Multifibre beams in large displacements

Summarized:

This document by the key word presents the processing of multifibre large displacements of beam element POU_D_TGM activated DEFORMATION='GROT_GDEP'.
It makes it possible to combine at the same time non-linearity material (description of the fiber section) and geometrical non-linearity. The description of large displacements remains approximate and makes the assumption of small time step.
This element is adapted for the characterization of the structure truss failure.

Note: the processing of large displacements for the other beam elements is described in [R3.08.01].

Contents

1 Position of the problem

In the field of the modelization of the beam elements it exists a large number of theories to describe kinematics (Eulerian-Bernoulli, Timoshenko, Vlassov,…) and one naturally counts in *Code_Aster* a variation of these theories in as many model finite elements (`POU_D_E`, `POU_D_T`, `POU_D_TG`).
In addition to the assumptions on which they are founded, these models can be nouveau riches of a nonlinear behavior that it is of origin material (plasticity criterion for the beams, behavior 1D leaning on a description by fiber) or geometrical (large displacements and three-dimensional large rotations).

This document describes the element `POU_D_TGM` which leans on a model of Timoshenko[1] with constrained torsion of Vlassov[2]. This element associates a description by fiber of the section to profit from a behavior 1D nonlinear in traction and compression/bending and a kinematics from the second order to allow a description in large displacements and large rotations[3].
One will concentrate on the geometrical aspect, the behavior multifibre being approached in detail in [4].

2 Notations

the notations used here correspond to those of [2] and [4].

3 Presentation of the approach

3.1 Introduction

the taking into account of large displacements, which they result from a rigid body motion or from an unspecified transformation of studied structure introduced an additional non-linearity (besides that introduced by behavioral non-linearity for example). This non-linearity results in the fact that initial configuration of structure and final configuration (or "deformed shape") cannot be confused more as it is usually the case for the processing of the problems in small disturbances.
For of the finite elements of structures (*i.e beams* , plates, shells), it poses an additional problem: rotations are not any more one vectorial, one cannot thus any more transform them like vectors during the transition of the local coordinate system to the total reference. Indeed
, when vectors displacements are expressed, in local coordinate system of finite element (reference attached to the SEG2 `of` the beam in which all computations are carried out to simplify the writing of the various terms), it is completely physical to transform them in a total reference attached to structure. When one is interested in structural elements, those also carry degrees of freedom of rotation; when these rotations are "small", one can show that they are identified with vectors and thus to transform them like displacements. That
is not any more the case when rotations are "large" and that results in the NON-commutation of rotations. For lack of a description taking into account this characteristic, the continuity of the degrees of freedom to the nodes of the elements is not then assured any more [5][5][6]

3.2 Element geometrically exact POU_D_T_GD

the finite element of beam POU_D_T_GD , `already` integrated into the code since many years [7][7] a beam of Timoshenko in large displacements in the sense that the field of displacements used in the formulation (during the transition 3D → beams) is written in an exact way. He takes in particular account of the exact operator rotation between two configurations of the element (i.e. one makes no simplifying assumption on displacements).
The behavior, only elastic, is always written in small strains; this element had indeed been introduced for treating "almost rigid" large displacements of a structure in dynamics (motion of the spacers between the conducteurs1Les[1] of an airline). This
element is formulated with a Total Lagrangian approach relatively complex, the exact processing of large rotations requiring to call on the theory of the quaternions to update displacements correctly. The

1spacers are used to guarantee a minimal spacing between the electrical drivers of a loom behavior is directly formulated on the forces generalized (without passing by the stresses) and really does not lend itself to an extension multifibre which by nature works on stresses 2 [2]

3.3 Element multifibre POU_D_TGM It

is a beam element of Timoshenko with warping of the multifibre cross-sectional area and approach to give an account of the progression of plasticity in the section [8][8] element already had a generic option "PETIT_REAC " (now removed) able to reactualize the geometry with each iteration of the algorithm of resolution by step. That made it possible to deal with problems in large displacements under the assumption of small rotations. However the geometrical absence of stiffness in the formulation makes in the case of convergence very difficult instability and this same for plane problems. Moreover, one does not exploit all the possibilities of the reactualization of the geometry (cf 4.4 16

3.4 A small history It

is as from the Seventies that the geometrical nonlinear analysis of formed structures by beams developed with the use of the finite element method. The analysis was initially concerned plane structures [9][9] on three-dimensional structures with the work founders of Bathe in 1979 [10[10] is indeed at that time that for the first time a formulation known as "Updated Lagrangian *appears" (UL*), i.e. a formulation which updates the geometry of structure at each iteration of the algorithm of resolution, making it possible to obtain a formulation simplified while remaining robust. Thereafter
by many works took as a starting point those of Bathe and the enriched, in particular with the processing of the thorny problem of the large rotations highlighted by Argyris [11[11] can also quote works of Yang and McGuire [5][5][12] those of Conci [6][6][13]
important point in nonlinear analysis of structures is the need for estimating the residue well to get right results [14[14] in the case as of elements beams, the interpolation functions geometrical are first order (the elements beams are not isoparametric). That involves errors in the computation of the residue if the structure becomes deformed much and if enough elements are not used.
A solution obviously consists in especially increasing the number of finite elements for curved structures. However that can become expensive and of the authors thus proposed techniques known as of "force *recovery3littéralement*[3] to circumvent this problem and to make it possible to use one finite element by beam. Some as Conci for that separated rigid body motion and motion involving a non-zero work of strain [6][6] approach caused a new formulation which one also describes as "CO-rotational" formulation [15[15]
Others undertook to correct the value of the residue by adding an additional term in the formulation [16[16][17]

