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Modelization élasto (visco) plastic taking into account of the metallurgical transformations

Summarized:

This document presents the modelization installation in *Code_Aster* for the mechanical analysis of operations generating of the metallurgical transformations. One presents the various mechanical effects resulting from structure transformations to take into account and their modelizations.

Contents

1 Introduction

Certain materials undergo structure transformations when they are subjected to particular thermal evolutions [bib1], [bib2], [bib3]. It is for example the case of the low alloy steels during operations of type welding and heat treatment or the alloys of zircaloy of the fuel sheaths for certain cases of accidental situation (APRP).
These transformations have a more or less strong influence on the thermal and mechanical evolutions.

From a thermal point of view, structure transformations are accompanied by a modification of the thermal characteristics (voluminal heat capacity, thermal conductivity) of the material which undergoes them, as well as production or of an energy absorption (latent heats of transformation) [bib2]. However, the latent heats of transformation in a solid state relatively weak are compared with the latent heats of change of state liquidate-solid and one can thus, at first approximation, regard the thermal and structural evolutions as decoupled. C `is currently the case of the thermal and metallurgical computation options established in *Code_Aster*. [bib16]

From a mechanical point of view, the consequences of structure transformations (at the solid state) are of four types [bib2]:

- the mechanical characteristics of the material which undergoes them are modified. More precisely, the elastic characteristics (modulus of YOUNG and Poisson's ratio) are not very affected whereas plastic characteristics (elastic limit in particular) and the thermal coefficient of thermal expansion are it strongly,
- the expansion or the voluminal contraction which accompanies structure transformations translates by a strain (spherical) “of transformation” which is superimposed on the purely thermal strain of origin. This effect is highlighted on a test of dilatometry and, in general, one gathers it with that due to the modification of the coefficient of thermal expansion and one speaks overall about the influence of the transformations on the thermal strain,
- a transformation being held under stresses can give rise to an unrecoverable deformation and this, even for levels of stresses much lower than the elastic limit of the material (with the temperature and in the structural state considered). One calls “**plasticity of transformation**” this phenomenon,
- one can have at the time of the metallurgical transformation a phenomenon of restoration of hardening. The hardening of the mother phase is not transmitted to the phases lately created. Those can then be born with a virgin state of hardening or inherit only one part, possibly totality, hardening of the mother phase.

In addition, the mechanical state also influences the metallurgical behavior. The stress state can in particular accelerate or slow down the kinetics of the transformations and modify the temperatures to which they occur. However, the experimental characterization of this influence, in particular in the case of complex situations (three-dimensional, under temperature and stress state variables) remains very delicate and it is very frequent to regard the structural evolution as independent of the mechanical state. C `is the case of the model of structure transformations established in *Code_Aster*.

If one neglects the various couplings of mechanical origin, the determination of the mechanical evolution associated with a process bringing into play structure transformations thus requires two successive and uncoupled computations:

- a thermo-metallurgical computation (decoupled) allowing the determination of the thermal evolutions then structural,
- a mechanical computation (élasto-viscoplastic) which takes account of the effects due to the thermal and structural evolutions.

This document presents the mechanical modelization established in *Code_Aster*. The modelization is available for two materials:

- the steel which undergoes around $850°$ a austénito-ferritic transformation (transition of cold α phases of cubic structure face centered (cfc) with a hot γ centered cubic structure phase (cc)). Steel presents 4 possible ferritic phases; ferrite, pearlite, the bainite and martensite,
- the alloys of Zircaloy which undergo around $800°C$ a transformation of cold phase α of compact hexagonal structure to a hot structure β phase cc .

The models are identical for the two materials, only the number of phase changes.

The model thus comprises 5 phases for steel and 3 phases for the zircaloy. The modelization behavior of the zircaloy indeed requires to consider 2 cold phases of different structural mechanics behavior; a phase α considered as pure and a phase α mixed with β [bib16], [bib17]. The various characteristics relative to the various phases are noted:

Steels	Zircaloy
Ferrite: F1_ ***	*** Alpha: F1_ ***
Pearlite: F2_ ***	mixed Alpha: F2_ ***
Bainite: F3_ ***	Beta: C_ ***
Martensite: F4_ ***	
Austenite: Pure C_	

Nota bene:

The metallurgical notions of bases necessary to the comprehension of the general problem are gathered in [bib1].

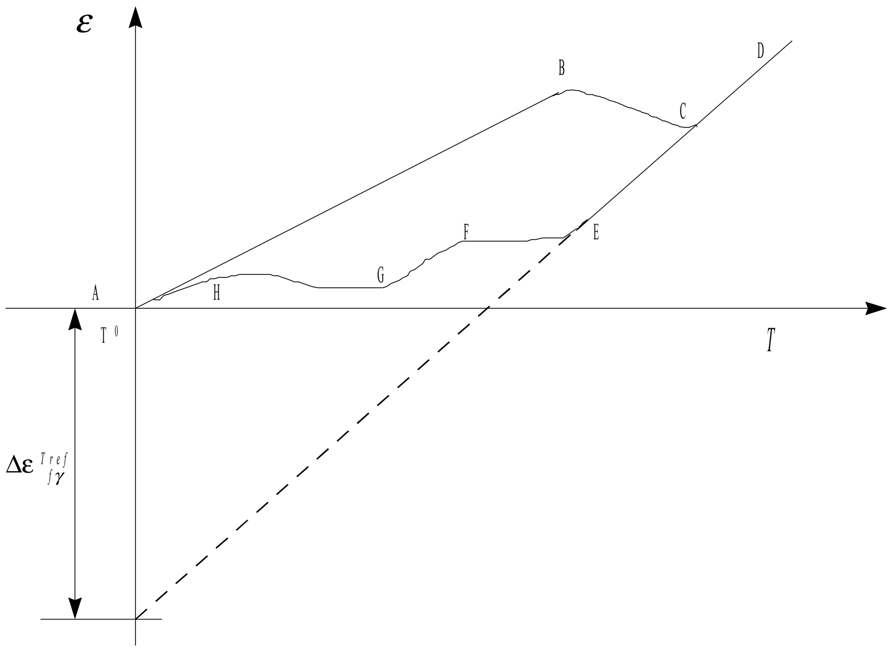
The elastoplastic algorithm of resolution, without taking into account of the effects due to structure transformations is clarified in [bib4].

This document to some extent is extracted from [bib5] and [bib14] where one makes a more detailed presentation of the model and some elements of validation.

The presentation of the models which one makes in this document is mainly illustrated with the case of steel.

2 Influence structure transformations on the thermal strain

a test of dilatometry consists in during measuring the strain (homogeneous) of a low-size test-tube according to the temperature (or of time) an imposed thermal cycle (presumedly identical in all the points of the test-tube). One presents [Figure 2-a] a test of dilatometry of a steel. The thermal cycle comprises a heating beyond the temperature of austenitization (that is to say $850°C$ approximately), then a maintenance with this temperature and, finally, a cooling controlled until the room temperature. One then obtains an evolution of the strain (variable according to the kinetics of cooling imposed) as represented on the figure.



Appear 2-a: Schematic statement of dilatometry

2.1 Zones of thermal strain

the various zones put in obviousness on the figure [Figure 2-a] can be interpreted as follows:

- A-B: thermal thermal expansion of metal in its initial metallurgical structure (of type ferrito - perlitic $(F+P)$, bainitic (B) and/or martensitic (M)) until the initial temperature of austenitization $T(B)$,
- B-C: austenitization and contraction of the test-tube (specific volume of the austenitic phase (γ) smaller),
- CD: thermal thermal expansion of austenite (with a coefficient of thermal expansion different from that of the phases known as " α " (F) (P) (B), (M)),
- OF: thermal of austenite, E-F
- contraction: first transformation (partial) of the austenite (for example $\gamma \rightarrow F+P$) which is accompanied by a voluminal expansion,
- F-G: zone without transformation with thermal contraction of remaining the austenite mixture - formed phase (with a certain apparent thermal coefficient of thermal expansion),
- G-H: second transformation of remaining austenite (for example $\gamma \rightarrow M$) which is accompanied by a voluminal expansion,
- ha: thermal contraction of final structure (with the same coefficient of thermal expansion as with the heating).

2.2 Assumptions and notations

- the structures ferritic, perlitic, bainitic and martensitic have an identical thermal coefficient of thermal expansion (noted α_f) different from that of austenite (noted α_γ).

One defines a state of reference for which one considers that the thermal strain is null: one chooses for that a metallurgical phase of reference (phase austenitic or ferritic phase) and a reference temperature T_{ref} .

- That is to say ε_γ^{th} thermal strain of the austenitic phase, and ε_f^{th} the thermal strain of the phases ferritic, perlitic, bainitic and martensitic, we will take:

$$\varepsilon_f^{\gamma}=\alpha_\gamma(T)\left(T-T_{ref}\right)-\left(1-Z_\gamma^R\right)\Delta\varepsilon_{f\gamma}^{T_{ref}}$$
$$\varepsilon_f^{th}=\alpha_f(T)\left(T-T_{ref}\right)+Z_\gamma^R\,\Delta\varepsilon_{f\gamma}^{T_{ref}}$$

where:

T^{ref} : Reference temperature

$\alpha_\gamma(T)$: average coefficient of thermal expansion of the austenitic phase to the current temperature T, compared to the reference temperature.

$\alpha_f(T)$: average coefficient of thermal expansion of the phases ferritic, perlitic, bainitic and martensitic with the current temperature T , compared to the reference temperature.

Z_γ^R : characterize the metallurgical phase of reference;

Z_γ^R = 1 when the phase of reference is the austenitic phase,

Z_γ^R = 0 when the phase of reference is the ferritic phase.

$\Delta\varepsilon_{f\gamma}^{T_{ref}}=\varepsilon_f^{th}(T_{ref})-\varepsilon_\gamma^{th}(T_{ref})$ translated the difference in compactness between cubic crystallographic structures with centered sides (austenite) and cubic centered (ferrite) with the reference temperature T_{ref} .

That is to say $\mathbf{Z}(M,t)=\{Z_1,Z_2,Z_3,Z_4\}$ respective proportions of ferrite, pearlite, bainite and martensite present in a material point M at time t . With the help of the assumption of a model of mixture to define the thermal strain of a multiphase mixture (characterized by Z) one a:

$$\varepsilon^{th}(\mathbf{Z},t)=\left(1-\sum_{i=1}^{i=4}Z_i\right)\left[\alpha_\gamma(T-T_{ref})-(1-Z_\gamma^R)\Delta\varepsilon_{f\gamma}^{T_{ref}}\right]+\left(\sum_{i=1}^{i=4}Z_i\right)\left[\alpha_f\cdot(T-T_{ref})+Z_\gamma^R\Delta\varepsilon_{f\gamma}^{T_{ref}}\right]$$

For the computation of the thermal strain it is thus necessary to be given:

- the coefficient of thermal expansion of the cold phases,
- the coefficient of thermal expansion of the hot phase,
- a metallurgical phase of reference and a reference temperature,
- the difference in compactness between the hot and cold phase with the reference temperature.

