Stochastic approach for the seismic analysis

Summarized:

This document presents a probabilistic method of calculating to determine the response of a structure subjected to a random excitation of seismic type starting from the interspectrums of the excitation to the something to lean on of structure. The response itself is expressed in the form of interspectrums.

Contents

1 Introduction

Classically the response of a structure subjected to a seismic excitation can be calculated by two approaches:

- computation of transient dynamics if the excitation is defined by an accelerogram (cf [R4.05.01]).
- computation by the classical spectral method if the excitation is defined by a response sprectrum of oscillator (SRO) (cf [R4.05.03]).

However a seismic excitation is by random nature. These two methods are not envisaged initially to take account of it: in a case it is necessary to reiterate for various excitations of many temporal computations then to make a statistical average of it (important cost computation), in the other case one carries out very conservative assumptions by considering averages (of quadratic type simple or supplements for example) for the maximum of the responses.

Also it was developed a method of calculating of the probabilistic type, also called “stochastic approach of seismic computation”, based on the computation of the dynamic response expressed in interspectrums of power starting from the power spectral densities of the excitation. This method has in particular the advantage of better taking into account the correlations between the excitations to the various bearings of structure.

The discussion of the various advantages of this method can be thorough in the reference [bib1].

We thus present the principle of the method and the notations retained starting from the classical approachs, then in third part probabilistic computation itself.

Finally in fourth part the various methods will be presented to obtain the exiting interspectrum.

2 Principle of the approach

2.1 Position of the problem considered and general principle

One in the case of places **a multi-supported structure**, i.e. the structure has m degree of freedom-bearings, each one being subjected to its own excitation (not necessarily equal everywhere). It is supposed that the structure is represented by a model finite elements comprising n degrees of freedom. One seeks the response in a number finished (and low) of l degrees of freedom.

It is supposed that **the quantity excitation is of standard imposed motion** and results in a family of accelerograms $g_j(t)$ for each degree of freedom-bearings j $j=1,m$.

The absolute motion of structure **is broken up** classically moving **of training and relative motion.**

The computation of the response in interspectrums of power is realized by **modal recombination**.

Following this modal computation, a computation of dynamic response random breaks up into three parts:

- definition of the interspectrum of power exiting,
- computation of the interspectrum of power response.

These the first two parts are of the command the subject DYNA_ALEA_MODAL [U4.53.22].

The restitution of the interspectrum of power response on physical base is carried out with command REST_SPEC_PHYS [U4.63.22].

- computation of statistical parameters from the interspectrum of power result.

This last stage is treated by the command POST_DYNA_ALEA [R7.10.01] [U4.84.04].

2.2 Decomposition of motion

following decompositions and projections are detailed in documentation of reference relating to the resolution by transient computation of a seismic computation [R4.05.01]. We retain only the broad outlines here of them.

That is to say $\mathbf{X_a}$ the vector absolute displacement (of dimension n) of all the degrees of freedom of structure.

The total response known as **absolute** $\mathbf{X_a}$ of structure is expressed as the sum of a relative **contribution** $\mathbf{X_r}$ and contribution **of training** $\mathbf{X_e}$ due to displacements of anchorage (subjected to the accelerations represented by an accelerogram $g_j(t)$ of each degree of freedom-bearings j, $j=1,m$).

$$\mathbf{X}_a(t) = \mathbf{X}_r(t) + \mathbf{X}_e(t)$$

Are $\mathbf{M}, \mathbf{K} \text{ et } \mathbf{C}$ the mass matrixes, of stiffness and damping of the problem, restricted with the not supported degrees of freedom.

The equation of motion is written then in the reference related to relative motion:

$$\mathbf{M}\ \ddot{X}_r(t)+\mathbf{C}\ \{\dot{X}_r(t)+\mathbf{K}\ \mathbf{X}_r(t)= -\mathbf{M}\ \ddot{X}_e(t)+\mathbf{F}_{ext}$$
$$F_{ext} \text{ vecteur des forces extRieures}$$

In general the external forces are null during a computation of seismic responses.

2.3 Decomposition on the basis of the response

modal base The computation in interspectrums of power is carried out by **modal recombination** and is appealed, moving imposed, with a modal base which understands at the same time dynamic modes and static modes.

That is to say $\Phi=\{\phi_{i,i=1,n}\}$ the matrix (n,n) of the dynamic modes calculated for the associated conservative system, by maintaining the m blocked bearings.

That is to say $\Psi=\{\psi_{j,j=1,m}\}$ the matrix (n,m) of the static modes. The mode Ψ_j corresponds to the deformed shape of structure under a unit displacement imposed on the degree of freedom-bearing j, the other degrees of freedom - bearings being blocked.

The imposed displacement of the anchorages $\mathbf{X_s}(t)$ is connected to $\mathbf{X_e}(t)$ by the relation: $\mathbf{X_e}(\mathbf{t})=\boldsymbol{\Psi}\,\mathbf{X_s}(\mathbf{t})$.

The components of the acceleration of the points of anchorage $\ddot{\mathbf{X}}_\mathbf{s}(t)$ are the accelerograms $g_j(t)$ $j=1,m$.

