

Anisotropic elasticity

Abstract

This document treats anisotropic elasticity, used for the modelizations of continuums 3D and 2D (`C_PLAN`, `D_PLAN`, `AXIS`), or the layers of the composite shells.

The springy medium can be anisotropic according to the 3 directions (orthotropic elasticity is spoken), or in isotropic in two directions (one speaks about transverse isotropic elasticity).

Contents

1 Introduction.....	3
2 Topology of the matrixes of Hooke.....	3.2.1
transverse.....	orthotropy
3.2.2 Isotropy.....	4.2.3
Isotropy	4
3 Matrix of Hooke and flexibility.....	4.3.1
Notations.....	4.3.2
Cases 3D.....	
6.3.2.1 Orthotropie.....	6
3.2.1.1 Stamp flexibility.....	6
3.2.1.2 Stamps of Hooke.....	
6.3.2.2 Isotropy transverse.....	7
3.2.2.1 Stamps flexibility.....	7
3.2.2.2 Stamps of Hooke.....	
8.3.2.3 Elasticity cubic.....	
9.3.2.4 Isotropy	10
3.2.4.1 Stamps flexibility according to E and ν	10
3.2.4.2 Stamps of Hooke according to E and ν	10
3.2.4.3 Stamps flexibility according to the coefficients of Lamé λ and μ	11
3.2.4.4 Stamps of Hooke according to the coefficients of Lamé λ and μ	11.3.3
Cases 2D orthotropic in plane strains and axisymmetric.....	
11.3.3.1 Matrix of flexibility.....	
11.3.3.2 Stamps of Hooke.....	12.3.4
Cases 2D orthotropic in plane stresses.....	
12.3.4.1 Stamp flexibility.....	
12.3.4.2 Stamps of Hooke.....	12
4 Use in Code_Aster	13
5 Bibliography.....	14
6 Description of the versions of the document.....	14

1 Introduction

the purpose of this document is to give the form of the matrixes of flexibility and Hooke for elastic materials orthotropic, isotropic transverse and isotropic in 3D case, 2D-forced, 2D - plane strains and axisymetry.

We speak about "matrixes" of Hooke because, by preoccupation with a simplification, we did not adopt the notation of a tensor of order 4.

In any rigor, for the linear elastic materials, the stresses are linear functions of the strains.

One writes: $\sigma_{ij} = H_{ijkl} \varepsilon_{kl}$

The symmetric nature of $\{\sigma\}$ and $\{\varepsilon\}$ the adoption for these tensors of order 2 of a vectorial form make it possible to write:

$$\{\sigma\} = [H] \{\varepsilon\}$$

where $\{\sigma\}$ and $\{\varepsilon\}$ are the vectorial representation of the tensors of order 2 $\{\sigma\}$ and $\{\varepsilon\}$ where $[H]$ is a matrix 6×6 .

2 Topology of the matrixes of Hooke

2.1 the orthotropy

One can show the symmetry of the matrix of Hooke $[H]$.

We thus have twenty and one independent components in 3D case.

$$[H] = \begin{matrix} H_{11} & H_{12} & H_{13} & H_{14} & H_{15} & H_{16} \\ & H_{22} & H_{23} & H_{24} & H_{25} & H_{26} \\ & & H_{33} & H_{34} & H_{35} & H_{36} \\ SYM & & & H_{44} & H_{45} & H_{46} \\ & & & & H_{55} & H_{56} \\ & & & & & H_{66} \end{matrix}$$

An orthotropic material has two orthogonal planes of elastic symmetry.

This wants to say that if one calls $[H']$ the matrix $[H]$ after symmetry (S)
 $[H'] = [H]$.

The relations obtained between the coefficients make it possible to write that $[H]$ is defined by nine independent components.

In the axes of orthotropy:

$$[H] = \begin{matrix} H_{11} & H_{12} & H_{13} & 0 & 0 & 0 \\ & H_{22} & H_{23} & 0 & 0 & 0 \\ & & H_{33} & 0 & 0 & 0 \\ \text{SYM} & & & H_{44} & 0 & 0 \\ & & & & H_{55} & 0 \\ & & & & & H_{66} \end{matrix}$$

9 coefficients thus should be provided.

2.2 Transverse isotropy

the transverse isotropy is a restriction of the orthotropy in where one has the isotropy in one of the two orthogonal planes of elastic symmetry.

The matrix $[H]$ will have the same form as for the orthotropy but with additional relations between the components.

Five components are enough to determine $[H]$.

