Finite elements in acoustics

Summarized :

This document describes in low frequency steady acoustics the equations used, the variational formulations which result from this as well as the corresponding translation in finite elements, for each of the two methods used in *Code_Aster* : classical " formulation” with an unknown p (acoustic pressure), and “mixed” formulation with two unknowns p, v (pressure and velocity acoustics).

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1 Introduction

Of the options of modelization were developed in *Code_Aster*, making it possible to study the low frequency steady acoustic propagation, in closed medium, for fields of propagation to complex topology, i.e. there to solve under the quoted conditions the equation of Helmholtz.

The solution by finite elements of this equation can be realized according to two methods:

- a first method consists in being fixed like unknowns of the problem, only the nodal complex acoustic pressures, that is to say 1 degree of freedom per node [bib1]; it is that which one describes as formulation to the "classical" finite elements,
- in the second method, called to the "mixed" finite elements, one sets like unknowns at the same time the nodal acoustic pressures and the 3 components nodal vibratory velocity, that is to say on the whole 4 degrees of freedom per node [bib5].

To know the paths of propagation of energy in the fluid, the acoustics expert has 2 quantities: active acoustic intensity I and reactive acoustic intensity J ; these two quantities are defined like:

$$I=\frac{1}{2}\mathrm{Re}\left[p\,v^*\right] \text{ and } J=\frac{1}{2}\mathrm{Im}\left[p\,v^*\right]$$ **éq 1-1**

where v^* the combined complex one the vibratory velocity indicates. The knowledge of these quantities brings a very important further information in the resolution of problems of all kinds, such as for example the measurement of the powers radiated by the machines, the recognition and the localization of the sources.

The computation of the acoustic intensity by the finite element method mixed must provide values more precise than the classical method; indeed in the mixed case one ensures the continuity of derivatives first of the pressure and not simply the continuity of the latter.

However if it is more precise, the mixed formulation spends on the other hand more size memory and TEMPS CPU, while keeping the advantage of having, with number of degrees of freedom by wave length equal, a relative error increasingly weaker on the computation of the acoustic intensity.

2 Equations and boundary conditions of the problem

2.1 Equations and boundary conditions

the equation to solve is the equation of Helmholtz [bib2]:

$$(\Delta+k^2)\,p=s \qquad \text{éq 2.1-1}$$

- k indicates the wave number of with the dealt problem; it can be complex or real, according to whether the propagation is carried out or not in a porous field [bib6]:

$$k=\frac{\omega}{c} \qquad \text{éq 2.1-2}$$

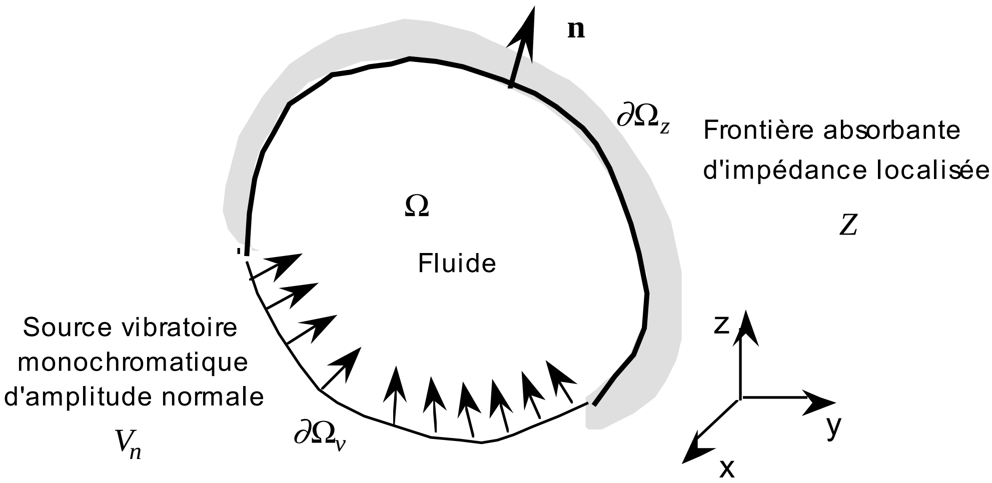
- c indicates the speed of sound, which can be complex in the case of a propagation in porous environment.
- p is a complex quantity indicating the acoustic pressure and s , also complex, represents the sources terms of the problem.
- ω is a reality in all the cases, which indicates the pulsation:

$$\omega=2\pi f \qquad \text{éq 2.1-3}$$

- f is the work frequency of the harmonic problem.