3.5

the approach described here leans on an exhaustive bibliographical study and more particularly on the work of two theses on the nonlinear analysis of formed structures by beams [18[18][19]The element selected leans on a Lagrangian formulation Brought up to date with each Iteration (FLAI) 4C'[4] one supposes the rotations "moderated" by iteration so that one can simply bring back oneself if rotations are commutative until the second order (from where the term "moderated" in opposition to "small" first order *i.e*). The approach

presented in detail in the following section can be broken up as follows: As

1)for all the structural elements, one formulates assumptions on the kinematics allowing to determine a three-dimensional field of displacements in any point of the section starting from the degrees of freedom considered. One writes
2)the principle of the virtual wors in the frame of a Lagrangian formulation Brought up to date with each Iteration. One is in small strains. One introduces

2exists other approaches to introduce plasticity but with the power of the current computers, the approach multifibre seems most adequate.
3to recover the force, i.e. to consider the work internal of structure"
4is the French equivalent of term UL introduces higher and

3)the field of displacement and the interpolation functions (which interpolate the displacements generalized according to the unknowns with the nodes) to obtain the forms of the tangent matrix which one will break up into several terms (each one returning to the various phenomena which one seeks to take into account). The taking
4)into account of large rotations of structure rests on a modification of the field of initial displacement, by supposing the rotations moderated between two iterations. Theoretical

4 description of the element Field

4.1 of displacement and strain field associated Assumptions

4.1.1 on the kinematics the kinematics

with adopted beam is the following one: The beam element

- •is supposed right, the constant section. The cross-sectional area
- •is indeformable in its plane but can warp the shear strains transverse
- •axially is not neglected and involves a rotation of the section around its axes the strains
- •are small, but displacements and the rotations can be large One

is placed on the level of a finite element, i.e. of a segment with 2 nodes, a local coordinate system with the element is defined by the axis of the segment and the principal axes of geometrical inertia. The center of the reference O coincides with the node n°1 (which is the center of gravity of the section), one also defines the center of torsion (or of shears) of C the section, where the application of shears does not generate work in torsion and vice versa. It is
easy to show [12] that[12] axial displacements are expressed starting from displacements of the point and the O displacements in the plane of the section from those of the point then C all the forces are uncoupled, i.e. the matrix of behavior (for the beam elements, it is the matrix obtained after statement of the Hooke's law and integration on the section) is diagonal. The restitution in the total reference is done then in two times: transition of all displacements in the local coordinate system of center then O transition of the local coordinate system to the total reference. Statement

4.1.2 of the field of displacement For

such a section, the displacement of a point of X-coordinate P , x coordinates in (y, z) the reference of center is expressed O as the sum of a term of translation and a term of rotation (assumption of indeformable cross-sections).
To take account of the warping of the sections, one adds a term of nonuniform axial displacement. The field of displacement is written $\vec{\xi}$ then: éq

$$\vec{\xi}(x,y,z)=\begin{cases} u(x,y,z) &=& u_O(x)+\left(z\times\theta_y(x)\right)-\left(y\times\theta_z(x)\right)+\left(\omega(y,z)\times\theta_{x,x}(x)\right)\\ v(x,y,z) &=& v_C(x)-\left((z-z_C)\times\theta_x(x)\right)\\ w(x,y,z) &=& w_C(x)+\left((y-y_C)\times\theta_x(x)\right)\end{cases}$$

4-1 4-1

refer C to the center of torsion which N C "is not confused with the center of gravity in the case of O nonbi--symmetric sections. The transition of displacements in to those C is carried out by it O by a simple change of reference. The function commonly ω called function of warping, and which depends only on the form of the section, makes it possible to describe nonuniform axial displacement. The degrees of freedom of L" element are carried to the nodes and are interpolated in the length of the element. There are 14 degrees of freedom, that is to say 7 with each node which are: . One $(u, v, w, \theta_x, \theta_y, \theta_z, \theta_{x,x})$naturally finds 3 degrees of translation and 3 degrees of rotation to describe the kinematics, moreover rate of torsion $\theta_{x,x}$ is also taken as measures warping. When

rotations cannot be regarded as small any more, it is then necessary to enrich the field by displacement of a nonlinear term. Indeed, at the end of warping close and while placing itself in the reference of center, O*the statement* (éq 4-14-1 4-1 be written vectoriellement like: éq

$$\vec{\xi}(x,y,z)=\vec{\xi}_O(x)+\left(\boldsymbol{R}(x)-\boldsymbol{I}\right)\begin{pmatrix}0\\y\\z\end{pmatrix}$$ **4-2 This 4-2**

statement reveals the operator rotation. If $\boldsymbol{R}$ is P the point of coordinates then (y,z) makes it possible $\boldsymbol{R}$ to take along the vector in $\overrightarrow{OP}$ its final configuration. depends $\overrightarrow{OP}'$ $\boldsymbol{R}$ on the skew-symmetric matrix associated with the vector rotation which is $\vec{\theta}={}^t(\theta_x,\theta_y,\theta_z)$ an unknown of the problem as well as, one $\vec{\xi}_O$can even show that is written $\boldsymbol{R}$ like exponential of matrix [7]: [7]

$$\boldsymbol{R}=e^{\boldsymbol{\theta}}=\boldsymbol{I}+\boldsymbol{\theta}+\frac{(\theta)^2}{2!}+\cdots+\frac{(\theta)^p}{p!}+\cdots \text{ avec } \boldsymbol{\theta}=\begin{pmatrix}0 & -\theta_z & \theta_y\\ \theta_z & 0 & -\theta_x\\ -\theta_y & \theta_x & 0\end{pmatrix}$$ **4-3 In 4-3**

