\Theta (\frac{I}{2} + [AR^2 + 3\frac{I}{2}][\frac{1}{2} - \frac{\sin\Theta}{4\Theta}])$$ Connection
$$I^c_{zz}(O) = \rho R\Theta (AR^2 + 3\frac{I}{2})$$

6 pipe-pipe In order to

be able to represent correctly line pipework where the elbows are not coplanar, it is necessary to thus choose an origin common Des. φ for two elbows belonging to two perpendicular planes between them, it is necessary to be able to take into account the fact that displacements in the plane of the first elbow are equal to displacements except plane of the second in the cross-section of connection. Appear

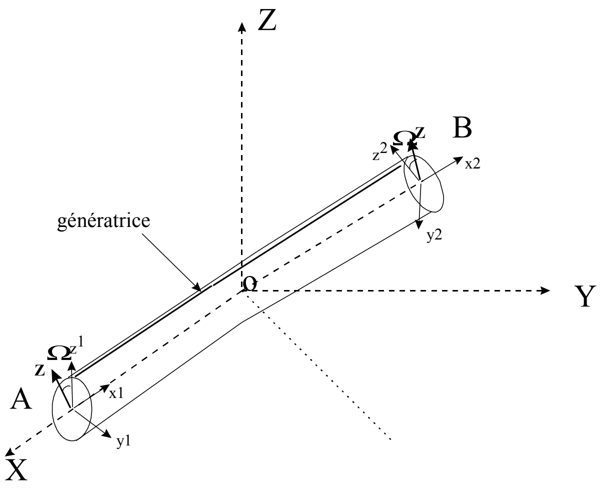


6-a: Representation of two noncoplanar elbows connected by a right pipe In

[bib12], this common origin is defined by generating line continuous along the pipework as indicated above. This generator intersects each cross-sectional area in a point. The angle between definite Z on [Figure 2.1.1-a] and the line passing by the center of the cross-sectional area and this point are worth. Ω Construction

6.1 of a particular generator For

a cross-sectional area end of line of pipework, one defines a unit vector origin z_1 in the plane of this section. The intersection between the direction of this vector and the mean surface of the elbow determines the trace of the generator on this section. One calls x_1, y_1, z_1 the direct trihedron associated with this section where is x_1 the unit vector perpendicular to the cross-sectional area builds with [Figure 2.1.1-a]. For all the other cross-sectional areas, the trihedron x_k, y_k, z_k is obtained either by rotation of the trihedron in the case of $x_{k-1}, y_{k-1}, z_{k-1}$ the bent parts, or by translation of the trihedron for $x_{k-1}, y_{k-1}, z_{k-1}$ the right parts of the pipework. The intersection between the cross-sectional area and the line resulting from the center of this section directed by is z_k the trace of a generator represented below Ci -. Appear



6.1-a: Representation of the generator of reference

the origin of the commune φ to all the elements is defined compared to the trace of this generator on the cross-sectional area. The angle between the trace of the generator and the current position on the cross-sectional area is then called. ψ Connection

6.2 from one element to another

the kinematics of [§3.1] is given in the plane of the elbow. This one is determined by the arc of a circle generated by the axis of the elbow. The origin of the angles is the norm with the plane chosen as with [§2.1]. To define the origin from a generator makes it possible to raise the problems of continuity of displacements of an element to another. Indeed if one applies that relative displacements of the cross-sectional areas are of the type where $\sum_{p=1}^{M} u_p^i \cos p\psi + u_p^o \sin p\psi$ is ψ the angle with the trace of the generator on the cross-sectional area, the continuity of displacements is automatically ensured of one element the other. One
notes $\mathbf{Z}$ the vector perpendicular to the plane of the elbow corresponding at the origin of the angles chosen up to now. It is noticed that the vectors and $\mathbf{Z}$ are $\mathbf{z}_k$ in the plane of the section. k is φ the angle defined compared to. $\mathbf{Z}$ If one introduces $\boldsymbol{\psi}$ the angle counted from the trace of the generator on the cross-sectional area (thus compared to) $\mathbf{z}_k$ there is the following relation: where $\psi=\varphi-\Omega_k$ angle $\Omega_k=(\mathbf{Z},\mathbf{z_k})$ enters and Z z_k the plane of the cross-sectional area. Thus displacements are from now on type. $\sum_{p=1}^{M} u_p^i \cos p(\varphi-\Omega_k)+u_p^o \sin p(\varphi-\Omega_k)$ It should be noted that for an elbow given the angle whatever the Ω_k selected cross-sectional area is identical. It is during the transition from one elbow to another which the exchange value Ω_k . Note:

When

the pipework consists of right elements colinéaires, one chooses arbitrarily. $\Omega=0$
Numerical

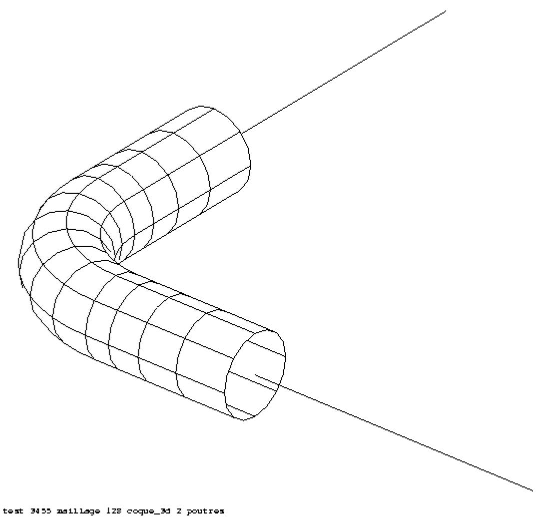
6.3 establishment

line of pipework is with a grid by right or curved elements to order. The first element indicates the beginning of line of pipework. One determines for this element the associated trihedron. x_1, y_1, z_1 If this element is right, one chooses, $\Omega_1=0$ if not one calculates as indicated in Ω_1 the preceding paragraph. If the first element is right the trihedron associated with the first cross-sectional area with the second element x_2, y_2, z_2 is obtained by translation of. x_1, y_1, z_1 If the first element is curved, the associated trihedron x_2, y_2, z_2 is obtained by rotation of in x_1, y_1, z_1 the plane of the elbow. In this case if $\Omega_2=0$ the second element is right and if $\Omega_2=(z^2, z_2)$ the second element is curved where z^2 is built like z [Figure 2.1.1-a]. The continuation of construction results easily by recurrence from the preceding diagram. Connections

