# Elements of absorbing border

**Summarized**

This document describes the establishment in *Code_Aster* of the elements of absorbing border. These elements of the paraxial type, which one describes the theory here, are assigned to borders of elastic domains or fluids for dealing with problems 2D or 3D of interaction soil-structure or soil-fluid-structure. They make it possible to satisfy the condition with Sommerfeld checking the assumption of anechoicity: the elimination of the elastic or acoustic plane waves diffracted and not physiques coming from the infinite one.

# 1.1Problematic

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## 1.1 Introduction of a semi-infinite medium for the ISS

the standard problems of seismic response and interaction soil-structure or soil-fluid-structure bring to consider infinite or supposed fields such. For example, in the case of stoppings subjected to the seisme, one often deals with reserves of big size which enable us to make the assumption of anechoicity: the waves which leave towards the bottom reserve “do not return” not. The purpose of this is reducing the size of structure to be netted and making it possible to pass from complex computations with the current computer resources. One proposes on [Figure 1.1-a] below diagram which describes the type of situations considered.

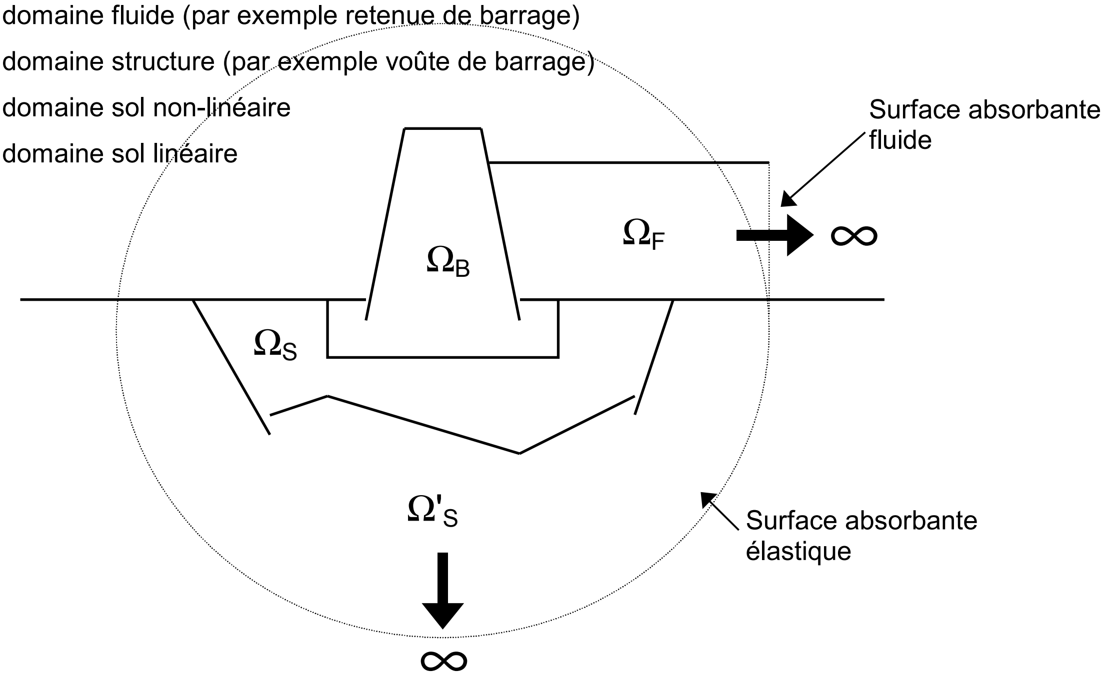
**Domaines modélisés aux éléments finis :**

$\Omega_F$ domaine fluide (par exemple retenue de barrage)

$\Omega_B$ domaine structure (par exemple voûte de barrage)

$\Omega_S$ domaine sol non-linéaire

$\Omega'_S$ domaine sol linéaire




**Appear 1.1-a: Field for the interaction soil-fluid-structure**

In all the document, one considers that the border of the mesh finite elements of the soil is in a field with the elastic behavior.

The elliptic system theory ensures simply the existence and the unicity of the solution of the acoustic or elastoplastic problems in the limited fields, under the assumption of boundary conditions ensuring the closing of the problem. It goes from there differently for the infinite fields. One must resort to a condition particular, known as of Sommerfeld, formulated in the infinite directions of the problem. This condition in particular ensures, in the case of the diffraction of one plane wave (elastic or acoustic) by a structure, the elimination of the diffracted waves not physiques coming from infinite that the classical conditions on edges of the field remotely finished are not enough to ensure.

## 1.2 State of the art of the numerical approaches

the privileged method for treating infinite fields is that of the finite elements of border (or integral equations). The fundamental solution used checks the condition of Sommerfeld automatically. Only, the use of this method is conditioned by knowledge of this fundamental solution, which is impossible in the case of a soil with complex geometry, for example, or when the soil or the structure is nonlinear. It is thus necessary then to resort to the finite elements. Consequently, conditions particular to the border of the mesh finite elements are necessary to prohibit the reflection of the outgoing diffracted waves and thus artificially to reproduce the condition of Sommerfeld.

Several methods make it possible to identify boundary conditions answering our requirements. Some lead to an exact resolution of problem: they are called “consistent borders”. They are founded on a precise taking into account of the wave propagation in the infinite field. For example, if this field can be presumedly elastic and with a simple stratigraphy far from structure, one can consider a coupling finite elements - integral equations. One of the problems of this solution is that it is not local in space: it is necessary to make an assessment on all the border separating the field finished from the infinite field, which obligatorily leads us to a problem of under - structuring. This NON-locality in space is characteristic of the consistent borders.
To lead in the local terms of border in space, one can use the theory of the infinite elements [bib1]. They are elements of infinite size whose elementary functions reproduce the elastic or acoustic wave propagation as well as possible ad infinitum. These functions must be close to the solution because the classical mathematical theorems do not ensure any more convergence of computation result towards the solution with such elements. In fact, one can find an analogy between the search for satisfactory elementary functions and that of a fundamental solution for the integral equations. The geometrical stresses are rather close but especially, this search presents a disadvantage of size: it depends on the frequency. Consequently, such borders, local or not spaces some, can be used only in the field of Fourier, which prohibits a certain category of problems, with non-linearities of behavior or large displacements for example.
One thus arrives at having to find borders absorbing powerful who are local in space and time for treating with the finite elements of the transitory problems posed on infinite fields.

We will present in the continuation the theory of the paraxial elements which carry out the absorption sought with an effectiveness inversely proportional to their simplicity of implementation as well as the description of the stresses of implementation in *Code_Aster*. One presents the developments for dealing with problems 3D. Those for the cases 2D were carried out and their theory results simply from the modelization 3D.

# 2 Theory of the paraxial elements

One presents in this part the principle of the paraxial approximation in the case of elastodynamic linear. Two theoretical approaches make it possible to determine the spirit and the practical application elastic paraxial elements: one owes the first in Cohen and Jennings [bib2] and the second with Modaressi [bib3]. The application of the theory of the paraxial elements to the fluid case will be made in the following part.
Subsequently, as presented on [Figure 1.1-a], one supposes that the border of the mesh of the soil is located in a field at the elastic behavior.
The approach of Modaressi established in *Code_Aster* at the same time makes it possible to build absorbing borders and to introduce the incidental seismic field.

