Elements MEMBRANE and GRILLE_MEMBRANE

Summarized:

This document describes the formulation and the establishment in *Code_Aster* of the elements MEMBRANE and GRILLE_MEMBRANE. The elements MEMBRANE make it possible to model a linear behavior of unspecified membrane. Elements GRILLE_MEMBRANE of the finite elements are more specifically dedicated to the representation of steel reinforcements in a solid mass (for applications of Civil Engineering standard reinforced concrete). The main features of these elements are the following ones:

- elements of membrane, without torsion stiffness;
- no degrees of freedom of rotation, but on the other hand not of possibility of eccentring;
- geometrical support surface (triangle, quadrangle; linear or quadratic);

Elements GRILLE_MEMBRANE make it possible to model a nonlinear behavior of the bars of reinforcement, which is not the case of the elements MEMBRANE.

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1 Introduction

The modelization MEMBRANE makes it possible to represent the structural mechanics behavior of a possibly anisotropic membrane. Let us specify that it is currently restricted with linear behaviors. It makes it possible to model structural elements whose flexural rigidity is negligible.

Elements GRILLE_MEMBRANE make it possible to represent the possibly nonlinear behavior of bars of reinforcement in a reinforced concrete structure. The principal stress is that the bars of reinforcement must be periodically distributed on a surface, and directed all in the same direction. Let us specify however that bars of cross reinforcements can be modelled by superposition of two modelizations GRILLE_MEMBRANE (see further).

The elements of the type GRILLE_MEMBRANE come to supplement the possibilities of modelization of reinforcement in *Code_Aster*, in complement of modelization GRILLE_EXCENTRE. One presents below the differences between modelizations GRILLE_MEMBRANE and GRILLE_EXCENTRE.

It is pointed out that modelization GRILLE_EXCENTRE is based on a kinematics of shell DKT with only one layer in the thickness [R3.07.03]. This base DKT implies the presence of degrees of freedom of rotation to the nodes of elements GRILLE_EXCENTRE: if it allows the notion of eccentring, it is useless when one does not need eccentring (in this case it weighs down in an useless way the model, because not only the degrees of freedom of rotation lengthen the vector of unknowns, but it is necessary moreover block one considerable number of these degrees of freedom by double multiplier of Lagrange). The modelization GRILLE_MEMBRANE is a modelization based on a “surface” kinematics, it does not require other degrees of freedom that usual displacements (on the other hand, obviously, this modelization does not make it possible to use the notion of eccentring).

The modelization GRILLE_EXCENTRE, based on DKT, requires geometric standards of support of type triangle or linear quadrangle; modelization GRILLE_MEMBRANE is developed starting from the geometrical supports surface triangle or quadrangle, linear or quadratic.

For the two types of modelization, on the other hand, only a direction of reinforcement is available by finite elements. That makes it possible to model any type of reinforcement to several directions, by superimposing an element by direction; the cost of computation generated by these duplications is weak: no the duplication of the degrees of freedom (thus constant cost of inversion of matrix), duplication of elementary computations (but they remain simple, reduced number in 3D – surface elements against volume elements – and the elementary computations for structures with great numbers of degrees of freedom are of weak cost compared to cost of inversion).

2 Formulation of the elements of MEMBRANE

For an element of membrane, strain energy can be put in the form:

$$\Phi=\frac{1}{2}\int \boldsymbol{\sigma}:\boldsymbol{\varepsilon}\, ds$$

with $\boldsymbol{\sigma}$ the membrane stress and $\boldsymbol{\varepsilon}$ the membrane strain.

The only difficulty is to obtain a statement of the type $\boldsymbol{\varepsilon}=BU_{nodal}$.

