Parametric and NON-parametric probabilistic models in dynamics

Summarized:

This document describes two probabilistic approaches, one parametric and other the nonparametric one in order to take into account uncertainties of model and modelization for the dynamic systems in structural mechanics. The non-parametric approach is specific with the resolution on modal base of the linear systems (out of transient or harmonic) and with the resolution on modal base of the systems with non- localised linearities (out of transient). The parametric approach is a priori universal, but it is more particularly presented if it is combined with the nonparametric approach.

Contents

1 General information

uncertainties, even when they are reduced, can significantly change the prediction of the vibratory behavior of structure ([bib13], [bib16]). It is thus necessary to take them into account in a quantifiable and explicit way to increase the realism and the robustness of the forecasts. In this context, a probabilistic model of uncertainties contributes to the realism of the approach.

The probabilistic approach classical, known as parametric, makes it possible to incorporate in the mechanical analysis uncertainties on the data, that is to say for example parametric uncertainties on the geometry, the boundary conditions or the properties of the materials. In this approach, each identified parameter as source of random uncertainties is modelled by a random variable. The parameters of entry of the model being thus characterized, the probabilistic numerical methods seek to characterize in a probabilistic way the results quantities of the model. For complex structures, for which the vibratory behavior depends on a large number of parameters, this kind of probabilistic analysis is limited by the great amount of information necessary to characterize the parameters of entry and the difficulties of implementation of the propagation of variability.

A new approach, known as probabilistic approach nonparametric of random uncertainties in dynamics of structures was recently proposed by C. Soize ([bib20] with [bib24]). This approach makes it possible and to take into account uncertainties of model (uncertainties on the geometry for example) uncertainties of modelization (uncertainties on the kinematics of beam or plate for example). It is based on the construction of random matrixes of the linear dynamic systems, after projection on modal base.

These two probabilistic approaches, one parametric and other the nonparametric one, are complementary. Thus a mixed, parametric and nonparametric approach, can be is developed (original method having caused publications ([bib6] and [bib11]). In particular, this mixed method is well adapted to the taking into account of uncertainties in the analysis of a nonlinear dynamic system composed of a structure linaire reduced on modal base and of localised non-linearities. Indeed, uncertainties on the level of linear structure can be treated naturally by the nonparametric approach and the uncertainties on non-linearities can be treated naturally by the parametric approach.

The model numerical basic of the nonlinear dynamic system is a model finite elements which will be called “model fine elements layer”. That it is the or not parametric parametric approach, the models of probability must be defined in an adequate way and most objectively possible from this average model. A gaussian model of the matrixes random is not adapted to the dynamics in low frequency (negative eigenfrequencies). In order to build the model of corresponding probability, one uses the principle of the maximum of entropy of Jayne ([bib14], [bib15], [bib17]) as well as information available (models fine elements layer, algebraic properties of the matrixes, etc)

In this document, we present the nonparametric approach for resolutions transient or harmonic of the dynamic system. The parametric approach is more particularly presented if it combined with the non-parametric method.

The readers seeking the fundamental results of the stochastic dynamics will be able to refer and the [18] to readers searching the theoretical details of the probabilistic approach presented in this document will be able to refer to [bib19]. Examples of uses of the approach are given in [bib5], and [bib7]. In [bib10], experimental tests made it possible to show the predictive character of the approach.

2 Modelizations of the dynamic system

2.1 Models finite elements transitory

2.1.1 Resolution layer in absolute coordinates

In the absolute coordinate system, the mechanical system is modelled by the method with the finite elements. This basic model (in general that which would have been used in the deterministic study) is indicated under the name of "model finite elements layer". All the quantities relating to the average models are underlined.

That is to say $t \quad \underline{\mathbf{y}}(t)$ the transient response in the absolute coordinate system of the "model finite elements layer" defined on the interval of study $[0,T]$ and at value in $\mathbb{R}^k$ where k is the number of degrees of freedom. The mass matrixes, of damping and stiffness are respectively noted $[\underline{M}]$, $[\underline{D}]$ and $[\underline{K}]$.
The transient response $\underline{\mathbf{y}}(t)$ of the "model finite elements layer" checks the discretized nonlinear differential equation following:

$$[\underline{M}]\ddot{\underline{\mathbf{y}}}(t)+[\underline{D}]\dot{\underline{\mathbf{y}}}+\mathbf{f_c}(t,\dot{\underline{\mathbf{y}}}(t),\underline{\mathbf{y}}(t),\underline{\mathbf{w}})=\mathbf{f}(t) \quad , \quad T \square \text{ , éq 2.1.1-1}$$

with the initial conditions

$$\underline{\mathbf{y}}(0)=\dot{\underline{\mathbf{y}}}(0)=0 \text{ , éq 2.1.1-2}$$

• $\mathbf{f}(t) \in \mathbb{R}^m$ represent the discretization by finite elements of the external forces.
• $\mathbf{f}_c(t,\underline{\mathbf{y}}(t),\dot{\underline{\mathbf{y}}}(t),\underline{\mathbf{w}})\in\mathbb{R}^m$ corresponds to nonthe localised linearities (for example due to elastic thrusts of shock). The elements $\underline{w}_1,\ldots,\underline{w}_\nu$ of the vector $\underline{\boldsymbol{w}}\in\mathbb{R}^\nu$ represent a set of parameters defining these nonlinearities (for example clearance, stiffness of shock, damping of shock, etc).

2.1.2 Transitory resolution in relative coordinates (seisme)

As in the transitory case in absolute coordinates, the mechanical system is modelled by a basic model, the "model finite elements layer".
One notes $t\rightarrow\underline{\mathbf{z}}(t)$ the transient response in absolute coordinates of this model on the interval of study $[0,T]$ at value in $\mathbb{R}^k$ (**attention:** notice the change of notation compared to the preceding paragraph.).