One can thus write $\ddot{\mathbf{X}}_\mathbf{e}(t)=\boldsymbol{\Psi}\,\ddot{\mathbf{X}}_\mathbf{s}(t)=\sum_{j=1}^{m}\boldsymbol{\psi}_\mathbf{j}\,g_j(t)$

One carries out the change of variable $\mathbf{X_r}(t)=\boldsymbol{\Phi}\,\mathbf{q}(t),\mathbf{q}(t)$ is the vector of the generalized coordinates. By prémultipliant the equation of motion by ${}^T\boldsymbol{\Phi}$ ' one obtains - in the absence of external forces other than the seismic excitation - the equation projected on the basis of dynamic mode:

$${}^T\boldsymbol{\Phi}\,\mathbf{M}\,\boldsymbol{\Phi}\,\ddot{\mathbf{q}}(t)+{}^T\boldsymbol{\Phi}\,\mathbf{C}\boldsymbol{\Phi}\,\dot{\mathbf{q}}(t)+{}^T\boldsymbol{\Phi}\,\mathbf{K}\,\boldsymbol{\Phi}\,\mathbf{q}(t)=-{}^T\boldsymbol{\Phi}\,\mathbf{M}\boldsymbol{\Phi}\,\ddot{\mathbf{X}}_\mathrm{s}(t)$$

It is supposed that the damping matrix is a linear combination of the mass matrixes and stiffness (assumption of damping of constant Rayleigh on the structure or assumption of Basile allowing a diagonal damping). The base $\boldsymbol{\Phi}$ which orthogonalise matrixes M and K, orthogonalise thus also the matrix $\mathbf{C}$.

Taking into account this assumption, the preceding equation breaks up into n decoupled scalar equations in the form:

$$\ddot{\mathbf{q}}_\mathrm{i}+2\xi_\mathrm{i}\omega_\mathrm{i}\dot{\mathbf{q}}_\mathrm{i}+\omega_\mathrm{i}^2\mathbf{q}_\mathrm{i}=-\sum_{j=1}^{m}\mathbf{p}_\mathrm{ij}g_\mathrm{j}(t) \quad \text{for } i=1,n$$

Where one noted:

$\mu_i={}^T\boldsymbol{\Phi}_\mathrm{i}\mathbf{M}\boldsymbol{\Phi}_i$ la masse modale

$k_i={}^T\boldsymbol{\Phi}_\mathrm{i}\mathbf{K}\boldsymbol{\Phi}_\mathrm{i}$ la rigidité modale

$\omega_i=\sqrt{\frac{k_i}{m_i}}$ la pulsation modale

$\xi_i=\frac{{}^T\boldsymbol{\Phi}_\mathrm{i}\mathbf{C}\boldsymbol{\Phi}_\mathrm{i}}{2\mu_i\omega_i}$ l'amortissement modal réduit

$\mathrm{p_{ij}}=\frac{{}^T\boldsymbol{\Phi}_i\mathbf{M}\boldsymbol{\Psi}_j}{\mu_i}$ le facteur de participation modale de l'appui j sur le mode dynamique i

The solution $\mathbf{q}_\mathrm{i}(t)$ of this equation corresponds to the response of the dynamic mode i with the group of the seismic excitation.

One can still break up the problem by introducing the unknown $\mathbf{d}_{ij}(t)$ solution of the differential equation: $\ddot{\mathbf{d}}_{ij}+2\boldsymbol{\xi}_i\boldsymbol{\omega}_i\dot{\mathbf{d}}_{ij}+\omega_i^2\mathbf{d}_{ij}=g_j(t)$, this last equation corresponds to the response of the dynamic mode i with acceleration $g_j(t)$. Relative displacement on physical base is expressed then:

$$\mathbf{X}_\mathrm{r}(\mathrm{t})=-\sum_{\mathrm{i}=1}^{\mathrm{n}}\sum_{\mathrm{j}=1}^{\mathrm{m}}\mathbf{p}_{ij}\mathbf{d}_{ij}(\mathrm{t})\boldsymbol{\varphi}_\mathrm{i}$$

Information on the position of the something to lean on is contained in the participation factor modal.

2.4 Harmonic response

One thus broke up the total response of structure into a relative contribution and a differential contribution due to displacements of the anchorages such as:

$$\mathbf{X}_\mathrm{a}(t)=\mathbf{X}_\mathrm{r}(t)+\mathbf{X}_\mathrm{e}(t)$$
$$\text{avec}$$
$$\begin{cases}\ddot{\mathbf{X}}_e(t)=\boldsymbol{\Psi}\ddot{\mathbf{X}}_\mathrm{s}(t)=\sum_{j=1}^{m}\boldsymbol{\psi}_j g_j(t)\\ \mathbf{X}_\mathrm{r}(t)=-\sum_{i=1}^{n}\sum_{j=1}^{m}\mathbf{p}_{ij}\mathbf{d}_{ij}(t)\boldsymbol{\varphi}_\mathrm{i}\quad \text{où } \mathbf{d}_{ij(t)} \text{ est solution de } \ddot{\mathbf{d}}_{ij}+2\xi_i\omega_i\dot{\mathbf{d}}_{ij}+\omega_i^2\mathbf{d}_{ij}=g_j(t)\end{cases}$$

