Seismic Response by spectral method

Abstract:

The study of the response of a structure under the effect of imposed motions of seismic type, with a single imposed motion (mono-bearing) or multiple (multi-bearing) is possible in transient analysis (time history). One will refer to the note [R4.05.01].

For studies of design, one can be interested only in one estimate of the maximum forces induced by the requests, to evaluate the safety margin with regulations of construction, without resorting to a transient analysis.

The spectral method leans on the notion of oscillator spectrum of an accelerogram of seisme. One details the method of development of this response sprectrum available in operator CALC_FONCTION [U4.32.04].

It is shown how this oscillator spectrum can be used to evaluate one raising of the response in relative displacement of a simple oscillator. This approach is justified if one does not wish to know the history of displacements and the forces, while being limited to the analysis of the inertial effects.

The spectral method uses general notions of the method of modal recombination [R5.06.01].

One describes the various combination rules usable to obtain one raising realistic but conservative maximum response of structure. These methods are available in operator COMB_SISM_MODAL [U4.84.01].

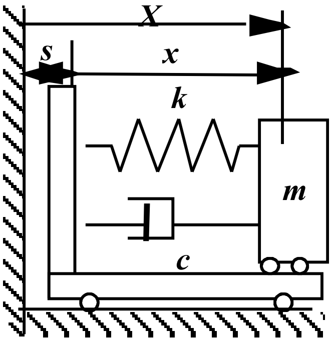
Contents

1 Notion of oscillator spectrum

the spectral method for the study of the response of a structure under the effect of imposed motions of seismic type leans on the notion of oscillator spectrum of an accelerogram of seisme.

1.1 Imposed motion defined by an accelerogram A (T)

For an imposed motion s of seismic type, one can deal with the problem in absolute displacement X or relative displacement x such as: $X=x+s$. The general equations of the motion of a simple oscillator are written then:



Absolute motion	Relative motion
$m\ddot{X}+c\dot{X}+kX=c\dot{s}+ks$	$m\ddot{x}+c\dot{x}+kx=-m\ddot{s}$

One retains the formulation starting from **relative motion** for two primary reasons:

- the seismic analysis of structures uses the stresses induced by the inertial effects of the seisme, stresses calculated starting from the structural deformations which are expressed starting from relative displacements;
- the characterization of the signal of excitation can be reduced in this case to the accelerogram of the seisme $\ddot{s}=A(t)$, quantity provided directly by the seismographs. The signals of displacement s and velocity $\dot{s}$ are in general not available in data bases geotechnics.

For the determination of the response of a simple oscillator with a motion imposed and the conventional notations, one will refer to appendix 2 [R4.05.03 Appendix 2].

If the seisme is defined by an accelerogram $A(t)$, absolute acceleration applied to the base, the reduced equation is in this case:

$$\ddot{x}+2\xi\ \omega_0\dot{x}+\omega_o^2 x=-\ddot{s}=-A(t) \quad \text{éq 1.1-1}$$

the solution of this problem is the integral of DUHAMEL presented to the appendix A [éq A3.3-1]:

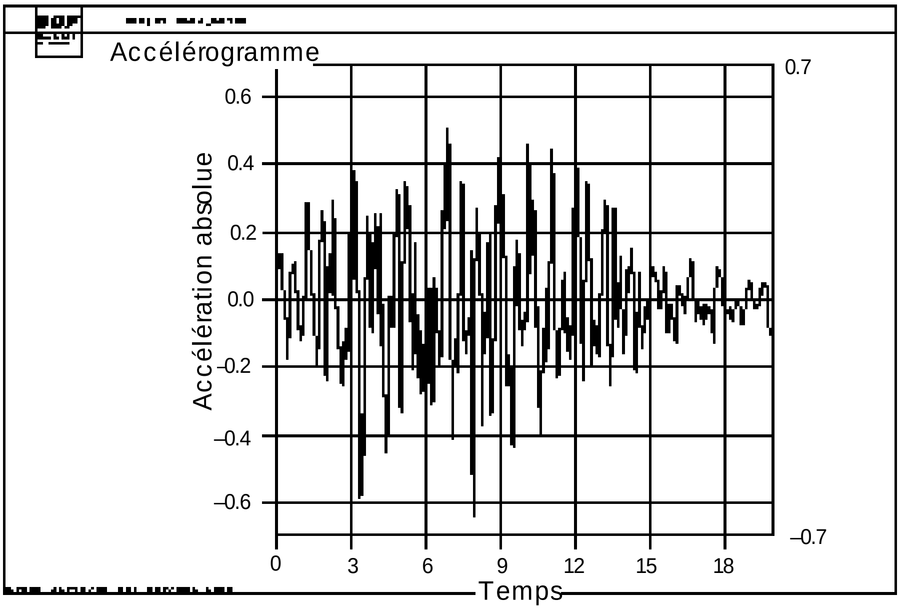
$$x(t)=\frac{1}{\omega'_0}\int_0^t A(\tau)e^{-\xi\omega_0(t-\tau)}\sin\omega'_0(t-\tau)\,d\tau=f(A,\xi,\omega'_0) \quad \text{éq 1.1-2}$$