These data are provided by the user in operator `DEFI_MATERIAU` [U4.23.01] under key word `ELAS_META_FO` except the reference temperature which one defines in `AFFE_MATERIAU`.

α Depend on the temperature and are calculated for the temperature of the current Gauss point.

3 Plasticity of transformation

In experiments, one notes that the dilatometric statement of a test-tube in the course of structure transformation is strongly influenced by the stress state and that the application of pressure even lower than the elastic limit of the material can nevertheless cause an unrecoverable deformation (cf [Figure 3-a]).

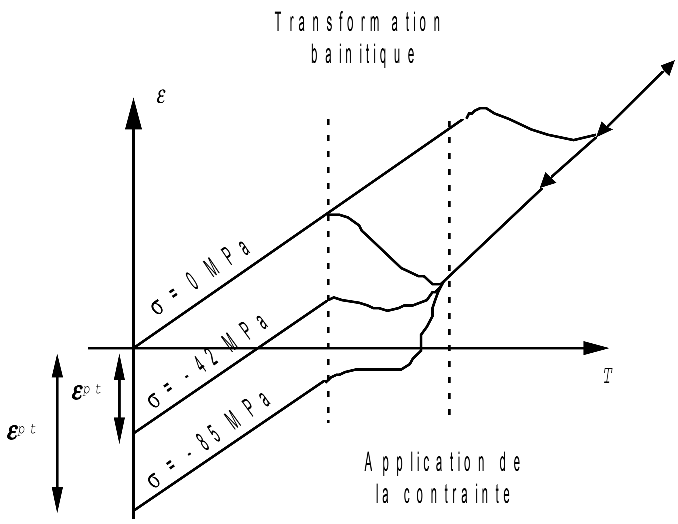


Figure 3-a : Swelling behaviors under uniaxial stresses of compression for a steel 16 MND5

One calls **plasticity of transformation** this phenomenon and one notes ε^{pt} **the corresponding unrecoverable** deformation.

The model of plasticity of transformation most frequently used is, at the origin, the three-dimensional generalization of the unidimensional phenomenologic model established by DESALOS [bib12]. If, from a dilatometric test, one traces the difference between the lengthening ε obtained for a pressure applied different from zero and that obtained for a stress null according to advance from the transformation, one notes that:

$$\varepsilon^{pt}(\sigma, b)=\varepsilon(\sigma, b)-\varepsilon(0,b)=kF(b)\sigma$$

where : k is a homogeneous constant contrary to a stress,
F is a standardized function ($F(0)=0$ and $F(1)=1$),
and b is the proportion of the transformed phase.

A three-dimensional and temporal generalization of the preceding experimental model, for only one transformation, was proposed by LEBLOND [bib8], [bib9], [bib10], [bib11], in the form:

$$\dot{\varepsilon}_{ij}^{pt}=\frac{3}{2}K\,\tilde{\sigma}_{ij}F'(b)\dot{b}$$

On the basis of experimental tests and for transformation of a bainitic type of a steel 16MND5 for example: K is taken equalizes with $10^{-4}\,MPa^{-1}$ and $F(b)=b(2-b)$.

It leans on the following heuristic considerations:

- the relation must be "incremental", i.e. to connect plastic strain rate to the rate of transformation,
- the plastic strainrate of transformation must be, as for classical plasticity, proportional to the deviatoric part $\tilde{\boldsymbol{\sigma}}$ of the tensor forced $\boldsymbol{\sigma}$ $\left(\tilde{\boldsymbol{\sigma}}=\boldsymbol{\sigma}-\frac{1}{3}tr\,\sigma\,\mathbf{Id}\right)$, (the plasticity of transformation occurs without change of volume, from where a dependence compared to the deviator of the stresses rather than at the stress field itself),
- the plastic strain rate of transformation must be null apart from the beaches of transformations,
- the integration of this relation in the uniaxial case with constant stress σ must give again the experimental relation.

The phenomenon of plasticity of transformation can during exist structure transformations under stresses of type the ferritic, perlitic, bainitic and martensitic, which can possibly appear simultaneously. On the other hand, it is considered that this phenomenon does not exist at the time of the austenitic transformation. The model general established in *Code_Aster* is thus:

$$\dot{\boldsymbol{\varepsilon}}^{pt}(\boldsymbol{\sigma},\mathbf{Z})=\sum_{i=1}^{i=4}\dot{\varepsilon}_i^{pt}(\boldsymbol{\sigma},\mathbf{Z})=\frac{3}{2}\tilde{\boldsymbol{\sigma}}\sum_{i=1}^{i=4}K_i F'_i\left(\sum_{i=1}^{i=4}Z_i\right)<\dot{Z}_i>$$

where: $\langle X\rangle$ indicate the positive part of a quantity.

The donnéeset K_i F'_i are provided by the user in `DEFI_MATERIAU` under key word `META_PT`.

In *Code_Aster,* it is possible not to take into account the phenomenon of plasticity of transformation. If this phenomenon is taken into account, it appears as soon as there are transformation and that even if the structure plasticizes. The model is more particularly dedicated to steel.

4 Restoration of hardening

In a usual way the state of hardening of a phase *I am* characterized by its plastic history. Thus for example in the case of plasticity with linear isotropic hardening, one generally takes as variable of hardening the noted cumulated plastic strain p . The term of hardening is written then: $R_i = R_{oi}\, p$ where R_{oi} is the linear coefficient of hardening of the phase i .

During the metallurgical transformations, there exists within the material of displacements of more or less important atoms. These displacements of atoms can destroy dislocations which are at the origin of hardening. In these cases, the hardening of the mother phase is not transmitted to the produced phase, it is the restoration of hardening. The new phase can then be born with a virgin plastic state or inherit only one part, possibly totality, hardening of the mother phase. The cumulated plastic strain p is not characteristic any more of the state of hardening and it is necessary to define other variables of hardening for each phase, noted r_i which take account of the restoration of hardening.

The term of hardening of the phase i is written then $R_i = R_{0i}\, r_i$.

4.1 Model with 2 phases with a meaning of transformation

to define the variables r_i , one chooses the model proposed by LEBLOND [bib11].
One considers a two-phase volume element V which undergoes a metallurgical transformation and a plastic strain.

Phase 1 is the mother phase, characterized by: $\begin{cases} \text{fraction volumique } V_1 \\ \text{proportion de phase (1-z)} \\ \text{variable d'écrouissage } r_1 \end{cases}$

Phase 2 is the phase produced, characterized by: $\begin{cases} \text{fraction volumique } V_2 \\ \text{proportion de phase z} \\ \text{variable d'écrouissage } r_2 \end{cases}$

The equations of evolution of r_i obtained by derivative compared to time are written:

$$\begin{cases} \dot{r}_1 = \dot{p} \\ \dot{r}_2 = \dot{p} - \frac{\dot{z}}{z} r_2 + \frac{\dot{z}}{z} \theta\, r_1 \end{cases} \qquad \text{éq 4.1-1}$$

θ characterizes the proportion of hardening transmitted of the mother phase to the produced phase.
$\dot{p}$ is plastic strain rate equivalent.

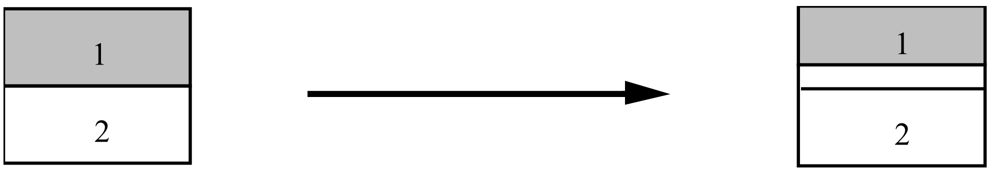
Note:

p here is not any more one local variable of the problem as such. The only meaning of $\dot{p}$ is here to be the plastic multiplier and it is equal to plastic strain rate equivalent.

SJÖSTRÖM obtains the same equations by means of a phenomenologic reasoning that one defers here to clarify the model [bib13].

That is to say an increment of time Δt, such as enters t and $t+\Delta t$:

- a fraction ΔV_2 of the mother phase is transformed into phase 2 and thus comes to be added with the volume V_2 of this phase produced,
- the volume element V undergoes a plastic strain Δp .



$$\begin{cases} V_2, r_2(t) \\ V_1, r_1(t) \end{cases} \qquad \begin{cases} \Delta t \\ \Delta p \\ \Delta V_2 \end{cases} \qquad \begin{cases} V_2, r_2(t)+\Delta p \\ \Delta V_2, \theta r_1(t)+\Delta p \\ V_1, r_1(t)+\Delta p \end{cases}$$

It is supposed that at the time of the metallurgical transformation, the transformed fraction ΔV_2 inherits only part θr_1 of hardening of the mother phase $0 \leq \theta \leq 1$.

Then the variables of hardening r_i at time $t+\Delta t$ are such as:

$$\begin{cases} r_1(t+\Delta t)=r_1(t)+\Delta p \\ r_2(t+\Delta t)=\dfrac{V_2(r_2(t)+\Delta p)+\Delta V_2(\theta r_1(t)+\Delta p)}{V_2+\Delta V_2} \end{cases}$$

Maybe, by considering that $r_i(t+\Delta t)=r_i(t)+\Delta r_i$

$$\begin{cases} \Delta r_1=\Delta p \\ \Delta r_2=\Delta p+\dfrac{\Delta V_2}{V_2+\Delta V_2}\theta r_1^- - \dfrac{\Delta V_2}{V_2+\Delta V_2} r_2^- \end{cases} \qquad \textbf{éq 4.1-2}$$

One obtains the equations [éq 4.1-1] while passing in extreme cases.

For the discretization of the laws of evolutions of r_i , one chooses a diagram of integration by means of clarifies directly the equations [éq 4.1-2].

4.2 Generalization of the model with *N* phases with transformations with double meaning

In the case of steel the existing phases are: Ferrite, Pearlite, Bainite, Martensite and Austenite γ of respective proportions Z_1, Z_2, Z_3, Z_4 and $1-\sum_{k=1}^{k=4} Z_k$.

- In the case of a cooling, the metallurgical transformations to consider are the transformations of (γ) in $(F),(P),(B),(M)$.
- In the case of a heating one considers the transformations in the other meaning: . $(F),(P),(B),(M)$ in (γ)

One can thus write in a general case (where $\langle x\rangle$ indicates the positive part of x .

$$\begin{cases} \text{Si}\, Z_\gamma>0 & \Delta r_g=\Delta p+\dfrac{\sum_{k=1}^{4}\langle -\Delta Z_k\rangle \theta_{k\gamma} r_k^- - \sum_{k=1}^{4}\langle -\Delta Z_k\rangle r_\gamma^-}{1-\sum_{k=1}^{4} Z_k} \\ \text{sinon} & r_\gamma^-=0 \text{ et } \Delta r_\gamma^-=0 \\ \text{Si}\, Z_k>0 & \Delta r_k=\Delta p+\dfrac{\langle \Delta Z_k\rangle \theta_{\gamma k} r_\gamma^- - \langle \Delta Z_k\rangle r_k^-}{Z_k} \\ \text{sinon} & r_k^-=0 \text{ et } \Delta r_k^-=0 \end{cases} \quad \textbf{éq 4.2-1}$$

$\theta_{\gamma k}$: proportion of restoration of hardening at the time of the transformation γ in k
$\theta_{k\gamma}$: proportion of restoration of hardening at the time of the transformation k in γ

For transformations with diffusion (ex: γ in F, P, B) implying important displacements of atoms one will be able to take $\theta=0$; dislocations at the origin of plastic hardening are completely destroyed by the transformation. For transformations without diffusion (ex: martensitic transformation), one will be able to take $\theta=1$, hardening being completely transmitted.

