Multifibre beam element (right)

Summarized :

This document presents the beam elements multifibre of *Code_Aster* based on a resolution of a problem of beam for which each section of a beam is divided into several fibers. Each fiber behaves then like a beam of Eulerian. Several materials can be affected on only one support finite element (SEG2) what avoids having to duplicate meshes (steel + concrete, for example).

The beams are right (element `POU_D_EM`). The section can be of an unspecified form, described by a "fiber mesh", to see [U4.26.01.
The assumptions selected are the following ones:

- assumption of Eulerian: the transverse shears are neglected (this assumption is checked for strong slenderness),
- the beam elements multifibre take into account the effects of thermal thermal expansion, drying and the hydration (terms of the second member) and in a simplified way torsion. The force-normal coupling bending is treated naturally, by integration in the section of the uniaxial responses of the models of behavior associated with each group with fibers. An enrichment of the axial strain, solved by local condensation in the case of nonlinear behaviors, allows numerical good performances, whatever the evolution in the thickness of centroid of the section.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

Contents

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

Notations

One gives the correspondence between the notations of this document and those of the documentation of use.

DX , DY , DZ and DRX , DRY , DRZ are in fact the names of the degrees of freedom associated with the components with displacement $u, v, w, \theta_x, \theta_y, \theta_z$.

E	Young's modulus	E
ν	Poisson's ratio	NU
G	modulates of Coulomb = $\frac{E}{2(1+\nu)}$	G
I_y, I_z	geometrical moments of constant bending compared to y, z	IY, IZ
J_x	the axes of torsion	JX
$\mathbf{K}$	stiffness matrix	
M	mass matrix	
M_x, M_y, M_z	moments around the axes x, y, z	MT, MFY, MFZ
N	normal force with the section	N
S	area of the section	A
u, v, w	translations on the axes x, y, z	$DX \quad DY \quad DZ$
V_y, V_z	shears along the axes y, z	VY, VZ
ρ	density	RHO
$\theta_x, \theta_y, \theta_z$	rotations around the axes x, y, z	$DRX \quad DRY \quad DRZ$
q_x, q_y, q_z	external linear forces	

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

1 Introduction

the structural analysis subjected to a dynamic loading requires models of behavior able to represent non-linearities of the material.

Many analytical models were proposed. They can be classified according to two groups:

- detailed models founded on the mechanics of solid and their description of the local behavior of the material (microscopic approach) and
- the models based on a total modelization of the behavior (macroscopic approach).

In the first type of models, we can find the models classical with the finite elements as well as "the fiber" models type (having element of type a beam how support).

While the "classical" models with the finite elements are powerful tools for the simulation of the nonlinear behavior of the complex parts of structures (joined, assemblies,…), their application to the totality of a structure can prove not very practical because of a prohibitory computing time or size memory necessary to the realization of this computation. On the other hand, a modelization of type multifibre beam (see [Figure 1-a]), has the advantages of the simplifying assumptions of a kinematics of type beam of Eulerian - Bernoulli while offering a practical solution and effective for a nonlinear analysis complexes composite structural elements such as those which one can meet for example out of reinforced concrete.

Moreover, this "intermediate" modelization is relatively robust and inexpensive in time computation because of use of nonlinear models of behavior 1D.

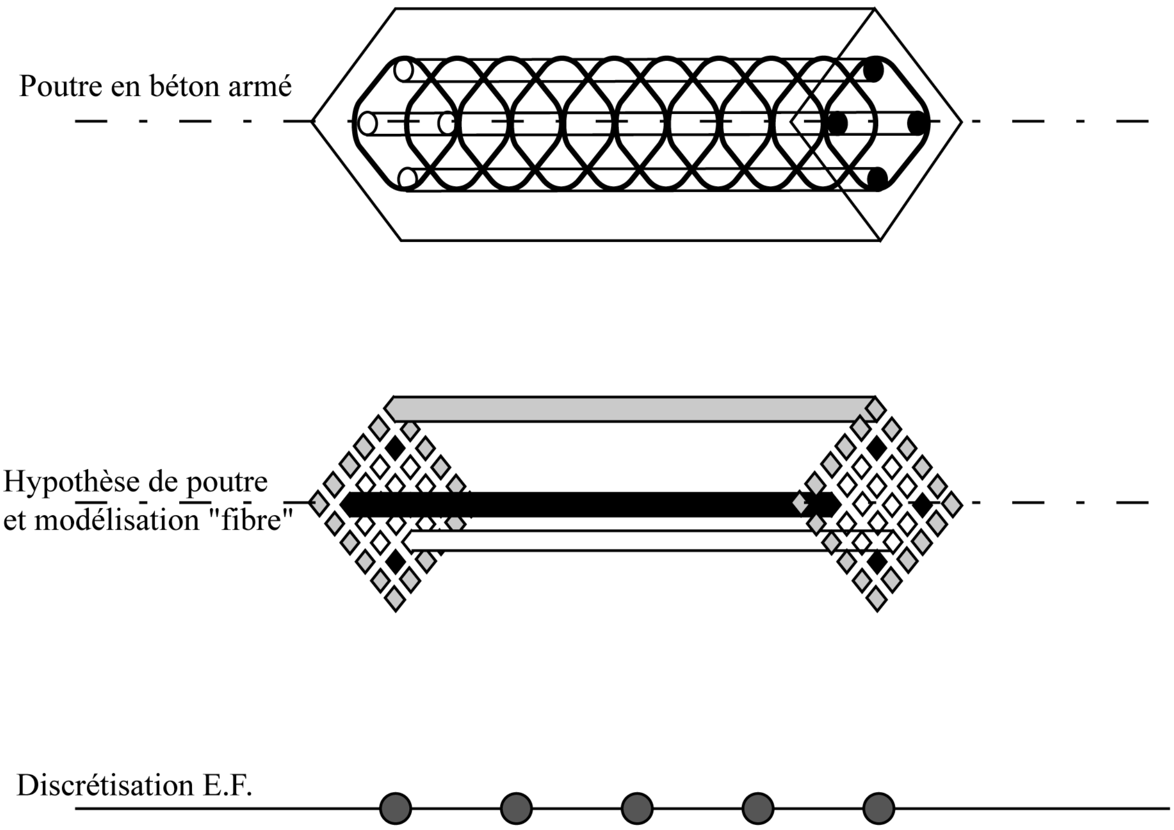

Figure 1-1 : Description of a modelization of type multifibre beam

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

2 Element of theory of the beams (recalls)

One takes again here the elements developed in the frame of the beam elements of Eulerian, [bib4].

A beam is a solid generated by a surface of area S whose geometrical center of inertia G follows a curve C called the average fiber or neutral fiber. The area S is the cross-section (cross section) or profile, and it is supposed that if it is evolutionary, its evolutions (size, form) continuous and progressive when G are described the line average one.

For the study of the beams in general, one makes the following assumptions:

the cross-section of the beam is indeformable,
transverse displacement is uniform on the cross-section.

These assumptions make it possible to express displacements of an unspecified point of the section, according to displacements of the point corresponding located on the line average one, and according to an increase in displacement due to the rotation of the section around the transverse axes.

The discretization in "exact" elements of beam is carried out on a linear element with two nodes and six degrees of freedom by nodes. These degrees of freedom are the three translations u, v, w and the three rotations $\theta_x, \theta_y, \theta_z$ [Figure 2-a]).

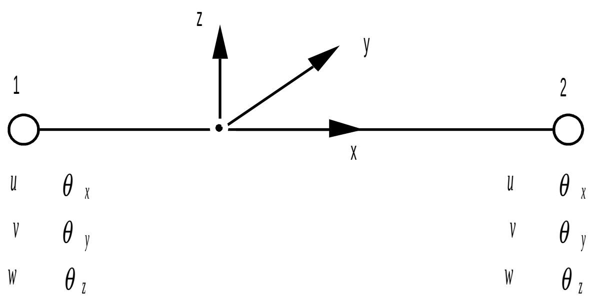

Figure 2-1 : Element beam

Expected that the strains are local, it is built in each top of the mesh a local base depending on the element on which one works. The continuity of the fields of displacements is ensured by a basic change, bringing back the data in the global database.

In the case of the straight beams, one traditionally places the line average one on axis X of the local base, transverse displacements being thus carried out in the plane (y, z) .