$$\omega'_0=\omega_0\sqrt{1-\xi^2}$$

1.2 Oscillator spectrum of an accelerogram

the notion of oscillator spectrum was introduced initially to compare between them the effects of various accelerograms. The spectrum of FOURIER of a signal $A(t)$ informs about its frequential contents. The response of a mechanical system with a motion imposed on the base depends largely on the dynamic characteristics of this system: eigenfrequencies and reduced damping (ξ, ω'_0). The appendix A details this aspect.

If one wishes to know the maximum value of the response of a simple oscillator with the parameters, (A, ξ, ω'_0) one must evaluate the integral of DUHAMEL which provides the response of the oscillator [éq 1.1-2] to an excitation imposed on the base.



Appear 1.2-a: Accelerogram

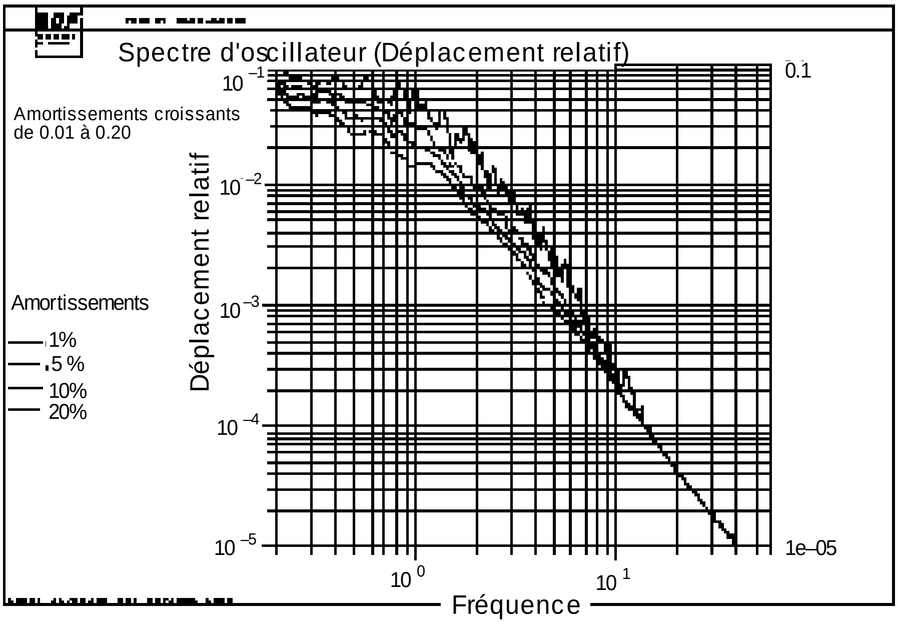
1.2.1 Oscillator spectrum in relative displacement

From the integral of DUHAMEL, one can define the oscillator spectrum of an accelerogram $A(t)$ like the function of the maximum values of relative displacement $x(t)=f(A, \xi, \omega'_0)$ for each value of (ξ, ω'_0) by recalling that: $\omega'_0=\omega_0\sqrt{(1-\xi^2)}$.

$$Srox(A, \xi, \omega'_0)=|x(t)|_{max}$$

$$x(t)=\frac{1}{\omega'_0}\int_0^t A(\tau)e^{-\xi\omega_0(t-\tau)}\sin\omega'_0(t-\tau)\,d\tau=f(A, \xi, \omega'_0)$$

One notes, on the figure [Figure 1.2.1-a], that beyond a certain frequency ($35\,Hz$ here), known as cut-off frequency of the spectrum, it does not have there significant dynamic amplification: relative displacement is null.