(éq 4-14-1 4-1 operator was replaced by his development with the first order because rotations were small. To take into account of large rotations, one of the possibilités5Il exists[5] to develop the operator rotation at least until the second order [19] [16[19][16] then about moderate rotations (finite *rotations in English*). It is the office plurality of these moderate rotations which will lead to large rotations. The field of displacement is then enriched by a nonlinear, quadratic term in. $\theta_x,\theta_y,\theta_z$The development with the second order is obtained by developing the exponential one until the term in. Associated $\boldsymbol{\theta}^2$

4.1.3 strain field in addition

Let us calculate the strains of Green-Lagrange of the field of displacement (éq 4 4-1 4-1 use right now the assumption of the small strains, indeed consider the complete statement of the strains of Green-Lagrange (the indeformable sections being supposed in their plane, half of the terms is already null): (tensor

$$\boldsymbol{F}=\boldsymbol{I}+\frac{\partial\vec{\xi}}{\partial\vec{x}}=\boldsymbol{I}+\nabla\vec{\xi}$$ gradient of the transformation) éq **4-4 éq 4-4**

$$\boldsymbol{E}=\frac{1}{2}\left({}^t\boldsymbol{F}\boldsymbol{F}-\boldsymbol{I}\right)=\begin{cases}E_{xx} &= u_{,x}+\frac{1}{2}(u_{,x}{}^2+v_{,x}{}^2+w_{,x}{}^2)\\ 2E_{xy} &= (u_{,y}+v_{,x})+(u_{,x}u_{,y}+w_{,x}w_{,y})\\ 2E_{xz} &= (u_{,z}+w_{,x})+(u_{,x}u_{,z}+v_{,x}v_{,z})\end{cases}$$ **4-5 4-5**

strains one from of being deduced that and consequently $|u_{,x}|\ll 1$ and. $|u_{,x}u_{,y}|\ll|u_{,y}|$ One $|u_{,x}u_{,z}|\ll|u_{,z}|$can thus simplify the complete statement of by neglecting $\boldsymbol{E}$ the quadratic terms in where $|u_{,i}|$. In $i=(x,y,z)$[12] [[12][16] are not neglected, they lead then to a form of the more complex tangent matrix and an improved convergence. On the other hand they do not modify the accuracy of the results. By injecting

the field of displacement (éq 4 4-1 4-1 this simplified statement one obtains finally the following form: éq $\boldsymbol{E}=\boldsymbol{\epsilon}+\boldsymbol{\eta}$

$$\boldsymbol{\epsilon}=\begin{cases}\epsilon_{xx} &= u_{O,x}(x)+\left(z\times\theta_{y,x}(x)\right)-\left(y\times\theta_{z,x}(x)\right)+\left(\omega(y,z)\times\theta_{x,xx}(x)\right)\\ 2\epsilon_{xy} &= \left(v_{C,x}(x)-\theta_z\right)+\left(\omega_{,y}-(z-z_C)\times\theta_{x,x}(x)\right)\\ 2\epsilon_{xz} &= \left(w_{C,x}(x)+\theta_y\right)+\left(\omega_{,z}+(y-y_C)\times\theta_{x,x}(x)\right)\end{cases}$$ **4-6 éq 4-6**

5about it the different one, to see [20] [20] a state of the art is

$$\eta = \begin{cases} \eta_{xx} = \frac{1}{2}\left(v_{C,x}{}^2 + w_{C,x}{}^2 + \left((y-y_C)^2 + (z-z_C)^2\right)\theta_{x,x}{}^2 \right. \\ \quad \left. +2(y-y_C)\,\theta_{x,x}\,w_{C,x} - 2(z-z_C)\,\theta_{x,x}\,v_{C,x}\right) \\ 2\,\eta_{xy} = \theta_x\left(w_{C,x} + (y-y_C)\,\theta_{x,x}(x)\right) \\ 2\,\eta_{xz} = -\theta_x\left(v_{C,x} - (z-z_C)\,\theta_{x,x}(x)\right) \end{cases}$$

4-7 where 4-7

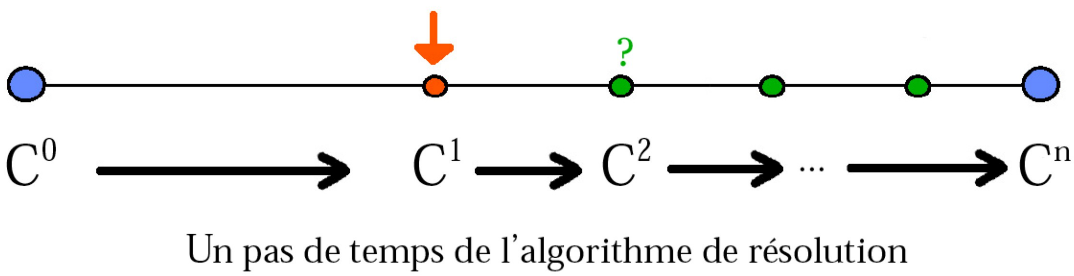
are ϵ the strains linearized and the strains η known as nonlinear (resulting from the quadratic terms in the strains of Green-Lagrange). If one then injects the nonlinear field of displacements for the processing of large rotations, an additional nonlinear term η_{gr} appears. Principle

4.2 of the virtual wors the assumptions

4.2.1 used

to write the principle of virtual wors (PTV), one will make the assumption of a Lagrangian formulation Brought up to date with each Iteration (FLAI) [10] [10] the geometry of studied structure will be constantly updating. Locally that results in the position of the nodes of the element which changes with each iteration of the algorithm of Newton [14] [14] to calculate a new transition matrix of the local coordinate system to the total reference. That is to say

an element in initial configuration. One C_0can describe the strains of the element from 3 configurations, the initiale6elle configuration[6] like 2 other configurations: who indicates C_1 a known deformed configuration but not inevitably balances some and who indicates C_2 an unknown deformed configuration near to. C_1The last known configuration is taken as reference, one will write the PTV in this configuration.
To be located in the algorithm of resolution, one can for example imagine that C_0 the configuration at the beginning of time step indicates. Moreover $(t=0)$ one already carried out a first iteration, one thus determined who N C_1 "is not in equilibrium. One will thus carry out a new iteration, while choosing like C_1 reference and by writing the PTV in an unknown configuration and so on… jusqu" to find a configuration in equilibrium and then to pass to time step according to. Illustration