Finally when we arrange quantities related to the degrees of freedom of an element in a vector or an elementary matrix (thus of dimension 12 or 12^2), one arranges initially the variables for the top 1 then those of the top 2 . For each node, one stores initially the quantities related to the three translations, then those related to three rotations. For example, a vector displacement will be structured in the following way:

$$\underbrace{u_1, v_1, w_1, \theta_{x_1}, \theta_{y_1}}_{\text{sommet 1}}, \theta_{z_1}, \ \underbrace{u_2, v_2, w_2, \theta_{x_2}, \theta_{y_2}}_{\text{sommet 2}}, \theta_{z_2}$$

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

3 The equations of the motion of the beams

We will not include in this document all the equations of the motion of the beams. For more complements concerning this part one can refer to documentation concerning elements POU_D_E and POU_D_T ([bib4]).

4 Multifibre beam element right

One describes in this chapter obtaining the elementary matrixes of stiffness and mass for the multifibre beam element right, according to the model of Eulerian. The stiffness matrixes are calculated with the options "RIGI_MECA" or "RIGI_MECA_TANG", and the mass matrixes with option "MASS_MECA" for the coherent matrix, and option "MASS_MECA_DIAG" for the diagonalized mass matrix.

We present here a generalization [bib3] where the reference axis chosen for the beam is independent of any geometrical consideration, inertial or mechanical. The element functions for an unspecified section (heterogeneous is without symmetry) and is thus adapted to a nonlinear evolution of the behavior of fibers.

One also describes the computation of the nodal forces for the nonlinear algorithms: "FORC_NODA" and "RAPH_MECA".

4.1 Element beam of reference

It [Figure 4.1-a] shows us the change of variable realized to pass from the real finite element [Figure 2-a] to the finite element of reference.

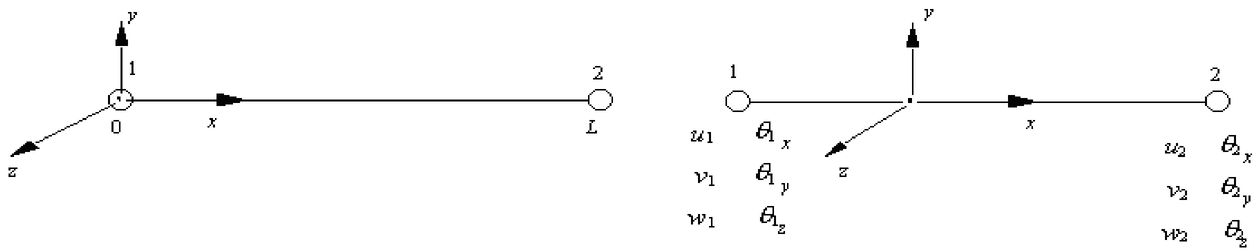

Figure 4.1-1 : Element of reference vs real Élément

One will then consider the continuous field of displacements in any point of line average compared to the field of displacements discretized in the following way:

$$\mathbf{U}_s=[\mathbf{N}]\{\mathbf{U}\}$$ **éq 4.1-1**

the index s indicates the quantities attached to average fiber.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

By means of the shape functions of the element of reference, the discretization of the variables $u_s(x), v_s(x), w_s(x), \theta_{sx}(x), \theta_{sy}(x), \theta_{sz}(x)$ becomes:

$$\begin{pmatrix} u_s(x) \\ v_s(x) \\ w_s(x) \\ \theta_{sx}(x) \\ \theta_{sy}(x) \\ \theta_{sz}(x) \end{pmatrix} = \begin{pmatrix} N_1 & 0 & 0 & 0 & 0 & 0 & N_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_3 & 0 & 0 & 0 & N_4 & 0 & N_5 & 0 & 0 & 0 & N_6 \\ 0 & 0 & N_3 & 0 & -N_4 & 0 & 0 & 0 & N_5 & 0 & -N_6 & 0 \\ 0 & 0 & 0 & N_1 & 0 & 0 & 0 & 0 & 0 & N_2 & 0 & 0 \\ 0 & 0 & -N_{3,x} & 0 & N_{4,x} & 0 & 0 & 0 & -N_{5,x} & 0 & N_{6,x} & 0 \\ 0 & N_{3,x} & 0 & 0 & 0 & N_{4,x} & 0 & N_{5,x} & 0 & 0 & 0 & N_{6,x} \end{pmatrix} . \begin{pmatrix} u_1 \\ v_1 \\ w_1 \\ \theta_{x1} \\ \theta_{y1} \\ \theta_{z1} \\ u_2 \\ v_2 \\ w_2 \\ \theta_{x2} \\ \theta_{y2} \\ \theta_{z2} \end{pmatrix}$$

éq 4.1-2

With the following interpolation functions, and their useful derivatives:

$$\begin{aligned} N_1 &= 1-\frac{x}{L} \quad &;\quad N_{1,x} &= -\frac{1}{L} \\ N_2 &= \frac{x}{L} \quad &;\quad N_{2,x} &= \frac{1}{L} \\ N_3 &= 1-3\frac{x^2}{L^2}+2\frac{x^3}{L^3} \quad &;\quad N_{3,xx} &= -\frac{6}{L^2}+12\frac{x}{L^3} \\ N_4 &= x-2\frac{x^2}{L}+\frac{x^3}{L^2} \quad &;\quad N_{4,xx} &= -\frac{4}{L}+6\frac{x}{L^2} \\ N_5 &= 3\frac{x^2}{L^2}-2\frac{x^3}{L^3} \quad &;\quad N_{5,xx} &= \frac{6}{L^2}-12\frac{x}{L^3} \\ N_6 &= -\frac{x^2}{L}+\frac{x^3}{L^2} \quad &;\quad N_{6,xx} &= -\frac{2}{L}+6\frac{x}{L^2} \end{aligned}$$

éq 4.1-3

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

4.2 Determination of the stiffness matrix of the multifibre element

4.2.1 general Case (beam of Eulerian)

Let us consider a beam Eulerian, line, directed in the direction x , subjected to distributed forces q_x, q_y, q_z [Figure 4.2.1-a].

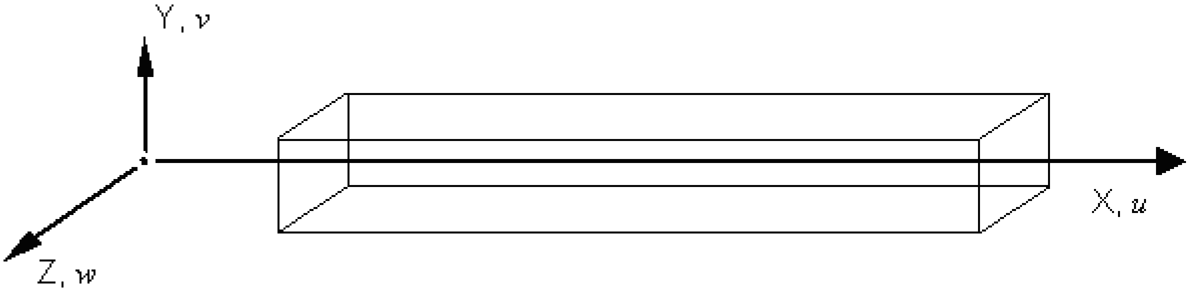

Figure 4.2.1-1 : Beam of Eulerian 3D.