Appear 1.2.1-a: Oscillator spectrum in relative displacement

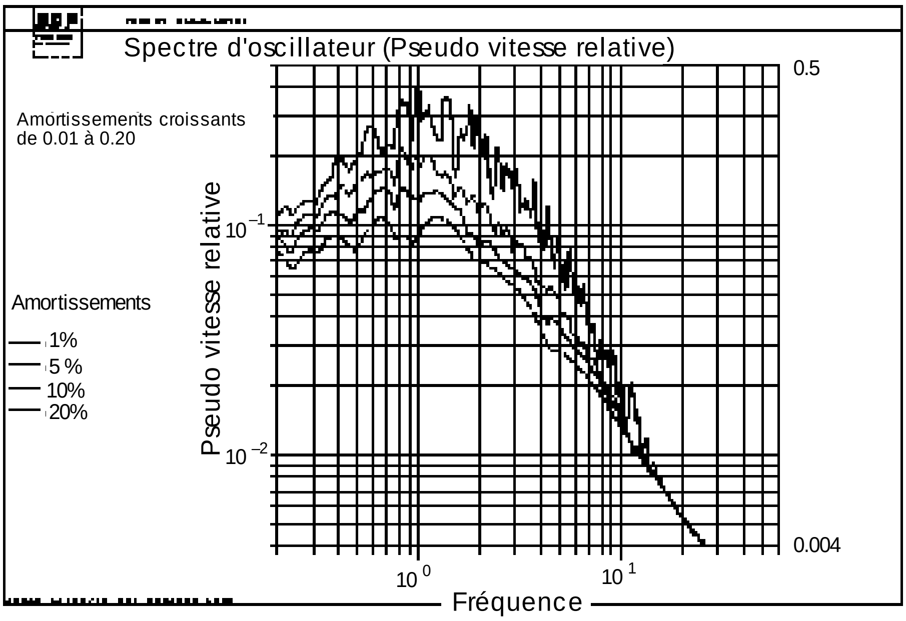
1.2.2 Oscillator spectrum in relative pseudovelocity

For structures with weak reduced damping $\xi < 0.2 = 20\%$, for which it is acceptable to assimilate ω_0 and ω'_0, one usually uses the spectrum of pseudovelocity defined by:

$$Sro\,\dot{x}(A, \xi, \omega_0) = \omega_0 Srox(A, \xi, \omega_0) = \omega_0 |x(t)|_{max}$$

The pseudonym velocity is the value the velocity which gives a value of the kinetic energy of the mass of the oscillator equal to that of the maximum strain energy of spring:

$$E_c = \frac{1}{2} m (\dot{x}(t))^2 = \frac{1}{2} m \left[Sro\dot{x}(A, \xi, \omega_0) \right]^2 = \frac{1}{2} m \omega_0^2 |x(t)|_{max}^2 = \frac{1}{2} k |x(t)|_{max}^2 = E_p$$

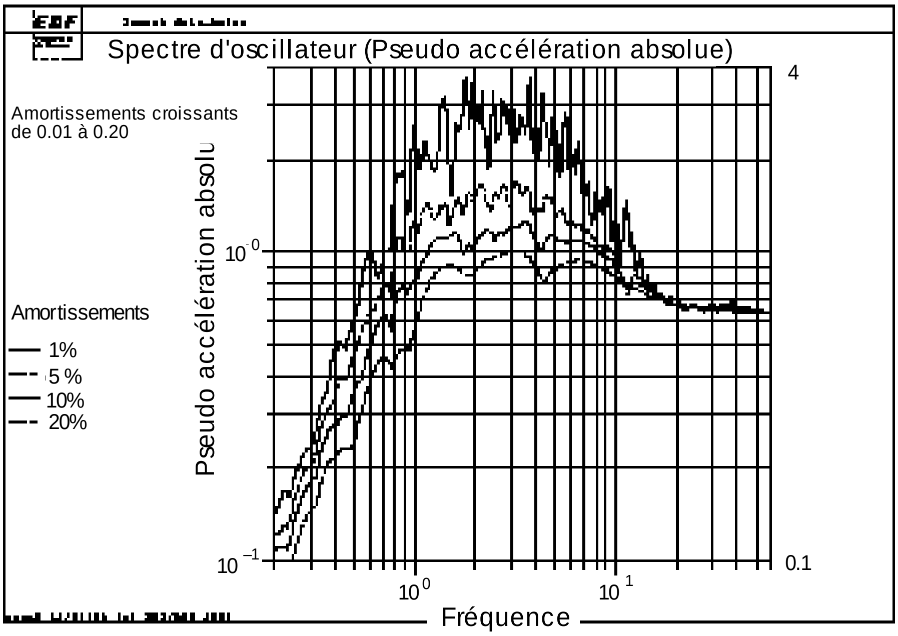


Appear 1.2.2-a: Oscillator spectrum in pseudonym relative velocity

1.2.3 Oscillator spectrum in absolute pseudo-acceleration

In the same way for a weak reduced damping, one can define the spectrum of pseudo-acceleration defined by:

$$Sro\,\ddot{x}(A,\xi,\omega_0)=\omega_O^2 Srox(A,\xi,\omega_0)=\omega_0^2|x(t)|_{max}$$