7 shell-pipe and 3D-pipe followed

7.1 Approach One

adopts here a approach similar to the cases 3D-beam [R3.03.03], and shell-beam [R3.03.06]: it is a question of characterizing connection between ending node of an element pipe and a mesh group of edge of shell elements or 3D. This makes it possible to net part of the pipework (for example an elbow) in shells or elements 3D, and the rest out of right pipes. Appear



7.1-a: Connection between a mesh COQUE_3D and right pipes [HI75-98/001] Thanks to

the kinematics introduced into the element pipe, connections shell - pipe and 3D - pipe must make it possible to net in shell elements or 3D only the elbow, without right parts, since the damping of ovalization (and warping) is taken into account in the element pipe.

Connection results in kinematic relations between the degrees of freedom of the nodes of (which S represents the section of connection, modelled by edge elements of shell or 3D), and the node of N pipe. So that connection is effective, it is necessary [R3.03.03] that it checks the following properties: to be able

[1] to transmit forces of beam to the mesh shells or 3D, and to be able to transmit all the degrees of freedom of the element pipe (or the forces duaux of those),
[2] not to generate in the shell elements or 3D of secondary stresses,
[3] not to support the static kinematic relations or conditions ones compared to the others, to admit
[4] unspecified behaviors and to function in dynamics.

The linear relations will have the same form as in the case shell-beam, with additional equations specific to the degrees of freedom of the pipe. One

already introduced with [§3.1] the space of $\mathbf{T}$ the fields associated with a torsor (defined by two vectors): where

$$\mathbf{T}=\{v\in V/\exists(T,\Omega)\text{ tel que } v(M)=T+\Omega\wedge\mathbf{GM}\}$$

for the fields of displacement of, $\mathbf{T}$ is T the translation of the section (or the point), G Ω infinitesimal rotation and the fields are $\mathbf{v}$ displacements preserving the plane section S and not deformed there (One uses still the assumptions of NAVIER-BERNOULLI).
The displacement of the pipe is worth then: where

$$u^t=u^p+u^s \quad u^p\in T \quad , u^s\in\mathbf{T}^\perp$$

:

$$\mathbf{T}^\perp=\left\{\mathbf{v}\in V/\int_S \mathbf{v}.\mathbf{w}=0\ \forall\,\mathbf{w}\in\mathbf{T}\right\}$$

The approach consists in breaking up the field of displacement of shell or u^c the field of three-dimensional displacement into u^{3d} three fields:

$$u^c=u^p+u^s+u^\varepsilon$$

- a field of displacement following a kinematics of beam (torsor u^p),
- a field of local displacement of the section according to the kinematics of pipe (Fourier series u^s) defined in [§3.1],
- an orthogonal additional field u^ε with the two first within the meaning of the scalar product. Note:

When

decomposition in Fourier series of [§3.1] is infinite one A. $u^\varepsilon=0$

to translate the equation above into linear relations, one shows that should be calculated the following integrals, for the shell (or 3D) and the pipe: average

- displacement: average $\int_S u^c\,dS$
- rotation: average $\int_S \mathbf{GM}\wedge u^c\,dS$
- swelling: modes $\int_S \frac{\mathbf{GM}}{\|\mathbf{GM}\|}.u^c\,dS$
- of Fourier: $\int_S u^c\cos p\varphi\,dS\,,\int_S u^c\sin p\varphi\,dS$ $\int_S u^c\wedge\frac{\mathbf{GM}}{\|\mathbf{GM}\|}\cos p\varphi\,dS$

  $\int_S u^c\wedge\frac{\mathbf{GM}}{\|\mathbf{GM}\|}\sin p\varphi\,dS$ $\int_S \frac{\mathbf{GM}}{\|\mathbf{GM}\|}.u^c\cos p\varphi\,dS$. $\int_S \frac{\mathbf{GM}}{\|\mathbf{GM}\|}.u^c\sin p\varphi\,dS$ For the modes of Fourier one will choose the relations simplest to exploit since some are redundant (see remark of [§6.7]). Note:

One

easily passes from the analytical statements of the connection 3D - pipe with those of the connection shell - pipe. It is enough to substitute for u^{3d} in u^c all the integrals of connection suggested above. One will thus speak again of this connection only to [§7.8] treating numerical establishment. Kinematics

7.2 of the pipe. In

the curvilinear base associated (o, x, y, z) with the cross-sectional area of [§2.1] one notes displacements where u_1, u_2 et u_3 : .

$$u_1(r,x,\phi)=u_x(x)-\theta_y(x) r\cos\phi+\theta_z(x) r\sin\phi+u(x,\phi)+\zeta\,\beta_\phi(x,\phi)$$
$$u_2(r,x,\phi)=u_y(x)+\theta_x(x) r\cos\phi-v(x,\phi)\cos\phi-w(x,\phi)\sin\phi+\zeta\,\beta_\theta(x,\phi)\cos\phi$$ Once
$$u_3(r,x,\phi)=u_z(x)-\theta_x(x) r\sin\phi+v(x,\phi)\sin\phi-w(x,\phi)\cos\phi-\zeta\,\beta_\theta(x,\phi)\sin\phi$$