4.1: 4.1Various reference configurations. One can

pass a first remark: the supports of the beam elements in the Code_Aster *are* segments with two nodes (SEG2), `consequently` their interpolation functions geometrical are linear . One will be able never with these elements to thus reproduce the "last known configuration exactly", that represents an approximation and it will thus be necessary to take care to have a sufficiently fine mesh for the problems where the curvature of the elements becomes considerable. The solution which consists in correcting the residue or using a CO-rotational formulation was not established because she would have asked profound changes of the code7La simple solution[7]

6is determined by the data of the mesh by the user thus,
7of the term of correction in the residue requires the safeguard of the tangent matrix to the preceding iteration, which is not possible simply with current architecture. The FLAI

present of many advantages, for example one will be able to confuse tensor stresses of Cauchy (expressed on the final configuration with the preceding iteration) and tensor of the stresses of Piola-Kirchoff of second species (expressed on the initial configuration with the current iteration) because between the two iterations, the geometry will have been brought up to date: because éq

$$\boldsymbol{S} = det(\boldsymbol{F})\boldsymbol{F}^{-1}\boldsymbol{\sigma}\ {}^t\boldsymbol{F}^{-1} = \sigma \qquad \boldsymbol{F} = \boldsymbol{I}$$ **4-8 That 4-8**

means that in the reference configuration (that in particular which makes it possible to express the virtual strains) the initial displacements already acquired by the element will be always null because the last calculated configuration will be taken as reference. Only the initial stress state in a given configuration will be in general non-zero. With regard to

the behavior, he is written in an incremental way and he is expressed in Lagrangian quantities thanks to the tensor of Piola-Kirchhoff and II $\boldsymbol{S}$ the strains of Green-Lagrange. $\boldsymbol{E}$ The small strains make it possible to linearize the constitutive law and consequently: éq

$$\Delta \boldsymbol{S} = \boldsymbol{C} : \Delta \boldsymbol{E} \approx \boldsymbol{C} : \Delta \boldsymbol{\epsilon}$$ **4-9 For 4-9**

beam elements, one makes the assumption of plane stresses in the section. That amounts supposing that the beam consists of longitudinal fibers working in traction and compression. This assumption () makes it possible$\sigma_{yy} = \sigma_{zz} = \sigma_{yz} = 0$ to express the constitutive law in the case as of beams in the form: éq

$$\boldsymbol{C} : \Delta \boldsymbol{\epsilon} = \begin{cases} E\ \delta\epsilon_{xx} \\ 2\mathrm{G}\ \delta\epsilon_{xy} \\ 2\mathrm{G}\ \delta\epsilon_{xz} \end{cases}$$ **the 4-10 4-10**

assumption of a field of displacement with moderate rotations will be licit to obtain large rotations between step and for the total loading. Indeed, the FLAI makes that one will add at the beginning of a step and each iteration the increases in displacements and stresses calculated. The total increase in displacement and rotation on a step could consequently be large. Writing

4.2.2 of the PTV and differentiation the principle

of the virtual wors which is the weak formulation of the models of the mechanics of the continuums writes: éq

$$\int_{C_2} \boldsymbol{\sigma} : \boldsymbol{\epsilon}^* dV = \int_{C_2} \vec{f} . \vec{\xi}^* dV + \int_{\partial C_2} \vec{t} . \vec{\xi}^* dS$$ **4-11 4-11**

the quantities are expressed on the unknown deformed configuration, is $\boldsymbol{\sigma}$ the tensor of the stresses of Cauchy. Moreover $\vec{\xi}^*$ a kinematically admissible field of virtual displacement within the meaning of the boundary conditions indicates.

To solve such a problem, one must initially change configuration thanks to a change of variable. The principle of the virtual wors writes in Lagrangian quantities (in the reference configuration thus and not deformed configuration) is expressed using the virtual strains of Green-Lagrange and $\boldsymbol{E}^*$ the tensor of the Piola-Kirchhoff stresses of second species (PKII). éq $\boldsymbol{S}$

$$W_{int} = \int_{C_1} \boldsymbol{S} : \boldsymbol{E}^* dV = \int_{C_1} \vec{f} . \vec{\xi}^* dV + \int_{\partial C_1} \vec{t} . \vec{\xi}^* dS$$ **4-12 4-12**

$\vec{f}$ the density of volume force represents, density $\vec{t}$ of force surface. Thereafter, the member of W_{ext} right will be noted, and it will be supposed that the loading is conservative and not follower so that the work of the external forces is constant on one time step. The tensor

of stresses PKII (éq 4 4-8 That 4-8 the same quantity as the tensor of the stresses of Cauchy but by taking as reference the initial configuration. The tensor of the virtual strains of Green-Lagrange is written: éq

$$\begin{cases} \boldsymbol{E}^* = \frac{1}{2}\left({}^t\boldsymbol{F}^*\boldsymbol{F}+{}^t\boldsymbol{F}\boldsymbol{F}^*\right) \\ \text{avec} \quad \boldsymbol{F} = \boldsymbol{I}+\frac{\partial\vec{\xi}^*}{\partial\vec{x}} \\ \text{et} \quad \boldsymbol{F}^* = \frac{\partial\vec{\xi}^*}{\partial\vec{x}} \end{cases}$$ **4-13 4-13**