## 2.1 Spectral impedance of the border

to obtain the paraxial equation, we should initially determine the shape of the field of displacement diffracted in the vicinity of the border. For that, one leaves the equations 3D elastodynamic:

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} - \mathbf{E}_{11} c^2 \frac{\partial^2 \mathbf{u}}{\partial x'^2} - \mathbf{E}_{12} c^2 \frac{\partial^2 \mathbf{u}}{\partial x' \partial x_3} - \mathbf{E}_{22} c^2 \frac{\partial^2 \mathbf{u}}{\partial x_3^2} = 0$$

With: $\mathbf{u} = \begin{Bmatrix} u' \\ u_3 \end{Bmatrix}$ $\quad \mathbf{E}_{11} = \frac{1}{c^2} \begin{bmatrix} c_P^2 & 0 \\ 0 & c_S^2 \end{bmatrix}$ $\quad \mathbf{E}_{12} = \frac{1}{c^2} \left( c_P^2 - c_S^2 \right) \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
$\mathbf{E}_{22} = \frac{1}{c^2} \begin{bmatrix} c_S^2 & 0 \\ 0 & c_P^2 \end{bmatrix}$

The constant $c$ , homogeneous at a velocity, is introduced to make certain quantities adimensional. The equations and their solutions are of course independent of this constant.
One and the $x'$ calls $u'$ directions and the component of displacement in the tangent plane and $x_3$ $u_3$ according to $\mathbf{e_3}$ , the normal direction at the border.
One proceeds to two transforms of Fourier, one compared to time, the other compared to the variables of space in the plane at the border. One limits oneself to the case of a plane border and without corner:
The equations are written then:

$$\left( c_P^2 - c_S^2 \right) \left[ -\boldsymbol{\xi}' . \hat{\mathbf{u}}' + i \frac{\partial \hat{\mathbf{u}}_3}{\partial x_3} \right] \boldsymbol{\xi}' + c_S^2 \left[ -|\boldsymbol{\xi}'|^2 + \frac{\partial^2}{\partial x_3^2} \right] \hat{\mathbf{u}}' + \omega^2 \hat{\mathbf{u}}' = 0$$

$$\left( c_P^2 - c_S^2 \right) \left[ -i \boldsymbol{\xi}' . \frac{\partial \hat{\mathbf{u}}'}{\partial x_3} + \frac{\partial^2 \hat{\mathbf{u}}_3}{\partial x_3^2} \right] + c_S^2 \left[ -|\boldsymbol{\xi}'|^2 + \frac{\partial^2}{\partial x_3^2} \right] \hat{\mathbf{u}}_3 + \omega^2 \hat{\mathbf{u}}_3 ¿ = 0$$

where $\hat{\mathbf{u}}$ and $\hat{\mathbf{u}}_3$ the transforms of Fourier and $\boldsymbol{\xi}'$ the vector of wave associated indicate with $x'$ .

It is about a differential connection in $x_3$ which one can solve by diagonalisant it. One from of deduced:

$$\frac{\left(\hat{\mathbf{u}}' \wedge \boldsymbol{\xi}'\right).\mathbf{e_3}}{|\boldsymbol{\xi}'|} = A\exp\left(-i\,\xi_S x_3\right)$$
$$\hat{\mathbf{u}}'.\boldsymbol{\xi}' = |\boldsymbol{\xi}'|\left[A_P\exp\left(-i\,\xi_P x_3\right) + A_S\exp\left(-i\,\xi_S x_3\right)\right]$$
$$|\boldsymbol{\xi}'|\hat{\mathbf{u}}_3 = -A_P\xi_P\exp\left(-i\,\xi_P x_3\right) - A_S\xi_S\exp\left(-i\,\xi_S x_3\right)$$

With: $\xi_P = \sqrt{\frac{\omega^2}{c_P^2} - |\boldsymbol{\xi}'|^2}$ and $\xi_S = \sqrt{\frac{\omega^2}{c_S^2} - |\boldsymbol{\xi}'|^2}$

to determine the constants $A, A_S \,\mathrm{et}\, A_P$, one supposes known $\hat{\mathbf{u}}\left(\boldsymbol{\xi}',0\right)$ on the border of the finite elements field. One expresses them according to $\hat{\mathbf{u}}'\left(\boldsymbol{\xi}',0\right) = \hat{\mathbf{u}}_0' \,\mathrm{et}\, \hat{\mathbf{u}}_3\left(\boldsymbol{\xi}',0\right) = \hat{\mathbf{u}}_{30}$.

One now will evaluate the vector forced on a facet of norm $\mathbf{e_3}$ in $x_3 = 0$, which will give us the impedance of the border. One subjects $t\left(x', x_3\right)$ the same transform of Fourier in space as for the equations of elastodynamic, so that:

$$\hat{\mathbf{t}}\left(\boldsymbol{\xi}', x_3\right) = \left[i\lambda\,\hat{\mathbf{u}}'.\boldsymbol{\xi}' + \left(\lambda + 2\mu\right)\frac{\partial\hat{\mathbf{u}}_3}{\partial x_3}\right]\mathbf{e_3} + \mu\left(\frac{\partial\hat{\mathbf{u}}'}{\partial x_3} + i\,\hat{\mathbf{u}}_3\,\mathbf{x}'\right)$$

One wishes to free into cubes $x_3 = 0$ terms containing from derivatives in $x_3$. The system obtained previously allows it to us according to $\hat{u}_0' \,\mathrm{et}\, \hat{u}_{30}$ :

$$\frac{\partial\hat{\mathbf{u}}_0'}{\partial x_3}.\boldsymbol{\xi}' = i|\boldsymbol{\xi}'|^2\,\hat{\mathbf{u}}_{30}$$
$$\left(\frac{\partial\hat{\mathbf{u}}_0'}{\partial x_3} \wedge \boldsymbol{\xi}'\right).\mathbf{e_3} = -i\,\xi_S\left(\hat{\mathbf{u}}_0' \wedge \boldsymbol{\xi}'\right).\mathbf{e_3}$$
$$\frac{\partial\hat{\mathbf{u}}_{30}}{\partial x_3} = i\left[-\xi_P\xi_S\frac{\hat{\mathbf{u}}_0'.\boldsymbol{\xi}'}{|\boldsymbol{\xi}'|^2} + \left(\xi_P + \xi_S\right)\hat{\mathbf{u}}_{30}\right]$$

One thus obtains the spectral impedance of the border:

$$\hat{\mathbf{t}}_0 = a^0\,\mathbf{e_3} + b^0\boldsymbol{\xi}' + c^0\boldsymbol{\xi}' \wedge \mathbf{e_3}$$

where $a^0, \mathrm{b}^0 \,\mathrm{et}\, c^0$ are functions of $|\boldsymbol{\xi}'|$ and $\omega$ which depend linearly on $\hat{\mathbf{u}}_0' \,\mathrm{et}\, \hat{\mathbf{u}}_{30}$

One can then write: $\hat{\mathbf{t}}_0 = A\left(|\boldsymbol{\xi}'|, \omega\right)\hat{\mathbf{u}}_0\left(\boldsymbol{\xi}', \omega\right)$
where $A$ the operator total spectral impedance indicates. One returns to physical space by two transforms of Fourier opposite.