Appear 1.2.3-a: Oscillator spectrum in pseudonym absolute acceleration

the interest of this spectrum of pseudo-acceleration lies in the fact that $Sro\,\ddot{x}(A,\xi,\omega_0)$ is a good approximation of the maximum of absolute acceleration $\ddot{X}(t)$. Indeed, at time when relative displacement is maximum, the relative velocity is cancelled and the reduced equation is written $\ddot{x}+0+\omega_0^2 x_{max}=-\ddot{s}$, which shows us that

$$|\ddot{X}|_{max}=|\ddot{x}+\ddot{s}|_{max}=|\omega_0^2 x_{max}|=\omega_O^2 Srox(A,\xi,\omega_0)=Sro\,\ddot{x}(A,\xi,\omega_0)$$

For this reason, this oscillator spectrum is called **spectrum of absolute pseudo-acceleration**.

The asymptote of this high frequency spectrum (acceleration at period null) corresponds to the response of a clean high frequency oscillator, i.e. very rigid. In this case, the mass tends to completely follow the imposed motion of the base. This asymptote thus corresponds to the maximum acceleration $|A(t)|_{max}$ of imposed motion (soil or anchor point of the oscillator). It is reached in practice from the cut-off frequency of the spectrum. For this reason, one says that an accelerogram is fixed, for example, on $0.15\,g$, when its maximum amplitude and its oscillator spectrum of absolute pseudo-acceleration at period null are equal to $0.15\,g$.

1.3 Determination of the oscillator spectrum

the determination of the oscillator spectrum of an accelerogram $A(t)$ is available in operator `CALC_FONCTION` [U6.34.04] with key word `SPEC_OSCI` : it is obtained by numerical integration of the equation of DUHAMEL by the method of NIGAM [R5.05.01]. This command provides the spectrum of absolute pseudo-acceleration and, on request, the spectrum of pseudovelocity or the spectrum of relative displacement.

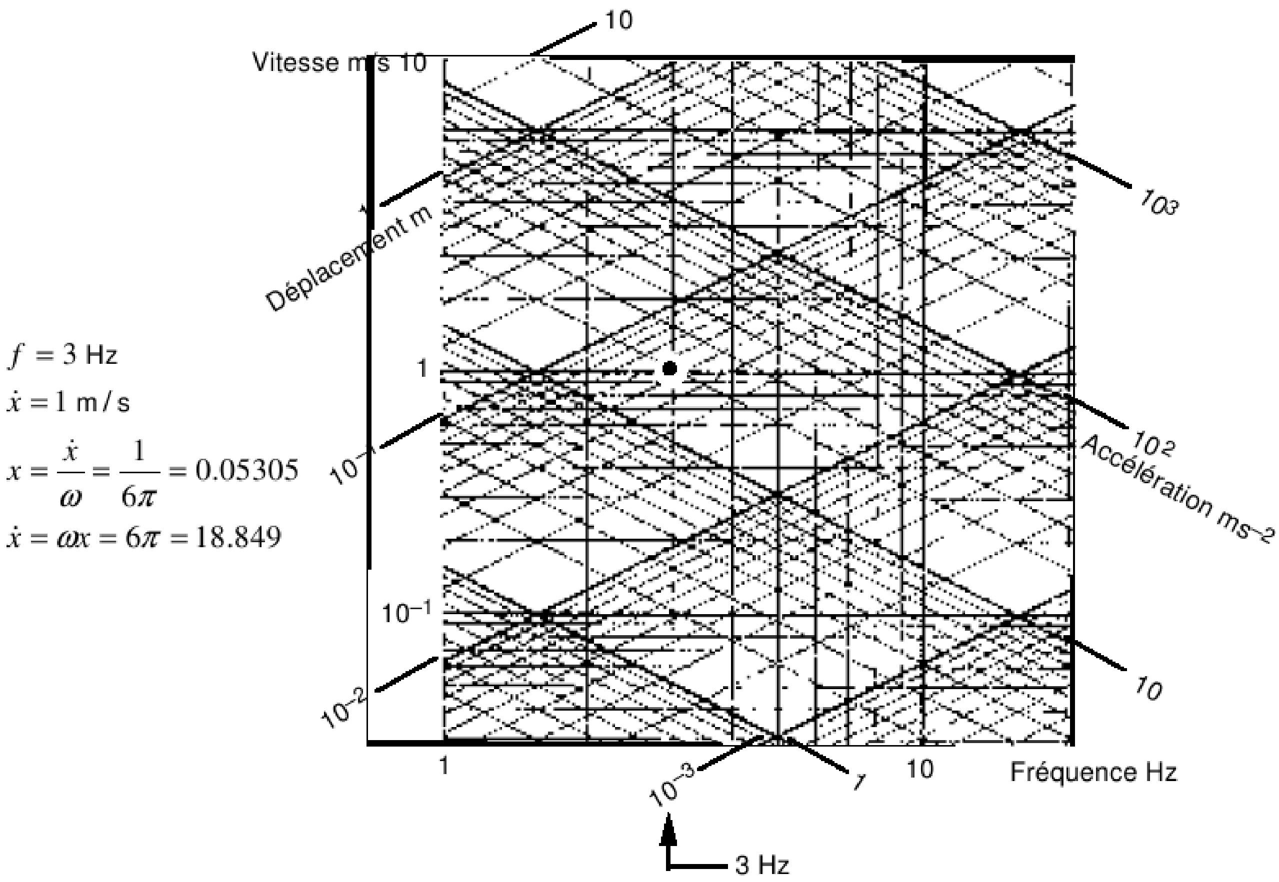
1.4 Representation and use of the oscillator spectrums

