Seismic Response by transient analysis

Summarized

the methods most frequently used for the seismic analysis of structures are the spectral methods and the transitory methods.

The transitory methods (direct linear or not, by modal synthesis) make it possible to calculate the structure response under the effect of imposed seismes: single excitation (identical of each point of anchorage of structure) or multiple and to take into account their possible nonlinear behavior.

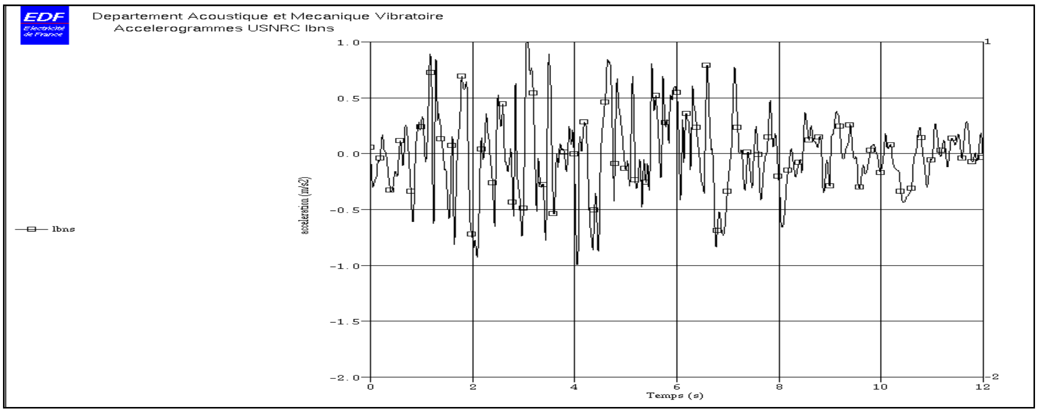
With regard to the spectral methods, one calculates the maximum response, for each mode of vibration, each point of anchorage. The maximum response of the group of structure is then determined by combination of the maximum responses of the modes. This kind of analysis is clarified in documentation of reference [R4.05.03].

Contents

1 Behavior seismic of a structure

1.1 Definitions

the analysis of the seismic behavior of a structure consists in studying its response with an imposed motion: an acceleration, in its various bearings. Imposed acceleration is a temporal signal $\gamma(t)$ called accelerogram (cf [Figure 1.1-a]).

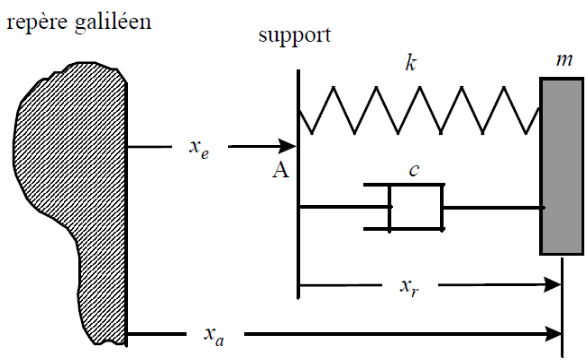


Appear 1.1-a: Accelerogram LBNS

the seismic motion considered in computation is either a real accelerogram known and read by the operator `LIRE_FONCTION` [U4.32.02] or a synthetic accelerogram calculated directly in the code, for example with the procedure `FORMULATES` [U4.31.05].

2 Seismic response of a system with a degree of freedom

Is a simple oscillator made up of a mass m connected to a fixed point by a spring k and a damper c which can move in only one direction x (cf [Figure 2-a]). This oscillator with a degree of freedom is subjected to a horizontal $\gamma(t)$ accelerogram in its support (not A).



Appear 2-a : simple oscillator subjected to a seismic request

displacements of the oscillator are measured or calculated, that is to say in a relative reference related to point: A relative displacement x_r , is in an absolute coordinate system (R_a) : absolute displacement x_a . Absolute displacement x_a breaks up into a uniform displacement of training in translation x_e and a relative displacement x_r :

$$x_a = x_r + x_e \quad \textbf{éq 2-1}$$

One from of deduced by derivative the relation between accelerations:

$$\ddot{x}_a = \ddot{x}_r + \gamma(t) \text{ with } \gamma(t) = \ddot{x}_e \quad \textbf{éq the 2-2}$$

mass is subjected to a horizontal force of recall which is proportional to relative displacement: $F_r = -k.x_r$ and with a horizontal force of damping presumedly proportional to the relative velocity: $F_v = -c.\dot{x}_r$.
The equation of the motion of the mass is written then: $-k.x_r - c.\dot{x}_r = m.\ddot{x}_a$.
Maybe, taking into account the equations [éq 2-1] and [éq 2-2]:

$$m.\ddot{x}_r + c.\dot{x}_r + k.x_r = -m.\gamma(t) = p(t) \quad \textbf{éq 2-3}$$

Note::

The study of the seismic response of an oscillator with a degree of freedom in the relative reference thus consists of the study of the response of an oscillator with a force $p(t)$ of an unspecified form. The solution of the equation of motion [éq 2-3] is then provided by the integral of Duhamel:

$$x_r = \frac{1}{m.\omega_D} \int_0^t p(\tau).e^{-\xi.\omega.(t-\tau)}.\sin\left[\omega_D(t-\tau)\right].d\tau$$

with:

$$p(t) = -m.\gamma(t)$$
$$\omega = \sqrt{\frac{k}{m}},\ \ \xi = \frac{c}{2.m.\omega} \text{ et } \omega_D = \omega.\sqrt{1-\xi^2}$$

3 Seismic response of a system with several Equations

3.1 degrees of freedom of motion in the absolute coordinate system

the equilibrium of a mechanical system consists in writing, some is the time of computation considered, that the sum of the internal forces, inertias and damping is equal to the external forces imposed on this known as system: $\mathbf{F}_{\mathbf{iner}} + \mathbf{F}_{\mathbf{amo}} + \mathbf{F}_{\mathbf{int}} = \mathbf{F}_{\mathbf{ext}}$.

In the case of a linear behavior, known the system is represented by a model of finite elements or discrete elements, one has (after discretization):

$$\begin{cases} \mathbf{F}_{\mathbf{iner}} = \mathbf{M}\,\ddot{\mathbf{X}}_a \\ \mathbf{F}_{\mathbf{int}} = \mathbf{K}\,\mathbf{X}_{\mathbf{a}} \end{cases}$$

- $\mathbf{X}_{\mathbf{a}}$ is the vector of nodal displacements of discretized structure, in the absolute coordinate system;
- $\mathbf{M}$ is the mass matrix of structure;
- $\mathbf{K}$ is the matrix stiffness of structure;
- $\mathbf{F}_{\mathbf{ext}} = \mathbf{F}_{\mathbf{e}} - \mathbf{F}_{\mathbf{c}}$ is the vector of the imposed forces on the structure studied, $\mathbf{F}_{\mathbf{c}}$ that of the possible shock forces (cf [R5.06.03]).
To simplify the presentation, it is considered that the structure is only requested by the displacements imposed on the level of its various supports. Thus $\mathbf{F}_{\mathbf{e}} = \mathbf{0}$.