## 2.2 Paraxial approximation of the impedance

the spectral impedance calculated previously is local neither spaces some nor in time since it utilizes $\hat{\mathbf{u}}_{\mathbf{0}}(\boldsymbol{\xi}',\omega)$ , transformed of Fourier of $\mathbf{u}_{\mathbf{0}}(x',t)$ for all $x'$ and all $t$ .

The idea is then to develop $\xi_P$ and $\xi_S$ according to the powers of $\frac{|\boldsymbol{\xi}'|}{\omega}$ . This approximation will be good either high frequency, or for $|\boldsymbol{\xi}'|$ small.

Let us examine the dependence in $x_3$, for example of $\hat{\mathbf{u}}_{\mathbf{3}}$ : one will have, for $\mathbf{u}_{\mathbf{3}}(x',x_3,t)$ terms of the form: $\exp\left[i\left(\xi' x' + \omega t - \xi_P x_3\right)\right]$

With the development of $\xi_P$ : $\xi_P = \frac{\omega}{c_P}\left[1 - \left(\frac{c_P|\boldsymbol{\xi}'|}{\omega}\right)^2 + \ldots\right]$

It is shown that for $|\boldsymbol{\xi}'|$ small, one will have waves being propagated according to directions close to the norm $\mathbf{e_3}$ at the border, because the exponential one is written:

$$\exp\left\{i\omega\left[\left(t - \frac{x_3}{c_P}\right) + i\,o\left(\frac{|\boldsymbol{\xi}'|}{\omega}\right)\right]\right\}$$

Consequently, with an asymptotic development of $\xi_P$ and $\xi_S$ , while multiplying by a power suitable $w$ to remove this quantity with the denominator, one obtains:

$$A_0(\boldsymbol{\xi}',\omega)\,\hat{\mathbf{t}}_0 = A_1(\boldsymbol{\xi}',\omega)\,\hat{\mathbf{u}}_0$$

where $A_0$ and $A_1$ are polynomial functions in $\boldsymbol{\xi}'$ and $\omega$ .

Maybe, after the two transforms of Fourier opposite:

$$A_0\left(\frac{\partial}{\partial x'},\frac{\partial}{\partial t}\right)\mathbf{t_0} = A_1\left(\frac{\partial}{\partial x'},\frac{\partial}{\partial t}\right)\mathbf{u_0}$$

One thus obtains the final form of the approximate transitory local impedance according to the last term in $\frac{|\boldsymbol{\xi}'|}{\omega}$ selected. One can find the computation detailed of $A_i$ in [bib5].

For example, for order 0:

$$\mathbf{t_0} = \rho\, c_P \frac{\partial u_3}{\partial t}\mathbf{e_3} + \rho\, c_S \frac{\partial \mathbf{u}'}{\partial t}$$

This corresponds to viscous dampers distributed along the border of the finite elements field.

With order 1:

$$\frac{\partial \mathbf{t_0}}{\partial t}=\rho c_P \frac{\partial^2 \mathbf{u_3}}{\partial t^2}\mathbf{e_3}+\rho c_S \frac{\partial^2 \mathbf{u}'}{\partial t^2}+\rho c_S\left[(2c_S-c_P)\frac{\partial^2 \mathbf{u_3}}{\partial x' \partial t}\mathbf{e_3}+(c_P-2c_S)\frac{\partial^2 \mathbf{u}'}{\partial x' \partial t}\right]$$
$$+\rho c_P^2\left(c_S-\frac{c_P}{2}\right)\frac{\partial^2 \mathbf{u_3}.\mathbf{e_3}}{\partial x'^2}+\rho c_S^2\left(c_P-\frac{c_S}{2}\right)\frac{\partial^2 \mathbf{u}'}{\partial x'^2}$$

One sees appearing derivative compared to the time of the vector forced. In the digital processing, it will be necessary to resort to an integration of this term on the elements of the border.
To conclude, it will be retained that the paraxial approximation led to a transitory local impedance utilizing only derivatives in time and in the tangent plane at the border. In way symbolic system, one writes:

$$\mathbf{t_0}=A_0\left(\frac{\partial \mathbf{u}}{\partial t}\right) \quad \text{l'ordre 0}$$
$$\frac{\partial \mathbf{t_0}}{\partial t}=A_1\left(\frac{\partial^2 \mathbf{u}}{\partial t^2},\frac{\partial \mathbf{u}}{\partial t},\mathbf{u}\right) \quad \text{à l'ordre 1}$$

## 2.3 Taken into account of the incidental seismic field

It is pointed out that the behavior of the soil is supposed to be elastic at least in the vicinity of the border. Ad infinitum, the total field $u$ must be equal to the incidental field $\mathbf{u_i}$ (one of the consequences of the condition of radiation of Sommerfeld). One thus introduces the field diffracted $\mathbf{u_r}$ such as:

$$\mathbf{u}=\mathbf{u_i}+\mathbf{u_r}$$
$$\lim_{x\to+\infty}\mathbf{u_r}=0$$

At the border of the mesh finite elements, one writes the condition of absorption for the field diffracted:

$$\mathbf{t_0}(\mathbf{u_r})=A_0\left(\frac{\partial \mathbf{u_r}}{\partial t}\right) \quad \text{with order 0}$$
$$\frac{\partial \mathbf{t_0}}{\partial t}(\mathbf{u_r})=A_1\left(\frac{\partial^2 \mathbf{u_r}}{\partial t^2},\frac{\partial \mathbf{u_r}}{\partial t},\mathbf{u_r}\right) \quad \text{order 1 One}$$

formulates from of deduced the total vector forced on the border from the mesh finite elements:

$$\mathbf{t_0}(\mathbf{u})=\mathbf{t_0}(\mathbf{u_i})+\mathbf{t_0}(\mathbf{u_r})=\mathbf{t_0}(\mathbf{u_i})+A_0\left(\frac{\partial \mathbf{u}}{\partial t}\right)-A_0\left(\frac{\partial \mathbf{u_i}}{\partial t}\right) \quad \text{with order 0}$$

One thus obtains the variational formulation of the problem in the vicinity of the border for order 0:

$$\rho \int_{\Omega} \frac{\partial^2 \mathbf{u}}{\partial t^2} \mathbf{v} + \int_{\Omega} \boldsymbol{\sigma}(\mathbf{u}) : \boldsymbol{\varepsilon}(\mathbf{v}) - \int_{S} A_0 \left( \frac{\partial \mathbf{u}}{\partial t} \right) \mathbf{v} = \int_{S} \left[ \mathbf{t}(\mathbf{u_i}) - A_0 \left( \frac{\partial \mathbf{u_i}}{\partial t} \right) \right] \mathbf{v}$$

For any kinematically admissible $\mathbf{v}$ field

For order 1, one preserves the classical formulation:

$$\rho \int_{\Omega} \frac{\partial^2 \mathbf{u}}{\partial t^2} v + \int_{\Omega} \boldsymbol{\sigma}(u) : \boldsymbol{\varepsilon}(v) - \int_{S} t(\mathbf{u}) v = 0$$

where $\mathbf{t}(\mathbf{u})$ the law of following evolution follows:

$$\frac{\partial \mathbf{t}(\mathbf{u})}{\partial t} = \frac{\partial \mathbf{t}(\mathbf{u_i})}{\partial t} + A_1 \left( \frac{\partial^2 \mathbf{u}}{\partial t^2}, \frac{\partial \mathbf{u}}{\partial t}, \mathbf{u} \right) - A_1 \left( \frac{\partial^2 \mathbf{u_i}}{\partial t^2}, \frac{\partial \mathbf{u_i}}{\partial t}, \mathbf{u_i} \right)$$

The request due to the incidental field appears explicitly in the case of order 0, but it is contained in the law of evolution of $\mathbf{t}(\mathbf{u})$ for order 1.