θ Are provided by the user in operator DEFI_MATERIAU under key word META_RE.

5 Models of strain (visco) plastic

the main feature of the evolutions thermal concerned in this kind of analysis is that they sweep a broad temperature range, which has an important effect on the structural mechanics behavior of the material which undergoes the thermal evolution. One is in particular in temperature ranges where the phenomena of viscosity can not be negligible more. It can thus be necessary to use a élasto-viscoplastic model of behavior especially when one remains in these fields for an important length of time; for example during processing of detensioning associated with welding.

One thus chooses a viscoplastic model whose characteristics are such as it makes it possible to describe with the same formalism, therefore without changing model:

- a classical plastic behavior; to model the cases at low temperature when the viscous effects are still negligible or to model the processes at high velocity (welding),
- a hammer-hardenable viscoplastic behavior at high temperature, to model the effects of creep and relaxation associated for example with the processing with detensioning or multirun weldings,
- a behavior of the fluid type viscous for the higher temperatures with the melting point, in order to have a reasonable description of the molten zone.

The model viscoplastic selected degenerates indeed for certain borderline cases out of model of plasticity independent of time, or out of viscous fluid model.

One places oneself here in the frame of the plasticity of von Mises with additive isotropic hardening. The use of a kinematic hardening being also possible (version 6.1.6).

Function threshold:

$$f=\sigma_{eq}-R(r;T,Z)-\sigma_c(T,Z)$$

σ_{eq} equivalent stress of von Mises $\sigma_{eq}=\left(\frac{3}{2}\tilde{\boldsymbol{\sigma}}:\tilde{\boldsymbol{\sigma}}\right)^{1/2}$

$R(r;T,Z)$: isotropic term of hardening

$\sigma_c(T,Z)$: initial critical stress; corresponds to the initial minimal stress to apply to have a viscoplastic flow.

Rate of yielding:

$$\dot{\boldsymbol{\varepsilon}}^{vp}=\dot{\lambda}\frac{\partial f}{\partial \boldsymbol{\sigma}}=\frac{3}{2}\dot{p}\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}$$

The cumulated plastic strain $\dot{p}$ is viscous and is written:

$$\dot{p}=\left(\frac{\langle\sigma_{eq}-R(r;T,Z)-\sigma_c(T;Z)\rangle}{\eta}\right)^n \quad \text{éq 5-1}$$

η, n : coefficients materials of viscosity.

Note:

> *One can rewrite the equation [éq 5 -1] in the form:* $\sigma_{eq}-R(r;T,Z)-\sigma_c(T,Z)-\eta\,\dot{p}^{1/n}=0$ *, it is - with - to say that in this model, the stress can be interpreted as the sum of an ultimate flow stress (which break up it even into an initial ultimate stress and a term of hardening) and a "viscous" stress depending the strainrate and null at velocity null.*

Viscous restoration of hardening

One also introduces into the modelization the phenomenon of viscous restoration of hardening which leads to an evanescence partial of hardening. Under the action of thermal agitation, it occurs a slow restoration of crystalline structure of metal by annihilation of dislocations and internal stress relaxation. The model used to describe this phenomenon is the following:

$$\begin{cases} R=R_0 r \\ \dot{r}=\dot{p}-(Cr)^m \end{cases}$$

The term of evolution of the variable of hardening r thus comprises a term of hardening due to the plastic strain and a term of restoration.
The model thus allows to describe the primary education phenomenon of creep (hardening) and secondary creep (stabilization of hardening).

Case of linear kinematic hardening:

In a way equivalent to the case with isotropic hardening the equations are written;

function threshold:
$f=(\tilde{\sigma}-X)_{eq}-\sigma_c$

flow Writing model
of strain rate (visco) plastic

$$\dot{\boldsymbol{\varepsilon}}^{vp}=\frac{3}{2}\dot{p}\frac{(\tilde{\boldsymbol{\sigma}}-\tilde{\mathbf{X}})}{(\sigma-X)_{eq}} \quad \text{with} \quad \dot{p}=\frac{\langle f\rangle^n}{\eta}$$

$$\mathbf{X}=\frac{2}{3}H_0\boldsymbol{\alpha}$$

$\boldsymbol{\sigma},\tilde{\boldsymbol{\sigma}}$: stress tensor and its deviator
$\boldsymbol{\alpha}$: variable tensor of kinematic hardening
$\mathbf{X}$: tensor of hardening associated with the variable tensor with hardening α
H_0 : model coefficient of

kinematic hardening of evolution of the tensor of hardening α of a material with n phases

$$\begin{cases} \dot{\boldsymbol{\alpha}}_\gamma=\dot{\boldsymbol{\varepsilon}}^{vp}+\dfrac{\sum_k(\langle\dot{z}_k\rangle\theta_{k\gamma}\alpha_k)-\sum_k(\langle\dot{z}_k\rangle\alpha_\gamma)}{z_\gamma}+\dfrac{3}{2}(C\alpha_{eq})^m\dfrac{\boldsymbol{\alpha}}{\alpha_{eq}} \\ \dot{\boldsymbol{\alpha}}_k=\dot{\boldsymbol{\varepsilon}}^{vp}+\dfrac{\dot{z}_k\theta_{\gamma k}\alpha_k-\dot{z}_k\alpha_k}{z_k}+\dfrac{3}{2}(C\alpha_{eq})^m\dfrac{\boldsymbol{\alpha}}{\alpha_{eq}} \end{cases}$$

By preoccupation with a simplification and in the same way that into isotropic, one takes for the term of viscous restoration:

$$\begin{cases} \boldsymbol{\alpha} = \sum_i z_i \boldsymbol{\alpha}_\mathbf{i} \\ C = \sum_i z_i C_i \\ m = \sum_i z_i m_i \end{cases}$$

θ_{ij} : coefficients of metallurgical restoration at the time of the transformation $i \rightarrow j$

C_i, m_i : coefficients of viscous restoration of the phase i .

5.1 Borderline case: Model plastic independent of time

One wants to describe an instantaneous elastoplastic behavior and to cancel the viscous effects. For that the viscous parameters η and C will be taken equal to zero. To free itself from the numerical problems that the taking into account of and null η can C pose, and in a way similar to the processing carried out for the model viscoplastic of Taheri [bib15], one rewrites the equation [éq 5-1] in the form:

$$f - \eta \dot{p}^{1/n} \leq 0 \qquad \textbf{éq 5.1-1}$$

the strict inequality being obtained in the case $\begin{cases} f < 0 \\ \dot{p} = 0 \end{cases}$ (elastic mode).

In the purely plastic field of behavior ($\eta \rightarrow 0$) the inequality [éq 5.1-1] is then reduced to: $f = \sigma_{eq} - R - \sigma_c(T) \leq 0$ and $\dot{p}$ can be given more only by the equation of consistency $\dot{f} = 0$. One thus finds oneself well in the frame of instantaneous plasticity independent of time, with a digital processing identical to that classically used for the processing of this one.

Note:

It will be noted that σ_c corresponds then to the classical definition of the yield stress σ_y .
The elastic limit will be noted σ_c in viscoplasticity and σ_y plasticity independent of time .

5.2 Borderline case: Model viscous fluid behavior A

very high temperature one a: $\begin{cases} R \rightarrow 0 \\ \sigma_c \rightarrow 0 \end{cases}$

if one takes $n \rightarrow 1$, then: $\dot{\varepsilon}^{vp} = \frac{3}{2} \frac{\sigma_{eq}}{\eta}$ maybe into unidimensional: $\dot{\varepsilon}^{vp} = \frac{\sigma}{\eta}$. One thus obtains a model of behavior of the fluid type viscous Newtonian, of viscosity η .

Note:

In Code_Aster, the behavior models available are either of the completely plastic models independent of time, or of the models with viscous effect. (cf [§6]).

5.3 Multiphase plasticity

the metallurgical transformations involve modifications of the mechanical characteristics of the material.

The elastic characteristics (modulus of YOUNG and Poisson's ratio) are affected little by the metallurgical structure changes. Only their dependence compared to the temperature is thus taken into account.

On the other hand, the plastic characteristics (elastic limit in particular) strongly depend on metallurgical structure. It is thus necessary to take into account the differences in characteristics plastic for each possible phase. In the modelization the strain and the stress are defined at the level of the material point (macroscopic) which can be multiphase. One seeks to define the plastic behavior are equivalent of the material when it has a multiphase structure, with in particular a single criterion of plasticity. The definition of the behavior of the material are equivalent is done using a model of the mixtures on the characteristics of the phases. More precisely the definition of this material equivalent would correspond in 1D to a rheological model of i bars in parallel such as:

$$\begin{cases} \dot{\varepsilon}^{vp} = \dot{\varepsilon}_i^{vp} \\ \sigma = \sum_i z_i \sigma_i \text{avec}\, \sigma_i = \sigma_{ci} + R_i + \eta\, \dot{\varepsilon}_i^{vp} \end{cases}$$

More precisely, in the case of the plasticity of von Mises with isotropic hardening;

- the function threshold is expressed by:

$$f(\sigma, R; T, \mathbf{Z}) = \sigma_{\text{eq}} - R(T, \boldsymbol{Z}, r) - \sigma_c(T, \boldsymbol{Z})$$

where: $$R(T, \boldsymbol{Z}, r) = \sum_i Z_i R_i(T, r_i)$$

is the hardening of the multiphase material, R_i being that of the phase i.

where $$\sigma_c(T, Z) = \sum_i Z_i \sigma_{ci}$$

is the elastic limit of the multiphase material, σ_{ci} that of the phase i.

- and plastic strain rate checks the condition of consistency $\tilde{f}=0$ given by this model of mixture. I.e. when one is in load, $\dot{p}$ is such that:

$$\tilde{f}=\sigma_{eq}-R(r;T,Z)-\sigma_c(T,Z)-\sum_i Z_i\eta_i\dot{p}^{1/n_i}=0$$

One also gives the possibility of using a nonlinear model of the mixtures [bib9] such as one has in 1D: $\sigma=(1-f_h(z))\sigma_\gamma+f_h(z)\sigma_\alpha$. One has then:

- $$\begin{cases} R=(1-f_h(z))R_\gamma+f_h(z)R_\alpha \\ \sigma_c=(1-f_h(z))\sigma_{c\gamma}+f_h(z)\sigma_{c\gamma} \end{cases}$$

$\sigma_y\gamma$ is the elastic limit of the austenitic phase,

$z=\sum_{k=1}^{4} Z_k$ is the total proportion of the phases "" α "" (F, P, B, M)

$\sigma_{c\alpha}=\frac{\sum_{k=1}^{4} Z_k\sigma_{c\alpha_k}}{z}$ is the equivalent elastic limit of the cold phases "" α ""

$R_\alpha=\frac{\sum_{k=1}^{4} Z_k R_{\alpha_k}}{z}$ is the average hardening of the cold phases.