With an aim of simplifying the presentation, one generally separates the degrees of freedom into two, according to their type:

- the degrees of freedom of structure not subjected to a motion imposed - also called active degrees of freedom - they are the unknowns of the problem;
- the degrees of freedom of structure subjected to a motion imposed - also called `DDL_IMPO` - they are the boundary conditions in displacement of the problem (limiting conditions of Dirichlet).

On the edges of structure where displacements $\mathbf{X}_{\mathbf{s}}$ are imposed, one a: $\mathbf{B}\,\mathbf{X}_{\mathbf{a}} = \mathbf{X}_{\mathbf{s}}$. $\mathbf{B}$ is the transition matrix of all the degrees of freedom of structure to the degrees of freedom of structure subjected to an imposed motion.

The equilibrium of the system is written then, some either $\mathbf{v}$ pertaining to the kinematically admissible space of displacements i.e., some or $\mathbf{v}$ such as $\mathbf{B}\,\mathbf{v} = 0$:

$$\begin{cases} \langle \mathbf{M}\ddot{\mathbf{X}}_a + \mathbf{F}_{\mathbf{amo}} + \mathbf{K}\,\mathbf{X}_{\mathbf{a}} - \mathbf{F}_{\mathbf{ext}}, \mathbf{v} \rangle = 0 \\ \mathbf{B}\,\mathbf{X}_{\mathbf{a}} = \mathbf{X}_{\mathbf{s}} \end{cases}$$

That is to say:

$$\begin{cases} \mathbf{M}\,\ddot{\mathbf{X}}_a + \mathbf{F}_{\mathbf{amo}} + \mathbf{K}\,\mathbf{X}_{\mathbf{a}} - \mathbf{F}_{\mathbf{ext}} = -\mathbf{B}^{\mathbf{T}}.\boldsymbol{\lambda} \\ \mathbf{B}\,\mathbf{X}_{\mathbf{a}} = \mathbf{X}_{\mathbf{s}} \end{cases} \quad \textbf{éq 3.1-1}$$

$\mathbf{F}_{\mathbf{a}} = -\,\mathbf{B}^{\mathbf{T}}.\boldsymbol{\lambda}$ is the vector of the reaction forces exerted by the bearings on the structure.

By taking account of the partition of the degrees of freedom, the vector of displacements in the absolute coordinate system is written: $\mathbf{X}_{\mathbf{a}} = \begin{Bmatrix} \mathbf{x}_{\mathbf{a}} \\ \mathbf{x}_{\mathbf{s}} \end{Bmatrix}$. The operators describing structure become: $\mathbf{M} = \begin{bmatrix} \mathbf{m} & \mathbf{m}_{xs} \\ \mathbf{m}_{sx} & \mathbf{m}_{ss} \end{bmatrix}$, $\mathbf{K} = \begin{bmatrix} \mathbf{k} & \mathbf{k}_{xs} \\ \mathbf{k}_{sx} & \mathbf{k}_{ss} \end{bmatrix}$ with $\mathbf{m}_{sx} = \mathbf{m}_{xs}^{\mathbf{T}}$ and $\mathbf{k}_{sx} = \mathbf{k}_{xs}^{\mathbf{T}}$ the vector of the external forces applied to structure is written: $\mathbf{F}_{\mathbf{ext}} = \begin{Bmatrix} -\mathbf{f}_{\mathbf{c}} \\ 0 \end{Bmatrix}$.

The fundamental equation of the dynamics in the absolute reference frame is written then, by taking account of the partition of the degrees of freedom:

$$\begin{bmatrix} \mathbf{m} & \mathbf{m}_{xs} \\ \mathbf{m}_{sx} & \mathbf{m}_{ss} \end{bmatrix} \cdot \begin{Bmatrix} \ddot{\mathbf{x}}_a \\ \ddot{\mathbf{x}}_s \end{Bmatrix} + \mathbf{F}_{amo} + \begin{bmatrix} \mathbf{k} & \mathbf{k}_{xs} \\ \mathbf{k}_{sx} & \mathbf{k}_{ss} \end{bmatrix} \cdot \begin{Bmatrix} \mathbf{x}_\mathbf{a} \\ \mathbf{x}_\mathbf{s} \end{Bmatrix} = \begin{Bmatrix} -\mathbf{f}_\mathbf{c} \\ \mathbf{f}_\mathbf{a} \end{Bmatrix}$$

Maybe, by considering only the active degrees of freedom:

$$\mathbf{m}\,\ddot{\mathbf{x}}_a + \mathbf{F}_{amo} + \mathbf{k}\,\mathbf{x}_\mathbf{a} = -\mathbf{f}_\mathbf{c} - \mathbf{m}_{xs}\,\ddot{\mathbf{x}}_s - \mathbf{k}_{xs}\,\mathbf{x}_\mathbf{s}$$

This approach requires the knowledge displacements and absolute velocities associated with the accelerogram $\gamma(t)$ but the recorders measure either of accelerations or velocities. One can go back to displacements by simple or double integration with command CALC_FONCTION [U4.32.04]. However, uncertainties of measurement give drifts which it is advisable to correct: displacements are thus less well-known than the velocities and accelerations. One will keep in memory the orders of magnitude of the maximum amplitudes following:

- some tenth of " g " for accelerations;
- a few tens of cm/s for the velocities;
- a few tens of cm for displacements.

One will also ensure oneself that at the end of the seisme the velocity and displacement are realistic i.e. with more few tens of cm for displacement, null for the velocity.

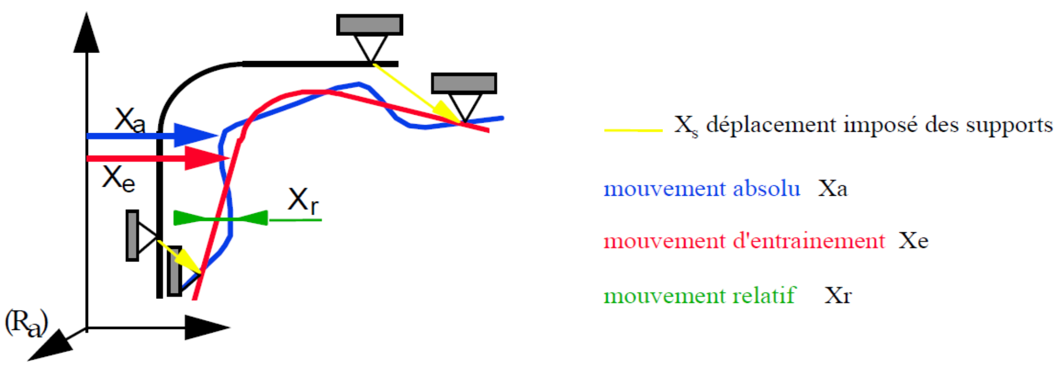
3.2 Equations of motion in the relative reference

3.2.1 Decomposition of absolute motion

the requests undergone by a structure during a seisme are classified in two types in the rules of construction (ASME, RCC-M):

- stresses induced by the relative motion of structure compared to its static deformed shape or **primary stresses**. These requests are due to the effects inertial of the seisme;
- stresses induced by differential displacements of the anchorages or **secondary stresses**.