The fields of displacements and strains take the following shape then when one writes the displacement of an unspecified point of the section according to displacement $(\mathbf{u}_s)$ and rotation $(\boldsymbol{\theta}_s)$ of line of average:

$$u(x, y, z) = u_s(x) - y\theta_{sz}(x) + z\theta_{sy}(x)$$ **éq 4.2.1-1**

$$v(x, y, z) = v_s(x)$$ **éq 4.2.1-2**

$$w(x, y, z) = w_s(x)$$ **éq 4.2.1-3**

$$\epsilon_{xx} = u_x^{'}(x) - y\theta_{sz}^{'}(x) + z\theta_{sy}^{'}(x)$$ **éq 4.2.1-4**

$$\varepsilon_{xy} = \varepsilon_{xz} = 0$$ **éq 4.2.1-5**

Note:

- *Torsion is treated overall by admitting an elastic assumption, except for, one does not calculate ε_{yz} here.*
- $f'(x)$ *indicate derivative from* $f(x)$ *ratio with* x *.*

By introducing the equations [éq 4.2.1-4] and [éq 4.2.1-5] in the principle of the virtual wors one obtains:

$$\int_{V_0} \sigma_{xx} . \delta \varepsilon_{xx} \, dV_0 = \int_0^L \left(\delta u_s(x) \; q_x + \delta v_s(x) \; q_y + \delta w_s(x) \; q_z \right) dx$$ **éq 4.2.1-6**

q_x, q_y, q_z indicating the linear forces applied.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

What gives by means of the equation [éq 4.2.1-1]:

$$\int_0^L \left(N\delta u_s'(x) + M_x \delta\theta_{sx}'(x) + M_y \delta\theta_{sy}'(x) + M_z \delta\theta_{sz}'(x) \right) dx = \int_0^L \left(q_x \delta u_s(x) + q_y \delta v_s(x) + q_z \delta w_s(x) \right) dx$$

éq 4.2.1-7

with:

$$N = \int_S \sigma_{xx} dS \quad ; \quad M_y = \int_S z\,\sigma_{xx} dS \quad ; \quad M_z = \int_S -y\,\sigma_{xx} dS$$

éq 4.2.1-8

Note:

•The twisting moment M_x is not calculated by integration but is not calculated directly from the stiffness in torsion (see [éq 4.2.2-4]).
•The theory of the beams associated with an elastic material gives: $\sigma_{xx} = E\,\varepsilon_{xx}$

4.2.2 Case of the multifibre beam

We suppose now that the section S is not homogeneous [Figure 4.2.2-a].
Without adopting particular assumption on the intersection of the axis X with the section S or on the directional sense of the axes Y Z, the relation between the "generalized" stresses and the "generalized" strains $\mathbf{D}_s$ becomes [bib2]:

$$\mathbf{F}_s = \mathbf{K}_s \cdot \mathbf{D}_s$$

éq 4.2.2-1

with:

$$\mathbf{F}_s = \left(N, M_y, M_z, M_x \right)^T$$
$$\mathbf{D}_s = \left(u_s'(x), \theta_{sy}'(x), \theta_{sz}'(x), \theta_{sx}'(x) \right)^T$$

éq 4.2.2-2

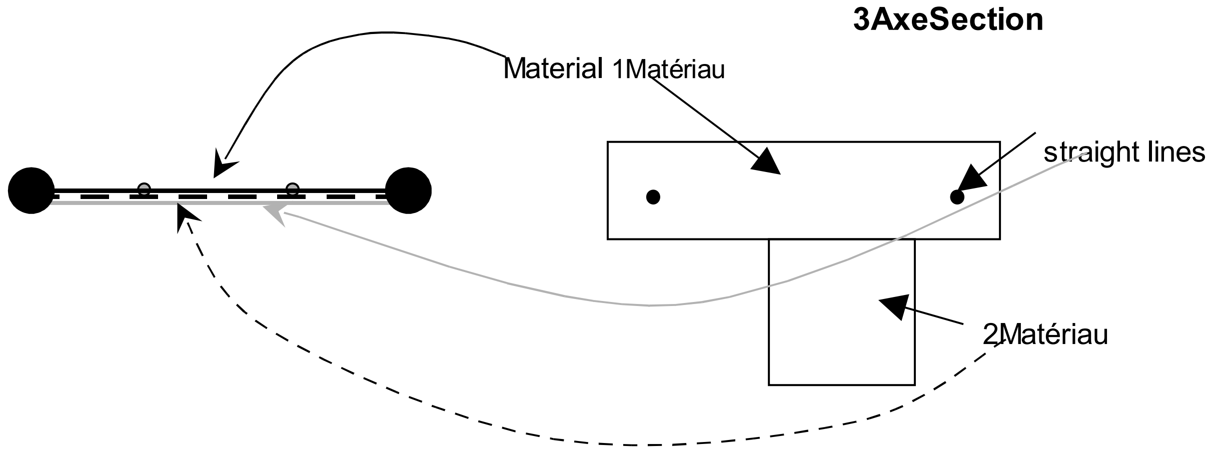

Figure 4.2.2-1 : Unspecified S section - multifibre beam

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

the matrix $\mathbf{K}_s$ can then be put in the following form:

$$\mathbf{K}_s=\begin{pmatrix} K_{s11} & K_{s12} & K_{s13} & 0 \\ & K_{s22} & K_{s23} & 0 \\ & & K_{s33} & 0 \\ sym & & & K_{s44} \end{pmatrix}$$ **éq 4.2.2-3**

with:

$$K_{s11}=\int_S EdS \;\; ; \;\; K_{s12}=\int_S Ezds \;\; ; \;\; K_{s13}=-\int_S Eyds$$
$$K_{s22}=\int_S Ez^2 dS \;\; ; \;\; K_{s23}=-\int_S Eyzds \;\; ; \;\; K_{s33}=\int_S Ey^2 ds$$ **éq 4.2.2-4**

where E can vary according to y and z . Indeed, it may be that in the modelization section planes [Figure 4.2.2-a]), several materials cohabit. For example, in a concrete section reinforced, there are at the same time concrete and reinforcements.
The discretization of the fiber section makes it possible to calculate the integrals of the equations [éq 4.2.2-4]. The computation coefficients of the matrix $\mathbf{K}_s$ is detailed in the paragraph [§4.2.3] according to.

Note:

> *The term of torsion $K_{s44}=GJ_x$ is given by the user using the data of J_x , using the command AFFE_CARA_ELEM .*

The introduction of the equations [éq 4.2.2-1] to [éq 4.2.2-4] in the principle of the virtual wors leads to:

$$\int_0^L \delta D_s^T . K_s . D_s \; dx-\int_0^L \left(\delta u_s(x)\; q_x+\delta v_s(x)\; q_y+\delta w_s(x)\; q_z\right)\; dx=0$$ **éq the 4.2.2-5**

generalized strains are calculated by (D_S is given to the equation [éq 4.2.2-2]):

$$\mathbf{D}_s=\mathbf{B}(\mathbf{U})$$ **éq 4.2.2-6**

With the following $\mathbf{B}$ matrix:

$$\mathbf{B}=\begin{bmatrix} N_{1,x} & 0 & 0 & 0 & 0 & 0 & N_{2,x} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -N_{3,xx} & 0 & N_{4,xx} & 0 & 0 & 0 & -N_{5,xx} & 0 & N_{6,xx} & 0 \\ 0 & N_{3,xx} & 0 & 0 & 0 & N_{4,xx} & 0 & N_{5,xx} & 0 & 0 & 0 & N_{6,xx} \\ 0 & 0 & 0 & N_{1,x} & 0 & 0 & 0 & 0 & 0 & N_{2,x} & 0 & 0 \end{bmatrix}$$
éq 4.2.2-7

the discretization of space $[0,L]$ with elements and the use of the equations [éq 4.2.2-5] makes the equation [éq 4.2.1-6] equivalent to the resolution of a classical linear system:

$$\mathbf{K}.\mathbf{U}=\mathbf{F}$$ **éq 4.2.2-8**

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

the stiffness matrix of the element [Figure 4.2.2-b] and the vector of the forces results are finally given by:

$$\mathbf{K}_{elem}=\int_0^L \mathbf{B}^T.\mathbf{K}_s.\mathbf{B}\ dx \qquad \text{éq 4.2.2-9}$$
$$\mathbf{F}=\int_0^L \mathbf{N}^T.\boldsymbol{Q}\ dx$$