- and in load $\dot{p}$ checks

$$\tilde{f}=\sigma_{eq}-R(r;T,Z)-\sigma_c(T,Z)-(1-f_h(z))\eta_\gamma\dot{p}^{1/n_\gamma}-\sum_k\frac{Z_k}{z}\eta_k\dot{p}^{1/n_k}=0 \ .$$

$f(z)$ is a function defined by the user under operand `SY_MELANGE` of the key word factor `ELAS_META_FO`.

The parameters η_i, n_i, C_i et m_i are defined in `DEFI_MATERIAU` under the key word factor `META_VISC`. The parameters elastic limits are defined under the key word factor `ELAS_META_FO` ; key word `*_SY` for the plastic models independent of time and key word `*_S_VP` for the viscoplastic models.

6 Behavior models

6.1 Partition of the strain:

The strain is written as the sum of four components:

$$\boldsymbol{\varepsilon}=\boldsymbol{\varepsilon}^{\mathbf{e}}+\boldsymbol{\varepsilon}^{\mathbf{th}}+\boldsymbol{\varepsilon}^{\boldsymbol{vp}}+\boldsymbol{\varepsilon}^{\mathbf{pt}}$$

where : $\boldsymbol{\varepsilon}^{\mathbf{e}},\boldsymbol{\varepsilon}^{\mathbf{th}},\boldsymbol{\varepsilon}^{\boldsymbol{vp}},\boldsymbol{\varepsilon}^{\mathbf{pt}}$ are respectively the elastic strain, plastic thermal, visco - and of plasticity of transformation,

6.2 Constitutive laws

6.2.1 Case with isotropic hardening

$$\begin{cases}
\varepsilon=\varepsilon^{e}+\varepsilon^{th}+\varepsilon^{vp}+\varepsilon^{pt} \\
\sigma=\mathbf{A}(T)\,\varepsilon^{e} \\
\dot{\varepsilon}^{pt}=\dfrac{3}{2}\tilde{\sigma}\displaystyle\sum_{i=1}^{4}K_i F'_i\left(1-Z_\gamma\right)\langle\dot{\boldsymbol{Z}}_{\boldsymbol{i}}\rangle \\
\varepsilon^{th}(Z,T)=Z_\gamma\left[\alpha_\gamma.\left(T-T_{ref}\right)-(1-Z_\gamma^R)\,\Delta\varepsilon_{f\gamma}^{T_{ref}}\right]+\left(\displaystyle\sum_{i=1}^{4}Z_i\right)\left[\alpha_f\left(T-T_{ref}\right)+Z_\gamma^R\,\Delta\varepsilon_{f\gamma}^{T_{ref}}\right] \\
\dot{\varepsilon}^{vp}=\dfrac{3}{2}\dot{p}\dfrac{\tilde{\sigma}}{\sigma_{eq}} \\
f=\sigma_{eq}-R(T,Z,r)-\sigma_c(T,Z)\ \text{ avec }\ R(T,Z,r)=\displaystyle\sum_{i=1}^{5}Z_i R_i(T,r_i) \\
\dot{p}=0\ \text{ si }\ f<0 \\
\dot{p}\geq 0\ \text{ si }\ f=0\ \text{ et vérifie }\ \tilde{f}=\sigma_{eq}-R(T,Z,r)-\sigma_c(T,Z)-\displaystyle\sum_i Z_i\eta_i\dot{p}^{1/n_i}=0 \\
\dot{r}_\gamma=\dot{p}+\dfrac{\displaystyle\sum_{k=1}^{4}\langle-\dot{Z}_k\rangle\theta_{k\gamma}r_k-\sum_{k=1}^{4}\langle-\dot{Z}_k\rangle r_\gamma}{1-\displaystyle\sum_{k=1}^{4}Z_k}-\left(C\,r_{moy}\right)^m\ \text{ si }\ Z_\gamma>0\ ,\ \dot{r}_\gamma=0\ \text{ si }\ Z_\gamma=0 \\
\dot{r}_k=\dot{p}+\dfrac{\langle\dot{Z}_k\rangle\theta_{\gamma k}r_\gamma-\langle\dot{Z}_k\rangle r_k}{Z_k}-\left(C\,r_{moy}\right)^m\ \text{ si }\ Z_k>0\ ,\ \dot{r}_k=0\ \text{ si }\ Z_k=0
\end{cases}$$

with $r_{moy}=\displaystyle\sum_{i=1}^{5}Z_i r_i$

f	the function threshold,
$r_i, Z_i R_i$	local variables of hardening and their thermodynamic forces associated,
$\mathbf{A}=(A_{ijkl})$	the tensor of elastic stiffness, depend on the temperature,
$T(t)$ and $\mathbf{Z}(t)$	the temperature and metallurgical structure.

6.2.2 Case with kinematic hardening

$$
\begin{cases}
\varepsilon=\varepsilon^{e}+\varepsilon^{th}+\varepsilon^{vp}+\varepsilon^{pt}\\
\sigma=\mathbf{A}(T)\,\varepsilon^{e}\\
\dot{\varepsilon}^{pt}=\frac{3}{2}\tilde{\sigma}\sum_{i=1}^{4}K_i F'_i(1-Z_\gamma)\langle\dot{\boldsymbol{Z}}_{\boldsymbol{i}}\rangle\\
\varepsilon^{th}(Z,T)=Z_\gamma\left[\alpha_\gamma(T-T_{ref})-(1-Z_\gamma^R)\Delta\varepsilon_{f\gamma}^{T_{ref}}\right]+\left(\sum_{i=1}^{4}Z_i\right)\left[\alpha_f(T-T_{ref})+Z_\gamma^R\Delta\varepsilon_{f\gamma}^{T_{ref}}\right]\\
\dot{\varepsilon}^{vp}=\frac{3}{2}\dot{p}\frac{(\tilde{\sigma}-\tilde{X})}{(\sigma-X)_{eq}}\\
f=(\sigma-X)_{eq}-\sigma_c(T,Z)\text{ avec }X(T,Z,\alpha)=\sum_{i=1}^{5}Z_i X_i(T,\alpha_i)\\
\dot{p}=0\quad\text{si }f<0\\
\dot{p}\geq 0\quad\text{si }f=0\quad\text{et vérifie }\tilde{f}=\sigma_{eq}-R(T,Z,r)-\sigma_c(T,Z)-\sum_i Z_i\eta_i\dot{p}^{1/n_i}=0
\end{cases}
$$

$$
\begin{cases}
\dot{\alpha}_\gamma=\dot{\varepsilon}^{vp}+\frac{\sum_k(<\dot{z}_k>\theta_{k\gamma}\alpha_k)-\sum_k(<\dot{z}_k>\alpha_\gamma)}{z_\gamma}+\frac{3}{2}(C\alpha_{eq})^m\frac{\alpha}{\alpha_{eq}} & \text{si } Z_\gamma>0\quad\dot{\alpha}_\gamma=0\text{ sinon}\\
\dot{\alpha}_k=\dot{\varepsilon}^{vp}+\frac{\dot{z}_k\theta_{\gamma k}\alpha_k-\dot{z}_k\alpha_k}{z_k}+\frac{3}{2}(C\alpha_{eq})^m\frac{\alpha}{\alpha_{eq}} & \text{si } Z_k>0\quad\dot{\alpha}_k=0\text{ sinon}
\end{cases}
$$

with $\alpha_{eq}=\left(\sum_i Z_i\alpha_i\right)_{eq}$

f	the function threshold,
$r_i, Z_i R_i$	local variables of hardening and their thermodynamic forces associated,
$\mathbf{A}=(A_{ijkl})$	the tensor of elastic stiffness, depend on the temperature,
$T(t)$ and $\mathbf{Z}(t)$	the temperature and metallurgical structure.

In term of behavior models of STAT_NON_LINE available, the modelization installation gives several opportunity:

- choice of the type of behavior for the plastic strain; plastic independent of time or with taking into account of the viscous effects,
- choices of a hardening isotropic linear, isotropic nonlinear or kinematical,
- taken into account or not of the plasticity of transformation,
- taken into account or not of the metallurgical restoration of hardening.

The choice of the material (steel or zircaloy) and thus amongst phase is done by informing the key word KIT of STAT_NON_LINE. "ACIER" for steel with 5 phases and "ZIRC" for the zircaloy with 3 phases.

6.3 Various behavior models elastoplastic META_P_ ***

There exist 12 behavior models elastoplastic independent of time META_P*.

- 8 relations with isotropic hardening according to whether one considers a linear isotropic hardening or not linear, that one takes into account or not the plasticity of transformations, that one takes into account or not the metallurgical restoration of hardening.
- 4 relations with linear kinematic hardening according to whether one takes into account or not the plasticity of transformations and/or the metallurgical restoration of hardening.

For these 12 behavior models one informs under key word ELAS_META_FO or ELAS_META the elastic parameters E and Nu , the coefficients of thermal expansion, as well as the elastic limits.

```
/ELAS_META_FO :   (   E: E
                       NU: ν
                      F_ALPHA         :   αf
                      C_ALPHA         :   αc
                      PHASE_REFE      :   "CHAUD"
                                          "FROID"
                       EPSF_EPSC_TREF :   Δε_fc^Tref

                      F1_SY  :  σ  yf1
                      F2_SY  :  σ  yf2
                      F3_SY  :  σ  yf3
                      F4_SY  :  σ  yf4
                      A_SY   :  σ  yc
                      SY_MELANGE: F)
```

with for steel:

α_f : α_f

αc : α_γ

$\Delta\varepsilon_{fc}^{T_{ref}}$: $\Delta\varepsilon_{f\gamma}^{T_{ref}}$

σyfi : elastic limit of phase i

6.3.1 Relation META_P_IL

This relation makes it possible to treat the behavior model in the case of the plasticity of Von Mises with linear isotropic hardening, applied to a material which undergoes metallurgical phase changes. The phenomena of plasticity of transformation and restoration of hardening are neglected. The coefficients of hardening are provided under key word META_ECRO_LINE of operator DEFI_MATERIAU.

```
/META_ECRO_LINE:
                    F1_D_SIGM_EPSI   : H0f1
                    F2_D_SIGM_EPSI   : H0f2
                    F3_D_SIGM_EPSI   : H0f3
                    F4_D_SIGM_EPSI   : H0f4
                    C_D_SIGM_EPSI    : H0c
```

with for steel :

H0fi : Linear coefficient of hardening of phase I.

f : function of z_γ defining the model of mixture for the plastic behavior.