Generally, one thus breaks up the study of structures into the study of the static deformed shape due to motions of the supports (it is the motion of training) and into the study of the vibrations induced by accelerations of the supports around this deformed shape (it is relative motion).



The absolute displacement of any point M of structure, not subjected to an imposed displacement, is equal to the sum of relative displacement and the displacement of training of this point:

$$\mathbf{X_a}(M)=\mathbf{X_r}(M)+\mathbf{X_e}(M)$$ **éq 3.2.1-1**

Is:

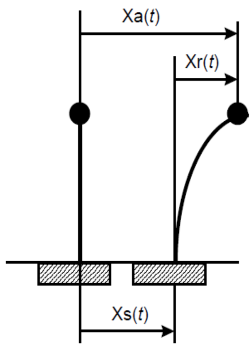
- $\mathbf{X_a}$, the vector of displacements in the absolute reference frame;
- $\mathbf{X_r}$, the vector of definite relative displacements as the vector of displacements of the structure compared to the deformed which it would have under the static action of the displacements imposed on the level of the supports. $\mathbf{X_r}$ is thus null at the points of anchorage: $\mathbf{B}\,\mathbf{X_r}=0$;
- $\mathbf{X_e}$, the vector of the displacements of training defined as displacements of structure requested statically by imposed displacements of the supports $\begin{cases}\mathbf{B}\,\mathbf{X_e}=\mathbf{X_s}\\ \mathbf{K}\,\mathbf{X_e}=-\mathbf{B^T}.\boldsymbol{\lambda}_\mathbf{e}\end{cases}$ with $\boldsymbol{\lambda}=\boldsymbol{\lambda}_\mathbf{r}+\boldsymbol{\lambda}_\mathbf{e}$ $\Leftrightarrow \mathbf{X_e}=\boldsymbol{\Psi}.\mathbf{X_s}$.
  - $\boldsymbol{\Psi}$ is the matrix of the static modes. The static modes represent, in the absence of external forces, the response of structure with a unit displacement imposed on each degree of freedom of connection (others being blocked).

3.2.2 Simple or multiple excitation

to clarify more in detail the approach moving relative motion, and more particularly the computation of the components of training, it is necessary to introduce notion of the simple or multiple excitation.

3.2.2.1 Simple excitation

One considers that imposed seismic motion is a motion of solid body. It is generally said that **the structure mono-is supported**.



The absolute displacement of any point M of structure, not subjected to an imposed displacement thus breaks up into **a relative displacement compared to a mobile coordinate system related to the support** where is imposed seismic motion and in **a displacement of rigid training**.

In this case, the static modes correspond to the six modes of rigid body. As the structure is linear elastic, one separately studies the effects of the six components of seismic motion. For each seismic direction, one simply writes the inertia forces induced by the seisme in the following form:

$$\mathbf{P}(t)=-\mathbf{M}\boldsymbol{\Psi}.\ddot{\mathbf{X}}_s=-\gamma(t).\mathbf{M}\boldsymbol{\Delta}$$

- $\gamma(t)$ is the accelerogram of seismic motion in a direction;

- Δ is the solid state mode and unit in this direction;
- The seismographs measure only signals of translation. To consider that the studied structure mono is supported amounts supposing that all its fulcrums undergo the same translation. In this case, the components of $[\Delta]$ are worth 1 for the degrees of freedom which correspond to displacement in the seismic direction considered and 0 for the degrees of freedom which correspond to displacement in seismic directions perpendicular to that considered or with rotations.
- However, considering the size of the models, the complete seismic analysis of equipment is generally carried out in several stages. The detailed seismic analysis of the equipment considered uses then like excitations, the accelerations calculated at the time of the first stage. They are composed of the six accelerograms of translation and rotation. One thus calculates the three modes corresponding to imposed displacements of translation and the three modes corresponding to imposed displacements of rotation. If seismic motion is an imposed Ω rotation, in a point M, $\Delta_M=\vec{OM}\wedge\vec{\Omega}$ for the degrees of freedom which correspond to displacement of translation and Ω for the degrees of freedom which correspond to rotations.

3.2.2.2 Multiple excitation

One cannot always only consider:

- the accelerations undergone by all the points of anchorage of studied structure are identical and in phase;
- the supports indeformable and are actuated by the same rigid body motion.

In this case, it is said that **the structure is multi - supported**. The static modes $\boldsymbol{\Psi}=\begin{Bmatrix}\Psi\\ \mathbf{Id}\end{Bmatrix}$ correspond then to $6.\mathrm{nb}_{supports}$ the static modes (or $3.\mathrm{nb}_{supports}$ modes) where $nb_{supports}$ is the number of different accelerograms undergone simultaneously by structure. They are calculated by the operator `MODE_STATIQUE` [U4.52.14] with option `DDL_IMPO`. They are solution of the following equation:

$$\begin{cases}\boldsymbol{\Psi}\,\mathbf{X_e}=\mathbf{X_s}\\ \mathbf{K}\,\mathbf{X_e}=-\mathbf{B^T}.\boldsymbol{\lambda}_\mathbf{e}\end{cases}\quad\text{either}\quad\begin{bmatrix}\mathbf{k} & \mathbf{k}_{xs}\\ \mathbf{k}_{sx} & \mathbf{k}_{ss}\end{bmatrix}.\begin{Bmatrix}\Psi\\ \mathbf{Id}\end{Bmatrix}=\begin{Bmatrix}0\\ \mathbf{f_{a\,e}}\end{Bmatrix}\qquad\textbf{éq 3.2.2.2 - 1}$$

Or, by considering only the active degrees of freedom: $\mathbf{k}.\boldsymbol{\Psi}+\mathbf{k}_{xs}.\mathbf{Id}=0$.

The inertia forces induced by the seisme are written then simply:

$$\mathbf{P}(t)=-\sum_{m=1}^{\text{nb\_supports}}\mathbf{M}.\boldsymbol{\Psi}_\mathbf{m}.\ddot{\mathbf{X}}_{s_m}(t)$$

3.2.3 Modelization of damping

One considers that the damping dissipated by structure is of viscous type i.e. the damping force is proportional to the relative velocity of structure:
$\mathbf{F_{amo}}=\mathbf{C}\dot{\mathbf{X}}_r$ where $\mathbf{C}$ is the damping matrix of structure.