1.4.1 Representation sort-logarithmic curve

the response sprectrums of oscillator are usually represented by graphics sort-logarithmic curves which make it possible to read on only one graph the three quantities: relative displacement, relative pseudovelocity, absolute pseudo-acceleration.

This representation is obtained by tracing the spectrum of relative pseudovelocity $Sro\,\dot{x}$ in coordinates $\log-\log$ such as $\log Sro\,\dot{x}=f(\log\omega_0)$, on which one defers two graduations complementary to $\pm 45°$ if the scale of the graduations logarithmic curves is the same one on the two axes:

- a graduation logarithmic curve with $+45°$ to measure displacements relative $\log Sro\,x=\log(\frac{Sro\,\dot{x}}{\omega_0})=\log Sro\,\dot{x}-\log\omega_0$
- a graduation logarithmic curve to $-45°$ measure absolute accelerations $\log Sro\,\ddot{x}=\log(\omega_0\,Sro\,\dot{x})=\log Sro\,\dot{x}+\log\omega_0$



Appears 1.4.1-a: Representation sort-logarithmic curve

1.4.2 Use of the oscillator spectrums

to evaluate the maximum response of a modal oscillator (ω_i,ξ_i) with an accelerogram $A(t)$, one uses **the spectrum of absolute pseudo-acceleration.**

It is represented in *Code_Aster* by a three-dimensions function made up of several functions $Sro\,\ddot{x}=f(freq)$ with $\xi_n=cte$.

One uses a linear interpolation on reduced damping for $\xi_n<\xi_i<\xi_{n+1}$ because dynamic amplification with resonance for $\omega=\omega_0$ (that is to say $\eta=1$) is equal to $\frac{x_m}{s_0}=\frac{1}{2\xi_i}$ [éq A2.2-3].

The variation of the modulus of the response in the vicinity of resonance also justifies an interpolation logarithmic curve for $\omega_m<\omega_i<\omega_{m+1}$. The oscillator spectrum must be represented with a discretization in sufficiently fine frequency to limit the effects of the interpolation.