2.3 Isotropy

the material is isotropic if $[H]$ remains invariant in any change of reference.

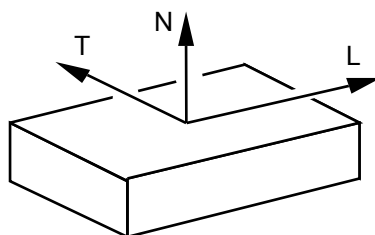
Two coefficients are enough to determine $[H]$.

3 Stamp of Hooke and from flexibility

3.1 Notations

Instead of using indices 1,2 and 3 to locate the axes, one will use the corresponding indices L , T and N :

1. L for longitudinal
2. T for transverse
3. N for normal



the coefficients which intervene are the following:

Key word	Notation	meaning
E_L	E_L	longitudinal Modulus Young
E_T	E_T	transverse Modulus Young
E_N	E_N	normal Modulus Young
G_LT	G_{LT}	Shear modulus in plane (L, T)
G_TN	G_{TN}	Shear modulus in plane (T, N)
G_LN	G_{LN}	Shear modulus in plane (L, N)
NU_LT	ν_{LT}	Poisson's ratio in plane (L, T)
NU_TN	ν_{TN}	Poisson's ratio in plane (T, N)
NU_LN	ν_{LN}	Poisson's ratio in the plane (L, N)

Notices very important:

ν_{LT} is different from ν_{TL} :

If one applies a tension according to L

$$\epsilon_{LL} = \frac{\overline{\sigma_{LL}}}{E_L} \quad (\text{Hooke's law according to a direction}).$$

This tension is accompanied, proportionally, of a contraction according to T, $-\nu_{LT} \cdot \frac{\sigma_{LL}}{E_L}$

and of a contraction according to N, $-\nu_{LN} \cdot \frac{\sigma_{LL}}{E_L}$.

The first index indicates the axis where the effect of the loading is exerted and the second index indicates the direction of the loading.

Then one exerts a tension according to T, then a tension according to N; one obtains:

$$\left. \begin{aligned} \epsilon_{LL} &= \frac{\sigma_{LL}}{E_L} - \nu_{TL} \frac{\sigma_{TT}}{E_T} - \nu_{NL} \frac{\sigma_{NN}}{E_N} \\ \epsilon_{TT} &= -\nu_{LT} \frac{\sigma_{LL}}{E_L} + \frac{\sigma_{TT}}{E_T} - \nu_{NT} \frac{\sigma_{NN}}{E_N} \\ \epsilon_{NN} &= -\nu_{LN} \frac{\sigma_{LL}}{E_L} - \nu_{TN} \frac{\sigma_{TT}}{E_T} + \frac{\sigma_{NN}}{E_N} \end{aligned} \right\} (S)$$

The matrix of flexibility $[H]^{-1}$ being symmetric; one from of deduced:

$$\frac{\nu_{LT}}{E_L} = \frac{\nu_{TL}}{E_T}$$

$$\frac{\nu_{LN}}{E_L} = \frac{\nu_{NL}}{E_N}$$

$$\frac{\nu_{TN}}{E_T} = \frac{\nu_{NT}}{E_N}$$

3.2 Case 3D

3.2.1 Orthotropie

3.2.1.1 Matrix of flexibility

$$\begin{bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\varepsilon_{LT} \\ 2\varepsilon_{LN} \\ 2\varepsilon_{TN} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_L} & -\frac{\nu_{TL}}{E_T} & -\frac{\nu_{NL}}{E_N} & 0 & 0 & 0 \\ -\frac{\nu_{LT}}{E_L} & \frac{1}{E_T} & -\frac{\nu_{NT}}{E_N} & 0 & 0 & 0 \\ -\frac{\nu_{LN}}{E_L} & -\frac{\nu_{TN}}{E_T} & \frac{1}{E_N} & 0 & 0 & 0 \\ & & & \frac{1}{G_{LT}} & 0 & 0 \\ & & & & \frac{1}{G_{LN}} & 0 \\ & & & & & \frac{1}{G_{TN}} \end{bmatrix} \begin{bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LT} \\ \sigma_{LN} \\ \sigma_{TN} \end{bmatrix}$$