The solution of this last differential equation by the method of the transformation of Fourier utilizes the modal transfer transfer functions $h_i(\omega)$ such as: . $h_i(\omega)=\dfrac{1}{\omega_i^2-\omega^2+2\mathrm{i}\xi_i\omega_i\omega}$

One thus obtains: $\mathbf{d}_{ij}(\omega)=h_i(\omega).g_j(\omega)\,\mathrm{et}\,\ddot{\mathbf{d}}_{ij}(\omega)=-\omega^2h_i(\omega).g_j(\omega)$

The total harmonic response of structure results from the preceding formulas by modal recombination.

$$\ddot{\mathbf{X}}_a(\omega)=\ddot{\mathbf{X}}_r(\omega)+\ddot{\mathbf{X}}_e(\omega)$$
$$\ddot{\mathbf{X}}_a(\omega)=\omega^2\sum_{i=1}^{n}\sum_{j=1}^{m}\mathbf{p}_{ij}\mathbf{h}_\mathrm{j}(\omega)\boldsymbol{\varphi}_j(\omega)j_i+\sum_{j=1}^{m}\boldsymbol{\Psi}_j g_j(\omega)$$

One then reveals the complex matrix (N, m), known as following matrix of $\mathbf{H}(\omega)$ transfer:

$$\mathbf{H}(\omega)=\omega^2\mathbf{p}\,\mathbf{h}(\omega)\boldsymbol{\Phi}+\boldsymbol{\Psi}$$

where $\mathbf{p}$ is the matrix of the participation factors, $\mathbf{h}(\omega)$ the vector of the modal transfer transfer functions $h_i(\omega)$.

The total response of structure is worth $\ddot{\mathbf{X}}_\mathrm{a}(\omega)=\mathbf{H}(\omega)\ddot{\mathbf{E}}(\omega)$, où $\ddot{\mathbf{E}}(\omega)$ is the vector of m lines made up of the transforms of Fourier of accelerations $g_j(t)$ to m the degrees of freedom-bearings.

It is seen that this statement determines the response in acceleration. This then forces to twice integrate the response to obtain displacement, this problem is presented in [bib4]. One of the additional interests of the method which we propose here is to abstract itself from this difficulty.

3 The dynamic response random

3.1 Recall on the power spectral densities [bib2]

3.1.1 Definitions

Is a probabilistic signal defined by its density of probability $p_x(x_1, t_1, \ldots, x_n, t_n)$. This density of probability makes it possible to calculate the functions moments of the signal.

Moment of order 1 or hope of the signal:

$$\mu_{\mathbf{X}}(t) = \mathrm{E}[\mathbf{X}(t)] = \int_{-\infty}^{+\infty} \mathbf{X}\, p_x(x, t)\, dx$$

Moments of order 2 or intercorrelation of two signals:

$$\rho_{\mathbf{XY}}(t_1, t_2) = \mathrm{E}\left[\mathbf{X}(t_1)\overline{\mathbf{Y}(t_2)}\right] = \int_{-\infty}^{+\infty} \mathbf{x}\,\overline{\mathbf{y}}\, p(x, t_1; y, t_2)\, dx dy$$

When the signal is steady, the intercorrelation depends only on $\tau = t_2 - t_1$.

It is written $R_{\mathbf{XY}}(t) = \mathrm{E}\left[\mathbf{X}(t)\overline{\mathbf{Y}(t-\tau)}\right]$

Power spectral density and interspectrum

One defines $\mathbf{S}_{\mathbf{XY}}(\omega)$ the interspectrum of power or density interspectral of power between two steady probabilistic signals by the transform of Fourier of the function of intercorrelation, which one writes:

$$\mathbf{S}_{\mathbf{XY}}(\omega) = \frac{1}{2\mathrm{p}} \int_{-\infty}^{+\infty} \mathbf{R}_{\mathbf{XY}}(\tau) e^{-i\omega\tau} d\tau$$

The opposite formula is written: $\mathbf{R}_{\mathbf{XY}}(t) = \int_{-\infty}^{+\infty} \mathbf{S}_{\mathbf{XY}}(\omega) e^{i\omega\tau} d\omega$

$\mathbf{S}_{\mathbf{XY}}(\omega)$ is generally complex and checks the relation of symmetry: $\mathbf{S}_{\mathbf{YX}}(\omega) = \overline{\mathbf{S}_{\mathbf{XY}}(\omega)}$.

When $\mathbf{X} = \mathbf{Y}$, $\mathbf{S}_{\mathbf{XX}}(\omega)$ is called **autospectrum of power or power spectral density (DSP)**. This function has the property to be real and always positive.