We represent on the figure [Figure 2.1-a] the unspecified confined field where the equation of Helmholtz [éq 2.1-1] applies and the conditions to the borders.

- Ω is open limited of R^3 regular $\partial\Omega$ border, partitionnée in $\partial\Omega_v$ and $\partial\Omega_z$; $\partial\Omega=\partial\Omega_v\cup\partial\Omega_z$



Appear 2.1-a: Configuration of the problem

the equation [éq 2.1-1] is to be solved in a closed field Ω. The boundary conditions to take into account on the border $\partial\Omega$ of the field Ω are expressed in their most general form like:

$$\alpha p+\beta\frac{\partial p}{\partial n}=\gamma \quad \textbf{éq 2.1-4}$$

$\partial/\partial n$ appoints the normal derivative operator.

α,β,γ are complex operators, who can be scalars, or integral operators according to whether the border of application of the boundary condition is with local reaction or nonlocal reaction (case of the interaction fluid-structure).

The developments currently carried out in *Code_Aster* relate to only boundary conditions with local reaction, for which α,β,γ are scalars; the cases spécifiables are the following:

- $\alpha=0,\beta\neq0,\gamma\neq0$ who indicates a border of the field at imposed vibratory velocity. Indeed, there exists a relation connecting the acoustic gradient of pressure complexes at the complex particulate vibratory speed.

$$\frac{\partial p}{\partial n}=-j\,\omega\,\rho_0\,V_n \quad \textbf{éq 2.1-5}$$

  ρ_0 indicates the density of the fluid considered, and one imposes V_n, the normal vibratory velocity with the wall ($V_n=v\cdot n$ where n the unit vector of the norm external with the border indicates $\partial\Omega$).

- $\alpha\neq0,\ \beta\neq0,\ \gamma=0$ relate to a border with imposed acoustic Z impedance. The acoustic impedance Z is defined like the ratio of the pressure at the particulate vibratory speed in the vicinity of the wall with imposed impedance:

$$Z=\frac{\mathrm{p}}{V_n} \quad \textbf{éq 2.1-6}$$

- $\alpha \neq 0, \beta = 0, \gamma \neq 0$ represents the case where one imposes the acoustic pressure p on a border (generally $\gamma = 0$, corresponding to $\mathrm{p} = 0$).

3 Classical formulation in mathematical

3.1 pressure Statement of the problem

the standard procedure aiming at posing the problem with the conventional finite elements is the following one:

- one supposes the sufficiently regular solution of the problem $\mathrm{p} \in H^2(\Omega)$. The equation is multiplied:

$$\left(\Delta + k^2\right) \mathrm{p} = 0 \qquad \text{éq 3.1-1}$$

by a function test $\tilde{\phi}$ One integrates on Ω and one uses the formula of Green. The border $\partial\Omega$ of the field Ω , is subdivided in 2 zones, a zone at imposed vibratory velocity, $\partial\Omega_v$ and a zone with imposed acoustic impedance $\partial\Omega_z$. The equation obtained can be rewritten in the form:

$$\int_\Omega \left(\mathbf{grad}\left(\mathrm{p}\right) . \mathbf{grad}\left(\phi\right) - \frac{\omega^2}{c^2} \mathrm{p}.\phi \right) dV + j \int_{\partial\Omega_z} \frac{\rho_0\, \omega}{Z} \mathrm{p}.\phi \; dS + j \int_{\partial\Omega_v} \rho_0\, \omega V_n .\phi \; dS = 0$$

éq 3.1-2

- dV represents a differential volume element in Ω and dS represents a surface element on $\partial\Omega$.
- the particulate vibratory velocity is then determined by:

$$\mathbf{v} = \frac{j}{\rho_0\, \omega} \mathbf{grad}\left(\mathrm{p}\right) \qquad \text{éq 3.1-3}$$

3.2 Discretization by finite elements

In the case of the conventional finite elements, the elementary integrals are four $\mathbf{K}^e, \mathbf{M}^e, \mathbf{C}^e, \mathbf{U}^e$ following the decomposition indicated in [éq 3.2-3] ($\mathbf{K}^e$ is the stiffness matrix, $\mathbf{M}^e$ the mass matrix, $\mathbf{C}^e$ the damping matrix and $\mathbf{U}^e$ the vector source). Two of them come from voluminal integrals, the two others are result of integrals respectively on a vibrating surface and a surface with imposed impedance.