discretized this statement becomes: where

$$u_1(r,x,\phi)=\sum_{k=1}^{N} H_k(x)(x_k\cdot x)u_x^k+H_k(x)(y_k\cdot x)u_y^k-\bar{H}_k(x)(x_k\cdot y)\theta_y^k r\cos\phi-\bar{H}_k(x)(y_k\cdot y)\theta_y^k r\cos\phi+\bar{H}_k(x)\theta_z^k r\sin\phi+u(x,\phi)+\zeta\,\beta_\phi(x,\phi)$$
$$u_2(r,x,\phi)=\sum_{k=1}^{N} \bar{H}_k(x)(x_k\cdot y)u_x^k+H_k(x)(y_k\cdot y)u_y^k+\bar{H}_k(x)(x_k\cdot x)\theta_x^k r\cos\phi+\bar{H}_k(x)(y_k\cdot x)\theta_y^k r\cos\phi-v(x,\phi)\cos\phi-w(x,\phi)\sin\phi+\zeta\,\beta_\theta(x,\phi)\cos\phi$$
$$u_3(r,x,\phi)=\sum_{k=1}^{N} H_k(x)u_z^k-\bar{H}_k(x)(x_k\cdot x)\theta_x^k r\sin\phi-\bar{H}_k(x)(y_k\cdot x)\theta_y^k r\sin\phi+v(x,\phi)\sin\phi-w(x,\phi)\cos\phi-\zeta\,\beta_\theta(x,\phi)\sin\phi$$

$u(x,\varphi), v(x,\varphi), w(x,\varphi), \beta_\theta(x,\varphi)$ et $\beta_\theta(x,\varphi)$ are discretized as with [§3.1].

Displacement in a node of k X-coordinate end x_k of pipe is written then: For

$$\begin{aligned}
u_1(r,x_k,\phi)= &\ u_x^k-\theta_y^k r\cos(\phi)+\theta_z^k r\sin(\phi)+\sum_{m=2}^{M}(u_{km}^i\cos m\phi+u_{km}^0\sin m\phi)+\zeta\,\beta_\phi(x_k,\phi)\\
u_2(r,x_k,\phi)= &\ u_y^k+\theta_x^k r\cos(\phi)-\cos\phi\sum_{m=2}^{M}(v_{km}^i\sin m\phi+v_{km}^0\cos m\phi)-\sin\phi\sum_{m=2}^{M}(w_{km}^i\cos m\phi+w_{km}^0\sin m\phi)\\
&-w_{k1}^i\sin 2\phi+w_{k1}^0\cos 2\phi-w_k^0\sin\phi+\zeta\,\beta_\theta(x_k,\phi)\cos\phi\\
u_3(r,x_k,\phi)= &\ u_z^k-\theta_x^k r\sin(\phi)+\sin\phi\sum_{m=2}^{M}(v_{km}^i\sin m\phi+v_{km}^0\cos m\phi)-\cos\phi\sum_{m=2}^{M}(w_{km}^i\cos m\phi+w_{km}^0\sin m\phi)\\
&-w_{k1}^i\cos 2\phi-w_{k1}^0\sin 2\phi-w_k^0\cos\phi-\zeta\,\beta_\theta(x_k,\phi)\sin\phi
\end{aligned}$$

the pipe the vector angular momentum and $\mathbf{GM}\wedge u(M)$ swelling have $\frac{\mathbf{GM}}{\|\mathbf{GM}\|}.u(M)$ as respective statements: ,

$$\mathbf{GM}\wedge u(M)=\begin{cases}-u_z(x) r\sin\varphi+u_y(x) r\cos\varphi+r^2\theta_x(x)-rv(x,\varphi)\\ -ru_1(r,x,\varphi)\cos\varphi\\ ru_1(r,x,\varphi)\sin\varphi\end{cases}$$ and

: .

$$\frac{\mathbf{GM}}{\|\mathbf{GM}\|}.u(M)=-u_z(x)\cos\varphi-u_y(x)\sin\varphi+w(x,\varphi)$$ Note:

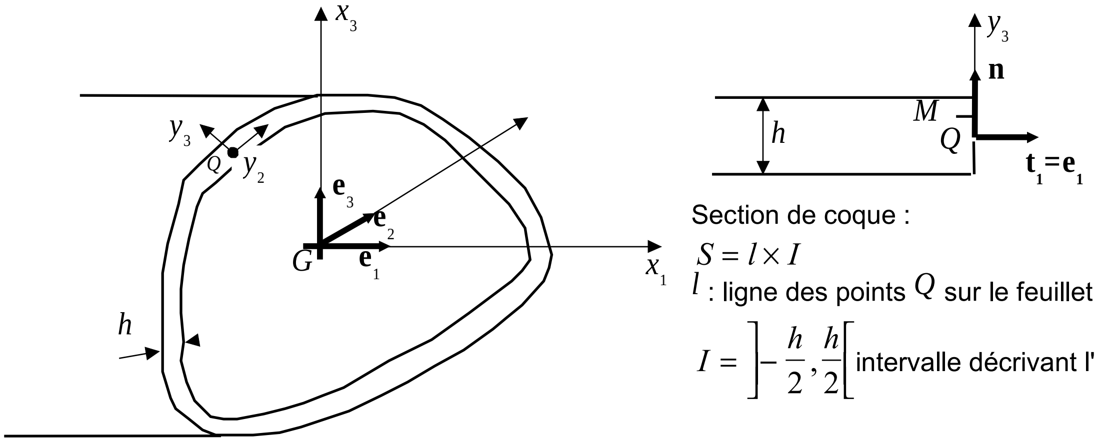
The first component of the field of displacement utilizes in $u(x,\varphi)$ *an isolated way. The same applies to the first component of the vector rotation with respect to and* $v(x,\varphi)$ *of swelling with respect to.* $w(x,\varphi)$ *This remark will be used with [§5.6] to bind the modes of Fourier to the degrees of freedom of edges of shell. Kinematics*

7.3 of shell

the kinematics of shell of Coils-Kirchhoff or of Naghdi-Mindlin is written in the thickness: constitute

$$u^c(M)=u^c(Q)+\left(\theta^c(Q)\wedge n\right).y_3$$

- $u^c(Q)$ the vector displacement of mean surface in, Q constitutes
- $\theta^c(Q)$ the vector rotation in Q norm according to the directions and t_1 of t_2 the tangent plane with. Q This



Section de coque :