The transient response of the "model finite elements layer" checks the discretized nonlinear differential equation following:

$$\begin{bmatrix}[\underline{M}] & [\underline{M}_{ls}]\\ [\underline{M}_{ls}]^T & [\underline{M}_s]\end{bmatrix}\begin{bmatrix}\ddot{\underline{\mathbf{z}}}(t)\\ \ddot{\underline{\mathbf{z}}}_s(t)\end{bmatrix}+\begin{bmatrix}[\underline{D}] & [\underline{D}_{ls}]\\ [\underline{D}_{ls}]^T & [\underline{D}_s]\end{bmatrix}\begin{bmatrix}\dot{\underline{\mathbf{z}}}(t)\\ \dot{\underline{\mathbf{z}}}_s(t)\end{bmatrix}+\begin{bmatrix}[\underline{K}] & [\underline{K}_{ls}]\\ [\underline{K}_{ls}]^T & [\underline{K}_s]\end{bmatrix}\begin{bmatrix}\underline{\mathbf{z}}(t)\\ \underline{\mathbf{z}}_s(t)\end{bmatrix}+\begin{bmatrix}F_c(t,\underline{\mathbf{z}}(t),\dot{\underline{\mathbf{z}}}(t);\underline{\mathbf{w}})\\ 0_d\end{bmatrix}=\begin{bmatrix}\mathbf{g}(t)\\ \mathbf{g}_s(t)\end{bmatrix}$$
$$t\in[\,0,T\,]\text{ .éq 2.1.2-1}$$

with the initial conditions,

$$\underline{\mathbf{z}}(0)=\dot{\underline{\mathbf{z}}}(0),\underline{\mathbf{z}}_s(0)=\dot{\underline{\mathbf{z}}}_s(0) \text{ éq 2.1.2-2}$$

• $g(t)\in\mathbb{R}^m$ represents the discretization by finite elements of the external forces and $\underline{g_s}(t)\in\mathbb{R}^d$ corresponds to the discretization of the reaction forces due to d the conditions of Dirichlet.

• $F_c(t,\mathbf{z}(t),\dot{\mathbf{z}}(t),\underline{\mathbf{w}})\in\mathbb{R}^m$ corresponds to nonthe linearities located with like previously $\underline{\boldsymbol{w}}\in\mathbb{R}^v$ representing a set of parameters defining these nonlinearities.

After static raising, the matric equations [éq 2.1.2-1] and [éq 2.1.2-2] in the absolute coordinate system are rewritten in "relative" coordinates:

$$[\underline{\mathbf{M}}]\ddot{\underline{\mathbf{y}}}(t)+[\underline{D}]\dot{\underline{\mathbf{y}}}+[\underline{K}]\underline{\mathbf{y}(t)}+\mathbf{f_c}(t,\underline{\dot{\mathbf{y}}(t)},\underline{\mathbf{y}(t)},\underline{\mathbf{w}})=\mathbf{f}(t)\ ,\ t\in[0,T]\ \text{.éq 2.1.2-3}$$

$$\underline{\mathbf{y}}(0)=\dot{\underline{\mathbf{y}}}(0)=0\ , \qquad \text{éq 2.1.2-4}$$

• $\underline{\mathbf{y}}(t)\in\mathbb{R}^m$ are the vector of the free degrees of freedom in the coordinate system "relative" such as

$\underline{\mathbf{z}}(t)=\underline{\mathbf{y}}(t)+[\underline{R}]\underline{\mathbf{z}}_s(t)$ with $[\underline{R}]=-[\underline{K}]^{-1}[\underline{K_{ls}}]$

•the function $t\to\underline{\mathbf{f}}(t)$ defined on $[0,T]$ and with value in $\mathbb{R}^m$ and the nonlinear application $(\mathbf{x},\mathbf{y})\quad\mathbf{f_c}(t,\mathbf{x},\mathbf{y}\ ;\ \underline{\mathbf{w}})$ of $\mathbb{R}^m\times\mathbb{R}^m$ in $\mathbb{R}^m$ are such as:

$$\mathbf{f}(t)=\mathbf{g}(t)-([\underline{M}][\underline{R}]+[\underline{M_{ls}}])\ddot{\underline{\mathbf{z}}}(t)-([\underline{D}][\underline{R}]+[\underline{D_{ls}}])\dot{\underline{\mathbf{z}}}(t) \qquad \text{éq 2.1.2-5}$$

$$\mathbf{f_c}(t,\mathbf{x},\mathbf{y}\ ;\ \underline{\mathbf{w}})=F_c(t,\mathbf{x}+[\underline{R}]\underline{\mathbf{z}}_s(t),\mathbf{y}+[\underline{R}]\dot{\underline{\mathbf{z}}}_s(t)\ ;\ \underline{\mathbf{w}}). \qquad \text{éq 2.1.2-6}$$

Note:

•In the continuation, according to whether or not a static raising were carried out, $\underline{\mathbf{y}}(t)$ corresponds either to the transient response in absolute coordinates defined by [the § 2.1.1], or the transient response in "relative" coordinates defined by [§ 2.1.2].
•It is supposed that if d the conditions of Dirichlet were homogeneous no rigid body motion could occur. Consequently, $[\underline{K}]$ is symmetric definite positive and its reverse $[\underline{K}]^{-1}$ is defined, which makes it possible to introduce the real matrix $[\underline{R}]=-[\underline{K}]^{-1}[\underline{K_{ls}}]$ of dimension $(m\times d)$.
•In Code_Aster the term of damping in [éq 2.1.2-5] is neglected.

2.1.3 Harmonic resolution

As in the transitory case, the mechanical system is modelled by a basic model, the "model finite elements layer". On a frequential tape $[\omega_1,\omega_2]$, the harmonic response $\mathbf{q}(\omega)$ of the "model finite elements linear layer" checks the following equation:

$$(-\omega^2[\underline{M}]+i\omega[\underline{D}]+[\underline{K}])q(\omega)=F(\omega)\quad,\quad\omega\in[\omega_1,\omega_2] \qquad \text{éq 2.1.3-1}$$

with $\mathbf{f}(\omega)$ representative discretization by finite elements of the external forces.