It will be supposed that the total coordinates of an element can be written thanks to the data of m elementary shape functions $\mathrm{H_i}$:

$$\mathbf{OM} = \sum_{i=1}^{m} \mathrm{H_i}\; \mathbf{OM}_i \qquad \text{éq 3.2-1}$$

One is given moreover, of the elementary functions $\mathrm{N_i}$, to describe the elementary pressure. The pressure inside an element will be able to be written:

$$\mathrm{p}^e(x,y,z)=\sum_{i=1}^{m}\mathrm{N_i}\;(\varepsilon,\eta,\xi)\;P_i^e \qquad \textbf{éq 3.2-2}$$

where P_i^e is the pressure with the node i of the element e.

In the case of the isoparametric finite elements, the elementary functions $\mathrm{N_i}$ are equal to the shape functions $\mathrm{H_i}$.

On each element of the field, the problem with the finite elements in pressure is written:

$$\left(\mathbf{K}^e-\omega^2\mathbf{M}^e+j\omega\mathbf{C}^e\right)_q^q\left(\mathbf{P}^e\right)_1^q=-j\omega\left(\mathbf{U}^e\right)_1^q \qquad \textbf{éq 3.2-3}$$

where $\left(\mathbf{P}^e\right)_1^q$ is the matrix column of the nodal values of the pressure on the element.

3.2.1 The stiffness matrix

the stiffness matrix $\mathbf{K}^e$ corresponds to the computation of: $\int_{\Omega^e}\mathbf{grad}(\mathrm{p})\,.\,\mathbf{grad}(\phi)\;dV$

She admits like general term:

$$K_{ij}^e=\int_{\Omega^e}\nabla\mathrm{N_i}\,\nabla\mathrm{N_j}\;dV \qquad \textbf{éq 3.2.1-1}$$

3.2.2 the mass matrix

the mass matrix $\mathbf{M}^e$ corresponds to the computation of: $\int_{\Omega^e}\frac{1}{c^2}\;\mathrm{p}.\phi\;dV$

She admits like general term:

$$M_{ij}^e=\int_{\Omega^e}\frac{1}{c^2}\;\mathrm{N_i}\,\mathrm{N_j}\;dV \qquad \textbf{éq 3.2.2-1}$$

3.2.3 the damping matrix

the damping matrix $\mathbf{C}^e$ corresponds to the computation of: $\int_{\partial\Omega_z^e}\frac{\rho_0}{Z}\;\mathrm{p}.\phi\;dS$

She admits like general term:

$$C_{ij}^e=\int_{\partial\Omega_z^e}\frac{\rho_0}{Z}\;\mathrm{N_i}\;\mathrm{N_j}\;dS \qquad \textbf{éq 3.2.3-1}$$

3.2.4 the vector source

the vector source $\mathbf{U}^e$ corresponds to the computation of: $\int_{\partial\Omega_v^e} \rho_0 V_n \phi \, dS$

He admits like general term:

$$U_i^e = \int_{\partial\Omega_v^e} \rho_0 V_n \mathrm{N_i} \, dS \qquad \textbf{éq 3.2.4-1}$$

4 mixed Formulation pressure-velocity

4.1 mathematical Statement of the problem

4.1.1 local Formulation

the equation of Helmholtz [éq 1-1] with the boundary conditions [éq 2.1-3] result by way of local equations below:

$$i\omega\chi \;\mathrm{p} + \mathrm{div}\;\mathbf{v} = 0 \quad \text{dans } \Omega \qquad \textbf{éq 4.1.1-1}$$
$$i\omega\rho_0 \;\mathbf{v} + \mathbf{grad}\;\mathrm{p} = 0 \quad \text{dans } \Omega \qquad \textbf{éq 4.1.1-2}$$
$$\mathbf{v} \cdot \mathbf{n} = \frac{1}{Z}\;\mathrm{p} \quad \text{sur } \partial\Omega_z \qquad \textbf{éq 4.1.1-3}$$
$$\mathbf{v} \cdot \mathbf{n} = V_n \quad \text{sur } \partial\Omega_v \qquad \textbf{éq 4.1.1-4}$$

where $\chi = 1/\rho_0 c^2$ is the adiabatic coefficient of compressibility of the fluid.