$S = l \times I$

l : ligne des points Q sur le feuillet moyen

$I = \left]-\frac{h}{2},\frac{h}{2}\right[$ intervalle décrivant l'épaisseur.

displacement and this rotation are calculated in the total reference. It is possible by change of reference to have their statements in the curvilinear base of (o,x,y,z) [§2.1] associated with the cross-sectional area of the junction between the shell and the pipe. For

each node, the program calculates the coefficients of the linear $9+6(M-1)$ relations which connect:

- 6 degrees of freedom of the node of beam of P the pipe,
- degrees of freedom $2+3\times2(M-1)$ of Fourier of the pipe,
- the degree of freedom of swelling of the pipe, with
- the degrees of freedom of all the nodes of the list of meshes of edge of shell. These

linear relations will be dualisées, like all the linear relations resulting for example from key word LIAISON_DDL `of` AFFE_CHAR_MECA . `They` are built as for connection 3D-beam from the assembly of elementary terms. Computation

7.4 of average displacement on the section S It

acts to calculate the integral, where $\int_S u^c\,dS$ is u^c the displacement of shell (comprising 6 d.o.f. per node), is S the edge of shell.

Average displacement on the section S is written: that is to say

$$\int_S u^c(M)\,dS = h\int_l u^c(Q)\,dl + \int_l \left(\theta^c(Q)\wedge n\right)\left(\int_{-h/2}^{h/2} y_3\,dy_3\right) dl$$

. $\int_S u^c(M)\,dS = h\int_l u^c(Q)\,dl$ In addition
one also has for the pipe part: .

$$\int_S u^c(M)\,dS = \int_S [u^p(M)+u^s(M)]\,dS = \int_S \begin{pmatrix} u_x^k \\ u_y^k \\ u_z^k \end{pmatrix} dS = S\begin{pmatrix} u_x^k \\ u_y^k \\ u_z^k \end{pmatrix}$$ One

establishes thus that the average displacement of the section of the pipe to the node is k equal to the displacement of beam of the node. k One can thus linearly bind the degrees of freedom of beam of translation to the node with k the average of the degrees of freedom of displacement of edge of the shell. One

neglects in this statement the variation of metric in the thickness of the shell. Computation

7.5 of the average rotation of the section S is

$$\begin{aligned}\int_S \mathbf{GM}\wedge u^c(M)\,dS &= \int_l \int_{-h/2}^{h/2} \left(\mathbf{GQ}+y_3\,n(Q)\right)\wedge\left(u^c(Q)+\theta^c(Q)\wedge n(Q)\,y_3\right) dl\,dy_3 \\ &= h\int_l \mathbf{GQ}\wedge u^c(Q)\,dl + \int_l \mathbf{GQ}\wedge\left(\theta^c(Q)\wedge n(Q)\right) dl \int_{-h/2}^{h/2} y_3\,dy_3 \\ &+ \int_l n(Q)\wedge u^c(Q)\left(\int_{-h/2}^{h/2} y_3\,dy_3\right) dl + \int_l n(Q)\wedge\left(\theta^c(Q)\wedge\ n(Q)\right)\int_{-\frac{h}{2}}^{\frac{h}{2}} y_3^2\,dy_3\,.\,dl\end{aligned}$$

. $\int_S \mathbf{GM}\wedge u^c(M)\,.\,dS = h\int_l \mathbf{GQ}\wedge u^c(Q)\,dl + \frac{h^3}{12}\int_l \theta^c(Q)\,dl$ In addition

one also has for the pipe part: where

$$\int_S \mathbf{GM}\wedge u^c(M)\,.\,dS = \int_S \mathbf{GM}\wedge\left[u^p(M)+u^s(M)\right].\,dS = \int_S \begin{pmatrix} r^2.\,\theta_x^k \\ r^2.\cos^2\phi\,\theta_y^k \\ r^2.\sin^2\phi\,\theta_z^k \end{pmatrix} = I\begin{pmatrix} \theta_x^k \\ \theta_y^k \\ \theta_z^k \end{pmatrix}$$

I am the tensor of inertia of the beam. One establishes thus that the average rotation of the section of the pipe to the node is k equal to the rotation of beam to the node. k One can thus linearly bind the degrees of freedom of beam of rotation to the node with k the degrees of freedom of rotation of edge of the shell. One

neglects in this statement the variation of metric in the thickness of the shell. Computation

7.6 of the average swelling of the section S It

acts to calculate the integral, $\int_S u^c . n\, dS = \int_S u^c . \frac{\mathbf{GM}}{\|\mathbf{GM}\|} dS$ where is $n = \frac{\mathbf{GM}}{\|\mathbf{GM}\|} = \frac{\mathbf{GQ}}{\|\mathbf{GQ}\|}$ the norm at the mean surface of shell.

Average displacement on the section S is written: .

$\int_S u^c(M).n\, dS = h\int_l u^c(Q).n\, dl + \int_l (\theta^c(Q)\wedge n).n\left(\int_{-h/2}^{h/2} y_3\, dy_3\right) dl = h\int_l u^c(Q).n\, dl$ In addition

one also has for the pipe part: One

$$\int_S \frac{\mathbf{GM}}{\|\mathbf{GM}\|}.u^c(M)\, dS = \int_S \frac{\mathbf{GM}}{\|\mathbf{GM}\|}.[u^p(M)+u^s(M)]\, dS = \int_S w_k^o\, dS$$

establishes thus that the average swelling of the section of the pipe to the node is k equal to the degree of freedom of swelling of the pipe to the node. k One can thus linearly bind the degree of freedom of swelling of pipe to the node with k the degrees of freedom of displacement of edge of the shell. One

neglects in this statement the variation of metric in the thickness of the shell. Computation