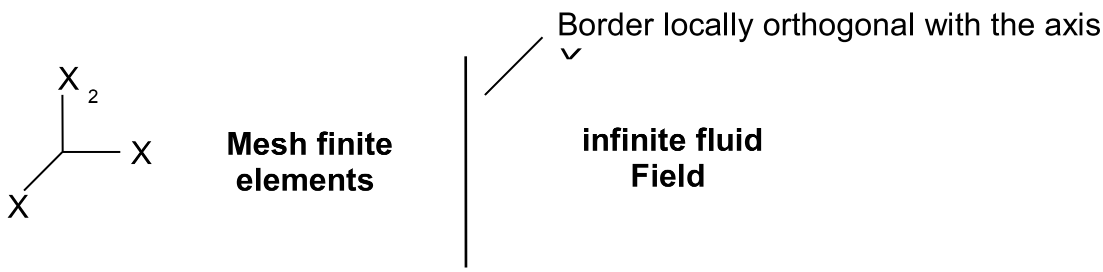
# 3 Anechoic fluid elements out of transient

This part presents the main part of the general stresses of implementation of anechoic fluid elements of border absorbents with the paraxial approximation of order 0 in *Code_Aster*. For reasons of simplicity related to the handling of scalar quantities such as the pressure or the potential of displacement, in opposition to the vector quantities like displacement, one is interested initially in the fluid elements.

## 3.1 Standard formulation

One takes again here the reasoning of Modaressi by adapting it to an acoustic fluid field. Initially, one is interested in the only data of the quantity pressure in this fluid. One will then reconsider this modelization to adapt to the stresses of *Code_Aster*, by underlining the adjustments to be made.

That is to say thus following configuration, by taking again conventions of the preceding part in the vicinity of the border:




the definition of a local coordinate system on the level of the element makes it possible to bring back for us systematically in such a situation.

### 3.1.1 Formulation finite elements

the pressure $p$ checks the equation of Helmholtz in all the field $\Omega$ modelled with the finite elements, which gives, for any virtual field of pressure $q$ :

$$-\int_{\Omega} \nabla p . \nabla q - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\int_{\Omega} pq + \int_{\Sigma} \frac{\partial p}{\partial n} q = 0$$

$\Sigma$ represent the border of the field $\Omega$.

The quantity to be estimated on $\Sigma$ thanks to the paraxial approximation is here $\frac{\partial p}{\partial n}$ .

### 3.1.2 Paraxial approximation

In the configuration suggested, the term $\frac{\partial p}{\partial n}$ corresponds to $\frac{\partial p}{\partial x_3}$ .
Let us consider a harmonic plane wave consequently being propagated in the fluid:

$$p = A \exp\left[ i \left( k_1 x_1 + k_2 x_2 + k_3 x_3 - \omega t \right) \right]$$

While replacing in the equation of Helmholtz, one obtains:

$$k_3 = \frac{\omega}{c}\sqrt{1 - \frac{c^2}{\omega^2}\left(k_1^2 + k_2^2\right)}$$

One obtains the following development then, for high frequencies ( $\omega$ large) or in the vicinity of the border ( $k_1$ and $k_2$ small):

$$k_3 = \frac{\omega}{c}\left(1 - \frac{c^2}{2\omega^2}\left(k_1^2 + k_2^2\right)\right)$$

Maybe, while multiplying by $\omega$ making disappear this quantity with the denominator and after a transform from Fourier reverses in space and time:

$$\frac{\partial^2 p}{\partial x_3 \partial t} = -\frac{1}{c}\frac{\partial^2 p}{\partial t^2} + \frac{1}{2} c \left( \frac{\partial^2 p}{\partial x_1^2} + \frac{\partial^2 p}{\partial x_2^2} \right)$$

As had presented it Modaressi, this equation utilizes the derivative compared to the time of the surface term. In the frame of this part, one is interested only at the end of order 0, that is to say, after an integration in time, which makes disappear the awkward derivative:

$\frac{\partial p}{\partial x_3} = -\frac{1}{c}\frac{\partial p}{\partial t}$ or more generally: $\frac{\partial p}{\partial n} = -\frac{1}{c}\frac{\partial p}{\partial t}$

It is this relation of impedance which we will discretize on the border of the finite elements field.

**Note:**

*Taking into account the disappearance of the term of order 1 in the development of the square root, the minimal order of approximation for the paraxial fluids is in fact 1 and not 0. We will preserve the name of elements of order 0 for coherence with solid. However, one speaks about fluid elements of order 2 at the time to consider elements of a strictly positive nature.*

## 3.2 Impedance of the vibro-acoustic elements in *Code_Aster*

*Code_Aster* has vibro-acoustic elements. One recalls in this paragraph the choices of formulation made at the time of their implementation. One is inspired to present existing it of the documentation of reference of *Code_Aster* [bib6].

### 3.2.1 Limits of the formulation out of p

In the frame of the interaction fluid-structure in harmonic, the formulation in pressure only of the acoustic fluid led to asymmetric matrixes. Indeed, the total system is expressed, in variational form, in the following way:

$$\int_{\Omega_s} \mathbf{C}_{ijkl} \, . \, \mathbf{u}_{k,l} \, \mathbf{v}_{i,j} - \omega^2 \int_{\Omega_s} \rho_s \mathbf{u}_i \mathbf{v}_i - \int_{\Sigma} p \, \mathbf{v}_i \, . \, \mathbf{n}_i = 0 \quad \text{for structure}$$

$$\frac{1}{\rho_f \omega^2} \int_{\Omega_f} \nabla p \, . \, \nabla q - k^2 \int_{\Omega_f} pq - \int_{\Sigma} \mathbf{u}_i \, . \, \mathbf{n}_i q = 0 \quad \text{for the fluid}$$

with $k = \frac{\omega}{c}$, wave number for the fluid, $v$ and $q$ two virtual fields in structure and the fluid respectively.

After discretization by finite elements, one obtains the following matric system:

$$\begin{bmatrix} \mathbf{K} & -\mathbf{C} \\ 0 & \mathbf{H} \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M} & 0 \\ \rho_f \mathbf{C}^{\mathbf{T}} & \dfrac{\mathbf{Q}}{c^2} \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = 0$$

where : $\mathbf{K}$ and $\mathbf{M}$ are the stiffness matrixes and of mass of structure

$\mathbf{H}$ and $\mathbf{Q}$ are the fluid matrixes obtained respectively starting from the bilinear forms: $\int_{\Omega_f} \nabla p \, . \, \nabla q$ and $\int_{\Omega_f} pq$

$\mathbf{C}$ is the matrix of coupling obtained from the bilinear form: $\int_{\Sigma} p \, \mathbf{u}_i \, . \, \mathbf{n}_i$

The asymmetric character of this system does not make it possible to use the algorithm of classics resolution of *Code_Aster*. This justifies the introduction of an additional variable into the description of the fluid.

