Beam élasto-acoustics

Summarized:

One presents an element of coupling élasto - acoustic right which applies to a structural element of type beam of Timoshenko. This element makes it possible to realize, in vibro - acoustic, the modal analysis of a right pipework containing of the fluid under pressure (water, vapor…). One can also carry out computations of response to fluid sources (flow masses, volume flow rate, pressure) by modal recombination. The boundary conditions applicable to the nodes of these elements are of Dirichlet type: displacement, pressure or potential can be imposed there.

Contents

1 Notations

P	:	instantaneous stagnation pressure in a point of the fluid
p	:	pressure realised on a cross-section
$\tilde{p}$	:	fluctuating pressure
$\mathbf{u}$	:	displacement of structure
Φ	:	potential of displacements of the fluid
ρ_0	:	density of the acoustic fluid
ρ_s	:	density of structure
ω, f	:	pulsation, frequency
c	:	speed of sound in the fluid
λ, k	:	wave length, wave number
s	:	tensor of the stresses of structure
$\boldsymbol{\varepsilon}$	:	tensor of the structural deformations
dV	:	volume element
dA	:	surface element
Σ	:	surface interaction between the pipework and the fluid
S_f	:	cross-section of the fluid
S_s	:	cross-section of the pipework

2 Introduction

In order to be able to carry out computations of dynamic response of structures filled with fluid to fluids, elements of fluid-structure coupling 3D were developed in *Code_Aster* (cf [bib2]).

These voluminal elements have the advantage of allowing a fine description of structure into cubes particular places like, for example, connection between a principal pipework and a bypass of instrumentation. On the other hand, their systematic use for the analysis of ramified and complex networks would lead to costs of modelization (realization of mesh) and of computation prohibitory.

For this reason, and in order to facilitate simplified studies of dynamic behavior of pipework, one developed a beam element right élasto-acoustics allowing to realize, with lower costs of computation and labor, of computations of overall behavior of the right parts of the pipework in low frequency.

One finds hereafter a presentation of the finite elements of pipework of type acoustic beam élasto -. The vibratory behavior of the networks of pipework is conditioned by the flow of the fluid which traverses them.

3 The model of beam élasto-acoustics

3.1 Assumptions

One studies low frequency vibrations of a pipework elastic, linear homogeneous and isotropic coupled to a compressible fluid.

The effects due to viscosity and flow of the fluid are neglected.

The pipework are lengthened bodies. Indeed, their transverse dimensions are much lower than their length: $D \ll L$, and the thickness is such as one can neglect the modes of swelling and ovalization of the pipe. One can use a model of beam.

Low frequency the acoustic wave lengths associated with the studied problems are large compared to transverse and small dimensions compared to the longitudinal dimension of the circuit: $\omega . L/c > 1$ and $\omega . D/c \ll 1$. Compressibility acts indeed mainly on longitudinal displacements. Transversely, it is considered that the fluid moves like an indeformable solid, i.e. it acts like an added mass. The pressure in a cross-section of the pipe being then constant, one says that the acoustic wave is plane.

3.2 Functional calculus of the coupled problem

One can write the variational formulation of the problem of the pipework filled of fluid starting from the balance equations and behavior of the fluid and the pipe as well as boundary conditions. From the general functional calculus of the three-dimensional coupled problem ([bib1], [bib2]), one can write the functional calculus applied to the typical case of the beams.

The variational formulation of the problem 3D amounts minimizing the functional calculus:

$$F(u, p, \Phi) = \frac{1}{2}\left\{\int_{\Omega_s}\left[\boldsymbol{\sigma}(\mathbf{u}):\boldsymbol{\varepsilon}(\mathbf{u}) - \rho_s \omega^2 \mathbf{u}^2\right] dV + \int_{\Omega_f}\left[\frac{P^2}{\rho_0 c^2}\left(\frac{2 \mathrm{P}\, \Phi}{\rho_0 c^2} - (\mathrm{grad}\, \Phi)^2\right)\right] dV\right\} - \rho_0 w^2 \int_S \Phi\, \mathbf{u} . \mathbf{n}\, dA$$

with:

Ωs , the field of structure

Ωf , the field of the fluid

Σ , the fluid surface of interaction - structure.