3.1.2 Relations between the DSP and the other characteristics of the signal

Note:

Most of the time, the signal is defined over a limited time, its transform of Fourier does not exist, one defines a transform of Fourier then estimated over one period length T by:

$$\hat{X}_T(\omega) = \frac{1}{2\pi} \int_{-T/2}^{T/2} X(t) e^{-i\omega\tau} d\tau \; .$$

One then has the following relationships to this estimated transform of Fourier:

$$\mathbf{S}_{\mathbf{XY}}(\omega) = \lim_{T \to +\infty} \frac{2\pi}{T} \mathrm{E}\left[\hat{\mathbf{X}}_{\mathrm{T}}(w)\overline{\hat{\mathbf{Y}}_{\mathrm{T}}(\omega)}\right]$$

$$\mathbf{S}_{\mathbf{XX}}(\omega) = \lim_{T \to +\infty} \frac{2\pi}{T} \mathrm{E}\left[\hat{\mathbf{X}}_{\mathrm{T}}(w)\overline{\hat{\mathbf{X}}_{\mathrm{T}}(\omega)}\right]$$

Restrain between the autospectrum of power and the power of the signal:

The power of a signal is equal to its variance. For a centered signal, the variance is worth: $\sigma_X^2 = R_{XX}(0)$.

One thus has: $\sigma_X^2 = R_{XX}(0) = \int_{-\infty}^{+\infty} S_{XX}(\omega) d\omega$.

3.2 The equations of motion

the total response of structure is determined by the relation: $\ddot{\mathbf{X}}_a(\omega) = \mathbf{H}(\omega)\ddot{\mathbf{E}}(\omega)$,

where $\ddot{\mathbf{E}}(\omega)$ is the vector of $\mathbf{m}$ lines made up of the excitations represented by the transforms of Fourier of the accelerograms $g_j(t)$ to $\mathbf{m}$ the degrees of freedom-bearings,

$\mathbf{H}(\omega)$ is the matrix of transfer defined by $\mathbf{H}(\omega) = \omega^2 \mathbf{ph}(\omega)\boldsymbol{\Phi} + \boldsymbol{\Psi}$

where $\mathbf{p}$ is the matrix of the participation factors,

$\mathbf{H}(\omega)$ the vector of the modal transfer transfer functions $h_i(\omega)$

$\boldsymbol{\Phi}$ bases dynamic modes

$\boldsymbol{\Psi}$ bases static modes

it comprises $\mathbf{n}$ lines (= many free degrees of freedom of structures) and $\mathbf{m}$ columns.

3.2.1 Stamp "interspectral-excitation"

NB:

This name "stamps interspectral-excitation" is abusive: it means "matrix of density interspectral of power of the excitation".

It is supposed that the seismic excitation can be regarded as a steady signal - taking into account the relationship between representative times - and centered. This makes it possible to use a certain number of result probabilistic analysis. One is interested then in the steady response of the system with a steady excitation.

One notes $\mathbf{S}_{\ddot{\mathbf{E}}\ddot{\mathbf{E}}}(\omega)$ the matrix of the interspectrums of power corresponding to the excitation. Its data is clarified in chapter 4.

For memory we recall here that it is calculated from transforms of Fourier of accelerations. It is a matrix ($m \times m$). The ij term corresponds to the interspectrum between the signals $\ddot{\mathbf{E}}^i$ and $\ddot{\mathbf{E}}^j$ is still between the transforms of Fourier of the accelerograms g_i and g_j .

3.2.2 Random dynamic response

One saw that the interspectrum of power between two probabilistic signals is the transform of Fourier of the function of intercorrelation of the two signals. One applies it to the total response of structure:

$$S_{\ddot{X}_a\ddot{X}_a}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} R_{\ddot{X}_a\ddot{X}_a}(\tau) e^{-i\omega\tau} d\tau = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{E}\left[\ddot{\mathbf{X}}_a(t)^T \overline{\ddot{\mathbf{X}}_a(t-\tau)}\right] e^{-i\omega\tau} d\tau$$

One works then in the temporal field to express the function of intercorrelation of the total response $R_{\ddot{X}_a\ddot{X}_a}(t,t')$.

One notes $h(t)$ the impulse response of the system: $\mathbf{h}(t) = \mathrm{TF}^{-1}[\mathbf{H}(\omega)]$

and $\ddot{\mathbf{e}}(t)$ the transform of Fourier reverses DSP exciter: $\ddot{\mathbf{e}}(t) = \mathrm{TF}^{-1}[\ddot{\mathbf{E}}(\omega)]$

By transform of Fourier relation reverses: $\ddot{\mathbf{X}}_a(\omega) = \mathbf{H}(w)\ddot{\mathbf{E}}(\omega)$

one has $\ddot{\mathbf{X}}_a(t)=\mathbf{h}\times\ddot{\mathbf{e}}(t)=\int_R \mathbf{h}(u)\ddot{\mathbf{e}}(t-u)du$

$$R_{\ddot{\mathbf{X}}_a\ddot{\mathbf{X}}_a}(t,t')=\mathrm{E}\left[\ddot{\mathbf{X}}_a(t)^T\,\overline{\ddot{\mathbf{X}}_a(t')}\right]$$
$$R_{\ddot{\mathbf{X}}_a\ddot{\mathbf{X}}_a}(t,t')=\mathrm{E}\left[\int_R \mathbf{h}(u)\ddot{\mathbf{e}}(t-u)du^T\,\overline{\int_R \mathbf{h}(v)\ddot{\mathbf{e}}(t'-v)dv}\right]$$
$$R_{\ddot{\mathbf{X}}_a\ddot{\mathbf{X}}_a}(t,t')=\mathrm{E}\left[\int_R\int_R \mathbf{h}(u)\ddot{\mathbf{e}}(t-u)^T\,\overline{\ddot{\mathbf{e}}(t'-v)}^T\,\overline{\mathbf{h}(v)}dv\,du\right]$$