to solve the equilibrium one must check (éq 4-124-12 4-12 any kinematically admissible virtual displacement with zero. One $\vec{\xi}^*$applies the method of Newton to the functional calculus and for this reason $W_{int}-W_{ext}$ one differentiates the functional calculus compared to the unknown, that $\vec{\xi}$allows to transform the resolution of a nonlinear problem (does not depend W_{int} linearly on) into $\vec{\xi}$ a continuation of resolution of linear systems. One thus

solves: éq

$$W_{ext}=W_{int}^1+\int_{C_1}\left(\boldsymbol{\Delta S}:\boldsymbol{E}^*+\boldsymbol{S}:\boldsymbol{\Delta E}^*\right)dV$$ **4-14 4-14**

represents W_{int}^1 the work already balanced in configuration 1. The principle is to balance this work with that of the internal forces using W_{ext} successive iterations. The first

term under the integral models the material stiffness. Indeed, as the constitutive law is linearized and as the reference configuration is the last known configuration (thus 8Au risk $\vec{\xi}=\vec{0}$ [8] can write by means of (éq 4 4-9 For 4-9 (éq 4 4-13 4-13 éq

$$\boldsymbol{E}^*=\boldsymbol{\epsilon}^*=\frac{1}{2}\left({}^t\boldsymbol{F}^*+\boldsymbol{F}\right)$$ **4-15 4-15**

$$\int_{C_1}\boldsymbol{\Delta S}:\boldsymbol{E}^*\,dV=\int_{C_1}\boldsymbol{\Delta\epsilon}:\boldsymbol{C}:\boldsymbol{\epsilon}^*\,dV$$ **the 4-16 4-16**

allows in fact here to free itself from an additional term which couple nonlinear strains virtual and linear strains and that one calls term of initial displacements (these displacements are the displacements already undergone by our structure compared to its reference configuration and which are always null here). In fact thanks to the FLAI, one off-sets the change of geometry and the non-linearity which results from this in the change from room-total reference suitable for the finite element method. The second
term under the integral models the geometrical stiffness of structure. It is thanks to this term that one will be able to translate the effects of the second order and to converge quickly towards the solution. This term indeed directly utilizes the nonlinear strains since according to (éq 4-134-13 4-13If $\boldsymbol{\Delta E}^*=\boldsymbol{\Delta\eta}^*$the field of displacement contains moreover of the nonlinear terms for the taking into

8to repeat itself, they are false in practice, because the shape functions of the elements beam are linear. In [17], [17] does not make this simplification and one uses the additional term to correct the residue.), one account of rotations moderated then of course, the nonlinear strains contain also an additional term. It remains

to be noticed that tensor of the stresses of Cauchy and PKII are identified because initial configuration and finale are confused following the reactualization. Assessment

4.2.3 One thus

has: éq

$$\Delta W=\int_{C_1}\left(\boldsymbol{\Delta\epsilon}:\boldsymbol{C}:\boldsymbol{\epsilon}^{*}+\boldsymbol{\sigma}:\boldsymbol{\Delta\eta}^{*}\right)dV=W_{ext}-W_{int}^{1} \quad \textbf{4-17 4-17}$$

, all occurs as Ci the residue is dissipated $W_{ext}-W_{int}^{1}$ iteration after iteration by a work partly ΔW due to the strain of the material (material stiffness) and partly due to large displacements of the structure (geometrical stiffness). The geometrical stiffness has a meaning since the structure undergoes large displacements. The term

•of material stiffness is a classical term which is already present in the current formulation of elements POU_D_TGM . It can possibly translate through the approach multifibre non-linearity material (plasticity for example). The geometrical
•term of stiffness that one usually finds in large displacements is new (in the reactualized reference configuration, there exists a non-zero stress state in general). It thus translates geometrical non-linearity and it is thanks inter alia in this term omitted in the formulation PETIT_REAC which convergence will be faster even finally possible in the case of instabilities. The second
•term in the member of right is the work of the internal forces to the preceding iteration, it is its exact computation which guarantees a convergence towards result just, this is why, one will propose an improvement in his computation. That

in the geometrical term of stiffness the unknown intervenes through the virtual strains, these last should well be noted indeed depend linearly on the unknowns. Determination

4.3 of the tangent matrix Recall

4.3.1 on the discretization From now on

it acts to determine the tangent matrix which is the combination of the various terms referred to above. For that it is necessary to utilize the discretization in finite elements, in particular the interpolation functions then to clarify the integrals of volume which intervene in (éq 4-174-17 4-17
structural elements like the beams, this discretization is carried out in two times: on the one hand integration on the sections located at Gauss points and on the other hand integration in the length. As one chose a behavior multifibre for the processing of plasticity, integration in the section is made thanks to fibers for the axial behavior (normal force, bending moments) and using constitutive laws in forces generalized for the rest of the forces [4]. [4]
integrals on the section in each Gauss point (of the beam element) because they are the quantities in these points which will then make it possible to integrate numerically in the length (using formulas of squaring of Gauss). The generalized strains are then expressed using a matrix of interpolation functions. Numerical integration then makes it possible to determine a tangent matrix: one will thus seek here only to express the matrix after integration on the section because it is its statement which will be necessary for the implementation in the code, numerical integration being transparent. Statement