6.3.2 Relation META_P_INL

This relation makes it possible to treat the behavior model in the case of the plasticity of von Mises with nonlinear isotropic hardening, applied to a material which undergoes metallurgical phase changes. In DEFI_MATERAU besides ELAS_META_FO one R returns under key word META_TRACTION the curves (R).

META_TRACTION:

F1_SIGM: $R_1(r)$

F2_SIGM: $R_2(r)$

F3_SIGM: $R_3(r)$

F4_SIGM: $R_4(r)$

C_SIGM: $R_c(r)$

6.3.3 Relation META_P_CL

This relation makes it possible to treat the behavior model in the case of the plasticity of Von Mises with linear kinematic hardening, applied to a material which undergoes metallurgical phase changes. The phenomena of plasticity of transformation and restoration of hardening are neglected. The coefficients of hardening are provided under key word META_ECRO_LINE of operator DEFI_MATERIAU.

```
/META_ECRO_LINE:
                    F1_D_SIGM_EPSI   : H0f1
                    F2_D_SIGM_EPSI   : H0f2
                    F3_D_SIGM_EPSI   : H0f3
                    F4_D_SIGM_EPSI   : H0f4
                    C_D_SIGM_EPSI    : H0c
```

with for steel:

H0fi : Linear coefficient of kinematic hardening of phase I.

f : function of z_γ defining the model of mixture for the plastic behavior.

6.3.4 Relation META_P_IL_PT, META_P_INL_PT , META_P_CL_PT

Compared to META_P_IL, META_P_INL or META_P_CL one take account besides the plasticity of transformation but the restoration of hardening is always neglected. Besides the data of the key word factor ELAS_META_FO and key word relating to the data of hardening, one must also inform those relating to the plasticity of transformation which are provided under the key word factor META_PT.

```
/META_PT       :  (  F1_D_F_META: F' 1F1_K      : K f1
                     F2_D_F_META: F' 2F2_K      : K f2
                     F3_D_F_META: F' 3F3_K      : K f3
                     F4_D_F_META: F' 4F4_K      : K f4
    )
```

with for steel:

F' f1 = F'_f K f1 = K_f

F' f2 = F'_p K f2 = K_p

F' f3 = F'_b K f3 = K_b

F' f4 = F'_m K f4 = K_m

6.3.5 Relation META_P_IL_RE , META_P_INL_RE and META_P_CL_RE

One takes account of the restoration of hardening but the plasticity of transformation is neglected. The data relating to the restoration of hardening are provided under the key word factor META_RE of operator DEFI_MATERIAU.

/META_RE : (C_F1_THETA : θ_{cf1} F1_C_THETA : θ_{cf2}

C_F2_THETA : q_{cf2} F2_C_THETA : θ_{cf2}

C_F2_THETA : θ_{cf2} F3_C_THETA : θ_{cf3}

C_F2_THETA : θ_{cf2} F4_C_THETA : θ_{cf4})

with for steel:

θ CF1 = $\theta_{\gamma F}$ θ F1C = $\theta_{F\gamma}$

θ CF2 = $\theta_{\gamma P}$ θ F2C = $\theta_{P\gamma}$

θ CF3 = $\theta_{\gamma B}$ θ F3C = $\theta_{B\gamma}$

θ CF4 = $\theta_{\gamma M}$ θ F4C = $\theta_{M\gamma}$

6.3.6 Relation META_P_IL_PT_RE , META_P_INL_PT_RE and META_P_CL_PT_RE

One takes account at the same time phenomena of plasticity of transformation and restoration of hardening. The data of the key keys factors ELAS_META_FO, META_PT and META_RE must be indicated.

6.4 Various behavior models élasto-viscoplastic META_V_ ***

One has in the same way that in classical plasticity, 12 behavior models which are available according to the type of hardening and according to whether one takes account or not phenomena of plasticity of transformation and/or metallurgical restoration of hardening. One uses the same terminology as in the case of classical plasticity to differentiate the 12 élasto-viscoplastic relations. For each relation one must inform in ELAS_META or ELAS_META_FO the yield stresses of flow viscous, instead of classical apparent elastic limits.

F1_SC: σ_{cf}

F2_SC: σ_{cp}

F3_SC: σ_{cb}

F4_SC: σ_{cm}

C_SC : $\sigma_{c\gamma}$

SC_MELANGE : function for the model of the mixtures

instead of the *_SY for the plastic case.

6.4.1 Relation META_V_IL and élasto-viscoplastic

META_V_INL Behavior model applied to a material which undergoes metallurgical transformations with or not linear linear hardening. One does not take account of the phenomena of plasticity of transformation and metallurgical restoration of hardening.

6.4.2 Relation élasto-viscoplastic

META_V_CL Behavior model applied to a material which undergoes metallurgical transformations with linear kinematic hardening. One does not take account of the phenomena of plasticity of transformation and metallurgical restoration of hardening.

6.4.3 Relation META_V_IL_PT, META_V_INL_PT and META_V_CL_PT

Idem that META_P_IL_PT , META_P_INL_PT and META_V_CL_PT but in viscoplasticity.

6.4.4 Relation META_V_IL_RE , META_V_INL_RE and META_V_CL_RE

Idem that META_P_IL_RE , META_P_INL_RE and META_V_CL_RE
but in viscoplasticity

6.4.5 Relation META_V_IL_PT_RE , META_V_INL_PT_RE and META_V_CL_PT_RE

Idem that META_P_IL_PT_RE , META_P_INL_PT_RE and META_V_CL_PT_RE but in viscoplasticity

Note:

- *For all relations META_ ** , the local variables produced in the Code_Aster are:*

  r_i *: variables of effective hardening for I phases*
  d *: indicator of plasticity (0 if the last calculated increment is elastic; 1 if not)*
  R *: the term of hardening of the function threshold*

- *In addition, these modelizations can be realized with the geometrical functionality of reactualization PETIT_REAC . For the relations with isotropic hardening, the model of large deformations SIMO_MIEHE is also available.*

7 Numerical formulation

One will treat the viscoplastic constitutive law with isotropic hardening.

7.1 Discretization

Knowing the fields $\boldsymbol{\sigma},\mathbf{u}$ and p at time t, one chooses an implicit scheme to discretize in time the equations of the continuous problem, except for the hardening parameters where one uses the equations [éq 4.2-1].

It is noticed that with an implicit discretization, only two points differentiate the two types of viscoplastic and plastic behavior independent of time:

- the form of the loading function, for which one has a complementary term in the case of viscosity,
- the presence of the term of restoration of hardening in the evolution of the variable of hardening for the viscoplastic case.

Moreover, incremental classical plasticity seems the borderline case (without associated numerical difficulty) of incremental viscoplasticity when $\begin{cases}\eta\to 0\\ C\to 0\\ \sigma_c\to\sigma_y\end{cases}$.

This kind of processing was already carried out by LORENTZ [bib15].

If one poses $\tilde{f}=f-\eta\left(\dfrac{\Delta p}{\Delta t}\right)^{1/n}$

$$\begin{cases}
\boldsymbol{\varepsilon}=\boldsymbol{\varepsilon}^{\mathbf{e}}+\boldsymbol{\varepsilon}^{\mathbf{th}}+\boldsymbol{\varepsilon}^{\mathbf{p}}+\boldsymbol{\varepsilon}^{\mathbf{pt}}\\
\boldsymbol{\sigma}=A(T)\boldsymbol{\varepsilon}^{\mathbf{e}}\\
\boldsymbol{\varepsilon}^{\mathbf{th}}(Z,T)=Z_\gamma\left[\alpha_\gamma.(T-T^\gamma)-(1-Z_\gamma^R)\Delta\varepsilon_{f\gamma}^{T_{ref}}\right]+\left(\sum_{i=1}^{4}Z_i\right)\left[\alpha_f(T-T^\gamma)+Z_\gamma^R\Delta\varepsilon_{f\gamma}^{\gamma}\right]\\
\Delta\boldsymbol{\varepsilon}^{\mathbf{pt}}=\dfrac{3}{2}\tilde{\boldsymbol{\sigma}}\sum_{i=1}^{4}K_i F'_i(1-Z_\gamma)\langle\Delta Z_i\rangle\\
\Delta\boldsymbol{\varepsilon}^{\mathbf{p}}=\dfrac{3}{2}\Delta p\dfrac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\\
\text{régime élastique : }\tilde{f}<0\text{ et }\Delta p=0\\
\text{régime (visco)plastique : }\tilde{f}=0\text{ et }\Delta p>0
\end{cases}$$

$$\begin{cases}\Delta r_\gamma=\Delta p+\dfrac{\sum_{k=1}^{4}<\text{-}\Delta Z_k>\theta_{k\gamma}r_k^- - \sum_{k=1}^{4}<\text{-}\Delta Z_k>r_\gamma^-}{1-\sum_{k=1}^{4}Z_k}-\left(Cr_{moy}^-\right)^m & \text{si } Z_\gamma>0, \Delta r_\gamma=0 \text{ si } Z_\gamma=0\\ \Delta r_k=\Delta p+\dfrac{\langle\Delta Z_k\rangle\theta_{\gamma k}r_\gamma^- \text{-} <\Delta Z_k>r_k^-}{Z_k}-\left(Cr_{\text{moy}}^-\right)^m & \text{si } Z_\gamma>0, \Delta r_k=0 \text{ si } Z_k=0\end{cases}$$

with:

$X=X(t+\Delta t)$

$X^-=X(t)$

$\Delta X=X(t+\Delta t)-X(t)$

7.2 Algorithm of resolution of the quasi-static problem

the incremental problem posed on the structure is a nonlinear problem. Its variational formulation, in the case of the small strains, is form:

To find $\Delta\mathbf{u}$ such as:

$$\begin{cases}\int_\Omega \boldsymbol{\sigma}\left(\boldsymbol{\varepsilon}\left(\mathbf{u}^-+\Delta\mathbf{u}\right),t\right)\boldsymbol{\varepsilon}(\mathbf{v})\,d\Omega=\mathbf{L}(t) \quad \forall\mathbf{v} \text{ cinématiquement admissible et } \forall t\\ \mathbf{B}\,\mathbf{u}=\mathbf{u}^{\mathbf{d}}(t)\end{cases}$$

where: $\mathbf{u}$ indicate the field of displacement

$\mathbf{B}\,\mathbf{u}=\mathbf{u}^{\mathbf{d}}(t)$ corresponds to the boundary conditions in displacement (kinematical connections)

and $\mathbf{L}(t)$ is the virtual wor of the mechanical loadings at the moment T.

In *Code_Aster*, this nonlinear problem is solved by a method of NEWTON [bib6], [bib7]. The algorithm of resolution comprises:

- a phase of prediction at the beginning of each time step,
- iterations of Newton inside one time step.

We do not detail here the algorithm implemented (one will refer for that to the reference documents [R5.03.01] and [R5.03.02]), but we endeavour to highlight the modifications made to the diagram of integration by the taking into account of the metallurgical evolution $\mathbf{Z}(t)$ and the plasticity of transformation.