2.2 Model matric average tiny room

One supposes that the energy of vibration of the dynamic response is mainly localised in the field of the low frequencies. One can thus the model build matric average tiny room while projecting $\underline{\mathbf{y}}(t)$ or $\underline{y}(\omega)$ on under clean space generated by N first modes of the system dynamic linear (infinite clearances) conservative homogeneous (blocked supports) associated which are written,

$$[\underline{K}]\underline{\varphi}=\lambda[\underline{M}]\varphi \ \text{.éq} \qquad \text{the 2.2-1}$$

matrixes $[\underline{M}]$ and $[\underline{K}]$ being definite positive (for $[\underline{K}]$ cf notices 2 [§2.1.2]), the eigenvalues are real and positive,

$$0 \leq \underline{\lambda_1} \leq \underline{\lambda_2} \leq \ldots \leq \lambda_n \qquad \text{éq the 2.2-2}$$

associated eigen modes of vibration $\{\phi_1, \phi_2, \ldots\}$ check the properties of orthogonality,

$$<[\underline{M}]\varphi_\alpha, \varphi_\beta> = \underline{\mu_\alpha}\,\delta_{\alpha\beta} \qquad \text{éq 2.2-3}$$

$$<[\underline{K}]\varphi_\alpha, \varphi_\beta> = \underline{\mu_\alpha}\,\underline{\omega_\alpha^2}\,\delta_{\alpha\beta} \qquad \text{éq 2.2-4}$$

with

$$\omega_\alpha = \sqrt{(\lambda_\alpha)} \qquad \text{éq 2.2-5}$$

One notes the generalized mass matrix respectively, the generalized stiffness matrix and damping stamps it generalized by:

$$[\underline{M_n}] = [\underline{\Phi_n}]^T [\underline{M}] [\underline{\Phi_n}] \qquad \text{éq 2.2-6}$$

$$[\underline{K_n}] = [\underline{\Phi_n}]^T [\underline{K}] [\underline{\Phi_n}] \qquad \text{éq 2.2-7}$$

$$[\underline{D_n}] = [\underline{\Phi_n}]^T [\underline{D}] [\underline{\Phi_n}] \qquad \text{éq 2.2-8}$$

2.2.1 Resolution out of transient

projection $\underline{y^n}(t)$ of $\underline{y}(t)$ on under space generated by N first modes of the associated homogeneous conservative system dynamic linear is written:

$$\underline{\mathbf{y^n}}(t) = [\underline{\Phi_n}]\underline{\mathbf{q^n}}(t) = \sum_{\alpha=1}^{n} \underline{q_\alpha^n}(t)\,\underline{\phi_\alpha}\,, \qquad \text{éq 2.2.1-1}$$

generalized displacements are solutions of the average matric model reduced (dynamic system not linaire),

$$[M_n]\ddot{q}^n(t) + [D_n]\dot{q}^n(t) + [K_n]\,q^n(t) + F_c^n(t, q^n(t), \dot{q}^n(t)\ ;\ W) = F^n(t)\,, \qquad \text{éq 2.2.1-2}$$

$$\underline{\dot{q}^n}(0) = \underline{q^n}(0) = 0\,, \qquad \text{éq 2.2.1-3}$$

with

$$\mathbf{F^n}(t) = [\underline{\Phi_n}]^T \mathbf{f}(t)\ , \qquad \text{éq 2.2.1-4}$$

$$\mathbf{F_c^n}(t, \mathbf{q}, \mathbf{p}\ ;\ \underline{\mathbf{w}}) = [\Phi_n]^T \mathbf{f_c}(t, [\Phi_n]\,\mathbf{q}, [\Phi_n]\,\mathbf{p}, \underline{\mathbf{w}}) \qquad \text{éq 2.2.1-5}$$

2.2.2 Resolution in harmonic

projection $\underline{y^n}(\omega)$ of $\underline{y}(\omega)$ on under space generated by N first modes of the associated homogeneous conservative system dynamic linear is written

$$\mathbf{q^n}(\omega) = \text{éq}\ [\Phi_n]^T \mathbf{f}(\omega)\,, \qquad 2.2.2\text{-}1$$

generalized displacements $\mathbf{q^n}(\omega)$ are solutions of the average matric model reduced

$$(-\omega^2[\underline{M_n}] + i\omega[\underline{D_n}] + [\underline{K_n}])\,\mathbf{q^n}(\omega) = \mathbf{F^n}(\omega) \qquad \text{éq 2.2.2-2}$$

with

$$\mathbf{F^n}(\omega) = [\underline{\Phi_n}]^T \mathbf{f}(\omega)\ , \qquad \text{éq 2.2.2-3}$$

3 Models probabilistic

3.1 Introduction of the probabilistic model into the dynamic problem

In order to and the take into account uncertainties of modelization uncertainties on the data, a mixed probabilistic formulation nonparametric – parametric is used. For that, the vector of n the generalized degrees of freedom $\mathbf{q}^{\mathbf{n}}(t)$ (resp. $\mathbf{q}^{\mathbf{n}}(\omega)$) is replaced by a random variable $Q^n(t)$ (resp. $Q^n(\omega)$).

Out of transient, the stochastic process $t \to \mathbf{Q}^{\mathbf{n}}(t)$ indexed by $[0,T]$ and with value in R^n is solution of the nonlinear dynamic system,

$$[M_n]\ddot{Q}^n(t)+[D_n]\dot{Q}^n(t)+[K_n]Q^n(t)+F_c^n(t,Q^n(t),\dot{Q}^n(t)\ ;\ W)=F^n(t), \qquad \text{éq 3.1-1}$$

$$\dot{Q}^n(0)=Q^n(0)=0\,, \qquad \text{éq 3.1-2}$$

and in harmonic, the stochastic process $t \quad Q^n(\omega)$ indexed on $[\omega_1,\omega_2]$ and with value in $\mathbb{R}^n$ is solution of the system:

$$(-\omega^2[\underline{M}_n]+i\omega[\underline{D}_n]+[\underline{K}_n])\,Q^n(\omega)=F^n(\omega) \qquad \text{éq 3.1-3}$$

where, in the two transitory and harmonic cases $[\underline{M}_n]$ $[\underline{D}_n]$, and $[\underline{K}_n]$ is random full positive matrixes real symmetric definite and where W is a random variable with value in $\mathbb{R}^\nu$. The introduction of random matrixes into the equations [éq 3.1-1] and [éq 3.1-3] makes it possible to model random uncertainties associated with the linear part of the dynamic system. The random variable W with vectorial value introduced into the equation [éq 3.1-1] makes it possible to model random uncertainties concerning the parameters of nonthe linearities of shock.

The parametric probabilistic approach and the nonparametric probabilistic approach introduce random matrixes (, and) and a random variable W whose models of probability are a priori nonknown. The choice of a probabilistic model rather than another must rest only on information available (algebraic properties of the generalized matrixes, mean values of the parameters and the generalized matrixes, etc). In order to objectively build the models of probability of the probabilistic model of uncertainties, ([bib20] with [bib24]), the principle of the maximum of entropy ([bib14], [bib15], [bib17]) is used with a system of stresses defined by this information available. The information available and the model probabilistic which results from this are presented in the next paragraph.