4.3.2 of the material stiffness We

can clarify the term of material stiffness, by initially clarifying the constitutive law (éq 4 the 4-10 4-10 by replacing the strains linearized by their $\boldsymbol{\epsilon}$ statement (éq 4 4-6 éq 4-6 finally while integrating on the section, thus preserving only one integral length. éq

$$\begin{aligned}\int_{C_1} \boldsymbol{\Delta\epsilon : C : \epsilon^*}\, dV &= \int_{C_1} \left(E\,\delta\,\epsilon_{xx}\,\epsilon^*_{xx} + 2\mathrm{G}\,\delta\,\epsilon_{xy}\,\epsilon^*_{xy} + 2\mathrm{G}\,\delta\,\epsilon_{xz}\,\epsilon^*_{xz}\right) dV \\ &= \int_{L_1} \left({}^t(\Delta\, D_s)\,\boldsymbol{K_s}\, D^*_s\right) dl\end{aligned}$$ **4-18 4-18**

a:, being $D_s = \boldsymbol{B}\, U$ $\boldsymbol{B}$ the interpolation matrix of the generalized strains and $\left(u_{O,x}, (v_{C,x} - \theta_z), (w_{C,x} + \theta_y), \theta_{x,x}, \theta_{y,x}, \theta_{z,x}, \theta_{x,xx}\right)$ the vector U of the nodal unknowns. is $\boldsymbol{K_s}$ a behavioral matrix which and the translates at the same time the geometrical characteristics of the material characteristic of the section (from where appearance of the constants and referring S'_y, S'_z, J I_ω respectively to the reduced sections, the constant of torsion and the warping constant): éq

$$\boldsymbol{K_s} = \begin{pmatrix} \int_S E\, ds & 0 & 0 & 0 & \int_S Ez\, ds & -\int_S Ey\, ds & 0 \\ & GS'_y & 0 & 0 & 0 & 0 & 0 \\ & & GS'_z & 0 & 0 & 0 & 0 \\ & & & GJ & 0 & 0 & 0 \\ & & & & \int_S Ez^2\, ds & -\int_S Eyz\, ds & 0 \\ & sym & & & & \int_S Ey^2\, ds & 0 \\ & & & & & & EI_\omega \end{pmatrix}$$ **4-19 4-19**

$$\boldsymbol{B} = \begin{pmatrix} -\frac{1}{L} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{L} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_1 & 0 & 0 & 0 & \phi_2 & 0 & 0 & \phi_3 & 0 & 0 & 0 & \phi_4 & 0 \\ 0 & 0 & \psi_1 & 0 & \psi_2 & 0 & 0 & 0 & 0 & \psi_3 & 0 & \psi_4 & 0 & 0 \\ 0 & 0 & 0 & N_{1,x} & 0 & 0 & N_{2,x} & 0 & 0 & 0 & N_{3,x} & 0 & 0 & N_{4,x} \\ 0 & 0 & \xi_{5,x} & 0 & \xi_{6,x} & 0 & 0 & 0 & 0 & \xi_{7,x} & 0 & \xi_{8,x} & 0 & 0 \\ 0 & -\xi_{5,x} & 0 & 0 & 0 & \xi_{6,x} & 0 & 0 & -\xi_{7,x} & 0 & 0 & 0 & \xi_{8,x} & 0 \\ 0 & 0 & 0 & N_{1,xx} & 0 & 0 & N_{2,xx} & 0 & 0 & 0 & N_{3,xx} & 0 & 0 & N_{4,xx} \end{pmatrix}$$

4-20 4-20

$$\begin{aligned} \phi_1 &= \xi_{1,x} + \xi_5 & \psi_1 &= \xi_{1,x} - \xi_5 \\ \phi_2 &= -\xi_{2,x} + \xi_6 & \psi_2 &= \xi_{2,x} - \xi_6 \\ \phi_3 &= \xi_{3,x} + \xi_7 & \psi_3 &= \xi_{3,x} - \xi_7 \\ \phi_4 &= -\xi_{4,x} + \xi_8 & \psi_4 &= \xi_{4,x} - \xi_8 \end{aligned}$$ **4-21 4-21**

of surface which still appear in the statement of refer $\boldsymbol{K_s}$ to the terms calculated thanks to the mesh of the fiber cross-sectional area. Except in the elastic case, the extra-diagonal terms are in general not null because the Young modulus is not E then homogeneous any more in the section. The statements of the interpolation functions $\xi_{i=(1,\ldots,8)}, N_{j=(1,\ldots,4)}$ are given in [2]. [2]

notice that the approach chosen to determine the material stiffness matrix is not that adopted in the Code_Aster *for* the beam elements of Eulerian POU_D_E and Timoshenko POU_D_T . Indeed , in our work, the approach is derived from the models of the mechanics of the continuums 3D and one 3D gives oneself for that a field of displacement, which enables us to utilize large displacements by writing the PTV in Lagrangian quantities.

To formulate the beam elements of Eulerian and Timoshenko in assumption of the small elastic disturbances, the approach used in Code_Aster *[1] consists in*[1] directly writing the basic principle of the dynamics on elementary sections of beam, then to write of it a weak formulation using functions tests, and finally introducing constitutive laws in generalized forces (of the type). One $N=ES\epsilon$ realizes whereas by choosing the functions tests well, it is possible to express the work developed in the field test only according to the unknowns with the nodes, that without having at any time discretized the unknowns. Thus
when one works in small elastic disturbances, it can be more interesting to use these exact elements rather than the element presented in this work which numerically integrates the stiffness matrix by squaring of Gauss. Statement

4.3.3 of the geometrical stiffness In the same way,

one can clarify the geometrical stiffness. For that, each component of can $\Delta\eta^*_{xi}$ $\boldsymbol{\Delta\eta}^*$ put in the form of a quadratic pseudo-form i.e where *is* ${}^tC\,L_{xi}\,C^*$ L_{xi} a matrix which does not depend on displacements and a vector C depend on the generalized unknowns and their derivatives: . With ${}^tC=\left(\theta_x, v_{C,x}, w_{C,x}, \theta_{x,x}\right)$obvious notations, one from of deduced: éq 4-22

$$\int_{C_1} \sigma : \Delta\eta^* dV = \int_{L_1}\left({}^t\Delta C\left(\text{[redacted]}\, L_{xy}\,\sigma_{xy}+2\,L_{xz}\,\sigma_{xz}\right)dS\right)C^*\Bigg)dl \quad R_s$$
$$= \int_{L_1}\left({}^t(\Delta C)\,\boldsymbol{R_s}\,C^*\right)dl \qquad \textbf{4-22}$$