One supposes in this analysis the deterministic system, one can thus leave the impulse response the computation of the expectation. It comes

$$R_{\ddot{\mathbf{X}}_a\ddot{\mathbf{X}}_a}(t,t')=\int_R\int_R \mathbf{h}(u)\mathrm{E}[\ddot{\mathbf{e}}(t-u)^T\,\overline{\ddot{\mathbf{e}}(t'-v)}]^T\,\overline{\mathbf{h}(v)}dv\,du$$

the excitation is supposed a steady process, the intercorrelation thus depends only on the variation of time $\tau=t\text{-}t'$:

$$R_{\ddot{\mathbf{E}}\ddot{\mathbf{E}}}(t-t'-u+v)=\mathrm{E}[\ddot{\mathbf{e}}(t-u)^T\,\overline{\ddot{\mathbf{e}}(t'-v)}]=R_{\ddot{\mathbf{E}}\ddot{\mathbf{E}}}(\theta)\,\text{pour}\,\theta=t-t'-u+v=\tau-u+v$$

from where $R_{\ddot{\mathbf{X}}_a\ddot{\mathbf{X}}_a}(t,t')=\int_R\int_R \mathbf{h}(u)\,R_{\ddot{\mathbf{E}}\ddot{\mathbf{E}}}(\theta)^T\,\overline{\mathbf{h}(v)}\,dv\,du=R_{\ddot{\mathbf{X}}_a\ddot{\mathbf{X}}_a}(\tau)$ what justifies a posteriori the approach.

One now defers this statement in the statement of the power spectral density of the response:

$$S_{\ddot{\mathbf{X}}_a\ddot{\mathbf{X}}_a}(w)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}R_{\ddot{\mathbf{X}}_a\ddot{\mathbf{X}}_a}(\tau)e^{-i\omega\tau}d\tau=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_R\int_R \mathbf{h}(u)R_{\ddot{\mathbf{E}}\ddot{\mathbf{E}}}(\tau-u+v)^T\,\overline{\mathbf{h}(v)}e^{-i\omega\tau}dv\,du\,dt$$

By distributing the dummy variables of integration one reveals the respective transforms of Fourier of $h(u), R_{\ddot{E}\ddot{E}}(t-u+v), {}^T\overline{h(v)}$, it comes finally:

$$S_{\ddot{X}_a\ddot{X}_a}=H(w)S_{\ddot{E}\ddot{E}}(w)\overline{^T H(w)}$$

with $\mathbf{H}(\omega)=\omega^2\ \mathbf{p}\ \mathbf{h}(\omega)\,\mathbf{\Phi}\ +\ \mathbf{\Psi}$

Taking into account the relations between the transforms of Fourier of displacement, velocity and acceleration, one has moreover:

$$S_{\dot{X}_a\dot{X}_a}=\frac{-1}{w^2}H(w)S_{\ddot{E}\ddot{E}}(w)\overline{^T H(w)}$$
$$S_{X_aX_a}=\frac{1}{w^4}H(w)S_{\ddot{E}\ddot{E}}(w)\overline{^T H(w)}$$

These relations make it possible to express the response of structure by the DSP of displacement or velocity.

Note:

- *According to the statement given to* $\mathbf{H}(\omega)$ *, one expresses the DSP of displacement (respectively the velocity or of acceleration) total, relative or differential:*

  absolute motion: $\mathbf{H}(\omega)=\omega^2\mathbf{p}\ \mathbf{h}(\omega)\mathbf{\Phi}\ +\ \mathbf{\Psi}$

  relative motion: $\mathbf{H}(\omega)=\omega^2\ \mathbf{p}\ \mathbf{h}(\omega)\ \mathbf{\Phi}$

  differential motion (IE of training): $\mathbf{H}(\omega)=\ \mathbf{\Psi}$

- *It is of use, during a computation with Code_Aster, to restrict the matrix of the transfer transfer function to the lines of l the degrees of freedom of observation. This makes it possible to reduce of as much computations as soon as l is small in front n .*

3.3 Application in *Code_Aster*

the group of the spectral approach for seismic computation is treated in command `DYNA_ALEA_MODAL` [U4.53.22]. The data are gathered under three key words factors and a simple key word.

Modal base is made up by dynamic modes calculated by the command `MODE_ITER_SIMULT` [U4.52.03] or `MODE_ITER_INV` [U4.52.04] stored in a concept of the mode_meca `type` recovered by the key word factor `BASE_MODALE`, on the one hand; static modes calculated by the command `MODE_STATIQUE` [U4.52.14] stored in a concept of the mode_stat `type` recovered by the single-ended spanner key - `MODE_STAT`, in addition. The key word factor `BASE_MODALE` also has the arguments which make it possible to determine the bande de fréquence or the modes retained for computation and the corresponding depreciation.