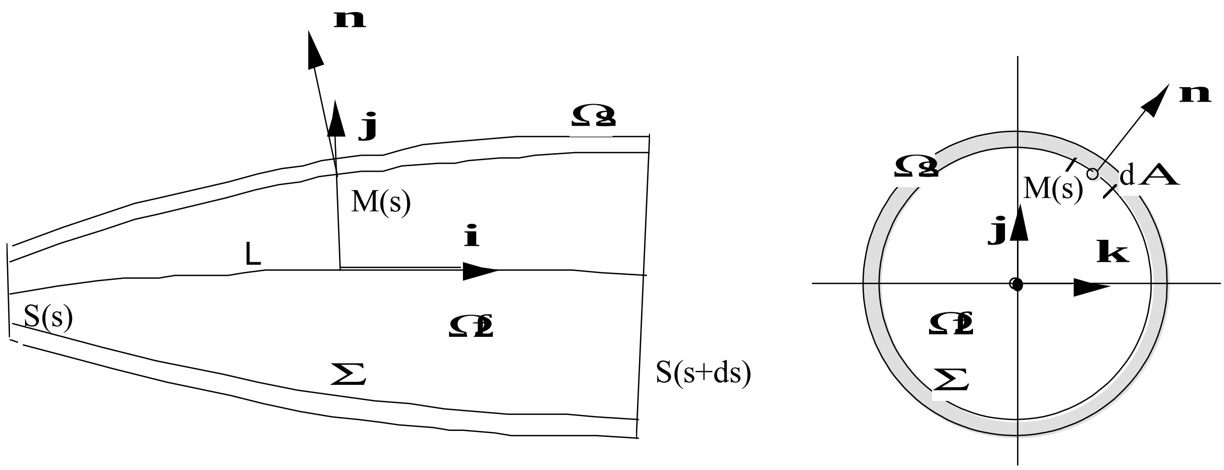
3.2.1 Contribution of the pipework

The model of beam used is that of Timoshenko with shear deformations and inertia of rotation of the cross section. It corresponds to the modelization `POU_D_T` of which it takes again elementary computations. One does not take into account the effects of ovalization [bib3].

The terms associated with the pipework in the variational formulation are written then:

$$\int_L \left[\boldsymbol{\sigma}(u):\boldsymbol{\varepsilon}(u) - \rho_s \omega^2 u^2\right] S_s\, ds$$

with: L , the average fiber of the pipework and S_s , the section of the pipework to the X-coordinate s (cf [Figure 3.2.1-a]).



Appear 3.2.1-a: geometry of the pipework

3.2.2 Contribution of the fluid

In this paragraph, one is interested only in the fluid part of the functional calculus, i.e., term of coupling put except for, at the end which is written in 3D:

$$\int_{\Omega_f}\left[\frac{P^2}{\rho_0 c^2}-\rho_0\omega^2\left(\frac{2P\Phi}{\rho_0 c^2}-\left(\operatorname{grad}\Phi\right)^2\right)\right]dV \qquad \text{éq 3.2.2-1}$$

One supposes that the pressure breaks up into two terms:

$$P(M(s),t)=p(s,t)+\tilde{p}(M(s),t)$$

where p is the value realised on a cross-section of the pressure:

$$p(s,t)=\frac{1}{S_f(s)}\int_{S_f(s)}P(M(s),t)\,dM$$

and $\tilde{p}$ is a term of fluctuating pressure which corresponds to the modal contribution transverses.

According to the assumptions of the paragraph [§1], p checks the equation 1-D of Helmholtz and $\tilde{p}$ the equation of Laplace (incompressible). The integral [éq 3.2.2-1] thus breaks up into two terms.

3.2.2.1 Term corresponding to the contribution of $\tilde{p}$

In motions perpendicular to the axis of the pipe, one considers that the fluid intervenes only by its added mass [bib4], the term related to $\tilde{p}$ is thus a term of inertia:

$$\int_L \rho_0 \omega^2 \left(\mathbf{u_t}\right)^2 S_f \, ds$$

$\mathbf{u_t}$ being transverse components of the vector displacement of structure and S_f the section of the fluid to the X-coordinate s .

3.2.2.2 Term corresponding to the contribution of p

$$\int_L \left[\frac{P^2}{\rho_0 c^2} - \rho 0 \, \omega^2 \left(\frac{2 \mathrm{P} \Phi}{\rho_0 c^2} - \left(\frac{\partial \Phi}{\partial s} \right)^2 \right) \right] S_f \, ds$$

3.2.3 Term of coupling

3.2.3.1 Current section

According to the references [bib4] and [bib5], one shows that the term of coupling C :

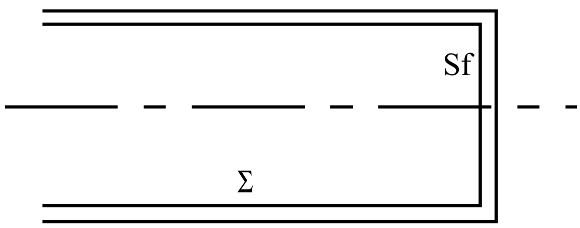
$$C = -\int_S \rho_0 \Phi \, \mathbf{u} . \mathbf{n} \, dA = \int_L -\frac{\rho_0 \Phi}{R} \mathbf{u} . \mathbf{j} S_f \, ds + \int_L \rho_0 \Phi \, \mathbf{u} . \mathbf{i} \frac{dS_f}{ds} ds$$