over the length is also carried out by interpolating the generalized unknowns, the unknowns becoming the degrees of freedom with the nodes. Thus one replaces par. Below C $\boldsymbol{B_\sigma}$ U, one gives the form of the interpolation matrix and $\boldsymbol{B_\sigma}$ the matrix: éq 4 $\boldsymbol{R_s}$ - 23

$$\boldsymbol{R_s}=\begin{pmatrix} 0 & -T_z & T_y & \frac{1}{2}K_{,x} \\ & N_x & 0 & -M_y+z_C N_x \\ & & N_x & -M_z-y_C N_x \\ sym & & & K \end{pmatrix} \qquad \textbf{éq 4-23}$$

$$\boldsymbol{B_\sigma}=\begin{pmatrix} 0 & 0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & 0 & N_3 & 0 & 0 & N_4 \\ 0 & \xi_{1,x} & 0 & 0 & 0 & -\xi_{2,x} & 0 & 0 & \xi_{3,x} & 0 & 0 & 0 & -\xi_{4,x} & 0 \\ 0 & 0 & \xi_{1,x} & 0 & \xi_{2,x} & 0 & 0 & 0 & 0 & \xi_{3,x} & 0 & \xi_{4,x} & 0 & 0 \\ 0 & 0 & 0 & N_{1,x} & 0 & 0 & N_{2,x} & 0 & 0 & 0 & N_{3,x} & 0 & 0 & N_{4,x} \end{pmatrix}$$

In 4-24

the geometrical stiffness matrix intervene the forces generalized (normal force, shears N and, bending moments T_y $T_z$and) as well as M_y M_z geometrical characteristics of the section. The coefficient is called K coefficient of Wagner and is written: éq 4-25

$$K=N_x\left(\frac{I_y+I_z}{S}+y_C^2+z_C^2\right)+M_y\left(\frac{I_{yr2}}{I_y}-2\,z_C\right)-M_z\left(\frac{I_{yr2}}{I_z}-2\,y_C\right)+M_\omega\frac{I_{\omega r2}}{I_\omega} \qquad \textbf{4-25}$$

characteristics of section are: éq 4-26

$$I_{yr2} = \int_S y(y^2+z^2)\,dS$$
$$I_{zr2} = \int_S z(y^2+z^2)\,dS$$
$$I_{\omega r2} = \int_S \omega(y,z)(y^2+z^2)\,dS$$

These 4-26

is in general non-zero for the sections not having double symmetry, it is the case for example angles equipping the supports truss. The last
term in the coefficient of Wagner is not taken into account in the code because one does not have in general the statement of the function of warping to compute: ω . Form $I_{\omega r2}$

4.3.4 of the matrix of correction for the taking into account of large rotations of structure To take into account

of large rotations of structure, it is necessary to express the matrix of correction made to the geometrical stiffness by the addition of the nonlinear strain field. The detailed $\Delta\boldsymbol{\eta}_{gr}$approach of derivative is given in [19] or [19] [16]. One [16] computations here, because they are long and tiresome. Other authors obtained the same matrix of correction from consideration on the nature of the bending moments and the twisting moment [6] [5]. It[6][5] interesting
to notice that the only modification made by the taking into account of large rotations relates to an addition with the tangent matrix. The residue is unchanged, i.e. without the taking into account of large rotations, one is not likely to get faux9sauf results of course[9] to encounter difficulties of convergence when with certain problems are dealt. Indeed
, the problem of large rotations within the meaning of the size is not strictly speaking an obstacle: for plane problems, it is absolutely not necessary to use the matrix of correction (more exactly its contribution cancels itself). The true problem during lies in the transformation of rotations a change of reference. However when rotations are infinitesimal, one can replace them by their development with the first order and then they are identified with vectors, which makes it possible during to transform them like displacements the transition of the local coordinate system to the total reference. On the other hand when they are large, it is not possible any more, but one is likely to encounter problems of convergence only for precise cases where the continuity of rotations to the nodes is lost. Thus
the problem arises for structures presenting a flexure-torsion coupling and of the finite elements not colinéaires. The tangent stiffness matrix

is thus enriched by a term of correction following $\boldsymbol{K}_c$ the taking into account of rotations moderated in the statement of displacements: éq 4-27

$$\boldsymbol{K}_c=\begin{pmatrix} [0]_{3\times3} & & & & \\ & -[k_c^1]_{3\times3} & & (\boldsymbol{0}) & \\ & & [0]_{4\times4} & & \\ & (\boldsymbol{0}) & & [k_c^2]_{3\times3} & \\ & & & & 0 \end{pmatrix}$$

with 4-27

4-28 $$[k_c^i]_{3\times3}=\frac{1}{2}\begin{pmatrix} 0 & -M_z^i & M_y^i \\ -M_z^i & 0 & 0 \\ M_y^i & 0 & 0 \end{pmatrix}$$

Computation 4-28

4.4 the residue to supplement

9if one uses the tangent matrix directly to make a search for mode of buckling but rather

the model, one presents in the continuation two improvements suggested by works of theses [18] and [18] [19] the accuracy of the results. The computation
residue, and more particularly of the work of the internal forces is carried out W^1_{int} by numerical integration after having integrated the constitutive law: éq 4-29

$$W^1_{int} = \int_{C_1} \boldsymbol{\sigma} : \boldsymbol{\epsilon}\, dV = \left(\int_{L_1} {}^t Q\, \boldsymbol{B}\, U\, dl \right)$$ **is 4-29**