That amounts neglecting the effect the imposed velocity. Indeed, one can more generally write:
$\mathbf{F_{amo}}=\mathbf{C}\dot{\mathbf{X}}_a=\mathbf{C}\dot{\mathbf{X}}_r+\mathbf{C}\boldsymbol{\Psi}.\dot{\mathbf{X}}_s$.

In the case of a uniform excitation at the base (case of the mono-bearing), damping intervenes only on relative displacements (the damping forces are null for a rigid displacement). In the case of a multiple excitation where the static solution is not any more one rigid displacement, to consider that the damping force is proportional to the relative velocity of structure is a simplifying assumption.

3.2.4 Fundamental equation of the dynamics

the fundamental equation of the dynamics [éq 3.1-1], in **the relative reference**, is written then, taking into account the equations [éq 3.2.1-1] and [éq 3.2.2.2 - 1]:

$$M\ddot{\mathbf{X}}_r+\mathbf{C}\dot{\mathbf{X}}_r+\mathbf{K}\mathbf{X}_\mathbf{r}=-\mathbf{M}.\boldsymbol{\Psi}.\ddot{\mathbf{X}}_s+\mathbf{F}_\mathbf{ext}-\mathbf{B}^\mathbf{T}.\boldsymbol{\lambda}_\mathbf{r} \qquad \textbf{éq 3.2.4-1}$$

Is, by partitionnant the degrees of freedom:

$$\begin{bmatrix} m & m_{xs} \\ m_{sx} & m_{ss} \end{bmatrix}.\begin{Bmatrix} \ddot{x}_r \\ 0 \end{Bmatrix}+\begin{bmatrix} c & c_{xs} \\ c_{sx} & c_{ss} \end{bmatrix}.\begin{Bmatrix} \dot{x}_r \\ 0 \end{Bmatrix}+\begin{bmatrix} k & k_{xs} \\ k_{sx} & k_{ss} \end{bmatrix}.\begin{Bmatrix} x_r \\ 0 \end{Bmatrix}=\begin{Bmatrix} -f_c \\ f_{a_r} \end{Bmatrix}-\begin{Bmatrix} \left(m.Y+m_{xs}.\mathbf{Id}\right)\ddot{x}_s \\ \left(m_{sx}.Y+m_{ss}.\mathbf{Id}\right)\ddot{x}_s \end{Bmatrix}$$

with $\mathbf{c}_{sx}=\mathbf{c}_{xs}^\mathbf{T}$

Is, by considering only the active degrees of freedom:

$$\mathbf{m}.\ddot{\mathbf{x}}_r+\mathbf{c}.\dot{\mathbf{x}}_r+\mathbf{k}.\mathbf{x}_\mathbf{r}=-\mathbf{f}_\mathbf{c}-\left(\mathbf{m}.\boldsymbol{\Psi}+\mathbf{m}_{xs}.\mathbf{Id}\right)\ddot{\mathbf{x}}_s$$

The main advantages of the approach in relative displacement compared to that in absolute displacement are the following:

- it is not necessary to integrate the accelerogram $\boldsymbol{\gamma}(t)$;
- relative displacements obtained make it possible to directly determine the induced primary stresses by the seisme.

3.3 Computation of the seismic loading

the seismic loading (cf [§3.2]) $-\mathbf{M}.\boldsymbol{\psi}$ is $-\left(\mathbf{m}.\boldsymbol{\Psi}+\mathbf{m}_{xs}.\mathbf{Id}\right)\ddot{\mathbf{x}}_s$ on the active degrees of freedom is built by the operator `CALC_CHAR_SEISME` [U4.63.01]. It is usable directly during a direct transient analysis with `DYNA_LINE_TRAN` [U4.53.02] or of a transient analysis by modal synthesis with `DYNA_TRAN_MODAL` [U4.53.21]. On the other hand, during a nonlinear direct transient analysis with `DYNA_NON_LINE` [U4.53.01], it should be transformed into a concept of the type `charges`. This is carried out from operator `AFFE_CHAR_MECA` [U4.44.01] in the following way:

```
char_sei=      CALC_CHAR_SEISME (...)
charge=            AFFE_CHAR_MECA (MODELS =…, VECT_ASSE = char_sei)
dyna_nlin=         DYNA_NON_LINE (
                 EXCIT=_F    ( CHARGE= con_lim,)
                         _F (   CHARGE= cham_no,
                                FONC_MULT= acceler)
                …)
```

In the case of a supported mono structure, it is enough to indicate the direction of the seisme:

```
mono_x = CALC_CHAR_SEISME (MATR_ASSE = mass,
              DIRECTION (...), MONO_APPUI = ' OUI')
```

In the case of a multimedia structure, should as a preliminary have been calculated the static modes. One calculates as many seismic loadings of supports which undergo a different acceleration.

```
multi_xi = CALC_CHAR_SEISME (MATR_ASSE = mass, DIRECTION (...),
                    NOEUD = NOI, MODE_STAT = mode_stat,)
```

3.4 Loading of type incident wave

It is also possible to impose a seismic loading by plane wave via the command AFFE_CHAR_MECA and the key word factor ONDE_PLANE. That corresponds to the loadings classically met during computations of interaction soil-structure by the integral equations.

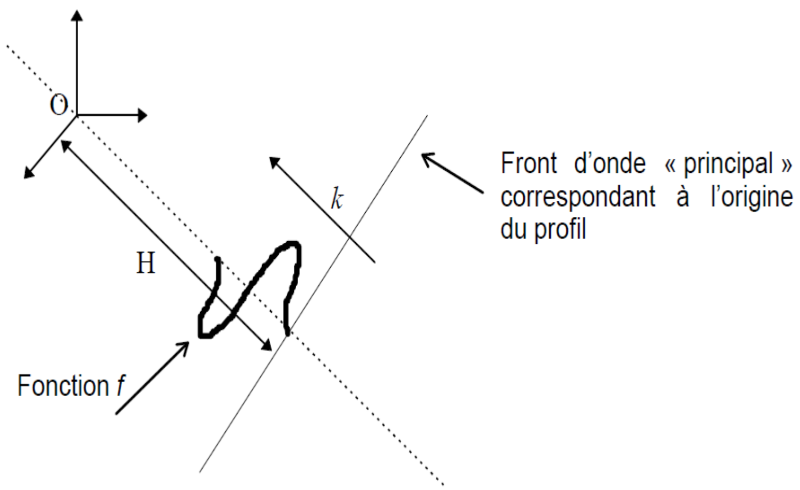
In harmonic, one plane wave elastic is characterized by its direction, its pulsation and its type (wave P for the compression waves, waves SV or SH for the waves of shears). Out of transient, the data of the pulsation, corresponding to one standing wave in time, must be replaced by the data of a profile of displacement which one will take into account the propagation in the course of time in the direction of the wave.

More precisely, one characterizes:

- one wave P by the function $\mathbf{u}(x,t)=f(k.x-C_p t)\mathbf{k}$
- one wave S by the function $\mathbf{u}(x,t)=f(k.x-C_s t)\wedge\mathbf{k}$

With:

- $\mathbf{k}$, unit vector of direction
- f then represents the profile of the wave given according to the direction $\mathbf{k}$.