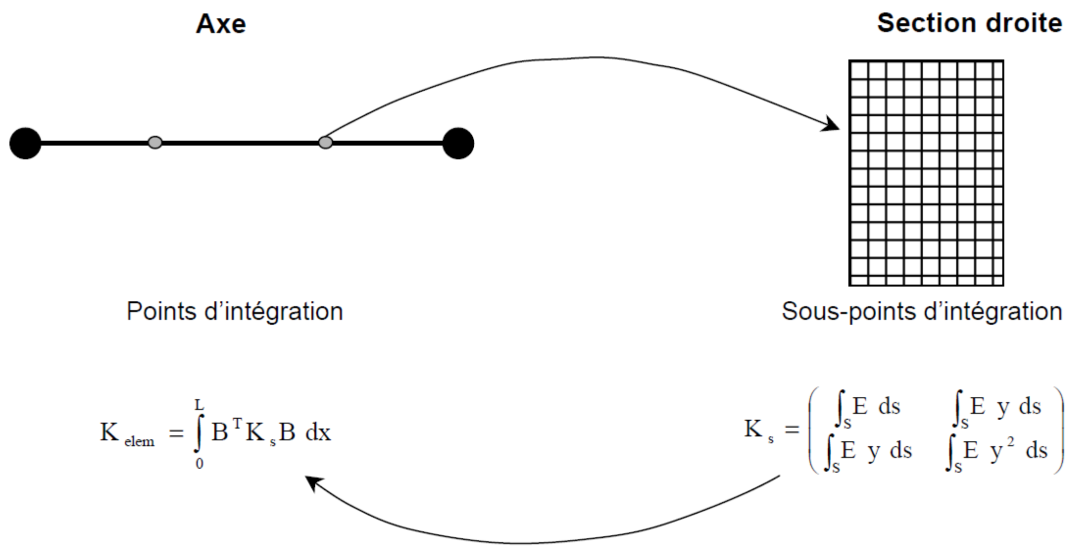

Figure 4.2.2-b: Multifibre beam – Computation of $\mathbf{K}_{elem}$

With the vector $\mathbf{Q}$ which depends on the external loading: $\mathbf{Q}=\left(q_x\ \ q_y\ \ q_z\ \ 0\ \ 0\ \ 0\right)^T$.

If we consider that the distributed forces q_x, q_y, q_z are constant, we obtain the vector nodal forces according to:

$$\mathbf{F}=\left(\frac{Lq_x}{2}\ \frac{Lq_y}{2}\ \frac{Lq_z}{2}\ 0\ -\frac{L^2 q_z}{12}\ \frac{L^2 q_y}{12}\ \frac{Lq_x}{2}\ \frac{Lq_y}{2}\ \frac{Lq_z}{2}\ 0\ \frac{L^2 q_z}{12}\ \frac{L^2 q_y}{12}\right)^T \qquad \text{éq 4.2.2-10}$$

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

4.2.3 Discretization of the fiber section – Computation of $\mathbf{K}_s$

the discretization of the fiber section makes it possible to calculate the various integrals which intervene in the stiffness matrix, and the other terms necessary.

The geometry of fibers gathered in groups of fibers, via operator DEFI_GEOM_FIBRE [U4.26.01]) contains in particular the characteristics (Y, Z, AREA) for each fiber. One can envisage with more the 10 groups of maximum fibers by element beam.

Thus, if we have a section which comprises n fibers we will have the following approximations of the integrals:

$$K_{s11}=\sum_{i=1}^{n} E_i S_i \quad ; \quad K_{s12}=\sum_{i=1}^{n} E_i z_i S_i \quad ; \quad K_{s13}=\sum_{i=1}^{n} E_i y_i S_i$$
$$K_{s22}=\sum_{i=1}^{n} E_i z_i^2 S_i \quad ; \quad K_{s23}=-\sum_{i=1}^{n} E_i y_i z_i S_i \quad ; \quad K_{s33}=\sum_{i=1}^{n} E_i y_i^2 S_i$$

éq 4.2.3-1

with E_i the initial or tangent modulus and S_i the section of each fiber. The stress state is constant by fiber.

Each fiber is also located using y_i and the z_i coordinated center of gravity of fiber compared to the axis of the section defined by the key word "COOR_AXE_POUTRE" (see command DEFI_GEOM_FIBRE [U4.26.01]).

The classification fibers depends on the choice of key word "FIBER" or "SECTION" (see command DEFI_GEOM_FIBRE [U4.26.01]).

4.2.4 Integration in the linear elastic case (RIGI_MECA)

When the behavior of the material is linear, if the element beam is homogeneous in its length, the integration of the equation [éq 4.2.2-9] can be made analytically.

The following stiffness matrix then is obtained:

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

$$
\mathbf{K}_{elem}=\begin{pmatrix}
\frac{K_{s11}}{L} & 0 & 0 & 0 & \frac{K_{s12}}{L} & \frac{K_{s13}}{L} & \frac{-K_{s11}}{L} & 0 & 0 & 0 & \frac{-K_{s12}}{L} & \frac{-K_{s13}}{L} \\
 & \frac{12K_{s33}}{L^3} & \frac{-12K_{s23}}{L^3} & 0 & \frac{6K_{s23}}{L^2} & \frac{6K_{s33}}{L^2} & 0 & \frac{-12K_{s33}}{L^3} & \frac{12K_{s23}}{L^3} & 0 & \frac{6K_{s23}}{L^2} & \frac{6K_{s33}}{L^2} \\
 & & \frac{12K_{s22}}{L^3} & 0 & \frac{-6K_{s22}}{L^2} & \frac{-6K_{s23}}{L^2} & 0 & \frac{12K_{s23}}{L^3} & \frac{-12K_{s22}}{L^3} & 0 & \frac{-6K_{s22}}{L^2} & \frac{-6K_{s23}}{L^2} \\
 & & & \frac{K_{s44}}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{K_{s44}}{L} & 0 & 0 \\
 & & & & \frac{4K_{s22}}{L} & \frac{4K_{s23}}{L} & \frac{-K_{s12}}{L} & \frac{-6K_{s23}}{L^2} & \frac{6K_{s22}}{L^2} & 0 & \frac{2K_{s22}}{L} & \frac{2K_{s23}}{L} \\
 & & & & & \frac{4K_{s33}}{L} & \frac{-K_{s13}}{L} & \frac{-6K_{s33}}{L^2} & \frac{6K_{s23}}{L^2} & 0 & \frac{2K_{s23}}{L} & \frac{2K_{s33}}{L} \\
 & & & & & & \frac{K_{s11}}{L} & 0 & 0 & 0 & \frac{K_{s12}}{L} & \frac{K_{s13}}{L} \\
 & & & & & & & \frac{12K_{s33}}{L^3} & \frac{-12K_{s23}}{L^3} & 0 & \frac{-6K_{s23}}{L^2} & \frac{-6K_{s33}}{L^2} \\
 & & & SYM & & & & & \frac{12K_{s22}}{L^3} & 0 & \frac{6K_{s22}}{L^2} & \frac{6K_{s23}}{L^2} \\
 & & & & & & & & & \frac{K_{s44}}{L} & 0 & 0 \\
 & & & & & & & & & & \frac{4K_{s22}}{L} & \frac{4K_{s23}}{L} \\
 & & & & & & & & & & & \frac{4K_{s33}}{L}
\end{pmatrix}
$$

éq 4.2.4-1

with the following terms $K_{s11}, K_{s12}, K_{s13}, K_{s22}, K_{s33}, K_{s23}, K_{s44}$ given to the equation [éq 4.2.2-4].

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

4.2.5 Integration in the nonlinear case (`RIGI_MECA_TANG`)

When the behavior of the material is nonlinear, to allow a correct integration of the internal forces (see paragraph [§4.4]), it is necessary to have at least two points of integration along the beam. We chose to use two Gauss points.

The integral of $\mathbf{K}_{elem}$ [éq 4.2.2-9] is calculated under digital form:

$$\mathbf{K}_{elem}=\int_0^L \mathbf{B}^T . K_s . \mathbf{B}\ dx = j\sum_{i=1}^{2} w_i \mathbf{B}(x_i)^T . \mathbf{K}_s(x_i)\ .\ \mathbf{B}(x_i) \qquad \textbf{éq 4.2.5-1}$$

- where x_i is the position of the Gauss point i in an element of reference length 1, i.e.: $(1\ \pm 0{,}57735026918963)/2$;
- w_i is the weight of the Gauss point i. One takes here $w_i=0{,}5$ for each of the 2 points; j is the Jacobian. One takes here $j=L$, the real element having a length L and the shape function to pass to the element of reference being $\dfrac{x}{L}$.

$\mathbf{K}_s$ is calculated using the equations [éq 4.2.2-3], [éq 4.2.2-4] (see paragraph [§4.2.3] for the numerical integration of these equations).