For that, a little differential geometry should be used. One leaves the form of the derivative contravariante:

$$\nabla u=\frac{\partial u^i}{\partial \xi_j}=u^i|_j a_i\otimes a^j=u^i|_j g^{jk} a_i\otimes a_k$$

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with $\{\xi_j\}$ an acceptable parameter setting of surface and $a_i = \frac{\partial x}{\partial \xi_i}$.

by noting a the natural base (nonorthogonal, only the 3rd vector, normal on the surface, is normalized) plane of reinforcement and g metric the contravariante associated with this base (cf [R3.07.04] .pour more details).
One then defines the direction of reinforcement by the vector normalized e_1 (which one supplements, for the facility of the talk in an orthonormal base $\{e_i\}$) and one calls R the operator of transition such as $a_i = R_i^p e_p$. One notes in Greek the indices taking only the values in $(1,2)$, and one obtains:

$$\varepsilon_{\alpha\beta} = (\nabla u)_{\alpha\beta} = \left(\frac{\partial u}{\partial \xi^j} \cdot a^i\right) R_i^\alpha R_k^\beta g^{jk}$$

By definition of R : $R_3^1 = 0$ and by definition of g : $g^{13} = g^{23} = 0$. One thus obtains:

$$\varepsilon_{\alpha\beta} = (\nabla u)_{\alpha\beta} = \left(\frac{\partial u}{\partial \xi^\delta} \cdot a^\gamma\right) R_\gamma^\alpha R_\theta^\beta g^{\delta\theta}$$

If one now notes $\hat{B}$ the derivative of the shape functions at the Gauss point considered, it comes:

$$\varepsilon_{\alpha\beta} = R_\gamma^\alpha R_\theta^\beta g^{\delta\theta} \hat{B}_{\delta n} (a^\gamma)_i U_{in}$$

from where formulates B sought.

$$B_{in} = R_\gamma^\alpha R_\theta^\beta g^{\delta\theta} \hat{B}_{\delta n} (a^\gamma)_i$$

From B , one then has all the classical statements of the strain, the nodal forces and the tangent matrix which are written:

$$\boldsymbol{\varepsilon} = BU$$
$$F = \int B^T \sigma$$
$$K = \int B^T \frac{\partial \sigma}{\partial \varepsilon} B$$

3 Formulation of the elements of GRILLE_MEMBRANE

For a three-dimensions function of uniaxial reinforcement, strain energy can put itself in the form:

$$\Phi = \frac{1}{2} \int S \sigma \varepsilon ds$$

with S the section of reinforcement per unit of length, σ the stress (scalar) and ε strain (scalar).

One seeks to obtain a statement of the type $\varepsilon = BU_{nodal}$.

By taking again the approach of the preceding section, one shows this time that:

$$\varepsilon = (\nabla u)_{11} = \left(\frac{\partial u}{\partial \xi^\beta} \cdot a^\alpha\right) R_\alpha^1 R_\gamma^1 g^{\beta\gamma}$$

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By introducing derivative $\hat{B}$ shape functions at the Gauss point considered, it comes:

$$\varepsilon = R^1_\alpha R^1_\gamma g^{\beta\gamma} \hat{B}_{\beta n} (a^\alpha)_i U_{in}$$

from where B sought. It will be noted that it has the shape of a vector, due to the scalar nature of the required strain.

$$B_{in} = R^1_\alpha R^1_\gamma g^{\beta\gamma} \hat{B}_{\beta n} (a^\alpha)_i$$

From B , one finds all the classical statements of the strain, the nodal forces and the tangent matrix which are written:

$$\begin{aligned} &\varepsilon = BU \\ &F = \int B^T \sigma \\ &K = \int B^T \frac{\partial \sigma}{\partial \varepsilon} B \end{aligned}$$

It will be noted that in fact the constitutive laws 1D are used to obtain the stress from the strain. All the constitutive laws available in 1D are usable. A default, one can also use the models 3D, thanks to the method De Borst.

4 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
7.4	P.Badel EDF-R&D/AMA	initial Text
9.5	J.M.Proix EDF-R&D/ AMA Modification	of GRILL in GRILLE_EXCENTRE 11.3 Mr.
David	EDF-R&D/ minor AMA Addition	of the modelization MEMBRANE and modifications

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