H_0 is the distance from the principal wave front in the beginning O , carried by the directing vector of the wave at the initial time of computation, H the distance from the principal wave front in the beginning O , one unspecified time.

Note:

This kind of load is available in a direct transient computation linear DYNA_LINE_TRAN or not DYNA_NON_LINE .
The use of this kind of loading will be detailed in a specific note.

4 Transitory seismic response by modal synthesis

4.1 Description of the method

the method of modal recombination consists in breaking up the relative motion of structure on the basis of the eigen modes. As this one is null on the level of the supports, one projects the equation of the dynamics on the basis of blocked eigen mode (eigen modes obtained by blocking all the degrees of freedom of connection).

$$\mathbf{X_r}=\boldsymbol{\Phi}.\mathbf{Q}$$

- $\boldsymbol{\Phi}$ is the matrix of the blocked eigen modes;
- $\mathbf{Q}$ the vector of the unknowns generalized on the basis of blocked eigen mode.

The blocked eigen modes are solution of:

$\left(\mathbf{K}-\Omega_i^2.\mathbf{M}\right)\boldsymbol{\Phi}_\mathbf{i}=\begin{Bmatrix}0\\\mathbf{F_i}\end{Bmatrix}$ where $\mathbf{F_i}$ are the modal reactions at the fulcrums.

The equation of motion projected on the basis of dynamic mode is written then:

$$\mathbf{M_G}\ddot{\mathbf{Q}}(t)+\mathbf{C_G}\dot{\mathbf{Q}}(t)+\mathbf{K_G}\mathbf{Q}(t)=-\boldsymbol{\Phi}^\mathbf{T}.\mathbf{M}.\boldsymbol{\Psi}.\ddot{\mathbf{X}}_s+\boldsymbol{\Phi}^\mathbf{T}.\mathbf{F_{ext}}-\boldsymbol{\Phi}^\mathbf{T}.\mathbf{B}^\mathbf{T}.\boldsymbol{\lambda}_\mathbf{r}$$

where $\mathbf{M_G}$, $\mathbf{C_G}$ and $\mathbf{K_G}$ are the mass matrixes, of generalized damping and stiffness. To simplify, it is considered that they are diagonal. The damping matrix as $\mathbf{C_G}$ generalized because one supposes as the assumption of Basile is checked (the damping matrix is a linear combination of the mass matrixes and stiffness).

Maybe, by considering only the active degrees of freedom:

$$\mathbf{m_G}.\ddot{\mathbf{q}}(t)+\mathbf{c_G}.\dot{\mathbf{q}}(t)+\mathbf{k_G}.\mathbf{q}(t)=-\boldsymbol{\Phi}^\mathbf{T}.\mathbf{f_c}-\boldsymbol{\Phi}^\mathbf{T}.\left(\mathbf{m}.\boldsymbol{\Psi}+\mathbf{m}_{xs}.\mathbf{Id}\right)\ddot{\mathbf{x}}_s$$

In the absence of shock, one is thus led to solve a set of decoupled equations (there is as much as eigen modes).

Note:

It is possible to calculate a modal base with nondiagonal matrixes. It is enough to specify it during construction to the classification generalized by the key word `STOCKAGE = "PLEIN"` of the command `NUME_DDL_GENE` [U4.65.03].

4.2 Choice of modal base

For the seismic analysis of a linear structure, it would be necessary in theory to retain all the modes of which the eigenfrequencies are lower than the cut-off frequency (generally about $33\,Hz$). In practice, one is often satisfied to preserve in modal base only the modes which contribute to a significant degree to the response. One then preserves only the modes of which the unit effective mass in a direction is higher than 1 ‰ and one also makes sure that, for the set of these modes selected, the unit effective mass cumulated in each direction is not very different from the total mass of structure (higher than 90%). The criterion of office plurality of the effective modal masses is reached by connecting the following operators:

- Computation of the total mass of structure: `POST_ELEM` [U4.81.22]
  ```
  masse_in = POST_ELEM (MASS_INER =_F (TOUT = "YES"))
  ```
- Computation of the blocked dynamic eigen modes: they are calculated in operator `MODE_ITER_SIMULT` [U4.52.03] or in `MODE_ITER_INV` [U4.52.04] according to the selected method.
  `mode = MODE_ITER_SIMULT ();` or `mode = MODE_ITER_INV ();`
- Standardization of the modes compared to the generalized mass: `NORM_MODE` [U4.52.11]
  ```
  NORM_MODE (MODE = mode, NORM = ' MASSE_GENE', MASSE_INER = masse_in);
  ```
- Extraction of the modal base of the modes of which the unit effective mass exceeds a certain threshold (1 ‰ for example) and checking which extracted modes represent at least 90% of the total mass: `EXTR_MODE` [U4.52.12]
  ```
  EXTR_MODE (
     FILTRE_MODE (MODE= mode, CRIT_EXTRE= "MASSE_EFFE_UN",
  SEUIL =1.e-3)
     PRINTING (CUMUL = ' OUI');
  ```

Note:

the sum of the effective modal masses is worth in fact the total mass which works on selected modal base. In other words, this working total mass is worth the total mass minus the contributions out of mass which are carried by clamped degrees of freedom (which thus do not work on modal base). Thus, for example, on a system with 1 mass-spring degree of freedom with a mass $M1$ at the top and another mass $M2$ at the level to erase it, then the working mass will be worth $M1$ and the total mass $M1+M2$. Consequently, the unit effective modal mass for the only mode of the system will be worth $M1/(M1+M2)$. The total office plurality will thus have the same value and, according to the ratio in $M1$ and $M2$, one will not be able thus inevitably to reach 90% of the total mass $(M1+M2)$, even by considering all the modes (there is only one only mode on this example). In practice, the model with the finite elements will be so and realistic, more the difference between the working mass and the total mass will be weak.

Macro command `MACRO_MODE_MECA` [U4.52.02] makes it possible to directly connect all the three last preceding commands.

Attention, certain local responses (in the typical case of nonlocalised linearities) can be strongly influenced by modes of a higher nature of which the frequency is beyond the cut-off frequency and of which the effective modal mass is low (lower than 1 ‰). Key word `VERI_CHOC` of the command `DYNA_TRAN_MODAL` [U4.53.21] makes it possible to check a posteriori that selected modal base is sufficient. If it is not the case, one highly advises to supplement it.

4.3 Computation of the dynamic response of structure studied by modal synthesis

After having calculated the base of the dynamic eigen modes and having built a classification generalized by `NUME_DDL_GENE` [U4.65.03], one projects then the mass matrixes, of damping and stiffness, on this same basis with operator `PROJ_MATR_BASE` [U4.63.12], the vectors second member with `PROJ_VECT_BASE` [U4.63.13].

Note:

Macro command `PROJ_BASE` [U4.63.11] makes it possible to directly connect all three operation.