The mathematical problem is the following: being given functions $Z \in L^\infty(\partial\Omega_Z)$ and $V_n \in \mathbf{H}^{\frac{1}{2}}(\partial\Omega_V)$, to find functions p and $\mathbf{v}$ defined in Ω and with values in C checking these equations. They describe, in harmonic mode of pulsation ω, the small fluctuations of pressure p and velocity $\mathbf{v}$ from at-rest state (c.à.d. acoustic pressure and acoustic particulate velocity) of a fluid compressible homogeneous, isotropic, nonviscous, confined in Ω and subjected to a distribution of normal velocity V_n on $\partial\Omega_V$.

ρ_0, χ and c the density, the adiabatic coefficient of compressibility and the speed of sound represent respectively relating to the fluid, in acoustic absence of disturbance; the coefficient $\alpha = 1/Z$ is the localised admittance of the material constituting $\partial\Omega_V$ with the pulsation considered.

To build a method of approximation by finite elements of this problem, it is necessary to put it in a variational form.

4.1.2 Mixed variational formulation

One takes the scalar product of the equation [éq 4.1.1-1] in $L^2(\Omega)$ with an unspecified function q in $H^1(\Omega)$ (it is the function-test).

The formula of Green and the fact that $\mathbf{v}$ checks the boundary conditions [éq 4.1.1-3] and [éq 4.1.1 - 4] enable us to lead to:

$$\int_\Omega i\omega\chi \;\mathrm{pq}^* + \int_{\partial\Omega_z} \alpha \;\mathrm{pq}^* - \int_\Omega \mathbf{v} \cdot \mathbf{grad}\,\mathrm{q}^* = -\int_{\partial\Omega_v} V_n \;\mathrm{q}^* \qquad \textbf{éq 4.1.2-1}$$

One proceeds in the same way with the equation [éq 4.1.1-1] by taking its scalar product in $L^2(\Omega)$ with an unspecified $\mathbf{u}$ function-test in $\left(L^2(\Omega)\right)^3$ one obtains:

$$\int_\Omega i\,\omega\,\rho_0 \mathbf{v}.\mathbf{u}^* + \int_\Omega \mathbf{grad}\,\mathrm{p}.\mathbf{u}^* = 0$$ **éq 4.1.2-2**

Now we multiply [éq 4.1.2-1] by $j\omega\rho_0$ and [éq 4.1.2-2] by $-j\omega\rho_0$, then we make **the change of function** :

$$j\omega\,\mathbf{v} \quad \square \quad \bar{\mathbf{v}}$$

Thus we obtain the mixed variational formulation [éq 4.1.2-3]:

To find $(\mathrm{p}, \bar{\mathbf{v}}) \in X \times M$ such as:

$$\begin{cases} \int_\Omega -\rho_0\,\mathbf{v}.\,\mathbf{grad}\,\mathrm{q}^* - \omega^2 \int_\Omega 1/c^2\,\mathrm{pq}^* + j\omega \int_{\partial\Omega_Z} \rho_0\,\alpha\,\mathrm{pq}^* = -j\omega \int_{\partial\Omega_V} \rho_0 V_n\,\mathrm{q}^* \quad \forall\,\mathrm{q} \in X \\ \int_\Omega \rho_0^2 \mathbf{v}.\mathbf{u}^* + \int_\Omega \rho_0\,\mathbf{grad}\,p.\bar{\mathbf{u}}^* = 0 \quad \forall \bar{\mathbf{u}} \in M \end{cases}$$ **éq 4.1.2-3**

where: $X = H^1(\Omega) = \left\{ \mathrm{p} \in L^2(\Omega)\ ;\ \partial\mathrm{p}/\partial x_i \in L^2(\Omega) \quad i = 1,2,3 \right\}$

and: $M = \left(L^2(\Omega)\right)^3 = \left\{ \bar{\mathbf{v}} = (\bar{\mathrm{v}}_i) \quad i = 1,2,3\ ;\ \bar{\mathrm{v}}_i \in L^2(\Omega) \right\}$

4.2 Discretization by finite elements.

The Ω field and its borders $\partial\Omega_V$ and $\partial\Omega_Z$ are cut out in elementary fields and borders:

$$\Omega^e, \partial\Omega^e{}_V, \partial\Omega^e{}_Z$$

on which are calculated elementary integrals.