SYM

$[H]^{-1}$ – Orthotropy

3.2.1.2 Stamps of Hooke

$$\begin{bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LT} \\ \sigma_{LN} \\ \sigma_{TN} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \frac{(1-\nu_{TN}\nu_{NT})}{E_T E_N} & \frac{(\nu_{TL}+\nu_{NL}\nu_{TN})}{E_T \cdot E_N} & \frac{(\nu_{NL}+\nu_{TL}\nu_{NT})}{E_T \cdot E_N} & 0 & 0 & 0 \\ \frac{(\nu_{LT}+\nu_{LN}\nu_{NT})}{E_L E_N} & \frac{(1-\nu_{NL}\nu_{LN})}{E_L \cdot E_N} & \frac{(\nu_{NT}+\nu_{NL}\nu_{LT})}{E_L \cdot E_N} & 0 & 0 & 0 \\ \frac{(\nu_{LN}+\nu_{LT}\nu_{TN})}{E_L \cdot E_T} & \frac{(\nu_{TN}+\nu_{TL}\nu_{LN})}{E_L \cdot E_T} & \frac{(1-\nu_{LT}\nu_{TL})}{E_L \cdot E_T} & 0 & 0 & 0 \\ & & & GLT * \Delta & 0 & 0 \\ & & & & GLN * \Delta & 0 \\ & & & & & GTN * \Delta \end{bmatrix} \begin{bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\varepsilon_{LT} \\ 2\varepsilon_{LN} \\ 2\varepsilon_{TN} \end{bmatrix}$$

SYM

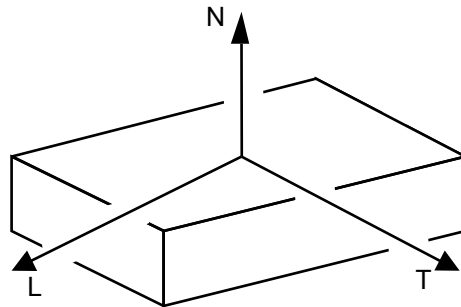
$[H]$ – Orthotropy with:

$$\frac{\nu_{TL}}{E_T} = \frac{\nu_{LT}}{E_L}; \frac{\nu_{NL}}{E_N} = \frac{\nu_{LN}}{E_L}; \frac{\nu_{NT}}{E_N} = \frac{\nu_{TN}}{E_T}$$

$$\frac{1}{\Delta} = \frac{E_L E_T E_N}{\begin{vmatrix} 1 - \nu_{TN} \nu_{NT} \\ -\nu_{NL} \nu_{LN} \\ -\nu_{LT} \nu_{TL} \\ -2\nu_{TN} \nu_{NL} \nu_{LT} \end{vmatrix}}$$

3.2.2 Transverse isotropy

3.2.2.1 Stamps flexibility

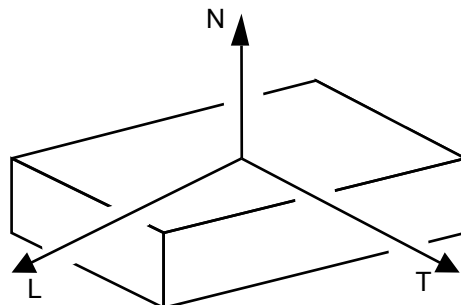


the matrix $[H]^{-1}$ can be deduced directly from the matrix $[H]^{-1}$ - Orthotropy by means of the properties of the transverse isotropy. In the plane (L, T) :

$$\begin{aligned} E_L &= E_T \\ \nu_{TL} &= \nu_{LT} \\ G_{LT} &= \frac{E_L}{2(1 + \nu_{LT})} \end{aligned}$$

In the planes (L, N) and (T, N) :

$$\begin{aligned} \nu_{NT} &= \nu_{NL} \\ \nu_{LN} &= \nu_{TN} \\ G_{TN} &= G_{LN} \end{aligned}$$



$$\begin{aligned}
 E_L &= E_T \\
 \nu_{LT} &= \nu_{TL} \\
 G_{LT} &= \frac{E_L}{2(1+\nu_{LT})} \\
 \nu_{NT} &= \nu_{NL} \\
 \nu_{LN} &= \nu_{TN} \\
 G_{TN} &= G_{LN} \\
 \frac{\nu_{NT}}{E_N} &= \frac{\nu_{LN}}{E_L}
 \end{aligned}$$