The data corresponding to the excitation are gathered under factor key word `the EXCIT` (cf paragraph [§4]): one specifies there the type of excitation within the meaning of `the QUANTITY` : excitation in displacement or force, the nodes `NOEUD` and component `NOM_CMP` excited, the name of the interspectrums or autospectrums `INTE_SPEC`, functions complex read beforehand or calculated, respectively by operators `LIRE_INTE_SPEC` [U4.36.01] or `CALC_INTE_SPEC` [U4.36.03] and stored in an array of interspectrum of concept `tabl_intsp` which apply in each excited degree of freedom.

Under factor key word `the response` are the data related to the choice of the discretization.

Command `DYNA_ALEA_MODAL` provides the response in the form of power spectral density on modal base. To obtain the restitution of the DSP on physical base, REST_SPEC_PHYS [U4.63.22] will be used which makes it possible to specify the type of quantity of the response (displacement or force), at the “points of observation” (node-component) of result. In the presence of a response of type displacement, one will specify here also if the response corresponds to absolute displacement, relative or differential.

`REST_SPEC_PHYS` provides an array of interspectrums which contains according to the request of the user, the interspectral matrix in displacement $S_{\mathbf{XX}}$, of velocity $S_{\dot{\mathbf{X}}\dot{\mathbf{X}}}$, **or** in acceleration $S_{\ddot{\mathbf{X}}\ddot{\mathbf{X}}}$ for a statement in the absolute coordinate system (index a), the relative reference (index r) or of training (index e).

Each preceding “combination” requires a call specific to command `REST_SPEC_PHYS`.

4 Definition of the interspectral matrix of exiting power

the seismic excitation is by nature, we said it, random. Also it can be known not by its temporal statement but in frequential form by one power spectral density also said interspectrum.

When there are several bearings, they can be excited by excitations identical or different, this last case is that of the multi-bearings.

For $\mathbf{m}$ bearings, one defines the matrix of density interspectral of power of order $\mathbf{m}$, or per abuse language the interspectrum of order $\mathbf{m}$, which is a matrix ($m \times m$) of complex functions depending on the frequency.

The diagonal terms represent the “auto-” densities spectral of powers - or autospectrums at the points of excitation, the extra-diagonal terms correspond to the densities interspectrals between the excitations in two distinct something to lean on (each line or column of the matrix represents in fact a something to lean on in physical mesh or a mode in modal computation). By definition of these terms, it from of deduced that the matrixes of density handled interspectrals of power are hermitian. (See [bib2] or documentation of reference associated with command POST_DYNA_ALEA [R7.10.01])

We present hereafter the various commands of Code_Aster which make it possible to obtain a matrix of density interspectral of power.

4.1 Reading on a file

the most elementary way to define a matrix of density interspectral of power is to give, “with the hand”, the values with the various steps of frequency.

Operator LIRE_INTE_SPEC [U4.36.01] is used then.

LIRE_INTE_SPEC reads in a file “interspectrum excitation”. The format of the file in which the interspectral matrix is consigned is simple: one describes successively the function of each term of the interspectral matrix; for each function, one line gives one by frequency by indicating the frequency, the parts real and imaginary of the complex number; or the frequency, the modulus and the phase of the complex number (key word FORMAT).

Example of file interspectrum excitation (for a matrix reduced in the term):

```
INTERSPECTRUM
DIM = 1
FONCTION_C
I = 1
J = 1
NB_POIN = 4
VALEUR =
       2.9999   0.  0.
       3.       1.  0.
       13.      1.  0.
       13.0001  0.  0.
FINSF
FIN
```

4.2 Obtaining an interspectrum from functions of time

One can deduce the matrix from density interspectral of power from functions of time. One uses then operator CALC_INTE_SPEC [U4.36.03] in *Code_Aster* [bib3].

From a list of N functions of time, this operator allows to calculate the interspectrum of power $N \times N$ which corresponds to them.
For each term of the interspectral matrix ($N \times N$) one uses the following approach [bib3].

To compute: the interspectrum of two signals one uses the relation of Wiener-Khichnine [bib7] which makes it possible to establish a formula of computation of the power spectral density by the transform of Fourier of finished samples of the signals $\mathbf{x}(t)$ and $\mathbf{y}(t)$.

It comes then:

$$S_{\mathbf{xy}}(f)=\lim_{T\to\infty}\frac{1}{T}\mathrm{E}[\mathbf{X}_{\mathrm{k}}(f,T).\mathbf{Y}_{\mathrm{k}}\times(f,T)]$$

where $\begin{cases} \mathbf{X}_k(f,T) = \mathrm{TF}[\mathbf{x}_k](f) = \int_0^T \mathbf{x}_k(t)\, e^{-i2\pi f} dt \\ \mathbf{Y}_k(f,T) = \mathrm{TF}[\mathbf{y}_k](f) = \int_0^T \mathbf{y}_k(t)\, e^{-i2\pi f} dt \end{cases}$

are the discrete transforms of Fourier of « x » and « y ».

When one is interested in signals resulting from measurements, one has most of the time only known signals in a discrete way, in the same way result of transient computation is a discrete signal.