To represent the fields of p^e and of $\mathbf{v}^e$ inside the element the same elementary functions are used $\mathrm{N_i}$.

Inside each element (comprising m nodes) one writes:

$$\mathbf{OM}^e = \sum_{i=1}^{m} \mathrm{N_i}(\varepsilon,\eta,\xi)\ \mathbf{OM}_i^e$$

$$\mathrm{p}^e = \sum_{i=1}^{m} \mathrm{N_i}(\varepsilon,\eta,\xi)\ p_i^e$$

$$\bar{\mathbf{v}}^e = \sum_{i=1}^{m} \mathrm{N_i}(\varepsilon,\eta,\xi)\ \bar{\mathbf{v}}_i^e$$

ε,η,ξ are the curvilinear coordinates of a three-dimensional element;

$\mathbf{OM}_i^e$ is the vector position of the node M_i of the element e with m nodes;

$\mathrm{N_i}, i=1, m$ are the elementary functions on the element e ;

$\overline{\mathbf{v}}_i^e$ “acceleration” with the node of the element is M_i the vector e.

In this case the system of equations [éq 4.1.2-3] is written matriciellement for each element e :

$$p^e\overline{\mathbf{v}}^e\ \mathbf{K}^e\begin{bmatrix} q^{e*} \\ \overline{\mathbf{u}}^{e*}\end{bmatrix} - \omega^2 p^e\overline{\mathbf{v}}^e\ \mathbf{M}^e\begin{bmatrix} q^{e*} \\ \overline{\mathbf{u}}^{e*}\end{bmatrix} + j\omega\, p^e\overline{\mathbf{v}}^e\ \mathbf{C}^e\begin{bmatrix} q^{e*} \\ \overline{\mathbf{u}}^{e*}\end{bmatrix} = -j\omega\mathbf{S}^e\begin{bmatrix} q^{e*} \\ \overline{\mathbf{u}}^{e*}\end{bmatrix} \qquad \textbf{éq 4.2-1}$$

where:

$p^e\overline{\mathbf{v}}^e = \begin{bmatrix} p^e \\ \overline{\mathbf{v}}^e\end{bmatrix}^t = \left\{ p_1^e, \overline{v}_{1x}^e, \overline{v}_{1y}^e, \overline{v}_{1z}^e \quad \cdots \quad p_m^e, \overline{v}_{mx}^e, \overline{v}_{my}^e, \overline{v}_{mz}^e \right\}$ is the vector solution in the element e ;

4.2.1 The stiffness matrix

$\mathbf{K}^e$ is the matrix of elementary “stiffness”, corresponding to the computation of the following part of [éq 4.1.2 - 3]:

$$\begin{cases} \displaystyle\int_{\Omega^e} -\rho_0\overline{\mathbf{v}}.\mathbf{grad}\ \mathrm{q}^* \\ \displaystyle\int_{\Omega^e} \rho_0{}^2\overline{\mathbf{v}}.\overline{\mathbf{u}}^* + \int_{\Omega^e} -\rho_0\,\mathbf{grad}\,\mathrm{p}.\overline{\mathbf{u}}^* \end{cases}$$

One can write it by breaking up it into mxm under matrixes $\mathbf{K}_{ij}^e$ of dimensions 4 X 4 like Ci - below:

$$\mathbf{K}^e = \begin{bmatrix} \mathbf{K}_{11}^e & \cdots\mathbf{K}_{1j}^e & \cdots\mathbf{K}_{1m}^e \\ \mathbf{K}_{i1}^e & \cdots\mathbf{K}_{ij}^e & \cdots\mathbf{K}_{im}^e \\ \mathbf{K}_{m1}^e & \cdots\mathbf{K}_{mj}^e & \cdots\mathbf{K}_{mm}^e \end{bmatrix} \quad \text{pour} \quad i,j=1,\ldots,m$$