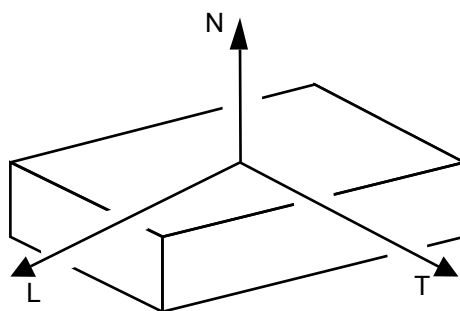
$$\begin{bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\varepsilon_{LT} \\ 2\varepsilon_{LN} \\ 2\varepsilon_{TN} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_L} & \frac{-\nu_{LT}}{E_L} & \frac{-\nu_{NL}}{E_N} & 0 & 0 & 0 \\ \frac{-\nu_{TL}}{E_L} & \frac{1}{E_L} & \frac{-\nu_{NT}}{E_N} & 0 & 0 & 0 \\ \frac{-\nu_{LN}}{E_L} & \frac{-\nu_{TN}}{E_L} & \frac{1}{E_N} & 0 & 0 & 0 \\ & & & \frac{2(1+\nu_{LT})}{E_L} & 0 & 0 \\ & & & & \frac{1}{G_{LN}} & 0 \\ & & & & & \frac{1}{G_{TN}} \end{bmatrix} \begin{bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LT} \\ \sigma_{LN} \\ \sigma_{TN} \end{bmatrix}$$

SYM

$[H]^{-1}$ - Transverse Isotropy

3.2.2.2 Stamps of Hooke

the matrix $[H]$ has same symmetries as $[H]^{-1}$.



$$\begin{bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LT} \\ \sigma_{LN} \\ \sigma_{TN} \end{bmatrix} = \frac{1}{\Delta'} \begin{bmatrix} \frac{1 - \nu_{NL} \cdot \nu_{LN}}{E_L \cdot E_N} & \frac{\nu_{LT} + \nu_{NL} \nu_{LN}}{E_L \cdot E_N} & \frac{\nu_{NL} + \nu_{LT} \nu_{NL}}{E_L \cdot E_N} & 0 & 0 & 0 \\ \frac{\nu_{TL} + \nu_{NL} \nu_{LN}}{E_L \cdot E_N} & \frac{1 - \nu_{NL} \cdot \nu_{LN}}{E_L \cdot E_N} & \frac{\nu_{LN} + \nu_{LT} \nu_{LN}}{E_L \cdot E_N} & 0 & 0 & 0 \\ \frac{\nu_{LN} + \nu_{LT} \cdot \nu_{LN}}{E_L^2} & \frac{\nu_{TN} + \nu_{LT} \cdot \nu_{TN}}{E_L^2} & \frac{1 - \nu_{LT}^2}{E_L^2} & 0 & 0 & 0 \\ & & & \frac{E_L \cdot \Delta'}{2(1 + \nu_{LT})} & & \\ & & & & G_{LN} \cdot \Delta' & \\ & & & & & G_{LN} \cdot \Delta' \end{bmatrix} \begin{bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\varepsilon_{LT} \\ 2\varepsilon_{LN} \\ 2\varepsilon_{TN} \end{bmatrix}$$

[H] – Transverse Isotropy

$$\frac{1}{\Delta'} = \frac{E_L^2 \cdot E_N}{\begin{bmatrix} 1 - 2\nu_{NL} \cdot \nu_{LN} - \nu_{LT}^2 \\ -2\nu_{NL} \nu_{LN} \nu_{LT} \end{bmatrix}}$$

3.2.3 cubic Elasticity

cubic elasticity corresponds to a matrix of elasticity of the form :

$$\begin{bmatrix} y_{1111} & y_{1122} & y_{1122} & & & \\ y_{1122} & y_{1111} & y_{1122} & & & \\ y_{1122} & y_{1122} & y_{1111} & & & \\ & & & y_{1212} & & \\ & & & & y_{1212} & \\ & & & & & y_{1212} \end{bmatrix}$$

Being given cubic symmetry, it remains to determine 3 coefficients:

$$E_L = E_N = E_T = E, G_{LT} = G_{LN} = G_{TN} = G, \nu_{LN} = \nu_{LT} = \nu_{TN} = \nu$$

To reproduce cubic elasticity with ELAS_ORTH, it is enough to calculate the coefficients of the orthotropy such that the matrix of elasticity obtained is form above:

$$\begin{aligned} y_{1111} &= \frac{E(1 - \nu^2)}{(1 - 3\nu^2 - 2\nu^3)} \\ y_{1122} &= \frac{E\nu(1 + \nu)}{(1 - 3\nu^2 - 2\nu^3)} \\ y_{1212} &= G_{LT} = G_{LN} = G_{TN} \end{aligned}$$

therefore, as long as $(1 - 3\nu^2 - 2\nu^3) \neq 0$ (i.e. ν different from 0.5).