### 3.2.2 Symmetric formulation out of p and phi

the new introduced quantity is the potential of displacements $\phi$ , such as $x=\nabla\phi$ . According to [bib6], one obtains the new variational form of the system coupled fluid-structure:

$$\int_{\Omega_s} \mathbf{C}_{ijkl} \cdot \mathbf{u}_{k,l} \mathbf{v}_{i,j} - \omega^2 \int_{\Omega_s} \rho_s \mathbf{u}_i \mathbf{v}_i - \rho_f \omega^2 \int_{\Sigma} \phi\, p\, \mathbf{v}_i \cdot \mathbf{n}_i = 0 \quad \text{for structure}$$

$$\frac{1}{\rho_f c^2} \int_{\Omega_f} pq - \rho_f \omega^2 \left[ \frac{1}{\rho_f c^2} \int_{\Omega_f} (\phi q + p\psi) - \int_{\Omega_f} \nabla\phi . \nabla\psi + \int_S \psi\, \mathbf{u}_i \mathbf{n}_i \right] = 0 \quad \text{for the fluid}$$

With: $p=\rho_f \omega^2 f$ in the fluid and $\psi$ a field of potential of virtual displacement

This leads us to the symmetric matric system:

$$\begin{bmatrix} \mathbf{K} & 0 & 0 \\ 0 & \dfrac{\mathbf{M_f}}{\rho_f c^2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ p \\ \phi \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M} & 0 & \rho_f \mathbf{M}_\Sigma \\ 0 & 0 & \dfrac{\mathbf{M}_{fl}}{c^2} \\ \rho_f \mathbf{M}_\Sigma^{\mathbf{T}} & \dfrac{\mathbf{M}_{fl}^{\mathbf{T}}}{c^2} & \rho_f H \end{bmatrix} \begin{bmatrix} u \\ p \\ \phi \end{bmatrix} = 0$$

where: $\mathbf{K}$ and $\mathbf{M}$ are the stiffness matrixes and of mass of structure

$\mathbf{M}_\Sigma$ is the matrix of coupling obtained from the bilinear form $\int_\Sigma \phi\, \mathbf{u}_i \mathbf{n}_i$

$\mathbf{M_f}, \mathbf{M}_{fl}$ and $\mathbf{H}$ are the fluid matrixes obtained starting from the bilinear forms: $\int_{\Omega_f} pq$ , $\int_{\Omega_f} pq$ (or $\int_{\Omega_f} \phi q$ ) and $\int_{\Omega_f} \nabla\phi . \nabla\psi$

### 3.2.3 Imposition of an impedance with the formulation out of p and phi

Generally, a relation of impedance at the border of the fluid is expressed as follows:

$$p = Z\, \mathbf{v} . \mathbf{n}$$

whe re: $Z$ is the outgoing normal

$\mathbf{v}.\mathbf{n}$ velocity of the fluid particles One is the imposed

impedance from of deduced, according to the constitutive law of the fluid, which connects the pressure to the displacement of the fluid particles for an acoustic fluid $\nabla p - \rho_f \frac{\partial^2 u}{\partial t^2} = 0$ :

$$\frac{\rho_f}{Z} \frac{\partial p}{\partial t} = \frac{\partial p}{\partial n}$$

The discretization of such an equation leads to an asymmetric term in a formulation in $p$ and $\phi$ . One prefers to formulate the condition compared to the potential of displacement, that is to say:

$$\nabla \phi+\frac{\rho_f}{Z}\frac{\partial \phi}{\partial t}=0$$

One obtains then like statement for the term of edge associated with the relation with impedance:

$$\rho_f \frac{\partial^2}{\partial t^2}\int_\Sigma \phi \frac{\partial \psi}{\partial n}=\frac{\partial^3}{\partial t^3}\int_\Sigma \frac{\rho_f^2}{Z}\phi \psi$$

One then notes the appearance (somewhat artificial) of a term in derived third compared to time. In harmonic, which is the privileged scope of application of the vibro-acoustic elements in *Code_Aster*, that does not pose a problem. One treats a term in $\omega^3$ without difficulty. For transient computation, rather than to introduce an approximation of a derivative third into the diagram of Newmark implemented in the operators of direct integration in dynamics in *Code_Aster* `DYNA_LINE_TRAN` [U4.53.02] and `DYNA_NON_LINE` [U4.53.01], one prefers to operate a simple correction of the second member, which returns in fact to consider the impedance explicitly. The stability conditions of the diagram of Newmark are not rigorously any more the same ones, but the experiment showed us that it is simple to arrive at convergence starting from the old conditions.

This choice of an explicit correction of the second member will be also justified at the time of the implementation of paraxial elements of order 1, which it makes easier definitely.

### 3.2.4 Detailed formulation

One proposes here the precise formulation for an acoustic fluid modelled on a field $\Omega$ with an anechoic condition on part $\Sigma_a$ of the border $\Sigma$ of the field. Apart from that, one breaks up the border into a free surface and a part in contact with a rigid solid. The introduction of requests external or the presence of an elastic structure is modelled easily by the current methods. The volume elements and of surface are formulated in $p$ and $\phi$ .
The equations in the fluid are:

$\rho_f \Delta \phi+\frac{1}{c^2}p=0$ in volume $\Omega$ 3.2.4-1

$p=\rho_f \frac{\partial^2 \phi}{\partial t^2}$ in volume $\Omega$ 3.2.4-2

$p=0$ on surface libreéq 3.2.4-3

$\frac{\partial \phi}{\partial n}=0$ on the wall rigideéq 3.2.4-4

$\frac{\partial p}{\partial n}=-\frac{1}{c}\frac{\partial p}{\partial t}$ on the part of the border with condition anéchoïqueéq 3.2.4-5

One multiplies the equation [éq 3.2.4-1] by a field of virtual potential $y$ and one integrates in $\Omega$ :

$$\int_{\Omega_f}\left[\frac{1}{c^2}p\,\psi+\rho_f\frac{\partial^2}{\partial t^2}(\nabla\phi.\nabla\psi)\right]+\int_{\Sigma}\psi\,\rho_f\frac{\partial^2}{\partial t^2}\left(\frac{\partial\phi}{\partial n}\right)=0$$ according to the formula of Green

Is, with the boundary conditions on $\Sigma$ and the equation [éq 3.2.4-2]:

$$\int_{\Omega_f}\left[\frac{1}{c^2}p\,\psi+\rho_f\frac{\partial^2}{\partial t^2}(\nabla\phi.\nabla\psi)\right]+\int_{\Sigma}\psi\,\rho_f\frac{\partial p}{\partial n}=0$$

One can consequently apply the condition of impedance formulated in pressure:

$$\int_{\Sigma_a}\psi\,\rho_f\frac{\partial p}{\partial n}=-\frac{1}{c}\int_{\Sigma_a}\psi\,\rho_f\frac{\partial p}{\partial t}$$

Moreover, to arrive to a symmetric formulation of the terms of volume, one multiplies the equation [éq 3.2.4-2] by a virtual field of pressure $q$ and one integrates in $\Omega$ :

$$\int_{\Omega_f}\frac{pq}{\rho_f c^2}-\frac{\partial^2}{\partial t^2}\int_{\Omega_f}\frac{\phi q}{c^2}=0$$