3.2 Model probabilistic for the matrixes of the dynamic system (nonparametric uncertainties)

3.2.1 Information available on the matrixes of the dynamic system

The model probabilistic nonparametric is built in substituent the matrixes $[\underline{M}_n]$ $[\underline{D}_n]$, and $[\underline{K}_n]$ by respectively noted random matrixes $[\mathbf{M}_n]$ $[\mathbf{D}_n]$, and $[\mathbf{K}_n]$. So that the probabilistic dynamic system thus built either mechanically and statistically correct, the construction of the random matrixes $[\mathbf{M}_n]$, $[\mathbf{K}_n]$ and $[\mathbf{D}_n]$ must be such as:

1) $[\mathbf{M}_n]$, $[\mathbf{K}_n]$ and $[\mathbf{D}_n]$ are of the random variables of the second order with values in all the positive definite real matrixes symmetric and dimension $(n \times n)$.

$[\mathbf{M}_n]$, $[\mathbf{K}_n]$ and $[\mathbf{D}_n] \in M_n^+$ P.S. (almost surely), éq 3.2.1-1 where

is M_n^+ all the positive definite symmetric matrixes real of dimension. $(n\times n)$ *This* algebraic property is absolutely required to have a random model of equation which corresponds to that of a dynamic system of the second damped order. 2)

Mean values of the random matrixes, etsont $\left[\mathbf{M}_n\right]$ $\left[\mathbf{K}_n\right]$ $\left[\mathbf{D}_n\right]$ respectively: $\left[\underline{M}_n\right]$ $\left[\underline{K}_n\right]$ $\left[\underline{D}_n\right]$,

$E\left(\left[\mathbf{M_n}\right]\right)=\left[\underline{M}_n\right]$ and $E\left(\left[\mathbf{K_n}\right]\right)=\left[\underline{K}_n\right]$, éq $E\left(\left[\mathbf{D_n}\right]\right)=\left[\underline{D}_n\right]$ 3.2.1-2 where E the expectation indicates. 3)

So tha t the solution of the probabilistic dynamic system is also a variable of the second order, one imposes at the times of the second order norms of Frobenius of the matrixes opposite, etd $\left[\mathbf{M}_n\right]^{-1}$ $\left[\mathbf{K}_n\right]^{-1}$ " $\left[\mathbf{D}_n\right]^{-1}$ to be finished: éq

$$E\{\left\|\left[\boldsymbol{M_n}\right]^{-1}\right\|_F^2\}<+\infty \qquad E\{\left\|\left[\boldsymbol{K_n}\right]^{-1}\right\|_F^2\}<+\infty \qquad E\{\left\|\left[\boldsymbol{D_n}\right]^{-1}\right\|_F^2\}<+\infty$$ 3.2.1-3 with

· $\|[A]\|_F=\left(\mathrm{tr}\left([A][A]^T\right)\right)^{1/2}$ Note:

The only property of positivity of the matrixes is not enough and it should be made sure that their opposite are of the second order, from where (3 (a random variable of the second order almost surely invertible does not have in the general case a random variable reverses of the second order). For more details to see [bib19]. Construction

3.2.2 of the probabilistic model by the principle of the maximum D" entropy

the entropy "measures" the level of uncertainty of a model of probability. Thus, if is $p_{[\mathbf{A}]}$ the function of density of probability corresponding to a random matrix (representing $[\mathbf{A}]$ the matrixes, $\left[\mathbf{M}_n\right]$ or $\left[\mathbf{K}_n\right]$) $\left[\mathbf{D}_n\right]$ of model given, then the entropy (or probabilistic uncertainty) of $S(p_{[\mathbf{A}]})$ $p_{[\mathbf{A}]}$ is defined by: éq

$$S(p_{[\mathbf{A}]})=-\int_{M_n^+} p_{[A]}([A])\ln(p_{[A]}([A]))d\tilde{A}$$ 3.2.2-1

the principle of the maximum of entropy of Jayne consists in building the function of density of probability which maximizes the probabilistic entropy while checking a system of stresses. In this case, the system of stresses is defined by information available corresponding to the equations [éq 3.2.1-1] with [éq 3.2.1-3]. For the random matrix, this system of stresses is written P.S.

$[\mathbf{A}]\in M_n^+$, .éq $E\{[\mathbf{A}]\}=[\underline{A}]$ $E\{\left\|\left[\boldsymbol{A}\right]^{-1}\right\|_F^2\}<+\infty$ 3.2.2-2 One shows

whereas the random matrix is such as $[\mathbf{A}]$ (see [bib20] with [bib24]) éq 3.2

$$[\mathbf{A}]=[\underline{L_A}]^T[\boldsymbol{G_A}][\underline{L_A}]$$ .2-3 where

is $[\underline{L_A}]$ the lower triangular matrix resulting from the factorization of Cholesky of the average matrix and where $[\mathbf{A}]$ the function of density of probability of the random matrix is defined $[\boldsymbol{G_A}]$ on the group compared to the M_n^+ measurement such as $\tilde{d}A$: éq 3.2

$$\tilde{d}G=2^{n(n-1)/4}\Pi_{1\le i\le j\le n}dG_{ij}$$ .2-4 éq 3.2

$$p_{[G_A]}([G]) = \mathbf{1}_{\mathbf{M}_n^+}([G]) \times C_{G_A} \times (\det[G])^{(1-\delta_A^2)(2\delta_A^2)^{-1}(n+1)} \times e^{-(n+1)(2\delta_A^2)^{-1} tr[G]}$$ .2-5 with

éq 3.2

$$C_{G_A} = \frac{(2\pi)^{-n(n-1)/4} \left(\frac{n+1}{2\delta_A^2} \right)^{n(n+1)(2\delta_A^2)^{-1}}}{\Pi_{j=1}^{n} \Gamma\left(\frac{n+1}{2\delta_A^2} + \frac{1-j}{2} \right)}$$ .2-6 where