$$\frac{y_{1122}}{y_{1111}} = \frac{\nu}{1-\nu} \text{ what provides } \nu = \frac{1}{1 + \frac{y_{1111}}{y_{1122}}} \text{ then } E = y_{1111} \frac{(1-3\nu^2-2\nu^3)}{(1-\nu^2)}$$

3.2.4 Isotropy

3.2.4.1 Stamps flexibility according to E and ν

$$\begin{bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\varepsilon_{LT} \\ 2\varepsilon_{LN} \\ 2\varepsilon_{TN} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ & \frac{1}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ & & \frac{1}{E} & 0 & 0 & 0 \\ & & & \frac{1}{G} = \frac{2(1+\nu)}{E} & 0 & 0 \\ & & & & \frac{1}{G} = \frac{2(1+\nu)}{E} & 0 \\ & & & & & \frac{1}{G} = \frac{2(1+\nu)}{E} \end{bmatrix} \begin{bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LT} \\ \sigma_{LN} \\ \sigma_{TN} \end{bmatrix}$$

$[H]^{-1}$ – complete Isotropy

3.2.4.2 Stamps of Hooke according to E and ν

$$\begin{bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LT} \\ \sigma_{LN} \\ \sigma_{TN} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ & 1-\nu & \nu & 0 & 0 & 0 \\ & & 1-\nu & 0 & 0 & 0 \\ & & & \frac{1-2\nu}{2} & 0 & 0 \\ & & & & \frac{1-2\nu}{2} & 0 \\ & & & & & \frac{1-2\nu}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\varepsilon_{LT} \\ 2\varepsilon_{LN} \\ 2\varepsilon_{TN} \end{bmatrix}$$

$[H]$ – complete Isotropy

3.2.4.3 Stamps flexibility according to the coefficients of Lamé λ and μ

the Hooke's law takes the following shape with the coefficients of Lamé λ and μ .

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}$$

By means of the system of equations (S), one obtains:

$$\begin{bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LT} \end{bmatrix} = \frac{1}{1 - \nu_{LT} \cdot \nu_{TL}} \begin{bmatrix} E_L & \nu_{TL} \cdot E_T & 0 & 0 \\ \nu_{LT} \cdot E_L & E_T & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{LT} \end{bmatrix} \begin{bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\varepsilon_{LT} \end{bmatrix}$$

[H] – Plane Orthotropy in plane stresses

3.2.4.4 Stamps of Hooke according to the coefficients of Lamé λ and μ

$$\begin{bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LN} \\ \sigma_{LT} \\ \sigma_{TN} \end{bmatrix} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ & & \lambda + 2\mu & 0 & 0 & 0 \\ & & & SYM & \mu & 0 \\ & & & & \mu & 0 \\ & & & & & \mu \end{bmatrix} \begin{bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\varepsilon_{LN} \\ 2\varepsilon_{LT} \\ 2\varepsilon_{TN} \end{bmatrix}$$

[H] – complete Isotropy with the coefficients of Lamé

3.3 Case 2D orthotropic in plane strains and axisymmetric

3.3.1 Matrix of flexibility

$$\begin{bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ 0 \\ 2\varepsilon_{LT} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_L} & -\frac{\nu_{TL}}{E_T} & -\frac{\nu_{NL}}{E_N} & 0 \\ -\frac{\nu_{LT}}{E_L} & \frac{1}{E_T} & -\frac{\nu_{NL}}{E_N} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{LT}} \end{bmatrix} \begin{bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LT} \end{bmatrix}$$

[H]⁻¹ – plane Orthotropy in plane strains and axisymetry

3.3.2 Stamps of Hooke

$$\begin{bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LT} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \frac{(1-\nu_{TN}\nu_{NT})}{E_T E_N} & \frac{(\nu_{TL}+\nu_{NL}\nu_{TN})}{E_T \cdot E_N} & \frac{(\nu_{NL}+\nu_{TL}\nu_{NT})}{E_T \cdot E_N} & 0 \\ \frac{(\nu_{LT}+\nu_{LN}\nu_{NT})}{E_L E_N} & \frac{(1-\nu_{NL}\nu_{LN})}{E_L \cdot E_N} & \frac{(\nu_{NT}+\nu_{NL}\nu_{LT})}{E_L \cdot E_N} & 0 \\ \frac{(\nu_{LN}+\nu_{LT}\nu_{TN})}{E_L \cdot E_T} & \frac{(\nu_{TN}+\nu_{TL}\nu_{LN})}{E_L \cdot E_T} & \frac{(1-\nu_{LT}\nu_{TL})}{E_L \cdot E_T} & 0 \\ 0 & 0 & 0 & GLT * \Delta \end{bmatrix} \begin{bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ 0 \\ \varepsilon_{LT} \end{bmatrix}$$