7.7 of the coefficients of Fourier on the section S It

acts to calculate the six integrals $\int_S u^c \cos p\varphi\, dS$ $\int_S u^c \sin p\varphi\, dS$ $\int_S u^c \wedge \frac{\mathbf{GM}}{\|\mathbf{GM}\|} \cos p\varphi\, dS$, $\int_S u^c \wedge \frac{\mathbf{GM}}{\|\mathbf{GM}\|} \sin p\varphi\, dS$ and $\int_S \frac{\mathbf{GM}}{\|\mathbf{GM}\|}.u^c \cos p\varphi\, dS$, $\int_S \frac{\mathbf{GM}}{\|\mathbf{GM}\|}.u^c \sin p\varphi\, dS$ where is u^c the displacement of shell (comprising 6 d.o.f. per node), is S the edge of shell. There

is the following relation for displacements on the section: S that is to say

$$\int_S u^c(M)\cos p\varphi\, dS = h\int_l u^c(Q)\cos p\varphi\, dl + \int_l (\theta^c(Q)\wedge n)\left(\int_{-h/2}^{h/2} y_3\, dy_3\right)\cos p\varphi\, dl$$

and $\int_S u^c(M)\cos p\varphi\, dS = h\int_l u^c(Q)\cos p\varphi\, dl$. $\int_S u^c(M)\sin p\varphi\, dS = h\int_l u^c(Q)\sin p\varphi\, dl$ In addition

one also has for the pipe part: .

$$\int_S u^c(M)\cos p\varphi\, dS = \int_S \begin{pmatrix} u_1(r, x_k, \varphi) \\ u_2(r, x_k, \varphi) \\ u_3(r, x_k, \varphi) \end{pmatrix} \cos p\varphi\, dS$$

The first component of this relation then enables us to connect linearly the coefficient of Fourier to u_{kp}^i the components of displacements of edge of shell in the following way: In the same way

$$h\int_l u_1^c(Q)\cos p\varphi\, dl = \int_S u_1(r, x_k, \varphi)\cos p\varphi\, dS = \begin{cases} -\int_S r\theta_y^k \cos^2\varphi\, dS & \text{si } p=1 \\ \int_S u_{kp}^i \cos^2 p\varphi\, dS & \text{si } p\neq 1 \end{cases}$$

from where $\int_S u^c(M)\sin p\varphi\, dS=\int_S \begin{pmatrix} u_1(r,x_k,\varphi) \\ u_2(r,x_k,\varphi) \\ u_3(r,x_k,\varphi)\end{pmatrix}\sin p\varphi\, dS$ one from of deduced that: There

$$h\int_l u_1^c(Q)\sin p\varphi\, dl=\int_S u_1(r,x_k,\varphi)\sin p\varphi\, dS=\begin{cases}\int_S r\theta_z^k\sin^2\varphi\, dS & \text{si } p=1\\ \int_S u_{kp}^o\sin^2 p\varphi\, dS & \text{si } p\neq 1\end{cases}$$

is the following relation for rotations on the section: S .

$$\int_S u^c(M)\wedge.\frac{\mathbf{GM}}{\|\mathbf{GM}\|}\cos p\varphi\, dS=h\int_l u^c(Q)\wedge\frac{\mathbf{GQ}}{\|\mathbf{GQ}\|}\cos p\varphi\, dl+\int_l(\theta^c(Q)\wedge n)\wedge n(\int_{-h/2}^{+h/2} y_3 dy_3)\cos p\varphi\, dl$$

The first component of this relation then enables us to linearly connect the coefficient of Fourier to v_{kp}^o the components of displacements and rotations of edge of shell in the following way: In the same way

$$h\int_l[u^c(Q)\wedge\frac{\mathbf{GQ}}{\|\mathbf{GQ}\|}]_1\cos p\varphi\, dl=\begin{cases}-\int_S[ru_y^k\cos^2\varphi+rw_{k1}^o\cos^2\varphi]dS & \text{si } p=1\\ \int_S rv_{kp}^o\cos^2(p\varphi)dS & \text{si } p\neq 1\end{cases}$$

there a: One

$$h\int_l[u^c(Q)\wedge\frac{\mathbf{GQ}}{\|\mathbf{GQ}\|}]_1\sin p\varphi\, dl=\begin{cases}\int_S[ru_z^k\sin^2\varphi+rw_{k1}^i\sin^2\varphi]dS & \text{si } p=1\\ \int_S rv_{kp}^i\sin^2(p\varphi)dS & \text{si } p\neq 1\end{cases}$$

is the following relation for swelling on the section: S .

$$\int_S\frac{\mathbf{GM}}{\|\mathbf{GM}\|}.u^c(M).\cos p\varphi\, dS=h\int_l\frac{\mathbf{GQ}}{\|\mathbf{GQ}\|}.\ u^c(Q)\cos p\varphi\, dl$$ This

relation enables us to connect linearly the coefficient of Fourier to w_{kp}^i the components of displacements of edge of shell in the following way: .

$$h\int_l\frac{\mathbf{GQ}}{\|\mathbf{GQ}\|}.u^c(Q)\cos p\varphi\, dl=\begin{cases}\int_S[-u_z^k\cos^2\varphi+w_{k1}^i\cos^2\varphi]dS & \text{si } p=1\\ \int_S w_{kp}^i\cos^2(p\varphi)dS & \text{si } p\neq 1\end{cases}$$ In the same way

, one a: One

$$h\int_l\frac{\mathbf{GQ}}{\|\mathbf{GQ}\|}.u^c(Q)\sin p\varphi\, dl=\begin{cases}\int_S -u_y^k\sin^2\varphi+w_{k1}^o\sin^2\varphi\, dS & \text{si } p=1\\ \int_S w_{kp}^o\sin^2(p\varphi)\, dS & \text{si } p\neq 1\end{cases}$$

uses for all these relations the fact that One $\int_S\cos p\varphi\cos q\varphi\, dS=0$ si $p\neq q$.

neglects in this statement the variation of metric in the thickness of the shell. Note:

One

can note that some of the relations established in this paragraph for p=1 are redundant with those established in the paragraphs [§7.4] and [§7.5]. On the six relations established starting from the computation of the integral forms $\int_S \boldsymbol{u}^{c.} \cos\phi\, dS$ $\int_S \boldsymbol{u}^{c.} \sin\phi\, dS$ $\int_S \boldsymbol{u}^c \wedge \frac{\boldsymbol{GM}}{\|\boldsymbol{GM}\|}.\cos\phi\, dS$, $\int_S \boldsymbol{u}^c \wedge \frac{\boldsymbol{GM}}{\|\boldsymbol{GM}\|}.\sin\phi\, dS$ *and* $\int_S \boldsymbol{u}^c \wedge \frac{\boldsymbol{GM}}{\|\boldsymbol{GM}\|}.\boldsymbol{u}^c \cos\phi\, dS$, $\int_S \boldsymbol{u}^c \wedge \frac{\boldsymbol{GM}}{\|\boldsymbol{GM}\|}.\boldsymbol{u}^c \sin\phi\, dS$ *only two among the four last are linearly independent of the others. Thus the two first were already established with [the §7.4] and of the combinations of the four last give again those of [§7.5]. Establishment*

7.8 of the method The computation

of the coefficients of the linear relations is done in three times: computation

- of elementary quantities on the elements of the list of meshes of edges of shells (mesh of type SEG2): surface
  - =. $\int_{elt} 1$; $\int_{elt} x$; $\int_{elt} y$; $\int_{elt} z$ summation
  - of these quantities on () from where S the computation of: position
  - $A=|S|$
  - of. G knowing
- , G elementary computation on the elements of the list of meshes of edges of shells of: It

$$\int_{elt} Ni \;;\; \int_{elt} xNi \;;\; \int_{elt} yNi \;;\; \int_{elt} zNi \quad \text{où :} \begin{array}{l}\mathbf{GM}=\{x, y, z\} \\ Ni=\text{fonctions de forme de l'élément}\end{array}$$

should be noticed that in the case of the connection shell - pipe, the integrals on the edge elements are to be multiplied by the thickness of the shell: where $\int_{elt} N_i = h \int_l N_i$ l the average fiber of the edge element represents of shell. Moreover, the additional term is added: . $\frac{h^3}{12}\int_l N_i$ “assembly

- ” of the terms calculated above to obtain of each node of the section of connection, coefficients of the terms of the linear relations, connection
- between the modes of Fourier and the displacements of shell as shown at the beginning of [§7].

More precisely: for

- the connection shell - pipe, one carries out elementary computations on all the edge elements of the section of connection of S the type: and

$$u_{\mathbf{cm}}=\frac{1}{\pi}\int_0^{2\pi} u^c \cos(m\varphi)\, d\varphi=\frac{1}{\pi}\int_0^{2\pi}\begin{pmatrix}u_{x^c}\\ u_{\varphi^c}\\ u_{r^c}\end{pmatrix}\cos(m\varphi)\, d\varphi=\frac{1}{\pi}\sum_{n=1}^{Nb\,\varphi\,\text{éléments}\in S}\int_{\varphi_1^n}^{\varphi_2^n}\cos(m\varphi)\, P\begin{pmatrix}u_{x^c}\\ u_{y^c}\\ u_{z^c}\end{pmatrix} d\varphi$$

if

$$u_{\mathbf{sm}}=\frac{1}{\pi}\int_0^{2\pi} u^c \sin(m\varphi)\, d\varphi=\frac{1}{\pi}\int_0^{2\pi}\begin{pmatrix}u_{x^c}\\ u_{\varphi^c}\\ u_{r^c}\end{pmatrix}\sin(m\varphi)\, d\varphi=\frac{1}{\pi}\sum_{n=1}^{Nb\,\varphi\,\text{éléments}\in S}\int_{\varphi_1^n}^{\varphi_2^n}\sin(m\varphi)\, P\begin{pmatrix}u_{x^c}\\ u_{y^c}\\ u_{z^c}\end{pmatrix} d\varphi$$

where $m>1$ is P the transition matrix of the local coordinate system of the element to the total reference and φ_1 the position orthoradiale of the element. By expressing displacement according to the nodal degrees of freedom: and

$$u_{\mathbf{cm}}=\begin{pmatrix} u_m^i \\ v_m^o \\ w_m^i \end{pmatrix}=\frac{1}{\pi}\sum_{n=1}^{Nb\varphi\, éléments\in S}\int_{\varphi_1^n}^{\varphi_2^n}\cos(m\varphi)\,P\sum_{n=1}^{Nb_{noeuds}}N_n(\varphi)\begin{pmatrix} U_{x^n} \\ U_{y^n} \\ U_{z^n} \end{pmatrix}d\,\varphi$$

where

$$u_{\mathbf{sm}}=\begin{pmatrix} u_m^o \\ v_m^i \\ w_m^o \end{pmatrix}=\frac{1}{\pi}\sum_{n=1}^{Nb\varphi\, éléments\in S}\int_{\varphi_1^n}^{\varphi_2^n}\sin(m\varphi)\,P\sum_{n=1}^{Nb_{noeuds}}N_n(\varphi)\begin{pmatrix} U_{x^n} \\ U_{y^n} \\ U_{z^n} \end{pmatrix}d\,\varphi$$

are to them N the shape functions of the element, one twice obtains for each computation 9 coefficients with the nodes of the element running of: S and

$$u_{\mathbf{cm}}=\begin{pmatrix} u_m^i \\ v_m^o \\ w_m^i \end{pmatrix}=\left\{\sum_{éléments\in S}\sum_{n=1}^{Nb_{noeuds}}\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\right\}\begin{pmatrix} U_{x^n} \\ U_{y^n} \\ U_{z^n} \end{pmatrix}.$$

$$a_{ij}^n=\frac{1}{\pi}\int_{\varphi_1}^{\varphi_2}\cos(m\varphi)\,P_{ij}(\varphi)\,N_n(\varphi)\,d\,\varphi=\frac{1}{\pi R}\int_0^l\cos(mx)\,P_{ij}(x)\,N_n(x)\,dx$$

an equivalent statement for where $\boldsymbol{u_{sm}}$ is I the length of the edge element of shell. for