The balance equations of structure and the equation of propagation the plane waves (Helmholtz) in the fluid are thus coupled with the level of the bent parts and the right parts where there is a change of section of the pipework. In the case of a pipe with constant cross-section:

$R \rightarrow \infty$ and $\dfrac{dS_f}{ds} = 0$ There $C = 0$

formulates is thus no coupling between motions of beam of structure and the longitudinal displacements of the fluid in the right parts of the circuit. In this case, the fluid is characterized only by its added mass related to transverse displacements.

3.2.3.2 Of pipe



a bottom of pipe Fund In the case of, one notes $\Sigma t = \Sigma + S_f$, the entire surface of interaction between the fluid and the pipework.

In the case of a right pipework with closed constant section, the term of coupling C is worth then:

$$C = -\int_{S+S_f} \rho_0 \Phi \, \mathbf{u} . \mathbf{n} \, dA = -\int_{S_f} \rho_0 \Phi \, \mathbf{u} . \mathbf{n} \, dA$$

It is the basic effect.

This term is added at the end of coupling of a current section. Thus, a free node which carries out a condition of null flow through the section [bib6] carries out an acoustic basic condition. Indeed, an incidental plane wave is completely considered on the bottom: the acoustic pressure in the conduit obeys the equation of Helmoltz with normal gradient of pressure no one (fluid displacements and solid being null).

$$\begin{cases} \dfrac{\partial^2 p}{\partial x^2}+k^2 p=0 \\ (\dfrac{\partial p}{\partial x})_{S_f}=0 \end{cases}$$

One seeks the solution in the form: $p=A\cos(\omega t-kx)+B\cos(\omega t+kx)$
i.e. in the form of a linear combination of one wave acoustic plane incidental and one wave considered.
The condition of null gradient on the bottom, checked for all times, imposes:

$$A=B$$

The considered wave is thus “equal” to the incident wave (coefficient of reflection equal to the unit).

3.2.4 Functional calculus of the system coupled in the case of the pipework

In the typical case which we treat of a not bent pipework, with constant section, the functional calculus of the coupled problem is thus written in the following form:

$$F(u,p,\Phi)=\frac{1}{2}\left\{\int_L \boldsymbol{\sigma}(\mathbf{u}):\boldsymbol{\varepsilon}(\mathbf{u})\,S_s\,ds-\omega^2\int_L\left[\rho_s S_s u^2+\rho_0 S_f(\mathbf{u_t})^2\right]ds\right.$$
$$\left.+\int_L\left[\frac{P^2}{\rho_0 c^2}-\rho_0\omega^2\left(\frac{2P\Phi}{\rho_0 c^2}-\left(\frac{\partial\Phi}{\partial\boldsymbol{\sigma}}\right)^2\right)\right]S_f\,ds\right\}-\omega^2\int_{S_f}\rho_0\Phi\,\mathbf{u}.\mathbf{n}\,dA$$

3.3 Discretization by finite elements

the sought (u,p,Φ) solution minimizes the functional calculus F . The approximation by finite elements of the complete problem leads then to the symmetric system:

$$\begin{bmatrix}\mathbf{K} & 0 & 0\\ 0 & \dfrac{\mathbf{K_f}}{\rho_0.c^2} & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{u}\\ p\\ \Phi\end{bmatrix}-\omega^2\begin{bmatrix}\mathbf{M}+\mathbf{M_f} & 0 & \mathbf{M_\Sigma}\\ 0 & 0 & \dfrac{\mathbf{M_{fl}}}{c^2}\\ \mathbf{M_\Sigma^T} & \dfrac{\mathbf{M_{fl}^T}}{c^2} & \rho_0.\mathbf{H}\end{bmatrix}\begin{bmatrix}\mathbf{u}\\ p\\ F\end{bmatrix}=0$$

$\mathbf{K}$ and $\mathbf{M}$ being respectively stiffness matrixes and of mass of structure

$\mathbf{K_f}$ $\mathbf{M_{fl}}$, $\mathbf{H}$ being fluid matrixes, respectively obtained starting from the quadratic forms:

$$\int_L p^2 S_f\,ds,\ \int_L p\Phi S_f\,ds,\ \int_L\left(\frac{\partial\Phi}{\partial s}\right)^2 S_f\,ds$$

$\mathbf{M_f}$ being the fluid matrix obtained from the quadratic form: $\int_L p_0 S_f(\mathbf{u_t})^2 ds$

$\mathbf{M}_\Sigma$ being the matrix of coupling obtained from the bilinear form: $\int_{S_f} p_0 \Phi \mathbf{u}\,\mathbf{n}\, dA$.