7.2.1 Integration of relations META ***

One gives the statement of $\boldsymbol{\sigma}$ according to;

- $\Delta\boldsymbol{\varepsilon}(\text{ou } \Delta\mathbf{u})$ unknown of the problem,
- the known terms such variables calculated with the preceding step ($\boldsymbol{\sigma}^-$, local variables…), the characteristic materials, $\boldsymbol{\varepsilon}^{th}$ …

$$\boldsymbol{\sigma}=\mathbf{A}(T)\boldsymbol{\varepsilon}^{\mathrm{e}}$$
$$\boldsymbol{\varepsilon}^{\mathrm{e}}=\boldsymbol{\varepsilon}^{tot}-\boldsymbol{\varepsilon}^{th}-\boldsymbol{\varepsilon}^{vp}-\boldsymbol{\varepsilon}^{pt}$$

On pose : $\boldsymbol{\varepsilon}=\boldsymbol{\varepsilon}^{tot}-\boldsymbol{\varepsilon}^{th}$

$$\begin{cases}\tilde{\boldsymbol{\sigma}}=2\mu\tilde{\boldsymbol{\varepsilon}}^{e}=\dfrac{\mu}{\mu^{-}}\boldsymbol{\sigma}^{-}+2\mu\left(\Delta\tilde{\boldsymbol{\varepsilon}}-\Delta\tilde{\boldsymbol{\varepsilon}}^{vp}-\Delta\tilde{\boldsymbol{\varepsilon}}^{pt}\right)\\ tr\,\boldsymbol{\sigma}=3\mathrm{K}\,tr\left(\boldsymbol{\varepsilon}^{\mathrm{e}}\right)=\dfrac{3\mathrm{K}}{3\mathrm{K}^{-}}tr\,\boldsymbol{\sigma}^{-}+3\mathrm{K}\,\mathrm{tr}\,\Delta\boldsymbol{\varepsilon}\end{cases}$$

$$\tilde{\boldsymbol{\sigma}}=\frac{\mu}{\mu^{-}}\boldsymbol{\sigma}^{-}+2\mu\left(\Delta\tilde{\boldsymbol{\varepsilon}}-\frac{3}{2}\tilde{\boldsymbol{\sigma}}F(Z,DZ)-\frac{3}{2}\Delta p\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\right)$$

from where

$$\tilde{\boldsymbol{\sigma}}=\frac{1}{1+3\mu F(Z,DZ)}\left(\frac{\mu}{\mu^{-}}\tilde{\boldsymbol{\sigma}}^{-}+2\mu D\tilde{\boldsymbol{\varepsilon}}-3\mu\Delta p\frac{\tilde{\boldsymbol{\sigma}}}{s_{eq}}\right)$$

with:

- **statement of** $\dfrac{\tilde{\boldsymbol{\sigma}}}{s_{eq}}$

$$\Delta\tilde{\boldsymbol{\varepsilon}}=\Delta\tilde{\boldsymbol{\varepsilon}}^{\varepsilon}+\Delta\tilde{\boldsymbol{\varepsilon}}^{pt}+\Delta\tilde{\boldsymbol{\varepsilon}}^{vp}$$

$$\Delta\tilde{\boldsymbol{\varepsilon}}=\left(\frac{\tilde{\boldsymbol{\sigma}}}{2\mu}-\frac{\tilde{\boldsymbol{\sigma}}^{-}}{2\mu^{-}}\right)+\frac{3}{2}\tilde{\boldsymbol{\sigma}}F(Z,DZ)+\frac{3}{2}\Delta p\frac{\tilde{\boldsymbol{\sigma}}}{s_{eq}}$$

$$2\mu\Delta\tilde{\boldsymbol{\varepsilon}}+\frac{\mu}{\mu^{-}}\tilde{\boldsymbol{\sigma}}^{-}=\frac{\tilde{\boldsymbol{\sigma}}}{s_{eq}}\left((1+3\mu F(Z,DZ))\sigma_{eq}+3\mu\Delta p\right)$$

one poses: $2\mu\Delta\tilde{\boldsymbol{\varepsilon}}+\dfrac{\mu}{\mu^{-}}\tilde{\boldsymbol{\sigma}}^{-}=\tilde{\boldsymbol{\sigma}}^{e}$

one a: $s_{eq}^{e}=(1+3\mathrm{mF}(Z,DZ))s_{eq}+3\mathrm{mDp}$

and

$$\frac{\tilde{\boldsymbol{\sigma}}^{e}}{\sigma_{eq}^{e}}=\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}$$

- **statement of** Δp

Is the loading function: $f=\dfrac{\sigma_{eq}^{e}}{1+3\mu F(Z,DZ)}-\overline{R}(r^{-};T,Z)-\sigma_{c}(t,Z)$

$\overline{R}(r^{-};T,Z)$ is the term of hardening $R(r;TZ)$ calculated for Δp = 0.

- If F < 0 then one is in elastic mode and Δp = 0
- If not one are in load and Δp check;

$$\eta\left(\frac{\Delta p}{\Delta t}\right)^{1/n}=\frac{\sigma_{eq}^{e}-3\mu\,\Delta p}{1+3\mu F(Z,\Delta Z)}-\overline{R}(r^{-};T,Z)-R_{0}\Delta p-\sigma_{c}(T,Z)$$

That is to say the function $\tilde{f}=\dfrac{\sigma_{eq}^{e}-3\mu\,\Delta p}{1+3\mu\,F(Z,\Delta Z)}-\overline{R}(r^{-}Z)-R_0\Delta p-\sigma_c(T,Z)-\eta\left(\dfrac{\Delta p}{\Delta t}\right)^{1/n}$, Δp is thus the solution of the nonlinear scalar equation $\tilde{f}=0$.

The resolution is made in *Code_Aster* by a method of the secants with interval of search [bib15].

Note:

> *Whenever the plasticity of transformation is not taken into account, the statements obtained are the same ones while taking* $F(Z,\Delta Z)=0$ *.*
>
> *Whenever it is the restoration of hardening which is neglected then one also has the same statements but by taking all* θ *the equal ones to 1.*
>
> H_0 *is the hardening slope of curve of tension. In the case of nonlinear isotropic hardening where curve of tension is linear per pieces ,* H_0 *is defined for the segment to which p belongs .*

7.2.2 Stamp tangent

7.2.2.1 Phase of prediction - Option `RIGI_MECA_TANG`

One linearizes the continuous problem compared to time, and one determines $\Delta\mathbf{u^0}$ like solution of the problem of velocity:

$$\int_\Omega \dot{\boldsymbol{\sigma}}\left(\boldsymbol{\varepsilon}\left(\Delta\mathbf{u^0}\right),t\right)\boldsymbol{\varepsilon}(\mathbf{v})\,d\,\Omega=\dot{\mathbf{L}}(t) \qquad \forall\,\mathbf{v}\ \text{cinématiquement admissible}$$

where $\dot{\mathbf{L}}(t)=\int_\Omega \dot{\mathbf{f}}\cdot\mathbf{v}\,d\,\Omega+\int_\Gamma \dot{\mathbf{g}}\cdot\mathbf{v}\,d\,\Gamma$

The problem from of velocity is obtained by deriving compared to time the equations from the continuous problem:

$$\dot{\tilde{\boldsymbol{\sigma}}}=2\,\mu\left(\dot{\tilde{\boldsymbol{\varepsilon}}}-\dot{\boldsymbol{\varepsilon}}^{vp}-\dot{\boldsymbol{\varepsilon}}^{pt}\right)$$

$$\dot{\boldsymbol{\varepsilon}}^{pt}=\frac{3}{2}F(Z,DZ)\,\dot{\tilde{\boldsymbol{\sigma}}}$$

In the case of the élasto-viscoplastic models, one uses, for the phase of prediction, the "elastic" matrix in the meaning where one will not take account of the term $\dot{\boldsymbol{\varepsilon}}^{vp}$. As for the plastic case one a:

$$\dot{\boldsymbol{\varepsilon}}^{vp}=\begin{cases}\dfrac{3}{2}\,\dot{p}\,\dfrac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}} & \text{si } \sigma_{eq}-R(T,Z,r)-\sigma_y(T,Z)=0\\ 0 & \text{si } \sigma_{eq}-R(T,Z,r)-\sigma_y(T,Z)<0\end{cases}$$

the derivative compared to the time of the equation $\sigma_{eq}-R(T,\mathbf{Z},r)-\sigma_y(T,\mathbf{Z})=0$ gives the statement of $\dot{p}$ (relation of consistency).

$$\frac{d\,\sigma_{eq}}{dt}-\frac{dR}{dt}-\frac{d\,\sigma_y}{dt}=\frac{3}{2}\,\frac{\tilde{\boldsymbol{\sigma}}:\mathbf{A}\left(\dot{\tilde{\boldsymbol{\varepsilon}}}-\dot{\tilde{\boldsymbol{\varepsilon}}}^{vp}-\dot{\tilde{\boldsymbol{\varepsilon}}}^{pt}\right)}{\sigma_{eq}}-\frac{dR}{dt}-\frac{d\,\sigma_y}{dt}$$

Note:

The derivative of A was neglected in this phase of prediction

$$\begin{aligned}\dot{R}(T, Z_i, \varepsilon_i^{eff}) &= \sum_{i=1}^{i=5} \dot{Z}_i R_{0i} r_i + \sum_{i=1}^{i=5} Z_i \dot{R}_{0i} r_i + \sum_{i=1}^{i=5} Z_i R_{0i} \dot{r}_i \\ &= \sum_{i=1}^{i=5} \dot{Z}_i R_{0i} r_i + \sum_{i=1}^{i=5} Z_i \dot{R}_{0i} r_i + \sum_{i=1}^{i=5} Z_i R_{0i} \dot{p} \\ &+ \sum_{k=1}^{k=4} \langle \dot{Z}_k \rangle R_{0k} \theta_{\gamma k} r_\gamma + \sum_{k=1}^{k=4} \langle -\dot{Z}_k \rangle R_{0\gamma} \theta_{k\gamma} r_k \\ &- \sum_{i=1}^{i=5} \langle \dot{Z}_i \rangle R_{0i} r_i \\ &= R_0 \dot{p} + B\end{aligned}$$

$$\dot{\sigma}_y = \frac{\partial \sigma_y}{\partial T} \dot{T} + \frac{\partial \sigma_y}{\partial Z} \dot{\boldsymbol{Z}} = C$$

from where:

$$\frac{d\sigma_{eq}}{dt} - \frac{dR}{dt} - \frac{d\sigma_y}{dt} = 3\mu \frac{\tilde{\boldsymbol{\sigma}} : \dot{\tilde{\boldsymbol{\varepsilon}}}}{\sigma_{eq}} - 3\mu \sum_{i=1}^{i=4} K_i F'_i \left(1 - Z_\gamma\right) \langle \dot{\mathbf{Z}} \rangle_i \sigma_{eq} - \left(3\mu + R_0\right) \dot{p} - B - C = 0$$

$$\dot{p} = \frac{\langle 3\mu \dfrac{\tilde{\boldsymbol{\sigma}} : \dot{\tilde{\boldsymbol{\varepsilon}}}}{\sigma_{eq}} - 3\mu \displaystyle\sum_{i=1}^{i=4} K_i F'_i \left(1 - Z_\gamma\right) \langle \dot{\boldsymbol{Z}}_i \rangle \sigma_{eq} - B - C \rangle}{3\mu + R_0}$$