- the connection 3D - pipe, one carries out elementary computations on all the edge elements of the section of connection of S the type: and

$$u_{\mathbf{cm}}=\frac{2}{S}\int_S u^{\mathbf{3d}}\cos(m\,\varphi)\,dS=\frac{2}{S}\int_S\begin{pmatrix} u_{x^{\mathbf{3d}}} \\ u_{\varphi^{\mathbf{3d}}} \\ u_{r^{\mathbf{3d}}} \end{pmatrix}\cos(m\varphi)\,r\,.\,d\,\varphi\,.\,d\zeta$$

$$=\frac{2}{S}\sum_{n=1}^{Nb\varphi\, éléments\in S}\int_{\varphi_1^n}^{\varphi_2^n}\int_{h_1^n}^{h_2^n}\cos(m\varphi)P\sum_{n=1}^{Nb_{noeuds}}N_n(\varphi,\zeta)\begin{pmatrix} U_x^{3\mathrm{d}} \\ U_y^{3\mathrm{d}} \\ U_z^{3\mathrm{d}} \end{pmatrix}^n r\,.\,d\,\varphi\,.\,d\zeta$$

if

$$u_{\mathbf{sm}}=\frac{2}{S}\int_S u^{\mathbf{3d}} c\sin(m\varphi)dS=\frac{2}{S}\int_S \begin{pmatrix} u_{x^{\mathbf{3d}}} \\ u_{\varphi^{\mathbf{3d}}} \\ u_{r^{\mathbf{3d}}} \end{pmatrix} \sin(m\varphi) r.d\varphi.d\zeta$$

$$=\frac{2}{S}\sum_{n=1}^{Nb\varphi\, éléments\in S}\int_{\varphi_1^n}^{\varphi_2^n}\int_{h_1^n}^{h_2^n}\sin(m\varphi)P\sum_{n=1}^{Nb_{noeuds}}N_n(\varphi,\zeta)\begin{pmatrix} U_x^{3d} \\ U_y^{3d} \\ U_z^{3d} \end{pmatrix}^n r.d\varphi.d\zeta$$

where $m>1$ is P the transition matrix of the local coordinate system of the element to the total reference, φ_1 the position orthoradiale of the element, its h_1 radial position and the are N the shape functions of the element. Establishment

8 of the element PIPE `in` Code_Aster *Description*

8.1 This

new element (of name METUSEG `3)` leans on a mesh SEG3 `or SEG 4 curvilinear` . It supposes that the section of the pipe is circular. Contrary to elements POU_D_E , POU_D_T , `[`R3.08.01] this element are not “exact” with the nodes for loadings or torsors concentrated at the ends. It is thus necessary to net with several elements to get correct results. Given

8.2 modelizations

the element is used in the following way: AFFE_MODELE

```
(MODELISATION = "TUYAU_3M"…)
```

Meshes with 4 nodes are generated from meshes with 3 nodes using: MAIL

```
=CRÉA_MAILLAGE (MAILLAGE=MAIL, MODI_MAILLE=_F (OPTION=' SEG3_4', TOUT='
OUI')) One
```

appeals has routine INI `090 for` the shape functions, their derivatives and their second derivative (for the shell part) with Gauss points, as well as the corresponding weights.

The characteristics of the section are defined in AFFE_CARA_ELEM `AFFE_CARA_ELEM`

```
(POUTRE
   = _F (SECTION = "CERCLE", CARA = ("R" "EP"), VALE = (.....),),
ORIENTATION
          = _F (GROUP_NO=D, CARA=' GENE_TUYAU', VALE= (X Y Z),), TUYAU_NCOU
   = ' COUCHES' many, TUYAU_NSEC = ' SECTEURS' many,),)
R
```

`and` EP `represent` , as for the classical beam elements, respectively the external radius and the thickness of the section. One also defines on one of the nodes end of line of pipework the vector whose projection on the cross-sectional area is the origin of the angles for decomposition in Fourier series. This vector should not be colinéaire at line average elbow with ending node considered. One also defines in this level the number of layers and angular sectors to use for numerical integration. AFFE_CHAR_MECA

```
(DDL_IMPO = _F (DX
=. , DY =. , DZ =. , DRX =. , DRY =. , DRZ =. , DDL of beam UI2
=. , VI2 =. , WI2 =. , UO2 =. , VO2 =. , WO2 =. , DDL related to mode 2 UI3
=. , VI3 =. , WI3 =. , UO3 =. , VO3 =. , WO3 =. , DDL related to mode 3 WO
```

=. , WI1 =. , WO1 =. , DDL of swelling and mode 1 on W

the loadings supported in AFFE_CHAR_MECA are :

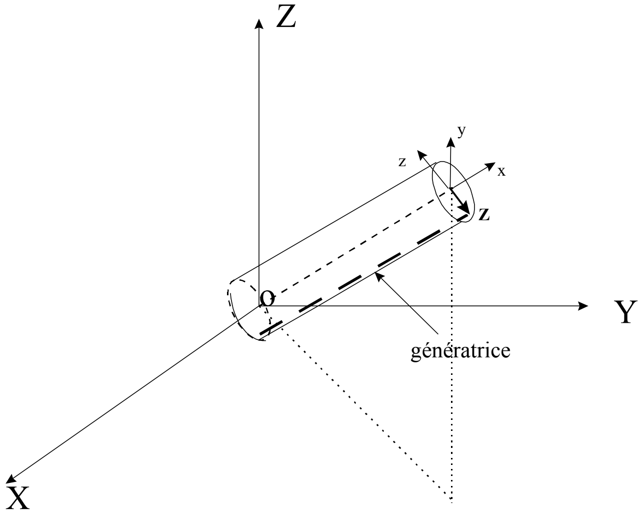
- the nodal forces (FORCE_NODALE), which work only on displacements of beam.
- the pressure interns (FORCE_TUYAU = _F (NEAR =.))
- gravity, (PESANTEUR)
- the linear forces (FORCE_POUTRE)

the internal pressure working on the degree of freedom of swelling, WO one calculates then: Computation

$$W_{pres}=\int_0^l \int_0^{2\pi} p\ w^o\ r_{\text{int}} d\phi\, dx=\int_0^l \int_0^{2\pi} p \sum_{k=1}^{N} H_k\ w_k^o\ r_{\text{int}} d\phi\, dx=\sum_{k=1}^{N} H_k \left[\int_0^l \int_0^{2\pi} p\, r_{\text{int}} d\phi\, dx\right] w_k^o$$