By adding the two variational equations, one obtains:

$$\frac{1}{\rho_f c^2}\int_{\Omega_f}pq+\rho_f\frac{\partial^2}{\partial t^2}\left[\frac{1}{\rho_f c^2}\int_{\Omega_f}(\phi q+p\psi)-\int_{\Omega_f}\nabla\phi.\nabla\psi\right]-\frac{1}{c}\int_{\Sigma_a}\psi\,\rho_f\frac{\partial p}{\partial t}=0$$

Matriciellement:

$$\begin{bmatrix}\mathbf{M_f} & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}p\\ \phi\end{bmatrix}-\frac{1}{c}\begin{bmatrix}0 & \mathbf{A}\\ 0 & 0\end{bmatrix}\begin{bmatrix}\dot p\\ \dot\phi\end{bmatrix}+\begin{bmatrix}0 & \frac{\mathbf{M}_{fl}}{c^2}\\ \frac{\mathbf{M}_{fl}^{\mathbf{T}}}{c^2} & \rho_f\mathbf{H}\end{bmatrix}\begin{bmatrix}\ddot p\\ \ddot\phi\end{bmatrix}=0$$

where the submatrices $\mathbf{M_f},\mathbf{M}_{fl}$ and $\mathbf{H}$ discretize the same bilinear forms as previously.

The submatrix $\mathbf{A}$ discretizes the term $\int_{\Sigma_a}\psi\,\rho_f\frac{\partial p}{\partial t}$ . The damping matrix obtained is not symmetric, as one had predicted higher. This is why one rejects this term with the second member.

### 3.2.5 Direct temporal integration

In our case, because of nonthe symmetry of the matrix of impedance, one chooses to consider the anechoic term explicitly as we evoked before. That amounts calculating it at time $t$ and placing it among the requests at the time of the statement of the dynamic equilibrium at time $t+\Delta t$ .
One solves:

$$\begin{bmatrix}\mathbf{M_f} & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}p_{t+\Delta t}\\ \phi_{t+\Delta t}\end{bmatrix}+\begin{bmatrix}0 & \dfrac{\mathbf{M}_{fl}}{c^2}\\ \dfrac{\mathbf{M}_{fl}^{\mathbf{T}}}{c^2} & \rho_f\,\mathbf{H}\end{bmatrix}\begin{bmatrix}\ddot{p}_{t+\Delta t}\\ \ddot{\phi}_{t+\Delta t}\end{bmatrix}=\frac{1}{c}\begin{bmatrix}0 & \mathbf{A}\\ 0 & 0\end{bmatrix}\begin{bmatrix}\dot{p}_t\\ \dot{\phi}_t\end{bmatrix} \qquad \text{éq 3.2.5-1}$$

Instead of:

$$\begin{bmatrix}\mathbf{M_f} & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}p_{t+\Delta t}\\ \phi_{t+\Delta t}\end{bmatrix}-\frac{1}{c}\begin{bmatrix}0 & \mathbf{A}\\ 0 & 0\end{bmatrix}\begin{bmatrix}\dot{p}_{t+\Delta t}\\ \dot{\phi}_{t+\Delta t}\end{bmatrix}+\begin{bmatrix}0 & \dfrac{\mathbf{M}_{fl}}{c^2}\\ \dfrac{\mathbf{M}_{fl}^{\mathbf{T}}}{c^2} & \rho_f\,\mathbf{H}\end{bmatrix}\begin{bmatrix}\ddot{p}_{t+\Delta t}\\ \ddot{\phi}_{t+\Delta t}\end{bmatrix}=0$$

Thus, there is not an asymmetric matrix to treat in the system giving $\mathbf{X}$ to time $t+\Delta t$ .

**Note:**

*In a nonlinear computation, one reactualizes the second member with each internal iteration. The computation can thus prove more exact and more stable in this case.*

## 3.3 Use in *Code_Aster*

the taking into account of anechoic fluid elements and computation of their impedance requires a specific modelization on the absorbing borders:

- in 2D with modelization "2D_FLUI_ABSO" on the finite elements of the MEFASEn type ( $n=2,3$ ) on the absorbing edges with $n$ nodes.
- in 3D with modelization "3D_FLUI_ABSO" on the finite elements of the MEFA_FACEn type ( $n=3,4,6,8,9$ ) on the absorbing sides with $n$ nodes.

In harmonic analysis with operator DYNA_LINE_HARM [U4.53.11], one calculates as a preliminary a mechanical impedance by option IMPE_MECA of operator CALC_MATR_ELEM [U4.61.01] and one informs it in DYNA_LINE_HARM (key word MATR_IMPE_PHI).
In transient analysis, the taking into account of the correct force due under the terms of impedance is automatic with the modelizations of elements absorbents in operators DYNA_LINE_TRAN [U4.53.02] and DYNA_NON_LINE [U4.53.01].

# 4 Elastic elements absorbents in *Code_Aster*

This part presents the main part of the general stresses of implementation of elastic elements of border absorbents with the paraxial approximation of order 0 in *Code_Aster*. One points out the relation of paraxial impedance of order 0 such as it was established by Modaressi for a linear elastic domain:

$$\mathbf{t}(\mathbf{u})=\rho\left(c_p\frac{\partial\mathbf{u}_\perp}{\partial t}+c_s\frac{\partial\mathbf{u}_{//}}{\partial t}\right)$$

$\mathbf{u_T}$ becomes $\mathbf{u_3}$ and $\mathbf{u}_{//}$ becomes $\mathbf{u}^{/}$

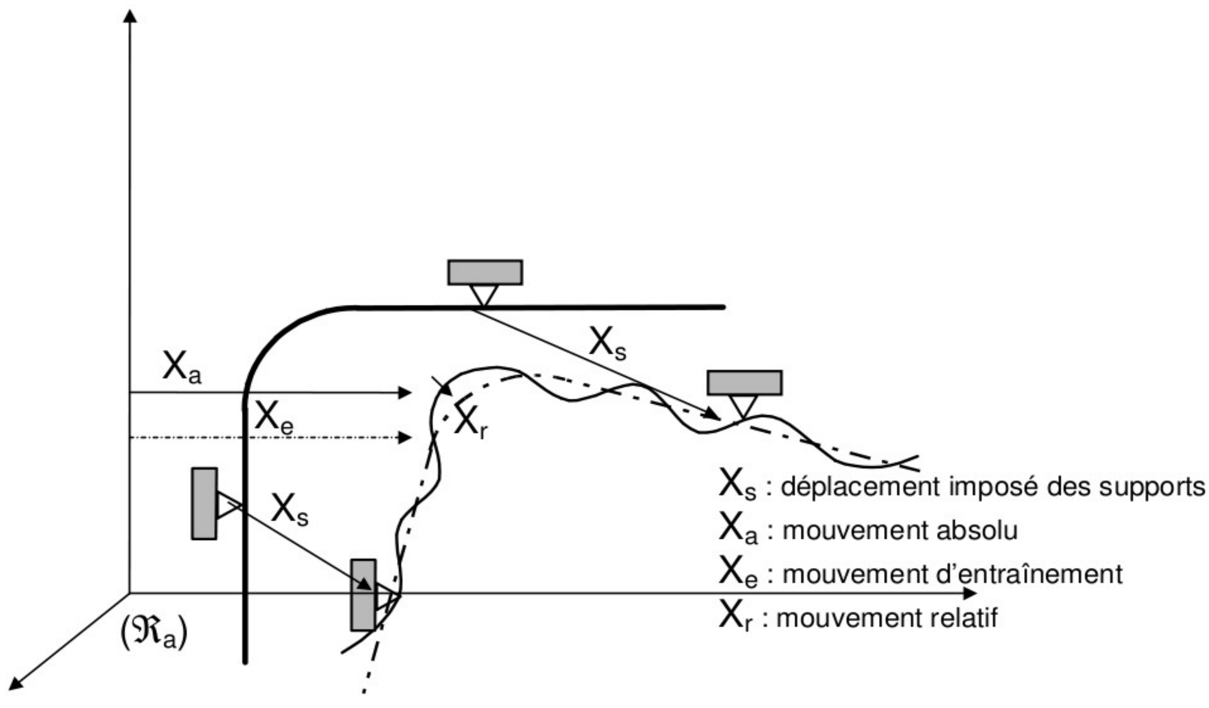
## 4.1 Adaptation of the seismic loading with the paraxial elements

One presented in the first part the principle of taking into account of the incidental field thanks to the paraxial elements. It is advisable here to present the methods of modelization of the seismic loading in *Code_Aster* to be able to adapt the data to the requirements of the paraxial elements.