The matrixes and vectors thus projected, one calculates the generalized response of the system mono or multi - excited using operator `DYNA_TRAN_MODAL` [U4.53.21].

4.4 Taken into account of the modes neglected by static correction

During the computation of the generalized response of an excited mono structure, it is possible to take into account, a posteriori, the static effect of the neglected modes. In this case, once returned on physical base one corrects the value of the relative displacement calculated (respectively relative velocity and relative acceleration) by the contribution of a pseudo-mode. The pseudo-mode is defined by the difference between the static mode associated with the unit loading of standard imposed constant acceleration and projection on the calculated dynamic modes of displacement (respectively relative velocity and relative acceleration).

One has then:

$$\begin{cases} \mathbf{X}_{\mathbf{r\_corrigé}} = \mathbf{X}_{\mathbf{r}} + \sum_i f_i(t).\left(\boldsymbol{\Psi}_{\mathbf{i}} - \sum_{j=1}^{p} \eta_j.\boldsymbol{\Phi}_{\mathbf{j}}\right) \\ \dot{\mathbf{X}}_{\mathrm{r\_corrigé}} = \dot{\mathbf{X}}_r + \sum_i \dot{f}_i(t).\left(\boldsymbol{\Psi}_{\mathbf{i}} - \sum_{j=1}^{p} \dot{\eta}_j.\boldsymbol{\Phi}_{\mathbf{j}}\right) \\ \ddot{\mathbf{X}}_{\mathrm{r\_corrigé}} = \ddot{\mathbf{X}}_r + \sum_i \ddot{f}_i(t).\left(\boldsymbol{\Psi}_{\mathbf{i}} - \sum_{j=1}^{p} \ddot{\eta}_j.\boldsymbol{\Phi}_{\mathbf{j}}\right) \end{cases}$$

The multiplicative functions of time $f_i(t)$ correspond to the accelerogram imposed $\gamma_i(t)$ in each direction i considered.

The approach to be followed is the following one:

- Computation of the unit loading of type forces imposed (constant acceleration) in the direction of the seisme: `AFFE_CHAR_MECA` [U4.44.01]. One will pay attention to permute the sign of the direction since the seismic inertia force is form $\mathbf{P}(t)$=- $\mathbf{M}\boldsymbol{\Psi}.\ddot{\mathbf{X}}_s$
```
cham_no = AFFE_CHAR_MECA (MODELE=modèle, PESANTEUR= (VALE,
DIRECTION)) ;
```
- Computation of the linear static response of structure to the preceding loading case: `MACRO_ELAS_MULT` [U4.51.02].
```
mode_cor = MACRO_ELAS_MULT (CHAR_MECA_GLOBAL = con_lim,…
               CAS_CHARGE =_F (NOM_CAS = ' xx', CHAR_MECA = cham_no)) ;
```
It will be noted that there is as much loading case than of direction of seisme
- Computation of derivatives first and second of the accelerogram: `CALC_FONCTION` [U4.32.04].
`deri_pre` and `deri_sec = CALC_FONCTION (OPTION = DERIVE);`
- Computation of the response generalized in taking into account the modes neglected by static correction:
```
dyna_mod = DYNA_TRAN_MODAL (MASS_GENE =… , RIGI_GENE =…
               MODE_CORR = mode_cor
               EXCIT =_F (CORR_STAT = "YES"
                      D_FONC_DT = deri_pre, D_FONC_DT2 = deri_sec.)
               …) ;
```
- Return towards physical base: static correction is not implicitly taken into account. It is necessary to specify `CORR_STAT=' OUI'` in `RECU_FONCTION` or `REST_GENE_PHYS` so that static correction is taken into account.

Note:

In the case of an multi-excited structure, the taking into account of the modes neglected by static correction is not developed. One post-draft absolute displacement in this case.

4.5 Taken into account of the multimedia character of a structure

It was seen previously (cf [§3.3]) that to compute: the seismic loading in the case of a multimedia structure, should as a preliminary have been calculated the static modes. If one wants to be able to restore the quantities calculated in the absolute coordinate system or if one wants to be able to take into account nonlocated linearities, it is also necessary to specify in DYNA_TRAN_MODAL which the studied structure is multi-excited. Indeed, in this last case, one compared to each time, the vector of absolute displacements of each point of shock considered, in order to determine if there is shock and to calculate the corresponding shock forces.

The approach to be followed is the following one:

- Computation of the static modes: MODE_STATIQUE [U4.52.14].
```
mode_stat = MODE_STATIQUE (DDL_IMPO = (...));
```
- Computation of the response generalized in taking into account the component of training:
```
dyna_mod = DYNA_TRAN_MODAL (MASS_GENE =… , RIGI_GENE =…
                MODE_STAT = mode_stat
                EXCIT =_F (MULT_APPUI = "YES"
                        ACCE = accelero, QUICKLY = velocity, DEPL =
                    moves
                        DIRECTION = (...), NOEUD =NO1
                        …)
                …) ;
```

4.6 Postprocessings

operators REST_GENE_PHYS [U4.63.31] or RECU_FONCTION [U4.32.03] can then restore in physical space the calculated evolutions:

- operator REST_GENE_PHYS restores overall (the complete field) displacements, velocities and accelerations;
- operator RECU_FONCTION locally restores (temporal evolution of a degree of freedom) displacements, velocities and accelerations.

One can restore the relative quantities while specifying (MULT_APPUI = "NON") or the absolute quantities by (MULT_APPUI = "YES").
One obtains then displacements of training necessary to the computation of the secondary quantities by withdrawing from absolute displacements relative displacements. This is carried out by the command CALC_FONCTION [U4.32.04] option COMB.

From the preceding evolutions, one can also extract the values maximum and RMS calculate the response sprectrum of associated oscillator. This is carried out by the command CALC_FONCTION options MAX, RMS and SRO.

5 Direct transitory seismic response

direct integration is realizable is with assumptions of linear behavior: operator DYNA_LINE_TRAN [U4.53.02] is with assumptions of nonlinear behavior: operator DYNA_NON_LINE [U4.53.01]. Put except for the way of taking into account the seismic loading (cf [§3.3]), syntaxes of DYNA_NON_LINE and DYNA_LINE_TRAN are identical.

5.1 Taken into account of a damping equivalent to modal damping

Generally, the most precise information that one has on damping comes from the tests of vibration which make it possible to determine, for a given resonance frequency f_i , the width of resonance corresponding and thus reduced damping ξ_i to this resonance. **It is thus necessary to be able to take into account, in a direct transient computation, a damping equivalent to modal damping**.