8.3 in linear elasticity

the stiffness matrix and the mass matrix (respectively options RIGI_MECA and MASS_MECA) are integrated numerically in TE 0582. The computation account holds owing to the fact that the terms corresponding to the degrees of freedom of beam are expressed classically in total reference, and that the degrees of freedom of Fourier are in the local coordinate system with the element. In the case or the element does not belong to any elbow, this local coordinate system is defined by the generator and the directing vector carried by average fiber of the element as indicated on [Figure 8.3-a]. If the element belongs to an elbow, the local coordinate system is defined starting from the plane of the elbow as mentioned with [§2.1]. Appear



8.3-a: Local coordinate system for a right pipe nonlinear

8.4 Computations

the tangent stiffness matrix (options RIGI _MECA_TANG and FULL _MECA) as well as plastic projection (options FULL _MECA and RAPH _MECA) are integrated numerically in TE 0586. All the models of plane stresses available in Code_Aster *can* be used: if they are not integrated directly, it is always possible to use a constitutive law formulated in plane strain, and to treat the assumption of plane stresses using the method of Borst.

The elements pipe should be used only in small strains and small displacements. Postprocessing

8.5

currently available elementary computations correspond to the options: SIEF_ELGA

- which provide the stresses to the points of integration in the reference user. One stores these values in the following way: for
  - each Gauss point in the length, () for $n=1,3$
    - each point of integration in the thickness, (, $n=1$) $2\,N_{COU}+1=7$ for
      - each point of integration on the circumference, (, $n=1$) $2\,N_{SECT}+1=33$ 6
        - stress components: SIXX SIYY SIZZ SIXY SIXZ SIYZ where
          indicates X the direction given by the two nodes tops of the element, represents Y the angle describing ϕ the circumference and represents Z the radius. SIZZ and SIYZ corresponding to σ_{rr}, $\sigma_{r\varphi}$ are taken equal to zero. EFGE_ELNO

- which makes it possible to obtain the forces generalized by element with the nodes in the reference of the beam. VARI_ELNO

- which calculates the field of local variables by element to the nodes for all the layers and all the sectors, in the local coordinate system of the element. EPSI_ELGA

-
- which provides the total deflections to the points of integration in the local coordinate system of the element. The computation is carried out in TE 0584, and currently gives the values to the 693 points of integration (for an element with 3 modes of Fourier). These fields are called fields at "subpoints" of integration. One stores these values in the following way: for
  - each Gauss point in the length, () for $n=1,3$
    - each point of integration in the thickness, (, $n=1$) $2\,N_{COU}+1=7$ for
      - each point of integration on the circumference, (, $n=1$) $2\,N_{SECT}+1=33$ 6
        - components of strain: EPXX EPYY EPZZ EPXY EPXZ EPYZ where
          indicates X the direction given by the two nodes tops of the element, represents Y the angle describing ϕ the circumference and represents Z the radius. EPZZ and EPYZ corresponding to ε_{rr}, $\varepsilon_{r\varphi}$ are taken equal to zero.

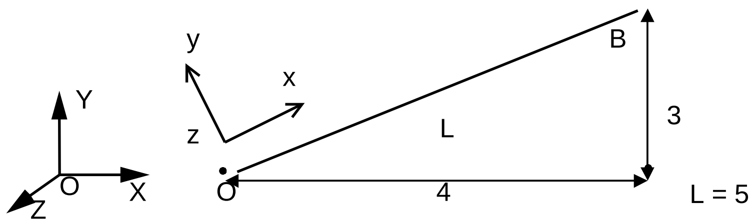
- Options SIEQ_ELGA and EPEQ_ELGA allow the computation of the invariants, (Von Mises, signed Von Mises, trace) in each point of integration (fields at "subpoints"). EFGE_ELNO

- provides the forces generalize of beam classics: N, VY, VZ, MT, MFY, MFZ. These forces are given in the local curvilinear reference of the element.

- Command POST _CHAMP/MIN_MAX_SP makes it possible to extract, of each of Gauss points linear of an element, the values maximum and minimum of a component of a field. The min/max is taken on all the subpoints of a point. Test

8.6 : SSLL106A It

acts of a right pipe of directing vector fixed $(4,3,0)$ in its end and O which is with a grid with 18 elements PIPE .



The pipe is subjected to different types of load:

- a 2 shears, tractive effort
- , 2
- bending moment, 1
- twisting moment,
- an internal pressure. One

calculates displacements at the point, B the strain and the forced in certain points of integration of the section containing, B as well as the first eigen modes. This
makes it possible to test the degrees of freedom of beam, the degree of freedom of swelling and the modes 1 of the development in Fourier series. Conclusion

9

the finite elements of elbow which we describe here are usable for computations of pipework in elasticity or plasticity. The pipework can be subjected to various combined loadings - internal pressure, cross-bendings and anti-plane, torsion, extension. For the moment

, the element carried out is a linear element of standard beam, right or curve, with three nodes, in small rotations and strains, with a local elastoplastic behavior in plane stresses. It makes it possible to take into account ovalization, warping and swelling. It combines the properties of shells and beams. The kinematics of beam for the axis of the elbow is increased by a kinematics of shell, of type Coils-Kirchhoff without transverse shears, for the description of the behavior of the cross-sectional areas. This last kinematics is discretized in modes M of Fourier, of which the number, M which the literature encourages us to choose equal to 6 [bib8], [bib13], must at the same time be sufficient to obtain good performances in plasticity and not too large to limit the computing time. In elasticity, for relatively thick pipework (the thickness ratio on radius of the cross-sectional area higher than 0.1), one can be satisfied with or $M=2$. $M=3$ Bibliography

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11 of the versions of the document Version

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EDF-R&	D/AMA Put of the computation options	