with the following terms for $\mathbf{K}_{ij}^e$:

$$\mathbf{K}_{ij}^e = \begin{bmatrix} 0 & -\displaystyle\int_{\Omega^e}\rho_0\frac{\partial \mathrm{N}_i}{\partial x}\mathrm{N}_j & -\displaystyle\int_{\Omega^e}\rho_0\frac{\partial \mathrm{N}_i}{\partial y}\mathrm{N}_j & -\displaystyle\int_{\Omega^e}\rho_0\frac{\partial \mathrm{N}_i}{\partial z}\mathrm{N}_j \\ -\displaystyle\int_{\Omega^e}\rho_0\frac{\partial \mathrm{N}_j}{\partial x}\mathrm{N}_i & \displaystyle\int_{\Omega^e}\rho_0{}^2\,\mathrm{N}_i\mathrm{N}_j & 0 & 0 \\ -\displaystyle\int_{\Omega^e}\rho_0\frac{\partial \mathrm{N}_j}{\partial y}\mathrm{N}_i & 0 & \displaystyle\int_{\Omega^e}\rho_0{}^2\,\mathrm{N}_i\mathrm{N}_j & 0 \\ -\displaystyle\int_{\Omega^e}\rho_0\frac{\partial \mathrm{N}_j}{\partial z}\mathrm{N}_i & 0 & 0 & \displaystyle\int_{\Omega^e}\rho_0{}^2\,\mathrm{N}_i\mathrm{N}_j \end{bmatrix}$$

4.2.2 The mass matrix

$\mathbf{M}^e$ is the matrix of elementary “mass”, corresponding to the computation of :

$$\int_{\Omega} 1/c^2 \, \mathrm{pq}^*$$

Its coefficients are the following:

$$M_{ij}^e = \int_{\Omega} 1/c^2 \, \mathrm{N}_i \mathrm{N}_j \qquad \text{pour } i, j = 1, \ldots, 4r-3, \ldots, 4m-3 \quad \text{avec } r = 1, \ldots, m$$

The other terms are **null**

4.2.3 the damping matrix

$\mathbf{C}^e$ is the matrix of elementary “damping”, coming from computation from:

$$\int_{\partial \Omega_V^e} \rho_0 \, \alpha \, \mathrm{pq}^*$$

Its coefficients are the following:

$$C_{ij}^e = \int_{\partial \Omega_V^e} \rho_0 \, \alpha \, \mathrm{N}_i \mathrm{N}_j \qquad \text{pour } i, j = 1, \ldots, 4r-3, \ldots, 4m-3 \quad \text{avec } r = 1, \ldots, m$$

The other terms are **null.**

4.2.4 The vector source

$\mathbf{S}^e$ is the vector “source” elementary, representing the computation of the terms :

$$\int_{\partial \Omega_Z^e} \rho_0 \, V_n \, \mathrm{q}^*$$

Its components are the following ones:

$$S_i^e = \int_{\partial \Omega_z^e} \rho_0 \, V_n \, \mathrm{N}_i \qquad \text{pour } i, j = 1, \ldots, 4r-3, \ldots, 4m-3 \quad \text{avec } r = 1, \ldots, m$$

The other terms are **null.**

After having obtained the field $\mathrm{p}, \overline{\mathbf{v}}$ on the field Ω by resolution of the equation [éq 4.2-1] assembled, one returns to the field $\mathrm{p}, \mathbf{v}$ by the opposite change of function; one can calculate the definite acoustic intensities by [éq 1-1] which are in this case continuous in all the field $\tilde{\Omega}$

5 Commands specific to the acoustic modelization

During a study by modelization in acoustic finite elements with *Code_Aster* one uses general commands and commands which are specific to the acoustics, or whose key words and options are particular with this discipline; we present the list below of it.

Definition of the characteristics of the propagation mediums

It is necessary to give the density (actual value) and the celerity of propagation (complex value); one uses for that the command:

`DEFI_MATERIAU` with the following key words:

factor key word:	`FLUIDE`	
key words:	`RHO`	(density ρ_0)
	`CELE_C`	(celerity c)

Example:

```
air = DEFI_MATERIAU (FLUIDE=_F (RHO= 1.3, CELE_C: IH 343.  0. ));
```

In this case $\rho_0 = 343. + j0$.