[H] – plane Orthotropy in plane strains and axisymetry

$$\frac{1}{\Delta} = \frac{E_L E_T E_N}{\begin{pmatrix} 1-\nu_{TN}\nu_{NT} \\ -\nu_{NL}\nu_{LN} \\ -\nu_{LT}\nu_{TL} \\ -2\nu_{TN}\nu_{NL}\nu_{LT} \end{pmatrix}}$$

3.4 Case 2D orthotropic in plane stresses

3.4.1 Stamps flexibility

$$\begin{bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\varepsilon_{LT} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_L} & -\frac{\nu_{TL}}{E_T} & 0 & 0 \\ -\frac{\nu_{LT}}{E_L} & \frac{1}{E_T} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{LT}} \end{bmatrix} \begin{bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LT} \end{bmatrix}$$

[H]⁻¹ – plane Orthotropy in plane stresses

3.4.2 Stamps of Hooke

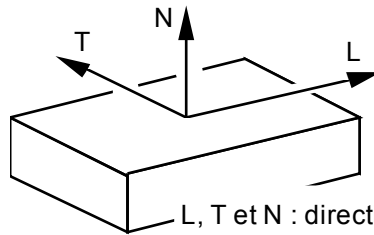
$$\begin{bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ 0 \\ \sigma_{LT} \end{bmatrix} = \frac{1}{1-\nu_{LT}\nu_{TL}} \begin{bmatrix} E_L & \nu_{TL} E_T & 0 & 0 \\ \nu_{LT} E_L & E_T & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{LT} \end{bmatrix} \begin{bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\varepsilon_{LT} \end{bmatrix}$$

[H] – Orthotropy in plane stresses

4 Use in Code_Aster

In *Aster*, the definition of the constant orthotropic elastic characteristics or functions of the temperature are carried out by the command `DEFI_MATERIAU`, key keys `ELAS_ORTH`, `ELAS_ISTR`, `ELAS_ISTR_FO` or `ELAS_ORTH_FO` for the isoparametric shell elements and the solid elements or layers constitutive of a composite (see command `DEFI_COMPOSITE`).

To define the orthotropic reference (L, T, N) related one to the elements, one can refer to documentations [U4.42.03] `DEFI_COMPOSITE` and [U4.42.01] `AFFE_CARA_ELEM`.



L, T et N : directions d'orthotropie
longitudinale, transversale et normale

```
/ELAS_ORTH = _F (
  ◆ E_L = ygl  longitudinal Modulus Young.
  ◆ E_T = ygt  transverse Modulus Young.
  ◇ E_N = ygn  normal Modulus Young.
  ◆ GL_T = glt  Shear modulus in the plane LT .
  ◇ G_TN = gtn  Shear modulus in the plane TN .
  ◇ G_LN = gln  Shear modulus in the plane LN .
  ◆ NU_LT = nult  Poisson's ratio in the plane LT .
  ◇ NU_TN = nutn  Poisson's ratio in the plane TN .
  ◇ NU_LN = nuln  Poisson's ratio in the plane LN .
```

Notice important:

The talk of this note of reference is based on the convention of the books of J.L.Batoz and D.Gay. The documentation U of DEFI_MATERIAU describes these choices, and coefficient NU_LT is interpreted in the following way in Aster: if one

exerts a tension according to the axis which causes L a strain according to this axis

equalizes with, one has $\varepsilon_L = \frac{\sigma_L}{ygl}$ a strain according to the axis equalizes with T : .

*Bibliography $\varepsilon_t = -nult * \frac{\sigma_l}{ygl}$*

5 J.C. MASSON

- 1) : Stamp of Hooke for the orthotropic materials, Ratio interns Applications in Mechanics, n°79-018, CiSi, 1979. D. GAY
- 2) : Composites, Hermes Edition, 1987 J.L. BATOZ
- 3) , G. DHATT: Modelization of structures by finite elements, Volume 1, Edition Hermes Description

6 of the versions of the document Version

Aster (S)	Author) Organization (S) Description	of the modifications 6.4 A. ASSIRE
		, EDF-R&D/ AMA initial Text	8.4 A. ASSIRE
		, X. DESROCHES, J.M. PROIX EDF-R&D/ AMA Corrections	tiny