From the spectral development of the matrix identity:

$$\mathbf{Id}=\sum_{i=1}^{n\_modes}\frac{\mathbf{X_i}\mathbf{X_i^T}\mathbf{K}}{\mathbf{X_i^T}\mathbf{K}\mathbf{X_i}}=\sum_{i=1}^{n\_modes}\frac{\mathbf{X_i}\mathbf{X_i^T}\mathbf{K}}{M_{G\_i}.\omega_i^2}$$

one shows:

- that one can develop the damping matrix of structure $\mathbf{C}$ in series of eigen modes:

$$\mathbf{C}=\sum_{i=1}^{n\_modes}a_i.\left(\mathbf{K}.\boldsymbol{\Phi}_\mathbf{i}\right)\left(\mathbf{K}.\boldsymbol{\Phi}_\mathbf{i}\right)^T$$

- and that, account held of the definition of the percentage of critical damping:

$$\boldsymbol{\Phi}_\mathbf{i}^\mathbf{T}.\mathbf{C}.\boldsymbol{\Phi}_\mathbf{i}=2.\mathbf{M_{G\_i}}.\omega_i.\xi_i.a_i=2.\frac{\xi_i}{K_{G\_i}.\omega_i}$$

It is thus advised with the user to specify (syntaxes of DYNA_NON_LINE and DYNA_LINE_TRAN are identical), the values of modal dampings for each eigenfrequency via the key word factor AMOR_MODAL.

That amounts imposing a damping force proportional to the relative velocity of structure:

$$\mathbf{F_{amo}}=\mathbf{C}\dot{\mathbf{X}}_r \quad \text{with} \quad \mathbf{C}=\sum_{i=1}^{n\_modes}2.\frac{\xi_i}{K_{G\_i}.\omega_i}.\left(\mathbf{K}.\boldsymbol{\Phi}_\mathbf{i}\right)\left(\mathbf{K}.\boldsymbol{\Phi}_\mathbf{i}\right)^T$$

5.2 Taking into account of a request multi-bearings with restitutions of the relative and absolute fields

By defaults, the quantities are calculated in the relative reference. In DYNA_NON_LINE and DYNA_LINE_TRAN, one uses a syntax identical to that of DYNA_TRAN_MODAL (presence of key keys MODE_STAT and MULT_APPUI = "YES") to calculate them in the absolute coordinate system.

6 Interaction soil-structure

the seismic behavior of a building depends on the characteristics of the soil on which it is posed since it depends on the seismic motion imposed on the ground and the dynamic behavior of the building and its foundations. The interaction soil-structure most frequently contributes to decrease the response of studied structure.

6.1 Impedance of a foundation

Is a massless surface rigid foundation, subjected to a harmonic force of pulsation ω : $P(t)=P_0.e^{iwt}$ It is thus actuated by of a the same $X(t)$ movement frequency. One calls **impedance of the foundation**, the complex number $K(\omega)$, function of the frequency ω such as: $K(\omega)=\frac{P(t)}{X(t)}$.

Several analytical or numerical methods make it possible to calculate the impedance of a foundation according to the complexity of the foundation and the soil on which it is posed or partially hidden. Among most frequently used, one quotes:

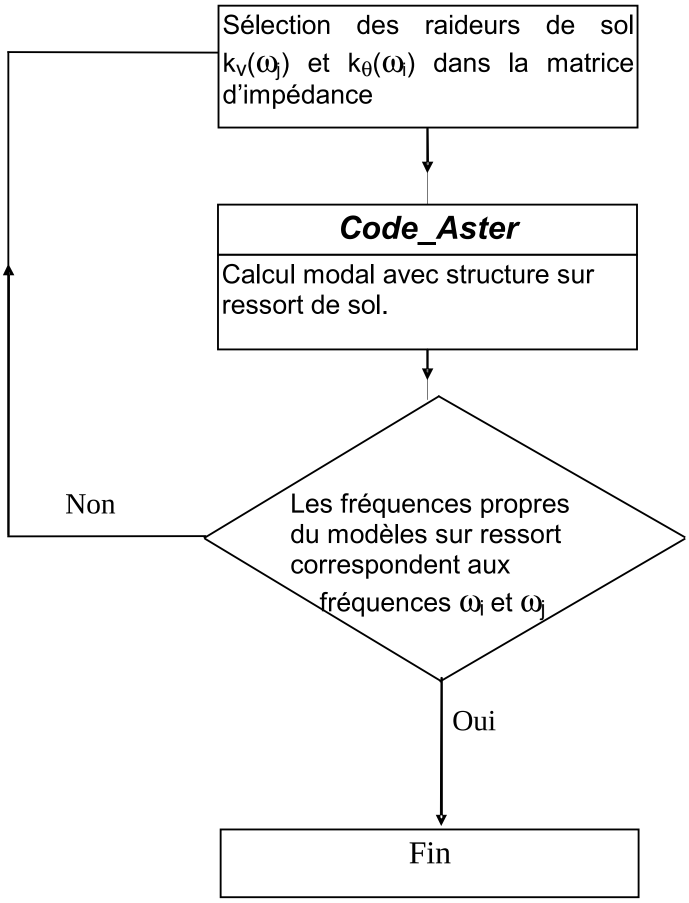
- analytical methods within the competences of WOLF or DELEUZE where it is supposed that to erase it is circular, rigid and posed on a homogeneous soil. The foundation must be surface;
- numerical method of the code CLASSI where it is supposed that to erase it is of an unspecified form, rigid and posed on a possibly stratified soil. The foundation must be surface;
- numerical method of the code MISS3D where to erase it can be of an unspecified form, possibly deformable and posed on a possibly stratified soil.

It is possible to treat the interaction soil-foundation by **the frequential method of coupling** (taken into account of the response frequency of the matrix of impedance) by carrying out a computation coupled MISS3D / *Code_Aster*. This kind of computations is not detailed in this documentation of reference. One presents here only the case more flow where the interaction soil-foundation is treated by **the method within the competences of soil** (it is considered that the terms of the matrix of impedance are independent of the frequency).

In the case of a surface rigid foundation, the impedance is calculated at the center of gravity of surface in contact in a reference related to the principal axes of inertia of this surface. For each frequency, it is expressed in the shape of a matrix of dimension $(6,6)$. One adjusts then the value of each term according to a particular eigen mode of the building studied in blocked base:

- frequency of the first mode of swinging ω_0 for the horizontal stiffness $Kx(\omega_0), Ky(\omega_0)$ and rotation $Krx(\omega_0), Kry(\omega_0)$;
- frequency of the first mode of pumping ω_1 for the vertical stiffness $Kz(\omega_1)$ and of torsion $Krz(\omega_1)$.

As the eigenfrequencies of the building depend on the soil stiffness, the computation of the global values within the six competences of soil results from an illustrated iterative process appears [Figure 6.1-a]. The first soil stiffness $Kx(\omega_0), Ky(\omega_0), Kz(\omega_1), Krx(\omega_0), Kry(\omega_0)$ and $Krz(\omega_1)$ are selected according to the first eigenfrequencies of swinging (ω_0) and pumping (ω_1) of structure in blocked base. The soil stiffness is then adjusted with the first significant eigenfrequencies of structure on spring until correspondence of the frequencies to which the functions of impedance are calculated with the values of the eigenfrequencies of the system coupled soil - building.