, éq $\Gamma(z) = \int_0^{+\infty} t^{z-1} e^{-t} dt$ 3.2.2-7 and where

is $\mathbf{1}_{\mathbf{M}_n^+}$ the indicating function of, and M_n^+ where the parameter controlling δ_A the dispersion of the random matrix is defined $[\mathbf{A}]$ by: , éq

$$\delta_A = \left\{ \frac{E\left\{ \left\| [G_A] - [\underline{G}_A] \right\|_F^2 \right\}}{\left\| [\underline{G}_A] \right\|_F^2} \right\}^{1/2}$$ 3.2.2-8 the theoretical

construction of the model provides an acceptable limit for the level of uncertainty introduced. must be selected so that, éq

$$0 < \delta_A < \sqrt{\frac{n_0+1}{n_0+5}}$$ 3.2.2-9 where

is $n_0 \in \mathbb{N}$ a constant of the probabilistic model selected so that. One shows

moreover than, under the only stresses of the equations [éq 3.2.1-1] to [éq 3.2.1-3], the principle of the maximum of entropy leads so that the random matrixes, or are statistically independent as a whole. This probabilistic

model for the positive definite symmetric matrixes random real and the differs from the more classical models of the random matrixes based on the Gaussian Sets Circular sets (references in [bib19]). The orthogonal gaussian whole used in addition for fields high frequencies would lead in the low frequencies field (in which one places oneself) to negative eigenvalues, which one cannot admit for the systems considered. Moreover, one matrix of the orthogonal whole gaussian does not have in the general case an opposite matrix of the second order, which would lead to a solution of the dynamic system of infinite variance, which one cannot admit either. Model

3.3 probabilistic for the real variables (parametric uncertainties) Information

3.3.1 available on the real variables In

the mixed probabilistic approach, the parametric probabilistic modelization consists in substituting the parameter of not $\underline{w}$ - linearities in the nonlinear dynamic systems given by [éq 2.1.1-1] or [éq 2.1.2-3] by a noted random variable. In $W = (W_1, \ldots, W_\nu)$ a purely parametric approach (i.e without making random the matrixes of the system dynamic), the parametric probabilistic modelization consists in substituting certain parameters of the matrixes $\underline{w}$, and $[\underline{M}_n(\underline{w})]$ the average $[\underline{K}_n(\underline{w})]$ $[\underline{D}_n(\underline{w})]$ dynamic system reduced by a random variable. These parameters W can be for example parameters of the material. It is supposed

that the components of are W independent real random variables between them and independent of the random matrixes of the dynamic system. In the continuation, to reduce the writing, an unspecified coordinate W is noted. The construction W_j of the probabilistic model requires to define the

information available, which constitutes a system of stress under which the entropy of the density of probability of the random variable is maximized W . Information

available is the following one: 1) The support

of the random variable is an interval W of éq 3.3 D .1 $\mathbb{R}^{\nu}$

$W \in D$, $p.s.$. - 1 2) The mean value

of the random variable is: . W éq 3.3 $\underline{w}$

$$E[W] = \underline{w} \quad .1\text{-}2$$

3) Possibly

, according to information indeed available, the moment of the second order of the random variable is finished $|W|^{-1}$: éq 3.3.1

$$E[|W|^{-2}] < +\infty \quad .$$

- 3 Construction

3.3.2 of the probabilistic model by the principle of the maximum of entropy If of

density of p_W probability corresponding to the random variable then the entropy (probabilistic W uncertainty) is the function of $S(p_W)$ is defined p_W by: , éq 3.3

$$S(p_W) = -\int_{-\infty}^{+\infty} p_W(w) \ln(p_W(w)) dw \quad .2\text{-}1$$

By means of

the principle of the maximum of entropy, one obtains three densities of probability according to nature of the support and according to whether D the stress corresponding to the equation [éq 3.2.2-6] is considered or not. Closed

3.3.3 support limited without information on the reverse If there exist

two realities and such as a b and if $D = [a, b]$ the information available is given by the equations [éq 3.3.1-1] and [éq 3.2 .2-5], then the random variable follows W an exponential model truncated whose function of density of probability is: éq 3.3.3

$$p_W(w) = 1_{[a,+\infty]}(w) \frac{k}{\alpha(k)} \exp(-kw)$$

- 1 where is

the indicating $1_{[a,b]}$ function of and where and are $\alpha(k)$ such as k : éq 3.3.3

$$(\underline{w_k} - 1)\alpha(k) - k\beta(k) = 0$$

- 2 with, éq

3.3

$$\alpha(k) = e^{-ak} - e^{-bk} \quad .3\text{-}3$$

and éq 3.3

.3

$$\beta(k) = a e^{-ak} - b e^{-bk}$$

- 4 closed

3.3.4 semi Support not limited without information on the reverse If there exists

a reality such as a and if $D = [a, +\infty[$ the information available is given by the equations [éq 3.3.1-1] and [éq 3.3.1-2] then the random variable follows W an exponential model whose function of density of probability is: , éq 3.3

$$p_W(w) = 1_{[a,+\infty]}(w) \frac{1}{\underline{w} - a} \exp\left(-\frac{w-a}{\underline{w}-a}\right) \quad .4$$

- 1 where is

the indicating $1_{[a,+\infty]}$ function of. Closed $[a, +\infty[$

3.3.5 semi support not limited with information on the reverse If there exists

a reality such as a and if $D=[a,+\infty[$ the information available is given by the equations. [éq 3.3.1-1], [éq 3.3.1-2] and [éq 3.3.1-3], then the random variable follows W a model gamma whose function of density of probability is, éq 3.3

$$p_W(w)=1_{[a,+\infty]}(w)\frac{(\underline{w}\,\delta^2-a\,\delta^2)^{-1/\delta^2}}{\Gamma(1/\delta^2)}(w-a)^{(1-\delta^2)/\delta^2}\exp\left\{-\frac{w-a}{(\underline{w}-a)\,\delta^2}\right\} \quad .5\text{-}1$$

where is a parameter δ controlling the level of uncertainty of the random variable which is written (in a way W similar to the nonparametric case [éq 3.2.2-8]): , éq *3.3*

$$\delta=\left\{\frac{E\left[(w-\underline{w})^2\right]}{\underline{w}^2}\right\}^{1/2} \quad .5\text{-}2$$

Construction

3.4 of the stochastic response and the statistics associated transitory Case

3.4.1 stochastic

3.4.1.1 Transient response the excitations

with the dynamic system are supposed to be deterministic, but in the paragraph [§3.1], of the matrixes and the random parameters were introduced into the model matric tiny room. Therefore, the transient response is a non $t\to Q_n(t)$ steady stochastic process indexed by with value $[0,T]$ in (by means of $\mathbb{R}^n$ some additional assumptions of existence, unicity and regularity of the deterministic solution, cf [bib19]). Consequently