From where, finally the statement of $\dot{\boldsymbol{\varepsilon}}^{vp}$:

$$\begin{aligned}&\dot{\varepsilon}^{vp} = \frac{3}{2\left(3\mu + H_0\right)} < 3\mu \frac{\tilde{\boldsymbol{\sigma}} : \dot{\tilde{\boldsymbol{\varepsilon}}}}{\sigma_{eq}} - 3\mu \sum_{i=1}^{i=4} K_i F'_i \left(1 - Z_\gamma\right) < \dot{\mathbf{Z}}_i > \sigma_{eq} - B - C > \frac{\tilde{\sigma}}{\sigma_{eq}} \\ &\text{si } \sigma_{eq} - R\left(T, \mathbf{Z}, \varepsilon_i^{eff}\right) - \sigma_y\left(T, \mathbf{Z}\right) = 0 \\ &\dot{\boldsymbol{\varepsilon}}^{\mathbf{vp}} = 0 \text{ si } \sigma_{eq} - R\left(T, \mathbf{Z}, \varepsilon_i^{eff}\right) - \sigma_y\left(T, \mathbf{Z}\right) < 0\end{aligned}$$

Taking into account the variations of H_{0i} and σ_y according to the temperature and of metallurgical structure, one chooses by convenience to neglect the term $(B + C)$ and one thus leads to a statement of $\dot{\tilde{\boldsymbol{\sigma}}}$ form:

$$\dot{\tilde{\boldsymbol{\sigma}}} = 2\mu \left[\dot{\tilde{\boldsymbol{\varepsilon}}} - \frac{3}{2\left(3\mu + R_0\right)} \left\langle 3\mu \frac{\tilde{\boldsymbol{\sigma}} : \dot{\tilde{\boldsymbol{\varepsilon}}}}{\sigma_{eq}} - \mu \sum_{i=1}^{i=4} K_i F'_i \left(1 - Z_\gamma\right) \left\langle \dot{\mathbf{Z}}_\mathbf{i} \right\rangle \sigma_{eq} \right\rangle \frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}} - \frac{3}{2} \sum_{i=1}^{i=4} K_i F'_i \left(1 - Z_\gamma\right) \left\langle \dot{\mathbf{Z}}_\mathbf{i} \right\rangle \tilde{\boldsymbol{\sigma}} \right]$$

The statement of $\dot{\tilde{\boldsymbol{\sigma}}}$ depends on the sign of the term (criterion of load-discharge) $3\mu \dfrac{\tilde{\boldsymbol{\sigma}} : \dot{\tilde{\boldsymbol{\varepsilon}}}}{\sigma_{eq}} - 3\mu \displaystyle\sum_{i=1}^{i=4} K_i F'_i \left(Z_i\right) \left\langle \dot{\mathbf{Z}}_i \right\rangle \sigma_{eq}$.

One approximates $\dot{\tilde{\boldsymbol{\sigma}}}$ by:

$$\dot{\tilde{\boldsymbol{\sigma}}}=2\mu\left[\dot{\tilde{\boldsymbol{\varepsilon}}}-\frac{9\mu}{2}\frac{\langle\tilde{\boldsymbol{\sigma}}:\dot{\tilde{\boldsymbol{\varepsilon}}}\rangle}{(3\mu+R_0)}\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}^2}-\frac{3}{2}\sum_{i=1}^{i=4}K_i F'_i(1-Z_\gamma)\langle\dot{\mathbf{Z}}_i\rangle\tilde{\boldsymbol{\sigma}}\left(1-d\frac{3\mu}{3\mu+R_0}\right)\right] \quad \textbf{éq 7.2.2.1 - 1}$$

with $d=1$ if one plasticizes and if one is in load at time t and $d=0$ in the contrary case.

It is noticed that $\dot{\tilde{\boldsymbol{\sigma}}}$ is a function closely connected of $\dot{\tilde{\boldsymbol{\varepsilon}}}$. The plasticity of transformation, like the thermal strain, generate in the problem of velocity a second member.

That introduced by the plasticity of transformation is form:

$$\Delta\mathbf{L}^{pt}=\int_\Omega 2\mu\left[\frac{3}{2}\sum_{i=1}^{i=4}K_i F'_i(1-Z_\gamma)\langle\dot{\mathbf{Z}}_\mathbf{i}\rangle\tilde{\sigma}\left(1-d\frac{3\mu}{3\mu+R_0}\right)\right]\boldsymbol{\varepsilon}(\mathbf{v})\,d\Omega$$

To determine $\Delta\mathbf{u}^\mathbf{0}$, it is necessary to solve after discretization spaces the following linear system of it:

$$\begin{pmatrix}\mathbf{K_0} & \mathbf{T_B}\\ \mathbf{B} & \mathbf{0}\end{pmatrix}\begin{pmatrix}\boldsymbol{\Delta}\mathbf{u^0}\\ \mathbf{0}\end{pmatrix}=\begin{pmatrix}\boldsymbol{\Delta}\mathbf{L}\\ \mathbf{0}\end{pmatrix}+\begin{pmatrix}\mathbf{0}\\ \boldsymbol{\Delta}\mathbf{u^d}\end{pmatrix}+\begin{pmatrix}\boldsymbol{\Delta}\mathbf{L^{th}}\\ \mathbf{0}\end{pmatrix}+\begin{pmatrix}\boldsymbol{\Delta}\mathbf{L^{pt}}\\ \mathbf{0}\end{pmatrix}$$

On simple cases tests for which there exists an analytical solution, one noted that the fact of neglecting the second member $(\Delta\mathbf{L^{pt}})$ could lead, to converge, with a significant number of iterations. This is why this term is taken into account for the phase of prediction.

7.2.2.2 Iterations of Newton - Option `FULL_MECA`

In the method of NEWTON, knowing $\Delta\mathbf{u^n}$, one determines as well as possible $\Delta\mathbf{u^{n+1}}$ checking:

$$\mathbf{F}(\boldsymbol{\varepsilon}(\Delta\mathbf{u^{n+1}}))=\int_\Omega\boldsymbol{\sigma}(\boldsymbol{\varepsilon}(\Delta\mathbf{u^{n+1}}),t)\boldsymbol{\varepsilon}(\mathbf{v})\,d\Omega-L(t)\approx 0$$

$$\mathbf{F}(\boldsymbol{\varepsilon}(\Delta\mathbf{u^{n+1}}))\approx\mathbf{F}(\boldsymbol{\varepsilon}(\Delta\mathbf{u^n}))+\left(\frac{\partial\mathbf{F}}{\partial\Delta\boldsymbol{\varepsilon}}\right)_{(\boldsymbol{\varepsilon}(\Delta\mathbf{u^n}),t)}(\boldsymbol{\varepsilon}(\Delta\mathbf{u^{n+1}})-e(\Delta\mathbf{u^n}))=0$$

From where:

$$\mathbf{K_n}\,\delta\boldsymbol{\varepsilon}=\left(\frac{\partial\mathbf{F}}{\partial\Delta\boldsymbol{\varepsilon}}\right)_{(\boldsymbol{\varepsilon}(\Delta u^n),t)}\delta\boldsymbol{\varepsilon}=-\mathbf{F}(\boldsymbol{\varepsilon}(\Delta\mathbf{u^n}))$$

A each iteration one solves the linear system:

$$\begin{pmatrix}\mathbf{K_n} & \mathbf{B^T}\\ \mathbf{B} & 0\end{pmatrix}\begin{pmatrix}\delta\mathbf{u^n}\\ \delta\boldsymbol{\lambda}^\mathbf{n}\end{pmatrix}=\begin{pmatrix}\mathbf{L}\\ \mathbf{0}\end{pmatrix}-\begin{pmatrix}\mathbf{F^n}\\ \mathbf{0}\end{pmatrix}\quad\text{avec}\quad\mathbf{F^n}=\int_\Omega\boldsymbol{\sigma}(\boldsymbol{\varepsilon}(\Delta\mathbf{u^n}),t)\boldsymbol{\varepsilon}(\mathbf{v})\,d\Omega$$

During iterations of one time step given, the method of NEWTON thus uses the computation of the tangent operator $\mathbf{K_n}$, who is given by derivative of the implicit problem according to the increment of strain $\Delta\boldsymbol{\varepsilon}$. The tangent operator $\mathbf{K_n}$ can be recomputed or not with each iteration.

One gives the statement of $\frac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{\varepsilon}}$ for the constitution of the consistent tangent matrix of the iterative method of Newton.

$$\frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\varepsilon}}=\frac{\partial \tilde{\boldsymbol{\sigma}}}{\partial \boldsymbol{\varepsilon}}+\frac{1}{3}\frac{\partial(tr\,\sigma)}{\partial \boldsymbol{\varepsilon}}\mathbf{Id} \quad \text{avec} \quad \frac{\partial \tilde{\sigma}}{\partial \boldsymbol{\varepsilon}}=\frac{\partial \tilde{\boldsymbol{\sigma}}}{\tilde{\partial} \boldsymbol{\varepsilon}}\frac{\partial \tilde{\boldsymbol{\varepsilon}}}{\partial \boldsymbol{\varepsilon}}$$

$$\frac{\partial \tilde{\boldsymbol{\sigma}}}{\partial \tilde{\boldsymbol{\varepsilon}}}=\frac{\partial}{\partial \tilde{\boldsymbol{\varepsilon}}}\left(2\mu\left(\Delta \tilde{\boldsymbol{\varepsilon}}-\Delta \boldsymbol{\varepsilon}^{\mathbf{pt}}-\Delta \boldsymbol{\varepsilon}^{\mathbf{vp}}\right)\right)$$