Appear 6.1-a : Process of adjustment of the soil stiffness

6.2 Taken into account of a modal damping calculated according to the rule of the RCC-G

One breaks up damping due on the ground into part of material origin and a geometrical part: damping due to the reflection of the elastic waves in the soil.
The rule of the RCC-G consists in adding, for each mode, depreciation of each under structure constitutive of the building considered and the depreciation structural and geometrical of the soil balanced by their respective rate of potential energy compared to total potential energy:

$$\eta_i = \frac{\sum_k E_{ki}.\eta_k + \sum_s E_{si}.\eta_{si}}{\sum_k E_{ki} + \sum_s E_{si}}$$

with:

- η_i , the average reduced damping of the mode i ;
- η_k , the reduced damping of k the ème element of structure;
- η_{si} , the reduced damping of the come out from soil S for the mode i ;
- E_{ki} , the potential energy of k the ème element of structure for the mode i ;
- and E_{si} , the potential energy of the come out from soil s for the mode i .

In the regulation, modal damping is restricted with a maximum value of 0,3.

The part of material origin of the damping of the soil is calculated by balancing damping of each under structure by the ratio: rate of potential energy on total potential energy. As for the geometrical part of damping, it is calculated by distributing the values of damping for each direction (three translations and three rotations) balanced by the rate of potential energy in the soil of the direction. The directional values of damping are obtained while interpolating, for each calculated eigenfrequency, the directional functions of damping exit of a code of interaction soil-structure (PARASOL, CLASSI or MISS3D). The ratio of the imaginary part on twice the real part of the matrix of impedance: $\frac{\mathrm{Im}(K(\omega))}{2.\mathrm{Re}(K(\omega))}$, provides the values of this radiative damping.

The approach to be followed is the following one:

- Computation of the potential energy dissipated in studied structure: `POST_ELEM` [U4.81.22]

  E_k = `POST_ELEM (ENER_POT=_F (TOUT = "YES")) ;`

- Computation of modal damping by the rule of the RCC-G: `CALC_AMOR_MODAL` [U4.52.13]

```
l_amor = CALC_AMOR_MODAL (
    ENER_SOL =_F (MODE_MECA = base_modale, GROUP_NO_RADIER =… ,
                  KX = Kx(ω0) , KY = Ky(ω0) , KZ = Kz(ω1) ,
                  KRX = Krx(ω0) , KRY = Kry(ω0) , KRZ: Krz(ω1) )) ;
    AMOR_INTERNE =_F (GROUP_MA =…, ENER_POT = Ek , AMOR_REDUIT = ηk )
    AMOR_SOL =F (FONC_AMOR_GEO = Im(K(ω)) / 2.Re(K(ω)) )
    );
```

The computation of the contribution of the soil to potential energy E_s (key word factor `ENER_SOL`) is calculated starting from the values of impedance of soil determined previously (cf [§6.1]). She can be calculated according to two methods different according to whether one average the modal forces (key word `RIGI_PARASOL`) or modal displacements with the nodes from the basemat.

The damping reduced of the come out from soil η_s (key word factor `AMOR_SOL`) is calculated starting from the values of radiative damping.

6.3 Distribution of the stiffness and damping of soil

If one wants to study the effect of a seisme on the possible separation of the basemat for example, one can have to model the soil either by a single spring at the center of gravity of the interface soil - building but by a carpet of springs. This is possible thanks to the command AFFE_CARA_ELEM [U4.42.01] option `RIGI_PARASOL`.

The approach consists in calculating in each node of the mesh of the basemat the elementary stiffness $\left(k_x, k_y, k_z, kr_x, kr_y, kr_z\right)$ to apply starting from the global values within the three competences of

translations: kx, ky, kz and within the three competences of rotations: krx, kry, krz exits of a code of interaction soil-structure (or calculated analytically).
It is supposed that the elementary stiffness of translation is proportional to the surface $S(P)$ represented by the node P and a function of distribution $f(r)$ depending on the distance r from the node P at the center of gravity from the basemat O :

$$\begin{cases} K_x=\sum_P k_x(P)=k_x.\sum_P S(P).f(OP) \\ K_y=\sum_P k_y(P)=k_y.\sum_P S(P).f(OP) \\ K_z=\sum_P k_z(P)=k_z.\sum_P S(P).f(OP) \end{cases}$$

One from of deduced then k_x $k_x(P)$ starting from computation:

$$k_x(P)=k_x.S(P).f(OP)=K_x.\frac{S(P).f(OP)}{\sum_P S(P).f(OP)}.$$

One from of deduced in the same way $k_y(P)$ and $k_z(P)$.

For the elementary stiffness of rotation, one distributes what remains after having removed the contributions due to the translations in the same way that translations:

$$\begin{cases} K_{rx}=\sum_P k_{rx}(P)+\sum_P \left[k_y(P).z_{OP}^2+k_z(P).y_{OP}^2\right]=k_{rx}.\sum_P S(P).f(OP)+\sum_P \left[k_y(P).z_{OP}^2+k_z(P).y_{OP}^2\right] \\ K_{ry}=\sum_P k_{ry}(P)+\sum_P \left[k_x(P).z_{OP}^2+k_z(P).x_{OP}^2\right]=k_{ry}.\sum_P S(P).f(OP)+\sum_P \left[k_x(P).z_{OP}^2+k_z(P).x_{OP}^2\right] \\ K_{rz}=\sum_P k_{rz}(P)+\sum_P \left[k_x(P).y_{OP}^2+k_y(P).x_{OP}^2\right]=k_{rz}.\sum_P S(P).f(OP)+\sum_P \left[k_x(P).y_{OP}^2+k_y(P).x_{OP}^2\right] \end{cases}$$

One from of deduced then k_{rx} $k_{rx}(P)$ starting from computation:

$$\begin{aligned} k_{rx}(P)&=k_{rx}.S(P).f(OP) \\ &=\left(K_{rx}-\sum_P \left[k_y(P).z_{OP}^2+k_z(P).y_{OP}^2\right]\right).\frac{S(P).f(OP)}{\sum_P S(P).f(OP)} \end{aligned}$$

One from of deduced in the same way $k_{ry}(P)$ and $k_{rz}(P)$.

Note:

By default, one considers that the function of distribution is constant and unit i.e. each surface is affected same weight.

One can distribute in the same way six global values of damping, analytical or calculated by a code of interaction soil-structure.

6.4 Taken into account of an absorbing border

If one wants to calculate the seismic response of a stopping, it is necessary, amongst other things, power to take into account it not reflection of the waves in the valley. This is possible thanks to elements at absorbing border: option `IMPE_ABSO` in `DYNA_NON_LINE` and `DYNA_LINE_TRAN`. This functionality is not detailed in this document. It will be the object of a specific note.

7 Bibliography

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8 of the versions of the document Version

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