, to the vector of the free m degrees of freedom corresponds $\underline{\mathbf{y}}^{\mathbf{n}}(t)$ the stochastic process indexed $Y^n(t)$ by and with value $[0,T]$ in such as $\mathbb{R}^m$ éq 3.4.1.1

$$Y^n(t)=[\underline{\Phi}_n]Q^n(t), \quad \text{- 1}$$

In the case of the transition in relative coordinates, to the stochastic process indexed $t\to\mathbf{Y_n}(t)$ by and with value $[0,T]$ in defined $\mathbb{R}^m$ by the equation [éq 3.4.1.1 - 1] corresponds the stochastic process of the free d.d.l $t\to\mathbf{Z_n}(t)$ of structure in absolute coordinates indexed by and with value in such as .éq $\mathbb{R}^m$ 3.4.1.1 -

$$\mathbf{Z_n}(t)=\mathbf{Y_n}(t)+[R]\mathbf{z_s}(t) \quad 2$$

elastic

3.4.1.2 Response sprectrum One notes

the component ème $Z_j^n(t)$ j of the vector corresponding $\mathbf{Z_n}(t)$ to a random realization of the stochastic response of the ème free j degree of freedom of structure. perhaps characterized $Z_j^n(t)$ by its elastic response sprectrum (also called oscillator spectrum in the documentation of Code_Aster) which *one notes* where and are $S_j(\xi,\omega)$ respectively ξ ω the associated rate of depreciation and the pulsation. With reasonable
assumptions, in particular on the regularity of the nonlinear application, one can show $(t,\mathbf{q},\mathbf{p}\ ;\ \mathbf{w})$ $\mathbf{F_c^n}(t,\mathbf{q},\mathbf{p}\ ;\ \mathbf{w})$ that is a process of the second order whose trajectories are almost surely continuous. Consequently, for all fixed in ξ a given interval, is J_ξ a stochastic $\omega\to S_j(\xi,\omega)$ process indexed on the tape of analysis to value in J_ω . It is admitted that $\mathbb{R}^+$ this process is of the second order, i.e.: .éq 3.4.1.2 -

$$E\{S_j(\xi,\omega)^2\}<+\infty, \quad \forall\,\omega\in J_\omega \quad 1$$ harmonic Case

3.4.2 In the paragraph

[§3.1], of the matrixes and the random parameters was introduced into the model matric tiny room. The harmonic response is thus a stochastic t $Q^n(\omega)$ process indexed on with value in $[\omega_1,\omega_2]$. Consequently R^n

, to the vector of the free m degrees of freedom corresponds $\underline{y}^n(\omega)$ the stochastic process indexed on $Y^n(\omega)$ and with value $[\omega_1,\omega_2]$ in such as éq 3.4 R^m .2-1

$$Y^n(\omega)=[\underline{\Phi}_n]\ Q^n(\omega)$$ the component ème

$Y_j^n(\omega)$ j of the vector is a random variable $Y^n(\omega)$ which one will admit of the second order. Construction

3.4.3 of the response stochastic by the Monte Carlo method Choice and put

3.4.3.1 in work of the Monte Carlo method the responses

and the response sprectrums correspond to strongly nonlinear transformations of the random matrixes and random parameters which result from the probabilistic modelization of uncertainties. Moreover, one can of course build only numerical approximations as of these responses and these response sprectrums. The statistics (first statistical moments, probability of going beyond a threshold,…) are written formally as multiple integrals of very great dimension because the number of random variables of the probabilistic model is by construction high. Lastly, the number of quantities observed is very large (several d.o.f. for several frequencies). For all these reasons, the most adapted method to compute: probabilistic solution (stochastic response and associated statistics) is the method of computational simulation of Monte Carlo. The method of

simulation of Monte Carlo has the advantage of giving results which one can control the accuracy (checking of convergence, cf [§4.1]), contrary to the majority of the methods based on approximations. It can be expensive in computing times, but the use of the techniques of reduction of the variance can make it possible to reduce the number of simulations necessary (cf [bib8] or [bib9]). The implementation

of the Monte Carlo method consists for the problem which concerns us to generate achievements n_s of the random matrixes, and of the dynamic $[\mathbf{M}_n]$ $[\mathbf{K}_n]$ system $[\mathbf{D}_n]$ and/or achievements n_s of the vectorial random variable. The resolutions W of the deterministic dynamic system for each achievement n_s of (,) produce $[\mathbf{M}_n]$ $[\mathbf{K}_n]$ $[\mathbf{D}_n]$ W achievements n_s of the stochastic process solution (resp.) and t $Q^n(t)$ in consequence of t $Q^n(\omega)$, of and ($t\to\mathbf{Y_n}(t)$ resp. $t\to\mathbf{Z_n}(t)$). $\omega\to S_j(\xi,\omega)$ The generation ω $Y_j^n(\omega)$ of the random matrixes is treated in the following paragraph; the generation of the random variable is more classical W and is not recalled. Generation

3.4.3.2 of the pseudo-random matrixes In order to generate

the achievements of the random matrix, one uses $[G_A]$ the following algebraic representation of the random matrix of which the model of $[G_A]$ probability is defined by the equations [éq 3.2.1 - 2], [éq 3.2.2-1]: , éq 3.4.3.2 -

$$[G_A]=[L]^T[L] \quad 1$$

the triangular random matrix being such as $[L]$: The random variables

- •are independent $[L]_{ij}, i \le j$. For, the real
- • $i<j$ random variables are written $[L]_{ij}$ where and where $[L]_{ij}=\sigma_n U_{ij}$ is $\sigma_n=\delta_A(n+1)^{-1/2}$ a gaussian U_{ij} real random variable of average 0 and variance 1. For, the real
- • $i=j$ random variables are written $[L]_{ij}$ where is defined $[L]_{ij}=\sigma_n(2V_j)^{1/2}$ previously σ_n and where is variable V_j real positive random of model gamma whose function of density of probability compared to the $p_{v_j}(v)$ measurement is written: dv , éq 3.4.3.2 -
- •

$$p_{v_j}(v)=\frac{\mathbf{1}_{[0,+\infty[}(v)}{\Gamma((n+1)/(2\delta_A^2)+(1-j)/2)}v^{(n+1)/(2\delta_A^2)+(1-j)/2}e^{-v} \quad 2$$

where is the indicating function of. Statistics $\mathbf{1}_{[0,+\infty[}$ $[0,+\infty[$

3.4.4 on the spectrums In this chapter

, one presents the definition of the statistics of the response sprectrums elastic, in the case of $S_j(\xi,\omega)$ a transitory resolution. In the harmonic case, the statistics on the random variables are defined $Y_j^n(\omega)$ in the same way and are thus not presented. Estimate of