Q containing the 7 generalized forces and the interpolation matrix $\boldsymbol{B}$ of the generalized strains already given in (éq 4-20)4-20 4-20 makes it possible to express the increments of strains associated with nodal displacements. The improvements U
suggested touch respectively with the statement of the constitutive law in torsion thus to and the computation Q of the linearized strains. They $\boldsymbol{\epsilon}$must allow a better accuracy but also an acceleration of convergence. Computation of

4.4.1 the increment of strain On the one hand

, it appears interesting to take account of the update with each iteration of the geometry, indeed, one can then calculate more astutely the increment of strain since the preceding step. Instead of calculating the increment of strain from the displacement increment of the step, one can calculate the strain due to the displacement of the last iteration of Newton and cumulate it with each iteration. As displacement between two iterations tends towards zero, one is more precise. Indeed
if δu_i the consecutive displacement increment with the iteration of the algorithm of i resolution by step indicates and if $\epsilon(u)$ the increment of generalized strain indicates associated with a displacement increment then with u the iteration, one proceeds n as follows: éq 4-30

$$\Delta U = \sum_{i=0}^{n} \delta u_i$$ **instead of 4-30**

writing: éq 4-31 $\Delta \epsilon = \epsilon(\Delta U)$ **one 4-31**

rather: éq 4-32 $\Delta \epsilon = \sum_{i=0}^{n} \epsilon(\delta u_i)$ **That 4-32**

a little more memory because it is necessary to be able to store the vector of the increments of strain, however $\epsilon(\delta u_i)$ this technique of computation is more relevant when the geometry is updating with each iteration. Constitutive law

4.4.2 in torsion In addition

, one chooses a more complete constitutive law in the statement of the torsion of a beam. Indeed [19] showed [19] that the new statement which translates the influence of warping on the normal stress, makes it possible in the case of to get corner results in perfect adequacy with a modelization shell. One thus adds at the twisting moment, the moment says "Moment of Wagner". Its computation utilizes the coefficient of Wagner already quoted K in (éq 4-25)4-25

$$M_x = (GJ + K)\,\theta_{x,x}$$ **Implementation 4-33**

5 in Code_Aster Use

5.1 One reaches

the processing of large displacements for multifibre beams POU_D_TGM `by selecting` strain "GROT_GDEP " `under COMP_INCR . All`
the nonlinear behaviors 1D are available [21]. Development[21]

5.2 Stamps

5.2.1 correction the form

of the matrix of correction due to the taking into account of moderate rotations is given in (éq 4-27)with 4-27 at the end of the computation of the tangent stiffness, since it is appeared as a term very integrated. The only obstacle relates to the bending moments which constitute this term very integrated, indeed, it acts of the bending moments to the nodes of the element, whereas one knows them only with Gauss points (following the integration of the constitutive law). It is thus
is possible to use the computation of to extract W^1_{int} the moments from them or then to interpolate the values with the nodes using a polynomial of order 2. It is this last solution which was chosen, because at the beginning of each time step, it is not possible to extract these values starting from work, because the latter is not calculated (at the beginning of each time step, only a tangent stiffness matrix of velocity is assembled for the phase of prediction). Once
the interpolated values, one is thus satisfied to correct the geometrical stiffness matrix. That is done just after the end of the loop on Gauss points. Computation of

5.2.2 the strains improved As one

announced, this computation is relevant only when one updates the geometry of structure at each iteration. To be able
to carry out this new computation, it is necessary to be able to store the increment of strain in each Gauss point and after each iteration (it is a vector of dimension 7). This information is stored in the field specific to structural elements STRX_ELGA . `This field`
is also used to store the generalized forces because the data of the only stress field by fibers cannot be enough to go with the shears, the twisting moment and the bi--moment. Validation

6 the validation

of the new element is carried out in 3 times. Initially one compares the results got on benchmarks having analytical solutions (primarily plane), then on academic benchmarks (3D) and finally on experimental results. Analytical

6.1 benchmarks Beam in

6.1.1 buckling (SSNL502) This test

represent a computation of stability of a cantilever beam subjected to a compressive force at an end. It makes it possible to validate the modelization in the nonlinear quasi-static field in large displacements and in the presence of instability (Buckling of Eulerian). Plate cantilever

6.1.2 subjected to one moment (SSNV138) This test

constitutes the quasi-static computation of an elastic plate embedded on a side and subjected to one bending moment at the other side, leading to large displacements in the plane. Academic

6.2 benchmarks Large displacements

6.2.1 of the arc with aperture 45° (SSNL136) In this

test, one study a plane arc of opening 45° embedded at an end and subjected to a bending stress perpendicular to its plane at the other end. This test, very severe, makes it possible to validate the beam element multifibre in the geometrical nonlinear field of large displacements and large rotations. Discharge

6.2.2 of a blade-square (SSNL133) an L-shaped

structure made up of two slender beams of mean rectangular section is subjected to a force at an end and is embedded at the other end. The field of the test is that of the nonlinear elastic mechanics in large displacements and large rotations, with instability of the standard discharge of beam. Elastoplastic the purpose of failure

6.2.3 of the gantry of Lee (SSNL134) This test

is simultaneously validating the nonlinear possibilities material and geometrical of the beam element multifibre. The element is implemented on a benchmark usually treated in the literature with regard to the elastic behavior because it presents a response complexes with snap-back *and* snap-through *: it is* the gantry of Lee. One supposes here an elastoplastic behavior of the gantry, which makes it possible to test the good integration of the constitutive law of the multifibre elements but also the correct processing of large displacements. Experimental

6.3 validation cantilever

6.3.1 MEKELEC (SSNL135) MEKELEC

indicates a structure carried out in 1991 from a cantilever of pylon P4T. Dimensions and the profiles used were adapted to the realization of tests. One wished at the time establishing a base of results experimental precis in order to validate the results of simulations. The validation
consists in comparing the experimental and numerical loads of failure of 3 cases of loading. Bibliography

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