The computation analytical of $\mathbf{B}(x_i)^T . \mathbf{K}_s(x_i)\ .\ \mathbf{B}(x_i)$ gives:

$$\begin{pmatrix}
B_1^2K_{s11} & -B_1B_2K_{s13} & B_1B_2K_{s12} & 0 & -B_1B_3K_{s12} & -B_1B_3K_{s13} & -B_1^2K_{s11} & B_1B_2K_{s13} & -B_1B_2K_{s12} & 0 & -B_1B_4K_{s12} & -B_1B_4K_{s13}\\
 & B_2^2K_{s33} & B_2^2K_{s23} & 0 & B_2B_3K_{s23} & B_2B_3K_{s33} & B_1B_2K_{s13} & -B_2^2K_{s33} & B_2^2K_{s23} & 0 & B_2B_4K_{s23} & B_2B_4K_{s33}\\
 & & B_2^2K_{s22} & 0 & -B_2B_3K_{s22} & -B_2B_3K_{s23} & -B_1B_2K_{s12} & B_2^2K_{s23} & -B_2^2K_{s22} & 0 & -B_2B_4K_{s22} & -B_2B_4K_{s23}\\
 & & & B_1^2K_{s44} & 0 & 0 & 0 & 0 & 0 & -B_1^2K_{s44} & 0 & 0\\
 & & & & B_3^2K_{s22} & B_3^2K_{s23} & B_1B_3K_{s12} & -B_2B_3K_{s23} & B_2B_3K_{s22} & 0 & B_3B_4K_{s22} & B_3B_4K_{s23}\\
 & & & & & B_3^2K_{s33} & B_1B_3K_{s13} & -B_2B_3K_{s33} & B_2B_3K_{s23} & 0 & B_3B_4K_{s23} & B_3B_4K_{s33}\\
 & & & & & & B_1^2K_{s11} & -B_1B_2K_{s13} & B_1B_2K_{s12} & 0 & B_1B_4K_{s12} & B_1B_4K_{s13}\\
 & & & & & & & B_2^2K_{s33} & -B_2^2K_{s23} & 0 & -B_2B_4K_{s23} & -B_2B_4K_{s33}\\
 & & & & & & & & B_2^2K_{s22} & 0 & B_2B_4K_{s22} & B_2B_4K_{s23}\\
 & & & & & & & & & B_1^2K_{s44} & 0 & 0\\
 & & & & & & & & & & B_4^2K_{s22} & B_4^2K_{s23}\\
 & & & & & & & & & & & B_4^2K_{s33}
\end{pmatrix}$$

éq 4.2.5-2

where them B_i are calculated with the X-coordinate x_i of the element of reference with:

$$\begin{aligned}
B_1 &= -N_{1,x} = N_{2,x} = \frac{1}{L}\\
B_2 &= -N_{3,xx} = N_{5,xx} = -\frac{6}{L^2}+\frac{12x_i}{L^2}\\
B_3 &= N_{4,xx} = -\frac{4}{L}+\frac{6x_i}{L}\\
B_4 &= N_{6,xx} = -\frac{2}{L}+\frac{6x_i}{L}
\end{aligned} \qquad \textbf{éq 4.2.5-3}$$

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

4.3 Determination of the mass matrix of the multifibre element

4.3.1 Determination of $\mathbf{M}_{elem}$

In the same way, the virtual wor of the forces of inertia becomes [bib2]:

$$\begin{aligned} W_{int} &= \int_0^L \int_S \rho \left(\delta u(x,y) \frac{d^2 u(x,y)}{dt^2} + \delta v(x,y) \frac{d^2 v(x,y)}{dt^2} + \delta w(x,y) \frac{d^2 w(x,y)}{dt^2} \right) dS\ dx \\ &= \int_0^L \delta \mathbf{U}_s . \mathbf{M}_s . \frac{d^2 \mathbf{U}_s}{dt^2}\ dx \end{aligned}$$

éq 4.3.1-1

with $\mathbf{U}_s$ the vector of “generalized” displacements.

What gives for the mass matrix:

$$\mathbf{M}_s = \begin{pmatrix} M_{s11} & 0 & 0 & M_{s12} & M_{s13} & 0 \\ & M_{s11} & 0 & 0 & 0 & -M_{s12} \\ & & M_{s11} & 0 & 0 & -M_{s13} \\ & & & M_{s22} & M_{s23} & 0 \\ & & & & M_{s33} & 0 \\ sym & & & & & M_{s22}+M_{s33} \end{pmatrix}$$

éq 4.3.1-2

with:

$$\begin{aligned} M_{s11} &= \int_S \rho\, ds \ ;\ M_{s12} = \int_S \rho\, zds \ ;\ M_{s13} = -\int_S \rho\, yds \\ M_{s22} &= \int_S \rho\, z^2 ds \ ;\ M_{s23} = -\int_S \rho\, yzds \ ;\ M_{s33} = \int_S \rho\, y^2 ds \end{aligned}$$

éq 4.3.1-3

with ρ which can vary according to y and z .

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

As for the stiffness matrix, we take into account the generalized strains and the discretization of space $[0,L]$. What gives finally for the elementary mass matrix:

$$\mathbf{M}_{elem}=\begin{vmatrix} M_{elem}^{1} \\ M_{elem}^{2} \\ M_{elem}^{3} \\ M_{elem}^{4} \\ M_{elem}^{5} \\ M_{elem}^{6} \\ M_{elem}^{7} \\ M_{elem}^{8} \\ M_{elem}^{9} \\ M_{elem}^{10} \\ M_{elem}^{11} \\ M_{elem}^{12} \end{vmatrix}$$

with:

$$M_{elem}^{1}=\left[\frac{LM_{s11}}{3}\ \frac{-M_{s13}}{2}\ \frac{M_{s12}}{2}\ 0\ \frac{LM_{s12}}{2}\ \frac{LM_{s13}}{12}\ \frac{LM_{s11}}{6}\ \frac{M_{s13}}{2}\ \frac{-M_{s12}}{2}\ 0\ \frac{-LM_{s12}}{12}\ \frac{-LM_{s13}}{12}\right]$$

$$M_{elem}^{2}=\left[sym\ \ \frac{13LM_{s11}}{35}+\frac{6M_{s33}}{5L}\ \frac{-6M_{s23}}{5L}\ \frac{-7LM_{s12}}{20}\ \frac{M_{s23}}{10}\ \frac{11L^2M_{s11}}{210}+\frac{M_{s33}}{10}\ \frac{-M_{s13}}{2}\ \frac{9LM_{s11}}{70}-\frac{6M_{s33}}{5L}\ \frac{6M_{s23}}{5L}\ \frac{-3LM_{s12}}{20}\ \frac{M_{s23}}{10}\ \frac{-13L^2M_{s11}}{420}+\frac{M_{s33}}{10}\right]$$

$$M_{elem}^{3}=\left[sym\ \ sym\ \ \frac{13LM_{s11}}{35}+\frac{6M_{s22}}{5L}\ \frac{-7LM_{s13}}{20}\ \frac{-11L^2M_{s11}}{210}-\frac{M_{s22}}{10}\ \frac{-M_{s23}}{10}\ \frac{M_{s12}}{2}\ \frac{6M_{s23}}{5L}\ \frac{9LM_{s11}}{70}-\frac{6M_{s22}}{5L}\ \frac{-3LM_{s13}}{20}\ \frac{13L^2M_{s11}}{420}-\frac{M_{s22}}{10}\ \frac{-M_{s23}}{10}\right]$$

$$M_{elem}^{4}=\left[sym\ \ sym\ \ sym\ \ \frac{LM_{s22}+LM_{s33}}{3}\ \frac{L^2M_{s13}}{20}\ \frac{-L^2M_{s12}}{20}\ 0\ \frac{-3LM_{s12}}{20}\ \frac{-3LM_{s13}}{20}\ \frac{LM_{s22}+LM_{s33}}{6}\ \frac{-L^2M_{s13}}{30}\ \frac{L^2M_{s12}}{30}\right]$$

$$M_{elem}^{5}=\left[sym\ \ sym\ \ sym\ \ sym\ \ \frac{L^3M_{s11}}{105}+\frac{2LM_{s22}}{15}\ \frac{2LM_{s23}}{15}\ \frac{-LM_{s12}}{12}\ \frac{-M_{s23}}{10}\ \frac{-13L^2M_{s11}}{420}+\frac{M_{s22}}{10}\ \frac{L^2M_{s13}}{30}-\frac{-L^3M_{s11}}{140}-\frac{LM_{s22}}{30}\ \frac{-LM_{s23}}{30}\right]$$