The fundamental equation of the dynamics associated with an unspecified model 2D or 3D discretized in finite elements with continuum or structure and in the absence of external loading is written in the absolute coordinate system:

$$\mathbf{M}\ddot{\mathbf{X}}_\mathbf{a}+\mathbf{C}\dot{\mathbf{X}}_\mathbf{a}+\mathbf{K}\mathbf{X}_\mathbf{a}=0$$

One breaks up the motion of structures into a motion of training $\mathbf{X_e}$ and a relative motion $\mathbf{X_r}$ .




**Appear 4.1-a: Decomposition of the motion of structures**

Thus, $\mathbf{X_a}=\mathbf{X_r}+\mathbf{X_e}$

- $\mathbf{X_a}$ is the vector of displacements in the absolute coordinate system,
- $\mathbf{X_r}$ is the vector of relative displacements, i.e. the vector of displacements of the structure compared to the deformed which it would have under the static action of the displacements imposed on the level of the supports $\mathbf{X_S}$ . $\mathbf{X_r}$ is thus null at the points of anchorage,
- $\mathbf{X_e}$ is the vector of displacements of training of structure produces statically by the imposed displacement of the supports $\mathbf{X_S}$ : $\mathbf{X_e}=\boldsymbol{\Psi}\mathbf{X_s}$ ,
- $\boldsymbol{\Psi}$ is the matrix of the static modes. The static modes represent the response of structure with a unit displacement imposed on each degree of freedom of connection (others being blocked), in the absence of external forces. Thus $\mathbf{K}\boldsymbol{\Psi}=0$ , i.e. $\mathbf{K}\mathbf{X}_e=0$ .
  In the case of the mono-bearing (all the bearings undergo same imposed motion), $\boldsymbol{\Psi}$ is a mode of rigid body.

**Assumption in *Code_Aster* :**

*It is supposed that the damping dissipated by structure is of viscous type i.e. the damping force is proportional to the relative velocity of structure. Thus* $\mathbf{C}\dot{\mathbf{X}}_e=0$ .

The fundamental equation of the dynamics in the relative reference is written then:

$$\mathbf{M}\ddot{\mathbf{X}}_\mathbf{r}+\mathbf{C}\dot{\mathbf{X}}_\mathbf{r}+\mathbf{K}\mathbf{X}_\mathbf{r}=-\mathbf{M}\boldsymbol{\Psi}\ddot{\mathbf{X}}_\mathbf{S}$$

Operator `CALC_CHAR_SEISME` [U4.63.01] calculates the term $-\mathbf{M}\boldsymbol{\Psi}$ , or more exactly, $-\mathbf{M}\boldsymbol{\Psi}\mathbf{d}$ where $\mathbf{d}$ is an unit vector such as $\mathbf{X_s}=\mathbf{d}.f(t)$ with $f$ a scalar function of time.

One distinguishes two types of seismic loadings introduced into *Code_Aster* thanks to operator `CALC_CHAR_SEISME` :

1) The loading of the type `MONO_APPUI`, for which $\boldsymbol{\Psi}$ is the matrix identity (the static modes are modes of rigid body),
2) the loading of the type `MULTI_APPUI`, for which $\boldsymbol{\Psi}$ is unspecified.

According to the method of taking into account of the incidental field with the paraxial elements presented in the first part, it is necessary for us to know on the border displacement and the forced due to the incidental field. For the loading of the type `MULTI_APPUI`, only displacement is directly accessible at any moment. It thus seems difficult to allow the use of such a load pattern with paraxial elements in the soil. Moreover, if such a loading models imposed displacements of the bearings, it does not require a modelization of the soil since all the influence is taken into account by these displacements.

Case `MONO_APPUI` can be perceived differently. It represents an overall acceleration applied to the model. Consequently, the wave propagation in the soil can play a role to play in the behavior of structure, since motions of the interface soil-structure are not imposed. Moreover, the paraxial elements are usable with this kind of loading because it does not create stresses at the border of the mesh (a mode of rigid body does not create strains). Consequently, one has all the data necessary to computation of the impedance absorbing on the border.

**Notice 1:**

*In the case of a seismic request `MONO_APPUI` , dynamic computation is done in the relative reference. If one amounts on the term discretizing on the paraxial elements (see first part), one notices that* $u_i$ *corresponds exactly to the displacement of training* $\mathbf{X_e}$ *presented higher. Thus,* $\mathbf{u}-\mathbf{u_i}$ *corresponds to the relative displacement calculated during* *computation. Consequently, the relation to be taken into account on the paraxial elements in such a configuration is simply:*

$$\mathbf{t}(\mathbf{u})=A_0\left(\frac{\partial \mathbf{u}}{\partial t}\right)$$

**Notice 2:**

*In the case of a computation of interaction soil-fluid-structure with infinite fluid, the pressure to be taken into account for the computation of the anechoic impedance in the fluid is well the absolute pressure, if there is not an incidental field in the fluid (what is often the case). The correction which one could exempt to make for the soil must then be made for the fluid paraxial elements.*

## 4.2 Implementation of the elements out of transient and harmonic

### 4.2.1 Implementation out of transient

the mode of implementation of the elastic paraxial elements out of transient is very close to that presented for the fluid elements. The difference comes primarily from the need for breaking up displacement into a component according to the norm with the element, corresponding to one wave $P$, and a component in the plane of the element, corresponding to one wave $S$. One is then capable to discretize the relation of impedance introduced into the first part:

$$\mathbf{t}(\mathbf{u})=\rho C_p \frac{\partial \mathbf{u_3}}{\partial t}+\rho C_s \frac{\partial \mathbf{u}^{/}}{\partial t}$$

One does not reconsider the diagram of temporal integration which one already described in the preceding part, knowing that one always considers the relation of impedance explicitly by a correction of the second member.

### 4.2.2 Implementation in harmonic

the fluid acoustic elements of *Code_Aster* propose already the possibility of taking into account an impedance imposed on the border of the mesh in harmonic. That corresponds to the processing of a term in $\omega^3$ the equations, as referred to above. It is trying to introduce the possibility of imposing an impedance absorbing for an elastic problem in harmonic.