Assignment of the model

It is obligatorily necessary to specify that it is **the “** acoustic” phenomenon and to choose one of the 3 **possible** modelizations of the acoustics; the command is thus used:

`AFFE_MODELE` with the following key words for which one specifies the possible values of assignment:

key word:	`PHENOMENE = “ACOUSTIC”`
	`MODELISATION = ' 3D'` or `“PLANE”`

Boundary conditions

One must assign values normal vibratory velocity per face (or edge into two-dimensional) to meshes defining the borders sources, and also values of acoustic impedance per face (edge into two-dimensional) with meshes defining the borders in imposed impedance.

One uses the command specific to acoustics `AFFE_CHAR_ACOU` with the following key words:

key word:	`MODEL`	
factor key word:	`VITE_FACE`	
key word:	`NET`	
	`GROUP_MA`	
	`VNOR`	(normal vibratory velocity V_n)
factor key word:	`IMPE_FACE`	
key word:	`NET`	
	`GROUP_MA`	
	`IMPE`	(acoustic impedance Z)
factor key word:	`PRES_IMPO`	
	`NODE`	
	`GROUP_NO`	
	`NEAR`	(pressure p imposed on the nodes)

Computation of the elementary matrixes

the various elementary matrixes (stiffness, mass and damping) are calculated by specific options. The command is employed:

CALC_MATR_ELEM with the key word OPTION for which one specifies the possible values of assignment:

key words: OPTION: "RIGI_ACOU"
"MASS_ACOU"
"AMOR_ACOU"

Foot-note:

the assembled matrixes can be obtained directly with the macro command ASSEMBLY and the same options.

Computation of the elementary vector source

the elementary vector is calculated by a specific option; the loading should obligatorily be indicated. The command is employed:

CALC_VECT_ELEM with the key word OPTION for which one specifies the only value of possible assignment:

key words: OPTION: "CHAR_ACOU"
key words: CHARGE

Computation of the solution

After assembly of the elementary matrixes and vector the harmonic solution can be calculated directly with the command: DYNA_LINE_HARM

Postprocessings

From result of the resolution of the matric transcription of the equations [éq 3.1-2] or [éq 4.1.2 - 3], of the commands of postprocessing make it possible to obtain the nodal fields of following acoustic quantities:

- acoustic L_p pressure level P partly dB : $L_p = 20 \log_{10}\left[\frac{|P|}{2.0\ 10^{-5}}\right]$
- real of the imaginary acoustic
- pressure left the acoustic pressure
- acoustic intensity activates $\mathbf{I} = \frac{1}{2}\mathrm{Re}\left[\mathrm{p}\,\mathbf{v}*\right]$
- acoustic intensity reactivates $\mathbf{J} = \frac{1}{2}\mathrm{Im}\left[\mathrm{p}\,\mathbf{v}*\right]$

These fields are calculated by use of the command of postprocessing CALC_CHAMP (the concept of result is of type "ACOU_HARMO" or "MODE_ACOU"):

CALC_CHAMP with key words RESULTAT and OPTION for which one specifies the possible values of assignment:

key word: RESULTAT
key word: OPTION: "PRAC_ELNO" (level of pressure in dB)
(left real the pressure)
(imaginary part of the pressure)
"INTE_ELNO" (intensity activates)
(intensity reactivates)

6 Conclusion

Of the moduli were thus integrated in *Code_Aster*, making it possible to do calculations of interior acoustics low frequency for complex geometries by two methods: classical acoustic finite elements and mixed acoustic finite elements. The two formulations were validated by comparison with the same analytical solution; cases tests are presented in the handbook of V7 validation under coding AHLV100. As it was envisaged, the accuracy, with identical mesh, is higher in the mixed case; if account of the obstruction memory is taken this superiority is advantageous only if we want to obtain the field of intensity: one should use the mixed E.F only in this case there.

7 Bibliography

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10) : “Integration of the finite elements acoustic mixed in Aster *” Ratio* DER/EDF – HP-61/92.081 Description

8 of the versions of the document Version

Aster Author	**(S) Organization (S) Description**	**of the modifications 3 F.**
STIFKENS	EDF-R&D/AMV initial	Text