$$M_{elem}^{6}=\left[sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ \frac{L^3M_{s11}}{105}+\frac{2LM_{s33}}{15}\ \frac{-LM_{s13}}{12}\ \frac{13L^2M_{s11}}{420}-\frac{M_{s33}}{10}\ \frac{M_{s23}}{10}\ \frac{-L^2M_{s12}}{30}\ \frac{-LM_{s23}}{30}\ \frac{-L^3M_{s11}}{140}-\frac{LM_{s33}}{30}\right]$$

$$M_{elem}^{7}=\left[sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ \frac{LM_{s11}}{3}\ \frac{M_{s13}}{2}\ \frac{-M_{s12}}{2}\ 0\ \frac{LM_{s12}}{12}\ \frac{LM_{s13}}{12}\right]$$

$$M_{elem}^{8}=\left[sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ \frac{13LM_{s11}}{35}+\frac{6M_{s33}}{5L}\ \frac{-6M_{s23}}{5L}\ \frac{-7LM_{s12}}{20}\ \frac{-M_{s23}}{10}\ \frac{-11L^2M_{s11}}{210}-\frac{M_{s33}}{10}\right]$$

$$M_{elem}^{9}=\left[sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ \frac{13LM_{s11}}{35}+\frac{6M_{s22}}{5L}\ \frac{-7LM_{s13}}{20}\ \frac{11L^2M_{s11}}{210}+\frac{M_{s22}}{10}\ \frac{M_{s23}}{10}\right]$$

$$M_{elem}^{10}=\left[sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ \frac{LM_{s22}+LM_{s33}}{3}\ \frac{-L^2M_{s13}}{20}\ \frac{L^2M_{s12}}{20}\right]$$

$$M_{elem}^{11}=\left[sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ \frac{L^3M_{s11}}{105}+\frac{2LM_{s22}}{15}\ \frac{2LM_{s23}}{15}\right]$$

$$M_{elem}^{12}=\left[sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ sym\ \ \frac{L^3M_{s11}}{105}+\frac{2LM_{s33}}{15}\right]$$

éq 4.3.1-4

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

with the following terms: $M_{s11}, M_{s12}, M_{s13}, M_{s22}, M_{s33}, M_{s23}$ who are given to the equation [éq 4.3.1 - 3].

Note:

> *The mass matrix is reduced by the technique of the lumped masses ([bib4]) . This diagonal mass matrix is obtained by option "*`MASS_MECA_DIAG`*" of operator* `CALC_MATR_ELEM` *[U4.61.01].*

4.3.2 Discretization of the fiber section - Computation of $\mathbf{M}_s$

the discretization of the fiber section makes it possible to calculate the various integrals which intervene in the mass matrix. Thus, if we have a section which comprises n fibers we will have the following approximations of the integrals:

$$M_{s11}=\sum_{i=1}^{n}\rho_i S_i \;\; ; \;\; M_{s12}=\sum_{i=1}^{n}\rho_i z_i S_i \;\; ; \;\; M_{s13}=-\sum_{i=1}^{n}\rho_i y_i S_i$$
$$M_{s22}=\sum_{i=1}^{n}\rho_i z_i^2 S_i \;\; ; \;\; M_{s23}=-\sum_{i=1}^{n}\rho_i y_i z_i S_i \;\; ; \;\; M_{s33}=\sum_{i=1}^{n}\rho_i y_i^2 S_i$$ **éq 4.3.2-1**

with ρ_i and S_i density and the section of each fiber. y_i and z_i are the coordinates of the center of gravity of fiber defined as previously.

4.4 Computation of the internal forces

The computation of the nodal forces $\mathbf{F}_{\text{int}}$ due in a stress state interns given is done by the integral:

$$\boldsymbol{F}_{\text{int}}=\int_0^L \boldsymbol{B}^{\boldsymbol{T}}.\boldsymbol{F}_{\boldsymbol{s}}\, dx$$ **éq 4.4-1**

where $\mathbf{B}$ is the matrix giving the generalized strains according to nodal displacements [éq 4.2.2-6] and where $\mathbf{F}_s$ is the vector of the generalized stresses given to the equation [éq 4.2.2-2],

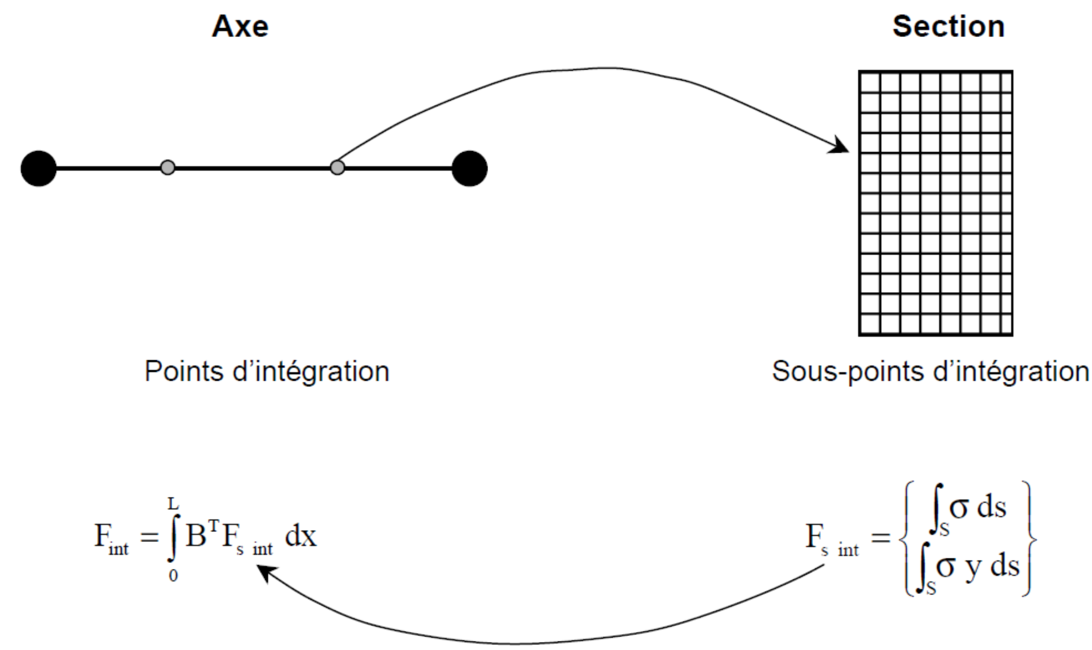

Figure 4.4-a: Multibre beam – Computation of $\mathbf{F}_{\text{int}}$

$$\mathbf{F}_s^T = \left(N \;\; M_y \;\; M_z \;\; M_x \right)$$ **éq 4.4-2**

the normal force N and the bending moments M_y and M_z are calculated by integration of the stresses on the section [éq 4.2.1-8].

Behavior in torsion being supposed to remain linear, the twisting moment is calculated with nodal axial rotations:

$$M_x = GJ_x \frac{\theta_{x2} - \theta_{x1}}{L}$$ **éq 4.4-3**

the equation [éq 4.4-1] is integrated numerically:

$$\mathbf{F}_i = \int_0^L \mathbf{B}^T . \mathbf{F}_s dx = j \sum_{i=1}^{2} w_i \mathbf{B}\left(x_i\right)^T . \mathbf{F}_s\left(x_i\right)$$ **éq the 4.4-4**

positions and weights of Gauss points as well as the Jacobian are given in the paragraph [§4.2.5].

The computation analytical of $B\left(x_i\right)^T . F_s\left(x_i\right)$ gives:

$$\left[\mathbf{B}(x_i)^T . \mathbf{F}_s(x_i)\right]^T = \left[-B_1 N \;\; B_2 M_z \;\; -B_2 M_y \;\; 0 \;\; B_3 M_y \;\; B_3 M_z \;\; B_1 N \;\; -B_2 M_z \;\; B_2 M_y \;\; 0 \;\; B_4 M_y \;\; B_4 M_z\right]$$ **éq 4.4-5**

where them B_i are given to the equation [éq 4.2.4-1].