3.4.4.1 the quantiles For all,

is a random variable $(\xi,\omega)\in J_\xi \times J_\omega$ $S_j(\xi,\omega)$ with value in. One seeks $\mathbb{R}^+$ to consider the quantile associated with the probability noted and defined α by $S_{j,\alpha}(\xi,\omega)$: éq 3.4.4.1 - 1

$$S_{j,\alpha}(\xi,\omega)=F^{-1}_{\xi,\omega}(1-\alpha)$$

where is the function $F_{\xi,\omega}$ of unknown distribution of. That is to say the sample $S_j(\xi,\omega)$

$(S_j(\xi,\omega;\theta_1),\ldots,S_j(\xi,\omega;\theta_r),\ldots,S_j(\xi,\omega;\theta_{n_s}))$ made up of the achievements n_s

of and the associated $S_j(\xi,\omega)$ $(S_j(\xi,\omega;\theta_{(1)}),\ldots,S_j(\xi,\omega;\theta_{(r)}),\ldots,S_j(\xi,\omega;\theta_{(n_s)}))$ ordered sample. A natural

estimator of the quantile for, is $S_{j,\alpha}(\xi,\omega)$: éq $\alpha=\frac{r}{n_S}$ 3.4.4.1 $1\le r\le n_S$ - 2

$$\hat{S}_{j,\frac{r}{n_S}}(\xi,\omega)=S_j(\xi,\omega;\theta_{(r)})$$

to obtain

a more robust estimator of the quantile, one can “realize” the estimator on several series of achievements n_s . If the desired probability is such as, or if one $\alpha<1/n_S$ wishes to reduce the number of simulations, it is possible to use more sophisticated estimators, for example by supposing

that the function of distribution belongs to $F_{\xi,\omega}$ a field of attraction given (extreme theory of value) or for example by means of a method of regularization bayésienne (cf [bib12]). Extreme values

3.4.4.2 of sample For a sample

of achievements n_s of noted one defines $S_j(\xi,\omega)$ $S_j(\xi,\omega;\theta_1)$, ... , $S_j(\xi,\omega;\theta_{n_s})$ the extreme values of sample by: éq 3.4.4.2 - 1

$$\omega \quad dB_{j,min}(\xi,\omega;n_S)=\log_{10}\left(\min_{r=1,\ldots,n_S} S_j(\xi,\omega;\theta_r)\right)$$ éq 3.4.4.2 - 2

$$\omega \quad dB_{j,max}(\xi,\omega;n_S)=\log_{10}\left(\max_{r=1,\ldots,n_S} S_j(\xi,\omega;\theta_r)\right)$$ "Field of confidence

3.4.4.3 " established from the inequality of Tchebychev For a sample

of achievements n_s of the process noted,... ω $S_j(\xi,\omega)$, one ω $S_j(\xi,\omega;\theta_1)$ can build ω $S_j(\xi,\omega;\theta_{n_s})$ the "field of confidence" of the random variable for all, by means of the inequality of Tchebychev associated on a level with probability: , éq 3.4.4.3 P_C -

$$\text{Proba}\left[dB_j^-(\xi,\omega)<dB_j(\xi,\omega)<dB_j^+(\xi,\omega)\right]\geq P_C$$ 1 where the lower

envelope and the higher envelope are defined by: , éq 3.4.4.3 -

$$dB_j^+(\xi,\omega)=\log_{10}\left(m_{1j}(\xi,\omega)+\frac{\sigma_j(\xi,\omega)}{\sqrt{1-P_C}}\right)$$ 2 .éq 3.4.4.3 -

$$dB_j^-(\xi,\omega)=2\log_{10}\left(m_{1j}(\xi,\omega)\right)-dB_j^+(\xi,\omega)$$ 3 with the average

and the standard deviation of: éq 3.4.4.3 $dB_j(\xi,\omega)$ - 4

$$m_{1j}(\xi,\omega)=E\{\,S_j(\xi,\omega)\}$$.éq 3.4.4.3 -

$$\sigma_j(\xi,\omega)=E\left\{\left(S_j(\xi,\omega)-m_{1j}(\xi,\omega)\right)^2\right\}^{1/2}$$ 5 the "field of

confidence" thus built proved to be a good approximation of the extreme values of sample for the case treated in [bib21]. However, this "field of confidence" utilizes only the first two moments of which consistent estimators more quickly with respect to the number of simulations n_s than the extreme values of sample. It can thus be interesting to rather use this construction of the "field of confidence" than a construction based on the more expensive estimate of the quantiles of many simulations. Note:

The term "field

of confidence", can be regarded by certain as an abuse language. One should rather use the less intuitive terminology "inter-quantiles field". Indeed, in the statistical literature, a confidence interval is theoretically the interval in which is the true value of a parameter of a random variable (for example its average) with a given probability. This terminology is employed in the very precise frame of the theory of the estimate ensemblist. The confidence interval is not a characterization of the variability of a random variable, contrary to a standard deviation or quantiles. One nevertheless uses " field of confidence" with parsimony in the continuation, because it is certainly a little more speaking for the non-specialists of the statistics. Put in work

4 in Code_Aster Study *of the stochastic*

4.1 convergence of the digital model transitory Case

4.1.1 the convergence

of the stochastic solution must be studied compared to the number of modes and n the number of simulations n_s of Monte Carlo. As the stochastic solution is a process of the second order (by assumption, cf [§ 3.4.1.2]), its convergence can be analyzed by studying the applications such as: , éq 4.1.1-1

$$|||\ddot{Z}_j^n|||^2 = \int_0^T E\left[\ddot{Z}_j^n(t)\right]^2 dt$$ where is a stochastic

process of the second order indexed by and with value in representative R the acceleration of the ème degree of freedom j of structure. In the frame of
simulations of Monte Carlo, this norm is considered for set from n a set of random n_s achievements by the approximation with éq 4.1.1 - 2

$$\mathrm{conv}_j(n, n_S)^2 = \int_0^T \left(\frac{1}{n_S} \sum_{i=1}^{n_S} \ddot{Z}_j^n(t;\theta_i) \right)^2 dt$$ the stochastic

convergence of the model is thus analyzed according to the dimension of the scale model (i.e. the number of mode of under clean n space of the model finite elements layer on which the stochastic nonlinear dynamic system was project in the paragraph [§2.2]) and the number of simulations n_s of Monte Carlo by studying the function. Harmonic case