An approximation of the interspectrum of the signals discrete $x[n]$ and $y[n]$ definite on L points spaced of Δt, cut out in p blocks of q points is obtained by the relation:

$$\hat{\mathbf{S}}_{xy}[k] = \frac{1}{pq\,\Delta t} \sum_{i=1}^{p} \mathbf{X}^{(i)}[k]\, \mathbf{Y}^{(i)}*[k]$$

$$\mathbf{X}^{(i)}[k] = \Delta t \sum_{n=0}^{q} \mathbf{x}^{(i)}[n]\, e^{-2i\pi kn/q}$$

$$\mathbf{Y}^{(i)}[k] = \Delta t \sum_{n=0}^{q} \mathbf{y}^{(i)}[n]\, e^{-2i\pi kn/q}$$

The various blocks can or not overlap. The values p et q are with the choice of the user.

This method is that of the periodogram of WELCH [bib8].

The computation is done on a window which moves on the field of definition of the functions. The user specifies in the command the length of the window of analysis, the shift between two successive windows of computation and the number of points per window.

4.3 Excitations preset or reconstituted from existing complex functions

One can wish to define a matrix of density interspectral of power in various ways:

- by a white vibration: the values are constant
- according to the analytical formula of useful KANAI-TAJIMI in seismic computation (white vibration filtered),
- or by taking again existing complex functions.

Operator DEFI_INTE_SPEC [`U4.36.02`] is used then.

4.3.1 Existing complex functions

It is enough under factor key word `the PAR_FONCTION` to give the name of the function for each pair of index `NUME_ORDRE_I, NUME_ORDRE_J`, corresponding to the higher triangular matrix (because of its hermiticity).

4.3.2 White vibration

a white vibration is characterized by a constant value on all the field of definition considered. Under factor key word `the CONSTANT`, one gives this value (`VALE_R` or `VALE_C`) on bande de fréquence `[FREQ_MIN, FREQ_MAX]` for each pair of index `INDI_I, INDI_J`, corresponding to the higher triangular matrix (because of its hermiticity). To define the function perfectly, one specifies the interpolation and the prolongations.

4.3.3 White vibration filtered by KANAI-TAJIMI [bib9]

For a structure leaned on the soil, it is current to take as excitation the power spectral density of Kanaï-Tajimi. This spectral concentration represents the filtering of a white vibration by the soil. The parameters of the formula make it possible to exploit the center frequency and the bandwidth of the spectrum.

The spectrum $G(\omega)$ is expressed by the following relation:

$$G(\omega)=\frac{\omega_g^4+4\xi_g^2\omega_g^2\omega^2}{(\omega_g^2-\omega^2)^2+4\xi_g^2\omega_g^2\omega^2}G_0$$

$\omega_g=2\pi f$ pulsation propre

ξ_g *amortissement total*

G_0 *niveau du bruit blanc avant filtrage*

The user must specify the eigenfrequency f_g of the filter, the modal damping ξ_g and the white level sound G_0 (= `VALE_R`) before filtering; like as for any function: the interpolation, profiles external and the field of definition (waveband).

By default a soil running is well represented by the values $f_g=2.5\,Hz$ and $\xi_g=0.6$.

Example of use for a white vibration filtered by KANAI_TAJIMI:

```
Interex =DEFI_INTE_SPEC     (
            DIMENSION: 1
            KANAI_TAJIMI: (
               NUME_ORDRE_I: 1indices              of the term of the matrix of density
               NUME_ORDRE_J: 1interspectrale                of power
               FREQ_MOY  : clean              2.5fréquence
               AMOR: 0.6amortissement                 modal
               VALE_R: 1niveau                of white vibration
               INTERPOL: "LIN"             linear interpolation
               PROL_GAUCHE: "CONSTANT" elongation
               PROL_DROIT: "CONSTANT"
               FREQ_MIN  : 0.domaine              of definition
               FREQ_MAX  : 200.
               NOT: 1.
            ));
```

4.4 Other types of excitation

computations of the preceding paragraphs were carried out in the frame of the assumption of an excitation moving **imposed** on a degree of freedom. With the help of some modifications it is possible to use the same approach for an excitation **in force** [§4.4.1] or **by fluid sources** [§ 4.4.2], this one being expressed in a finite element [§4.4.3] or on a shape function of the structure [§4.4.4].

In the continuation of this paragraph, one supposes the random excitation known and provided by the user in the form of a DSP, power spectral density.

4.4.1 Case of the excitation in forces imposed

Under key word `EXCIT` one has `QUANTITY = EFFO`.

When the excitation with the bearings is of type forces imposed, the general equation of motion is:

$$\mathbf{M}\ddot{\mathbf{X}}(t)+\mathbf{C}\dot{\mathbf{X}}(t)+\mathbf{K}\mathbf{X}(t)=\sum_{j=1}^{m}\mathbf{F}_j$$

The response of structure is then calculated on **a basis of dynamic modes** $\boldsymbol{\Phi}=\{\phi_{i,i=1,n}\}$, these modes being calculated by supposing **the free exiting bearings.** One does not distinguish, in this case of absolute motion, relative and differential and one does not use static modes.

One defines the participation factor modal in the form: $P_{ij}=\frac{{}^T\phi_i F_j}{\mu_i}$

The random transient responses, harmonics and have **the same statements** as the responses of **the relative motion** of the excitation multi-bearing in the general case [§3]. (What corresponds to the absence of static modes). The exiting force is represented in each degree of freedom-bearing by its DSP in the form of a term are equivalent to $\mathbf{S}_{\ddot{\mathbf{E}}\ddot{\mathbf{E}}}(\omega)$.