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

4.5 Formulation enriched in strain

With the interpolations by displacements by the equation [éq 4.1-1], the axial generalized strain is constant and the curvatures are linear (see equations [éq 4.2.2-6], [éq 4.2.2-7] and [éq 4.2.5-3]):

$$\begin{cases} \varepsilon_s(x)=\dfrac{u_2-u_1}{L} \\ \chi_{ys}(x)=-\left(-\dfrac{6}{L^2}+\dfrac{12\,x}{L^3}\right)w_1+\left(\dfrac{6x}{L^2}-\dfrac{4}{L}\right)\theta_{y1}-\left(-\dfrac{12\,x}{L^3}+\dfrac{6}{L^2}\right)w_2+\left(\dfrac{6x}{L^2}-\dfrac{4}{L}\right)\theta_{y2} \\ \chi_{zs}(x)=\left(-\dfrac{6}{L^2}+\dfrac{12\,x}{L^3}\right)v_1+\left(\dfrac{6x}{L^2}-\dfrac{4}{L}\right)\theta_{z1}+\left(-\dfrac{12\,x}{L^3}+\dfrac{6}{L^2}\right)v_2+\left(\dfrac{6x}{L^2}-\dfrac{4}{L}\right)\theta_{z2} \end{cases} \qquad \textbf{éq 4.5-1}$$

If there is no coupling between these two strains (elastic case, with the line average one of reference which passes by the barycenter of the section), that does not pose problems. But in the nonlinear general case, there is a shift of the neutral axis, and the terms K_{s12} and K_{s13} of $\mathbf{K}_s$ (equations [éq 4.2.2-3] and [éq 4.2.2-4]) are not null, there is coupling between the moments and the normal force. There is then an incompatibility in the approximation of the axial strains of a fiber:

$$\varepsilon=\varepsilon_s(x)-y\,\chi_{zs}(x)+z\,\chi_{ys}(x) \qquad \textbf{éq 4.5-2}$$

a layer to eliminate this incompatibility is to enrich the strain field axial:

$$\varepsilon_s(x) \qquad \varepsilon_s(x)+\tilde{\varepsilon}_s(x)\ \ ;\ \ \tilde{\varepsilon}_s(x)=\alpha.G(x)\ \ ;\ \ G(x)=\frac{4}{L}-\frac{8\,x}{L^2} \quad \textbf{for } x\in\left[-\frac{L}{2},\frac{L}{2}\right] \qquad \textbf{éq 4.5-3}$$

where $G(x)$ is an enriched strain which derives from a function “bubble” in displacement and α the degree of freedom of enrichment. The variational base of such an enrichment is provided by the principle of Hu-Washizu [bib5] which can be presented same way as the method of the incompatible modes [bib6].

4.5.1 Method of the incompatible modes

the regular field of generalized displacements $\mathbf{U}_s$ is defined by the equation [éq 4.1-0]. Strains generalized $\mathbf{D}_s$ and the generalized stresses $\mathbf{F}_s$ by the equation [éq 4.2.2-2].

The principle of Hu-Washizu consists in writing the weak form of the balance equations, but also of the computation of the strains and the constitutive law, in projection on the three virtual fields (generalized displacements $\mathbf{U}_s^*$, generalized strains $\mathbf{D}_s^*$ and generalized stresses $\mathbf{F}_s^*$):

$$\int_0^L \frac{dU_s^*}{dx}\cdot F_s\,dx-W_{ext}=0\ \ ;\ \ \int_0^L \mathbf{F}_s^*\cdot\left(\frac{d\,\mathbf{U}_s}{dx}-\mathbf{D}_s\right)dx=0\ \ ;\ \ \int_0^L \mathbf{D}_s^*\cdot\left(\mathbf{F}_s-\mathbf{K}_s.\mathbf{D}_s\right)dx=0 \qquad \textbf{éq 4.5.1-1}$$

One introduces the enrichment of the real strains, and one chooses to break up the virtual field of strains into a “regular” part exit of the virtual field of displacements and an enriched part:

$$D_s=\frac{dU_s}{dx}+\bar{D}_s \quad ; \quad D_s^*=\frac{dU_s^*}{dx}+\bar{D}_s^* \qquad \textbf{éq 4.5.1-2}$$

One defers [éq 4.5.1-2a] in [éq 4.5.1-1b], which justifies “enrichment” by orthogonality:

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

$$\int_0^L \mathbf{F}_s^*.\mathbf{D}_s \; dx=0 \quad \text{éq 4.5.1-3}$$

the equation [éq 4.5.1-1c] breaks up into two since one has two independent virtual fields in [éq 4.5.1-2b]:

$$\int_0^L \frac{dU_s^*}{dx}.\left(F_s-K_s.D_s\right) dx=0 \quad ; \quad \int_0^L \overline{\mathbf{D}}_s^*\cdot\left(\mathbf{F}_s-\mathbf{K}_s\cdot\mathbf{D}_s\right) dx=0 \quad \text{éq 4.5.1-4}$$

Lastly, the method of the incompatible modes consists in choosing the orthogonal space of the stresses to the space of the enriched strains, so that [éq 4.5.1-3] is automatically checked and [éq 4.5.1-4b] thus gives simply:

$$\int_0^L \overline{\mathbf{D}}_s^*.\mathbf{K}_s.\mathbf{D}_s \; dx=0 \quad \text{éq 4.5.1-5}$$

If one returns to the strong formulation of the constitutive law in [éq 4.5.1-4a] and [éq 4.5.1-5], the system [éq 4.5.1-1] becomes:

$$\int_0^L \frac{dU_s^*}{dx}.F_s \; dx-W_{ext}=0 \quad ; \quad \int_0^L \overline{\mathbf{D}}_s^*\cdot\mathbf{F}_s \; dx=0 \quad ; \quad \mathbf{F}_s=\mathbf{K}_s\cdot\mathbf{D}_s \quad \text{éq 4.5.1-6}$$

Note::

•*Here one enriches only the axial strain by a beam element of Eulerian-Bernoulli, with a continuous function [éq 4.5-3], therefore* $\overline{\mathbf{D}}=\left(\bar{\varepsilon}_s \quad 0 \quad 0 \quad 0\right)^T$.

4.5.2 Numerical establishment

From the point of view finite elements, one can write displacements and the strains in matric form, with the enriched part:

$$\mathbf{B}_s=\mathbf{N}.(\mathbf{U})+\mathbf{Q}.(\alpha) \quad ; \quad \mathbf{D}=\mathbf{B}.(\mathbf{U})+\mathbf{G}.(\alpha) \quad \text{éq 4.5.2-1}$$

where $\mathbf{N}$ and $\mathbf{B}$ are the classical matrixes of the interpolation functions and their derivatives (see [éq 4.1-1] and [éq 4.2.2-7]) and:

$$\mathbf{Q}=\left(\frac{4x}{L}-\frac{4x^2}{L^2} \quad 0 \quad 0 \quad 0\right)^T \text{ and } \mathbf{G}=\left(\frac{4}{L}-\frac{8x}{L^2} \quad 0 \quad 0 \quad 0\right)^T \quad \text{éq 4.5.2-2}$$

Note::

• $\mathbf{G}$ *was selected so that the element always passes the "patch test" (strain energy null for a solid motion)* :

$$\int_0^L \mathbf{G}(x)\,dx \;=\; \mathbf{0} \quad \text{éq 4.5.2-3}$$

After classical handling of transition of continuous to discrete, the system of equations [éq 4.5.1-6], written for the group of structure, is approximated by:

$$\begin{cases} A_{e=1}^{N_{elem}}\left(F_{\text{int}}-F_{ext}\right)=0 \\ h_e=0 \quad \forall e\in[1,\,N_{elem}] \end{cases} \quad \text{éq 4.5.2-4}$$

with:

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

$$\begin{cases} \mathbf{F}_{\text{int}} = \int_0^L \mathbf{B}^T . \mathbf{F}_s \, dx = \int_0^L \mathbf{B}^T . \mathbf{K}_s . \left(\mathbf{B} . \mathbf{U}_s + \mathbf{G} . \alpha \right) dx \\ \mathbf{F}_{ext} = \int_0^L \mathbf{N}^T . \mathbf{f} \, dx \\ h_e = \int_0^L \mathbf{G}^T . \mathbf{F}_s \, dx \end{cases}$$ **éq 4.5.2-5**

$A_{e=1}^{N_{elem}}$ indicates the assembly on all the elements of the mesh; $\mathbf{f}$ is the axial loading distributed on the element beam. The system of equations [éq 4.5.2-4] is nonlinear, it is solved in an iterative way (see `STAT_NON_LINE`).