4.1.2 convergence

in the case of a transitory resolution can transpose a harmonic resolution directly in the case of, with the norm: , éq 4.1.2-1

$$|||Z_j^n|||^2 = \int_{\omega_1}^{\omega_2} E\left[Z_j^n(\omega)\right]^2 d\omega$$ Choices of the parameters

4.2 of dispersion to use

the method, the parameters of dispersion must be δ built-in. Two approaches can be a priori used to fix the value of these parameters. The first

approach consists in identifying the value of the parameters for a given δ structure or a structure class using suitable methods. For that, one can use experimental results of the dynamic responses of structure. One can also use computational simulations built by means of a parametric approach of uncertainties. In this last case, it should be noted that only the errors on the data of the model are taken into account, since the errors of modelization cannot be taken into account by the parametric approach. The second

approach consists in not fixing a priori *a fixed value* of the parameters but varying them δ in a given beach (only 3 scalars to vary for the mass matrixes, of stiffness and damping on the NON-parametric part in comparison with the very a large number of parameters to vary simultaneously in a classical parametric study). This approach makes it possible to carry out a total analysis of sensitivity to uncertainties. In the case of absence of objective information on the parameters dispersion to be chosen, it is preferable to use such an approach. The non-parametric method suggested then seems a robust and simple approach of analysis of sensitivity to uncertainties. Main steps

4.3 the implementation

in Code_Aster is *made up* of three main steps: the construction of the average reduced matric model, the generation of the achievements of the response seen like a stochastic process, and finally the

statistical postprocessing of these achievements. The two last stages constitute in fact the method of computational simulation of direct Monte Carlo. Stage 1: construction

of the matric model reduced average The model matric
average tiny room is built using a classical sequence operators depending on the precise analysis carried out whose principal ones can be: ASSE_MATRICE, MODE_ITER_SIMULT , MODE_STATIQUE , CALC_CHAR_SEISME , PROJ_BASE… Stage 2 : generation

of the achievements of the transient response the achievements
n_s of the stochastic transient response are calculated in a loop in language Python made up of: Generation

•of the ième achievements p of the random generalized matrixes of mass, stiffness and damping by genE_matR_alea (DOC. [U4.36.06]). These matrixes are not diagonal and thus require a full storage. Generation
•of the ième achievements p of the random variables of the parameters of non-linearities by genE_VARI_alea (DOC. [U4.36.07]). Computation of
•the ième realization p or solution $Q^n(t\,;p)$ of the stochastic $Q^n(\omega\,;p)$ matric system. This realization s is the solution of the classical matric system whose matrixes and the second members are the achievements previously generated. The computation is thus carried out by dyna_tran_modal OR DYNA_LINE _HARM (with matr _asse_GENE _R and vect_asse _GENE as starter). 1 - Extraction
•of the temporal observations of the preset physical degrees of freedom (for example or, but also $\ddot{Z}_i^n(t\,;p)$ $Y_j^n(\omega\,;p)$ possibly fields of displacement, velocity, stresses, etc) via recu_fonction (AFTER A REST _GENE_PHYS for). 2 - Computation $Y_j^n(\omega\,;p)$ of
the spectrums corresponding (by calc_fonction (SPEC_OSCI) FOR AND CALC _fonction $\omega \to S_j(\xi,\omega\,;p)$ (MODULE) FOR). EVALUATING $Y_j^n(\omega\,;p)$,
•via calc_fonction KEY WORDS COMB or PUISSANCE or ENVELOPPE, of THE CONTRIBUTIONS to the estimators of the averages, the moments of order two, the max. extreme values and min. of sample for the standardized spectrums: . Stage

$$\hat{m}_{1j}(x,w\,;p)=S_j(x,w\,;p)+\hat{m}_{1j}(x,w\,;p-1)$$ 3

$$\hat{m}_{2j}(x,w\,;p)=S_j(x,w\,;p)^2+\hat{m}_{2j}(x,w\,;p-1)$$:

$$\hat{S}_{j,\max}(x,w\,;p)=\mathrm{Max}\left\{S_j(x,w\,;p),\hat{S}_{j,\max}(x,w\,;p-1)\right\}$$ post

$$\hat{S}_{j,\min}(x,w\,;p)=\mathrm{Min}\left\{S_j(x,w\,;p),\hat{S}_{j,\min}(x,w\,;p-1)\right\}$$

- statistical processing the averages,
the standard deviations, the max. extreme values and min. of sample for the standardized spectrums can be evaluated via calc_fonction (COMB): . THE CONFIDENCE

$$m_{1j}(x,w)=\frac{1}{n_s}\hat{m}_{1j}(x,w\,;n_s) \qquad m_{2j}(x,w)=\frac{1}{n_s}\hat{m}_{2j}(x,w\,;n_s)$$

intervals can then be traced starting from the extreme values of sample or the limits obtained by Tchebychev cf [§3.4.4]. In the transitory case

, an example is given by a case test of a flexbeam with nonlinearities of shock, cf Doc. [V5.06.001] [bib1]. Other details are given in Doc. [U2.08.05] [bib2]. Numerical effectiveness

4.4 of the nonparametric approach the non

parametric approach is more economic in computing times than a purely parametric approach in which the parameters of geometry, materials, etc are random variables. In the purely
parametric approach, the model finite elements depends on the dubious parameters. For each simulation of Monte Carlo, the model finite elements is different. One thus needs, for each simulation, compute the elementary matrixes, to carry out the assemblies, to pass in relative coordinates, to solve the problem with the eigenvalues, to project on modal base, to solve the reduced system and to return in physical base then in relative coordinates. In the non

parametric approach, only the reduced system is different with each simulation. It is thus simply necessary, with each simulation, to solve the reduced system and to return in physical base then in relative coordinates. In particular, the resolution of the problem to the eigenvalues of the model average finite elements is carried out once and for all, before simulations of Assembles - Carlo. The saving of time

of computation which results from it is variable, but it can be important. At first approximation, this saving of time computation depends on the ratio between the TEMPS CPU necessary to the resolution with the eigenvalues and the TEMPS CPU necessary to the resolution of the reduced system. The larger this ratio is, the more the nonparametric approach is advantageous compared to the purely parametric approach. In particular, the saving of time of computation can be very important for structures with one very a large number of degrees of freedom and a modal base low-size. Checking

5 the features

and the methods presented in this document are checked by the cases following tests: SDNS01 Models

probabilistic	nonparametric – parametric of a flexbeam with nonlocalised linearities of shock [V5.06.001] SHLS	200 Models
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,	*C. DESCELIERS EDF-R&D/AMA initial Text*	