One a:

$$\frac{\partial}{\partial \tilde{\boldsymbol{\varepsilon}}}(\Delta \tilde{\varepsilon})=\mathbf{Id} \quad \text{with} \quad Id_{ijkl}=\delta_{ik}\delta_{jl}$$

$$\frac{\partial}{\partial \tilde{\boldsymbol{\varepsilon}}}(\Delta \tilde{\boldsymbol{\varepsilon}}^{pt})=\frac{3}{2}F(Z\,,DZ)\frac{\partial \tilde{\boldsymbol{\sigma}}}{\partial \tilde{\boldsymbol{\varepsilon}}}$$

$$\frac{\partial}{\partial \tilde{\boldsymbol{\varepsilon}}}\left((\Delta \boldsymbol{\varepsilon}^{\tilde{\mathbf{v}}\mathbf{p}})=\frac{3}{2}\frac{(\partial \Delta p)}{\partial \tilde{\boldsymbol{\varepsilon}}}\otimes\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}+\Delta p\frac{\partial}{\partial \tilde{\boldsymbol{\varepsilon}}}\left(\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\right)\right)$$

$$\frac{\partial \tilde{\boldsymbol{\sigma}}}{\partial \tilde{\boldsymbol{\varepsilon}}}=2\mu\,\mathbf{Id}$$

$$\frac{\partial \tilde{\sigma}^{e}_{eq}}{\partial \tilde{\boldsymbol{\varepsilon}}}=3\mu\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}$$

$$\frac{\partial}{\partial \tilde{\boldsymbol{\varepsilon}}}\left(\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\right)=\frac{1}{\sigma^{e}_{eq}}\left(2\mu\,\mathbf{Id}-3\mu\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\otimes\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\right)$$

the statement of $\frac{\partial(\Delta p)}{\partial \tilde{\boldsymbol{\varepsilon}}}$ is obtained while deriving $\tilde{f}=0$ compared to $\delta\boldsymbol{\varepsilon}$, which gives:

$$\frac{\partial(\Delta p)}{\partial \tilde{\boldsymbol{\varepsilon}}}=\frac{3\mu}{3\mu+\left[R_0+\frac{\eta}{n\,\Delta t}\left(\frac{\Delta p}{\Delta t}\right)^{\frac{1-n}{n}}\right](1+3\mu F(Z\,,\Delta Z))}\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}$$

from where:

$$\frac{\partial \tilde{\boldsymbol{\sigma}}}{\partial \tilde{\boldsymbol{\varepsilon}}}=\frac{1}{1+3\mu F(Z\,,\Delta Z)}\left[2\mu\,\mathbf{Id}\left(1-\frac{3\mu\,\Delta p}{\sigma_{eq}^{\ e}}\right)-3\mu^2\left(\frac{1}{3\mu+\left(R_0+\frac{\eta}{n\,\Delta t}\left(\frac{\Delta p}{\Delta t}\right)^{\frac{1-n}{n}}\right)(1+3\mu F(Z\,,\Delta Z))}-\frac{\Delta p}{\sigma^{e}_{eq}}\right)\left(\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\otimes\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\right)\right]$$

éq 7.2.2.2 - 1

7.2.2.3 tangent Operator

Either $\Delta\boldsymbol{\sigma}^*=(\Delta\sigma^*_{11},\Delta\sigma^*_{22,}\Delta\sigma^*_{33},\sqrt{2}\Delta\sigma^*_{12},\sqrt{2}\Delta\sigma^*_{23,}\sqrt{2}\Delta\sigma^*_{13})$ the virtual increase in stress and or $\Delta\boldsymbol{\varepsilon}^*=(\Delta\varepsilon^*_{11},\Delta\varepsilon^*_{22,}\Delta\varepsilon^*_{33},\sqrt{2}\Delta\varepsilon^*_{12},\sqrt{2}\Delta\varepsilon^*_{23,}\sqrt{2}\Delta\varepsilon^*_{13})$ the virtual increase in strain, the operator who binds $\Delta\boldsymbol{\varepsilon}^*$ to $\Delta\boldsymbol{\sigma}^*$ is given by the following statement:

$$\begin{cases}\Delta\tilde{\sigma}^*_{ij}=\left[\frac{2\mu}{a}\left(1-c_3\frac{3\mu\Delta p}{\sigma^{e}_{eq}}\right)\delta_{ik}\delta_{jl}-\frac{c_p}{a}\tilde{\sigma}^{e}_{ij}\tilde{\sigma}^{e}_{kl}\right]\Delta\tilde{\varepsilon}^*_{kl}\\ tr(\Delta\sigma^*)=3\mathrm{K}\,tr(\Delta\varepsilon^*)\end{cases}$$

with

$$a=\begin{cases} 1 & \text{au début de chaque pas de temps (cf .[éq 7.2.2.1-1])(option 'RIGI\_MECA\_TANG')} \\ 1+3\mu F(Z,\Delta Z) & \text{lors des itérations courantes (cf .[éq 7.2.2.2-1])(option 'FULL\_MECA')} \end{cases}$$

$$c_3=\begin{cases} 0 & \text{au début de chaque pas de temps} \\ 1 & \text{lors des itérations courantes} \end{cases}$$

$$\text{et } c_p=\begin{cases} c_1\dfrac{(3\mu)^2}{(\sigma_{eq}^e)^2}\dfrac{1}{3\mu+R_0} & \text{au début de chaque pas de temps} \\ c_2\dfrac{(3\mu)^2}{(\sigma_{eq}^e)^2}\left[\dfrac{1}{3\mu+\left(R_0+\dfrac{\eta}{n\,\Delta t}\left(\dfrac{\Delta p}{\Delta t}\right)^{1-n/n}\right)(1+3\mu F(Z,\Delta Z))}-\dfrac{\Delta p}{\sigma_{eq}^e}\right] & \text{lors des itérations} \end{cases}$$

where:

$$c_1=\begin{cases} 1 & \text{si élasto-plastique et } 3\mu\dfrac{\tilde{\boldsymbol{\sigma}}:\dot{\tilde{\boldsymbol{\sigma}}}}{\sigma_{eq}}-3\mu\displaystyle\sum_{i=1}^{i=4}K_i F'_i(1-Z_\gamma)\langle\dot{\boldsymbol{Z}}_{\boldsymbol{i}}\rangle\sigma_{eq}\geq 0 \\ 0 & \text{sinon} \end{cases}$$

$$: c_2=\begin{cases} 1 & \text{si on est en charge } (\tilde{\boldsymbol{\sigma}}:\delta\tilde{\boldsymbol{\varepsilon}}\geq 0) \text{ et si on plastifie} \\ 0 & \text{sinon} \end{cases}$$

Let us note then $\mathbf{K}$ the operator such as $\Delta\boldsymbol{\sigma}^*=\mathbf{K}\,\Delta\boldsymbol{\varepsilon}^*$

and is S the vector of the deviator of the stresses: . $\mathbf{s}=(\tilde{\sigma_{11}},\tilde{\sigma_{22}},\tilde{\sigma_{33}},\sqrt{2}\tilde{\sigma_{12}},\sqrt{2}\tilde{\sigma_{23}},\sqrt{2}\tilde{\sigma_{13}})$

$$\frac{d\,\Delta\sigma^*}{d\,\Delta\varepsilon^*}=\frac{d\,\Delta\tilde{\sigma}^*}{d\,\Delta\varepsilon^*}+\mathbf{K}(\mathbf{1}\otimes\mathbf{1})$$

$$\frac{d\,\Delta\tilde{\sigma}^*}{d\,\Delta\varepsilon^*}=\frac{d\,\Delta\tilde{\sigma}^*}{d\,\Delta\varepsilon^*}\left(\mathbf{Id}-\frac{1}{3}\mathbf{1}\otimes\mathbf{1}\right)$$

$$\frac{d\,\Delta\tilde{\sigma}^*}{d\,\Delta\varepsilon^*}=\frac{2\mu}{a}\left(1-c_3\frac{3\mu\,\Delta p}{\sigma_{eq}^e}\right)\mathbf{Id}-\frac{c_p}{a}\boldsymbol{\sigma}\otimes\boldsymbol{\sigma}$$

from where:

$$\frac{d\,\Delta\sigma^*}{d\,\Delta\varepsilon^*}=\frac{2\mu}{a}\left(1-c_3\frac{3\mu\,\Delta p}{\sigma_{eq}^e}\right)\left(\mathbf{Id}-\frac{1}{3}\mathbf{1}\otimes\mathbf{1}\right)-\frac{c_p}{a}\boldsymbol{\sigma}\otimes\boldsymbol{\sigma}+K(\mathbf{1}\otimes\mathbf{1})$$

The operator $\mathbf{K}$ is written:

$$K_{11}=\left(K+\frac{2}{3}\left[\frac{2\mu}{a}\left(1-c_3\frac{3\mu\,\Delta p}{\sigma_{eq}^e}\right)\right]-\frac{c_p}{a}s_1 s_1\right)$$

$$K_{22}=\begin{pmatrix} K-\frac{1}{3}\left[\frac{2\mu}{a}\left(1-c_3\frac{3\mu\Delta p}{\sigma_{eq}^e}\right)\right]-\frac{c_p}{a}s_1 s_2 \\ K+\frac{2}{3}\left[\frac{2\mu}{a}\left(1-c_3\frac{3\mu\Delta p}{\sigma_{eq}^e}\right)\right]-\frac{c_p}{a}s_2 s_2 \end{pmatrix}$$

$$K_{33}=\begin{pmatrix} K-\frac{1}{3}\left[\frac{2\mu}{a}\left(1-c_3\frac{3\mu\Delta p}{\sigma_{eq}^e}\right)\right]-\frac{c_p}{a}s_1 s_3 \\ K-\frac{1}{3}\left[\frac{2\mu}{a}\left(1-c_3\frac{3\mu\Delta p}{\sigma_{eq}^e}\right)\right]-\frac{c_p}{a}s_2 s_3 \\ K+\frac{2}{3}\left[\frac{2\mu}{a}\left(1-c_3\frac{3\mu\Delta p}{\sigma_{eq}^e}\right)\right]-\frac{c_p}{a}s_3 s_3 \end{pmatrix}$$

$$K_{44}=\begin{pmatrix} -\frac{c_p}{a}s_1 s_4 \\ -\frac{c_p}{a}s_2 s_4 \\ -\frac{c_p}{a}s_3 s_4 \\ \frac{2\mu}{a}\left(1-c_3\frac{3\mu\Delta p}{\sigma_{eq}^e}\right)-\frac{c_p}{a}s_4 s_4 \end{pmatrix}$$

$$K_{55}=\begin{pmatrix} -\frac{c_p}{a}s_1 s_5 \\ -\frac{c_p}{a}s_2 s_5 \\ -\frac{c_p}{a}s_3 s_5 \\ -\frac{c_p}{a}s_4 s_5 \\ \frac{2\mu}{a}\left(1-c_3\frac{3\mu\Delta p}{\sigma_{eq}^e}\right)-\frac{c_p}{a}s_5 s_5 \end{pmatrix}$$

$$K_{66}=\begin{pmatrix} -\frac{c_p}{a}s_1 s_6 \\ -\frac{c_p}{a}s_2 s_6 \\ -\frac{c_p}{a}s_3 s_6 \\ -\frac{c_p}{a}s_4 s_6 \\ -\frac{c_p}{a}s_5 s_6 \\ \frac{2\mu}{a}\left(1-c_3\frac{3\mu\Delta p}{\sigma_{eq}^e}\right)-\frac{c_p s_6 s_6}{a} \end{pmatrix}$$

where the component i "" of the vector K_{ii} correspond to the "" i terms of the upper part of the ième column of the symmetric matrix $\mathbf{K}$

One notices that the operator $\mathbf{K_0}$ and the operator $\mathbf{K_n}$ are different. The plasticity of transformation does not intervene in the same way in the computation of the two operators.

Note:

These various terms were obtained by developing the case with isotropic hardening but one obtains the same thing in the case of L "kinematic hardening, R_0 is then replaced by the coefficient D" kinematic hardening H_0 .

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