With the iteration $(i+1)$, with $\Delta \mathbf{U}^{(i)} = \mathbf{U}^{(i+1)} - \mathbf{U}^{(i)}$ and $\Delta \alpha^{(i)} = \alpha^{(i+1)} - \alpha^{(i)}$, the linearization of the system gives (iterations of correction of Newton):

$$\begin{cases} A_{e=1}^{N_{elem}} \left(\left(\mathbf{F}_{\text{int}}^{(i+1)} - \mathbf{F}_{ext}^{(i+1)} \right) + \mathbf{K}_e^{(i)} . \Delta \mathbf{U}^{(i)} + \mathbf{X}_e^{(i)} \Delta \alpha^{(i)} \right) = 0 \\ h_e^{(i+1)} + \mathbf{X}_e^{(i)T} . \Delta \mathbf{U}^{(i)} + H_e^{(i)} \Delta \alpha^{(i)} = 0 \qquad \forall e \in [1, \ldots, N_{elem}] \end{cases}$$ **éq 4.5.2-6**

with:

$$\begin{cases} \mathbf{K}_e^{(i)} = \int_L \mathbf{B}^T . \mathbf{K}_s^{(i)} . \mathbf{B} \, dx \\ \mathbf{X}_e^{(i)} = \int_L \mathbf{B}^T . \mathbf{K}_s^{(i)} . \mathbf{G} \, dx \\ H_e^{(i)} = \int_L \mathbf{G}^T . \mathbf{K}_s^{(i)} . \mathbf{G} \, dx \end{cases}$$ **éq 4.5.2-7**

the second equation of the system [éq 4.5.2-6] is local. It makes it possible independently to calculate the degree of freedom of α enrichment on each element. One calculates it by a local iterative method (iterations (j) for a built-in $d^{(i)} = \Delta \mathbf{U}^{(i)}$ displacement):

$$\alpha_{(j+1)}^{(i)} = \alpha_{(j)}^{(i)} - \left(H_{e(j)}^{(i)} \right)^{-1} h_{e(j)}^{(i)}$$ **éq 4.5.2-8**

Thus, when one converged at the local level, one a:

$$h_e(d^{(i)}, \alpha^{(i)}) = 0$$ **éq 4.5.2-9**

And one can operate a static condensation to eliminate α at the total level.

$$\mathbf{K}_e^{(i)} = \mathbf{K}_e^{(i)} - \mathbf{X}_e^{(i)} \left(H_e^{(i)} \right)^{-1} \mathbf{X}_e^{(i)T}$$ **éq 4.5.2-10**

From a practical point of view, this technique makes it possible to treat enrichment at the elementary level without disturbing the number of total degrees of freedom. It is established with the level of the elementary routine charged to calculate options `FULL_MECA`, `RAPH_MECA` and `RIGI_MECA_TANG`.

Note:

- in the typical case exposed here, $H_e^{(i)}$ is a reality, therefore very easy to reverse!
- in the same way, h_e and α are also realities.
- The computation of $K_e^{(i)}$ is explained in the paragraph § 4.2.5, the other quantities of the equation [éq 4.5.2-8] are calculated according to the same technique.
- In the same way the computation of $\mathbf{F}_{\text{int}}$ is explained in the paragraph § 4.4, h_e in the equation [éq 4.5.2-5] is calculated according to the same technique.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

4.5.3 Warning

For an elastic behavior, the enrichment of the axial strains makes it possible to take account correctly coupling between the normal force and the bending moments, and to return the response of the beam independent of the position chosen for the reference axis (see key word `COOR_AXE_POUTRE` in operator `DEFI_GEOM_FIBRE`, [U4.26.01]). It thus makes it possible to treat the case of the offset beams.

But in the current version of *Code_Aster*, the enrichment of the axial strains was established only for nonlinear computations with `STAT_NON_LINE` or `DYNA_NON_LINE`, for options `FULL_MECA`, `RAPH_MECA` and `RIGI_MECA_TANG`.

On the other hand, it is not established yet for the option `RIGI_MECA` (matrix elastic, behavior model "`ELAS`"), because in this case it is necessary to write the explicitly condensed matrix (not of possible iterations).

Thus, if one wants to do a correct calculation for a beam offset with element `POU_D_EM`, it is necessary to use `STAT_NON_LINE` with option `MATRICE=' TANGENTE'`. All computations using `RIGI_MECA` are not correct (`STAT_NON_LINE` with option `MATRICE=' ELASTIQUE'`, `MECA_STATIQUE`, but also operators of computation of eigen modes…).

In the same way, the mass matrix (see § 4.4) was not modified and does not take account of the enrichment of axial displacement.

4.6 Nonlinear models of behavior usable

the supported models are on the one hand the behavior models 1D of type `VMIS_ISOT_LINE` , `VMIS_CINE_LINE`, `VMIS_ISOT_TRAC`, `CORR_ACIER` and `PINTO_MENEGOTTO` [R5.03.09] for steels, on the other hand the model `LABORD_1D` [R7.01.07] dedicated to the uniaxial behavior of the concrete in cyclic. One can thus have several materials by multifibre beam element.

In addition, if the behavior used is not available in 1D, one can 3D use the other models using the method of R. De Borst [R5.03.09]). For example, one can treat: `GRAN_IRRA_LOG`, `VISC_IRRA_LOG`. However in this case, one can treat one material by multifibre beam element.

Note:

> *The local variables, constants by fiber, are stored in the subpoints attached to the point of integration considered.*
>
> *The access to the postprocessing of the quantities defined in the subpoints is done via format MED3.0, of Salomé.*

5 Case of application

One will be able usefully to consult the cases test-following:

- ssll111a: Static response of a reinforced concrete beam (section in T) with thermo-elastic linear behavior , [V3.01.111];
- sdll130b: Seismic response of a reinforced concrete beam (rectangular section) with linear behavior, [V2.02.130];
- sdll132a : Eigen modes of a frame out of multifibre beams; [V2.02.132];
- ssnl119a, ssnl119b: Static response of a reinforced concrete beam (rectangular section) with nonlinear behavior, [V6.02.119];
- sdnl130a: Seismic response of a reinforced concrete beam (rectangular section) with nonlinear behavior, [V5.02.130];
- ssll102j: Clamped beam subjected to unit forces, [V3.01.102];
- ssnl106g, ssnl106h: Elastoplastic beam in tension and pure bending, [V6.02.106];

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

- ssnl122a: Cantilever beam Multifibre subjected to a force [V6.02.122];
- ssnl123a: Buckling of a beam Multifibre [V6.02.123] .

6 Bibliography

[1] J.L. BATOZ, G. DHATT: Modelization of structures by finite elements - HERMES.

[2] J. GUEDES, P. PEGON & A. PINTO: A fiber Timoshenko beam element in CASTEM 2000 – Ispra, 1994.

[3] P. KOTRONIS: Dynamic shears of reinforced concrete walls. Models simplified 2D and 3D – Thesis of Doctorate of the ENS Cachan – 2000.

[4] J.M. PROIX, P. MIALON, M.T. BOURDEIX: É léments “exact” of beams (right and curved), Documentation of reference of *the Code_Aster* [R3.08.01] .

[5] O.C ZIENKIEWICZ and R.L TAYLOR. The Finite Method Element. Butterworth-Heinemann, Oxford, the U.K., 5th ED. Zienkiewicz and Taylor – 2000.

[6] A. IBRAHIMBEGOVIC and E.L. WILSON. A modified method of incompatible modes. Commum. Numer. Methods Eng., 7:187-194 – 1991.

[7] [U4.26.01] Operator `DEFI_GEOM_FIBRE`.

[8] [R7.01.12] Modelization of thermohydration, the drying and the shrinking of the concrete. G.Debruyne, May 2005.

7 Description of the versions of the document

Index document	Version Aster	Author (S) Organization (S)	Description of the modifications
A	6.4	**S.Moulin** (EDF-R&D/AMA), L.Davenne (ENSC/LMT),	initial Version
B	9.5	L.Davenne (ENSC/LMT), F.Voldoire (EDF-R&D/AMA)	Enrichment of the axial strain by function bubble and static condensation in nonlinear, taken into account of torsion into linear, adaptation with the new data structure `GROUP_FIBRE`, cf drives REX 9141. List cases of applications.
C10	10	F.Voldoire (EDF-R&D/AMA)	Corrections of working Openoffice.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)