While discretizing linearly p and Φ , one thus has:

$p = p_1 \frac{L-x}{L} + p_2 \frac{x}{L} \text{et } \Phi = \Phi_1 \frac{L-x}{L} + \Phi_2 \frac{x}{L}$, L being the length of the element considered.

In this case, the elementary matrix of stiffness of the fluid is written:

$$\mathbf{K_f} = \frac{S_f L}{3} \{p_1\, p_2\} \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix} \begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix}$$

The elementary matrix of coupling is written:

$$\mathbf{M}_\Sigma = r_0 S_f \{\Phi_1 \Phi_2\} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

The various elementary mass matrixes fluid are written:

$$\mathbf{M}_\Sigma = r_0 S_f \{\Phi_1 \Phi_2\} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix}$$

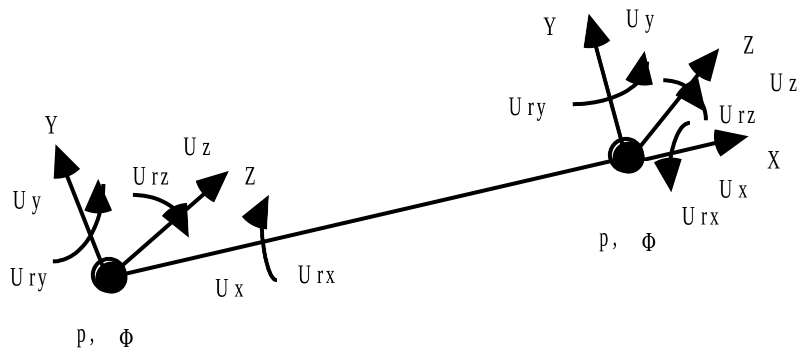
$$\mathbf{M}_\Sigma = -\frac{S_f}{L} \{\Phi_1 \Phi_2\} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} \Phi_1 \\ \Phi_2 \end{Bmatrix}$$

3.4 Establishment in *Code_Aster*

On the principles which we have just described, a vibro-acoustic beam element, of Timoshenko for the part pipework, right to constant section, was established in *Code_Aster*. It belongs to modelization "`FLUI_STRU`" of the " `MECHANICAL" phenomenon`.

This element has 8 degrees of freedom by the node is outside the field of definition with a right profile of the EXCLU type node: displacements and rotations of the pipework, pressure and potential of displacement of the fluid (cf [Figure 3.4-a]). The formulation is written for local **displacements** in the local coordinate system with the element made up of neutral fiber (axis X) and the principal axes of inertia (axis Y, axis Z) of the section. The two scalars p and Φ (pressure and potential of fluid displacements) are invariants by change of reference.

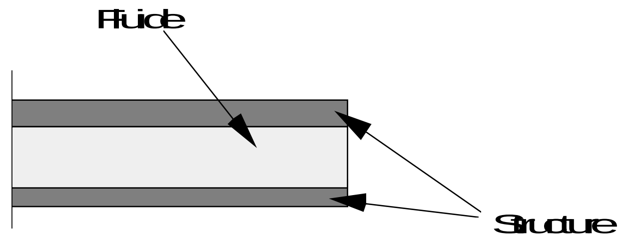
On each node of this element, one can impose boundary conditions of the Dirichlet type in pressure, potential of fluid displacements and displacements (translation or rotation).



Appear 3.4-a: beam element filled with fluid

This element to date makes it possible only to calculate the eigen modes of a right pipework filled with fluid and to do harmonic calculation of response. The effects of curvature or abrupt widening of section are not taken into account for the moment, but these effects fluid - structure, when one deals with not very dense fluids like the vapor of a pipework of admission, do not seem to have a determinant importance on the computation of the first modes: the correct mechanical representation of the elbow (coefficient of compliance) seems sufficient to compute: these frequencies [bib7].

In modal analysis, one can quote the case of a right pipework filled with fluid with loose lead:



Appear 3.4-b: beam filled with fluid clamped - free

the eigenfrequency of the mode of traction and compression of this fluid coupled system/structure is given by the relation:

$$\mathrm{tg}\left(\frac{\omega L}{c}\right)=\sqrt{\frac{S_s}{S_f}\frac{E}{\rho_0 c^2}}$$

One indicates by:

- E : Young modulus of the solid material
- S_S : section of solid
- S_f : section of the fluid

One supposes here that the velocity of speed of sound in the fluid is equal to the speed of sound in the solid $c_S=\sqrt{\frac{E}{\rho_S}}$ [bib7].

The computation of transient response for this kind of finite element $(\mathbf{u}, p, \varphi)$ is not available yet in *Code_Aster*.

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