For a harmonic computation of response of an infinite structure, the taking into account of the absorbing impedance as a correction of the second member is obviously not applicable. However, the relation of impedance to order 0 expresses the surface terms according to the velocity of the nodes of the element. One can thus build a pseudo-matrix of viscous damping translating the presence of the infinite field.

The decomposition of the relation of impedance according to the components norm or tangential of displacement on the constrained element us to build the matrix of impedance in a local coordinate system on the element. One defines this local coordinate system in the elementary routine as well as the transition matrix which allows the return to the global database.

**Note:**

*In the case of the elastic paraxial elements of order 0, to create a damping matrix could have enabled us to solve the problem out of transient without deteriorating the stability of the diagram of Newmark, contrary to the taking into explicit account which we retained. However, we showed the problems that created for the fluid elements and we wished to keep the homogeneity of the modes of implementation. Moreover, the processing of the elements of a nature 1 making the way explicit compulsory, the whole seems coherent. That also makes it possible to take into account at the same time an infinite field with paraxial elements and a damping of type modal damping for structure.*

## 4.3 Seismic load pattern per plane wave

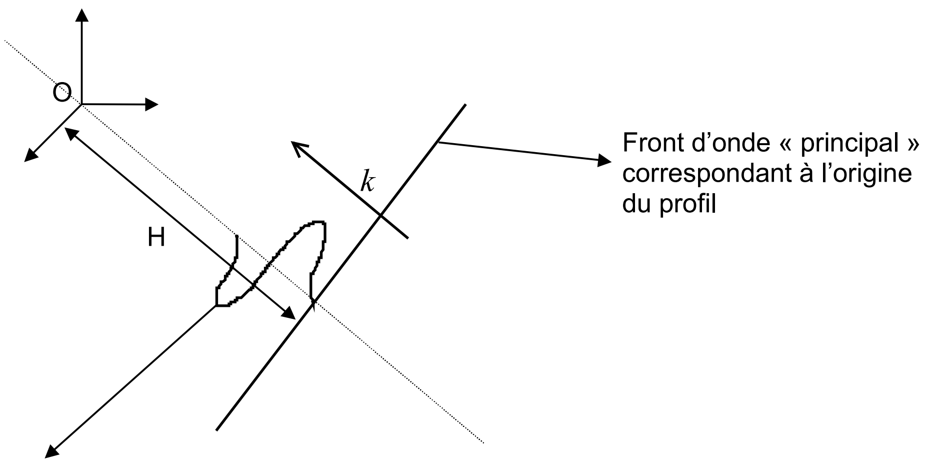
In complement of the methods of taking into account of the seismic loading already available and because of the inadequacy of mode `MULTI_APPUI` with the paraxial elements, it seems interesting to introduce a principle of loading per plane wave. That corresponds to the loadings classically met during computations of interaction soil-structure by the integral equations.

### 4.3.1 Characterization of one plane wave out of transient

In harmonic, one plane wave elastic is characterized by its direction, its pulsation and its type (wave $P$ for the compression waves, waves $SV$ or $SH$ for the waves of shears). Out of transient, the data of the pulsation, corresponding to one standing wave in time, must be replaced by the data of a profile of displacement which one will take into account the propagation in the course of time in the direction of the wave.
More precisely, one will consider one plane wave in the form:

$$\mathbf{u}(\mathbf{x},t)=f(\mathbf{k}.\mathbf{x}-C_p t)\mathbf{k} \quad \text{for one wave } P \text{ (with } \mathbf{k} \text{ unit)}$$
$$\mathbf{u}(\mathbf{x},t)=f(\mathbf{k}.\mathbf{x}-C_S t)\wedge\mathbf{k} \quad \text{for one wave } S \text{ (with always } \mathbf{k} \text{ unit)}$$

$f$ the profile of the wave represents then given according to the direction $\mathbf{k}$.




$H$ is the distance from the origin to the principal wave front.

### 4.3.2 User data for the loading by plane wave

In accordance with the theory exposed in first part, we should calculate the stress at the border of the mesh due to the incident wave and the term of impedance corresponding to incidental displacement, is:

$\mathbf{t}(\mathbf{u_i})$ and $A_0\left(\frac{\partial \mathbf{u_i}}{\partial t}\right)$

to express the stresses, it is necessary for us to have the strains due to the incident wave, the constitutive law of the material enabling us to pass from the ones to the others.
On the elements of border, one can express the tensor of the strains linearized in each node by the classical formula:

$$\boldsymbol{\varepsilon}(x,t)=\frac{1}{2}\left[\nabla\mathbf{u}(x,t)+{}^t\nabla\mathbf{u}(x,t)\right]$$

Finally, to consider the stresses due to the incidental field, we thus should determine derivatives $\frac{\partial (\mathbf{u_i})_j}{\partial x_k}$ for $j$ and $k$ traversing the three directions of space. One obtains these quantities from the definition of the incidental plane wave:

$$\frac{\partial (\mathbf{u_i})_j}{\partial x_k} = k_k f'(\mathbf{k}.\mathbf{x} - C_m t) k_j \text{ with } m=S \text{ or } P$$

With regard to the term of impedance, it is necessary for us $\frac{\partial \mathbf{u_i}}{\partial t} = -C_m \dot{f}(\mathbf{k}.\mathbf{x} - C_m t)\mathbf{k}$, always with $m=S$ or $P$.

It is seen whereas the important function for a loading by plane wave with paraxial elements of order 0 is not the profile of the wave $f$, but his derivative, either $f'$ or $\dot{f}$. The wave being plane, the wave front is characterized by the planes $\mathbf{k}.\mathbf{x} - C_m t = cte$, from where the relation: $\mathbf{k}.d\,\mathbf{x} = C_m dt$. There is thus following equivalence between the two derivatives of $f$ : $f' = \frac{1}{C_m}\dot{f}$. One chooses to request the function $\dot{f}$ from the user like data of computation.
One can consequently recapitulate the parameters to enter for the definition of a loading per plane wave out of transient:

| | |
|---|---|
| Type of the wave | : $P$, $SV$ or $SH$ |
| Direction of the wave | : $k_x$ $k_y$, $k_z$ |
| Derived from the profile of the wave | : $\dot{f}(t)$ for $t \in [0, +\infty[$ |

### 4.4 Use in *Code_Aster*

the taking into account of elastic elements absorbents and computation of their impedance requires a specific modelization on the absorbing borders:

- in 2D with modelization "D_PLAN_ABSO" on the finite elements of the MEPASEn type ( $n=2,3$ ) on the absorbing edges with $n$ nodes.
- in 3D with modelization "3D_ABSO" on the finite elements of the MEAB_FACEn type ( $n=3,4,6,8,9$ ) on the absorbing sides with $n$ nodes.

In harmonic analysis with operator DYNA_LINE_HARM [U4.53.11], one calculates as a preliminary a mechanical cushioning by option AMOR_MECA of operator CALC_MATR_ELEM [U4.61.01] and one informs it in DYNA_LINE_HARM (key word MATR_AMOR).

In transient analysis, the taking into account of the correct force due under the terms of impedance is automatic with the modelizations of elements absorbents in operators DYNA_LINE_TRAN [U4.53.02] and DYNA_NON_LINE [U4.53.01].

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# 6 Description of the versions of the document

| Version Aster | Author (S) Organization (S) | Description of the modifications |
|---|---|---|
| 05/01/00 | *G. DEVESA, V. TO MOW (EDF/RNE/AMV)* | initial Text |