4.4.2 Excitation by fluid sources

the fluid sources appear, for example, in the study of a network of pipework. They correspond to active bodies or connections of secondary pipework. They are generally sources of pressure or sources of flow. These various types of source are presented hereafter according to their mathematical working and what Code_Aster *in* each configuration makes.

These fluid sources are not directly seismic excitations but can be induced by a seisme. The resolution of the mechanical problem makes call with the very methods, because of their randomness, which justifies their presentation here.

The modelization network of pipework is supposed to be realized using vibro-acoustic beam of *Code_Aster*.

The response with fluid sources is calculated in the frame of the response to imposed forces (cf [§4.4.1]), in this frame one is interested in responses of quantity of type "displacement" (`QUANTITY = DEPL_R` under the key word `response`).

The sources of pressure and force, for reasons of modelization of the fluid sources are represented by dipoles [bib5], it is thus necessary to give **two points of application**.

Source of flow-volume: `QUANTITY = SOUR_DEBI_VOLU` under key word `EXCIT`

a volume flow rate is expressed in m^3/s , its power spectral density in $(m^3/s)^2/Hz$.

A source of flow-volume is considered, in the formulation $P-\phi$ of the pipes with fluid, like a force imposed on the degree of freedom ϕ of the node of application of the source [R4.02.02].

The user provides the DSP of volume flow rate $\mathbf{S}_{vv}(\omega)$, the DSP $\mathbf{S}'_{vv}(\omega)$ applied in force to the degree of freedom ϕ is: $\mathbf{S}'_{vv}(\omega)=(\rho\,\omega)^2\mathbf{S}_{vv}(\omega)$

where ρ is the density of the fluid.

Source of flow-mass: `QUANTITY = SOUR_DEBI_MASS` under key word `EXCIT`

a flow-mass is expressed in kg/s , its power spectral density in $(kg/s)^2/Hz$. The flow - mass is the product of flow-volume by the density of the fluid.

The user provides the DSP of flow-mass $\mathbf{S}_{mm}(\omega)$, the DSP $\mathbf{S}'_{mm}(\omega)$ applied in force to the degree of freedom ϕ is: $\mathbf{S}'_{mm}\omega=\omega^2\mathbf{S}_{mm}\omega$

Source of pressure: `QUANTITY = SOUR_PRESS` under key word `EXCIT`

a source of pressure is applied in *Aster* in **a dipole** $P_1 P_2$.

For a source of pressure whose DSP is $\mathbf{S}_{PP}(\omega)$, expressed in Pa^2/Hz , *Aster* builds a matrix of density interspectral of power $\mathbf{S}'_{pp}(\omega)$ which is applied in force imposed to the degree of freedom ϕ of the points P_1 and P_2 .

$$\mathbf{S}'_{PP}(\omega)=\mathbf{S}_{PP}(\omega)\begin{pmatrix} \left(\frac{S}{dx}\right)^2 & -\left(\frac{S}{dx}\right)^2 \\ -\left(\frac{S}{dx}\right)^2 & \left(\frac{S}{dx}\right)^2 \end{pmatrix}$$

where S is the fluid section, dx the distance between the two points P_1 and P_2 .

Source of force : `QUANTITY = SOUR_FORCE` under key word `EXCIT`

the force corresponds simply to the product of the pressure by the fluid section of the tube: $F=PS$. It thus is also applied to **a dipole** $P_1 P_2$.

For a source of force whose DSP is $\mathbf{S}_{FF}(\omega)$, expressed in N^2/Hz , *Aster* applies in force imposed to the degree of freedom f of the points P_1 and P_2 , (distant of dx), the matrix of density interspectral of power $\mathbf{S}'_{FF}(\omega)$ such as:

$$\mathbf{S}'_{FF}(\omega)=\mathbf{S}_{FF}(\omega)\begin{pmatrix} \left(\frac{1}{dx}\right)^2 & -\left(\frac{1}{dx}\right)^2 \\ -\left(\frac{1}{dx}\right)^2 & \left(\frac{1}{dx}\right)^2 \end{pmatrix}$$

4.4.3 Excitation distributed on a shape function

If the power spectral density of the excitation $E(\omega)$ corresponds to a force imposed on a shape function f_i , $E(\omega)$ gives the frequential dependence of the level of the excitation.

The spatial weighting of the force is represented in *Code_Aster* by a field at nodes which does not depend on the frequency: key word `CHAM_NO` under factor key word `the EXCIT`. This field at nodes is a "assembled vector". From the theoretical point of view the formalism of computation is the same one as previously (excitation in imposed force [§4.4.1]), for a force vector in second member equal to f_i .

4.5 Applications

These various types of excitation are included in the tests of validation, and are presented for examples in the ratio [bib6]. In particular the excitations of the fluid type are in the test: pipe subjected to random fluid excitations [V2.02.105] (SDLL105). The excitations on shape functions are tested in the case test: beam subjected to a random excitation distributed [V2.02.106] (SDLL106).

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6 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
4	A.Dumond EDF-R&D/AMA	initial Text