1.5 Oscillator spectrums used for studies

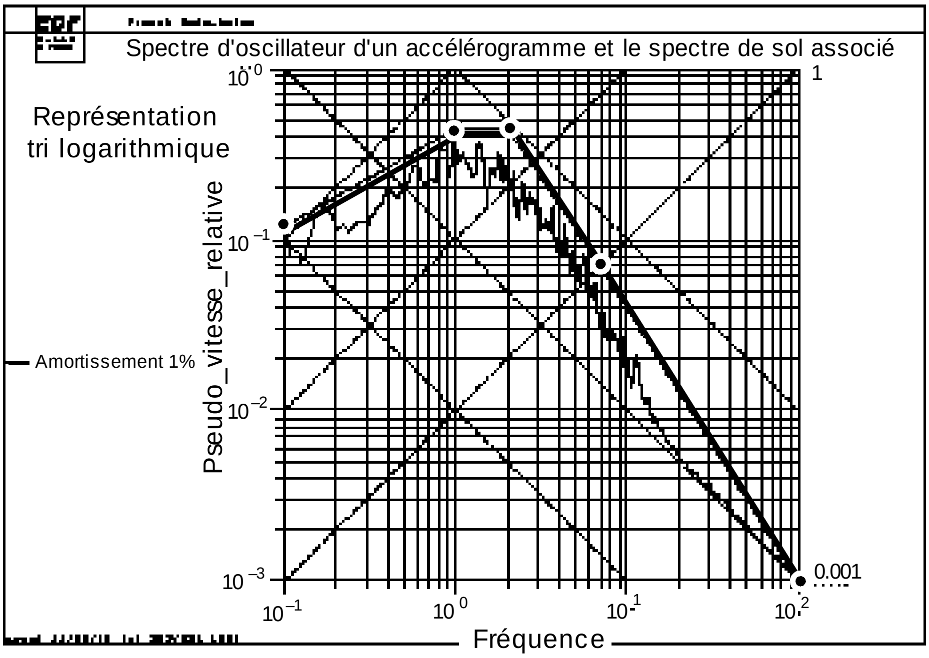
For the studies of industrial facilities, such as the nuclear power plants, the seismic analysis led to establish several models:

- a model of the civil engineer of design of the buildings to determine:
    - accidental requests for the computation of the frameworks of these buildings;
    - the motions imposed on the anchor points of the equipment (reactor vessel, bearings of the networks of pipework, electrical equipment boxes.) at various levels of the buildings;
- models of study of checking of each equipment subjected to the imposed motions amplified by the dynamic behavior of the buildings.

1.5.1 Spectrum of soil of design and checking of the buildings

A this stage, the equipment are known only like inertial overloads and one can admit that they do not bring any stiffness to the building. The structures in this case are subjected to a spectrum of soil.

The frequential contents of an oscillator spectrum reflect that of the accelerogram used and “are thus marked” by the properties of the soil instead of record. To work out the spectrum of soil at the stage of the project, it is thus recommended to establish the oscillator spectrums for several accelerograms and to build a spectrum envelope which smoothes antiresonances.



Appear 1.5.1-a: Spectrum of soil for a project

Note:

In many cases, one does not know the rotation movement imposed by the seisme, since the accelerograms of known seismes result from records of seismographs, sensors to a degree of freedom of translation.

1.5.2 Floor spectrum of checking of the equipment

the study of the dynamic behavior of the equipment subjected to the motions imposed by the structure support on the fulcrums is possible starting from the accelerograms of response in these points, results of the transient analysis of the behavior of the building: these accelerograms, known as of bottom, make it possible to build floor spectrums.

For a checking of the equipment, one can limit oneself to a spectral analysis starting from the floor spectrums and the differential displacements imposed on the bearings.

The floor spectrums are representative of the dynamic amplification brought by the structure support: a lissage of the spectrum can be useful to take into account uncertainty on the position of the eigenfrequencies of the building, but one will take care to preserve realistic margins, since the spectrum of soil is already one raising of the seismic request. The oscillator spectrum must be represented with a discretization in sufficiently fine frequency “to collect” resonances of structure.

Note:

Techniques of direct determination of the floor spectrums, starting from the spectrum of soil and modes of structure were developed [bib1], but are not currently available in Code_Aster.

2 Seismic response by modal recombination

2.1 Recalls of the formulation

the spectral method of seismic analysis leans on the formulation of the response transient dynamics by modal recombination presented in the documents “Methods of RITZ in dynamics linear and nonlinear” [R5.06.01] and “Analyzes seismic by direct method or modal recombination” [R4.05.01].

Let us summarize the principles of the approach detailed in the note [R4.05.01] for a structure represented in form discretized by the matric system:

$$\mathbf{M}\ddot{\mathbf{U}}+\mathbf{C}\dot{\mathbf{U}}+\mathbf{K}\mathbf{U}=\mathbf{F}(t) \qquad \text{éq 2.1-1}$$

Notations moving absolute motion

$\mathbf{U}$ *represents all the components of motion (internal degrees of freedom of structure and the degrees of freedom subjected to an imposed motion): one separates them in the form* $\mathbf{U}=\left(\dfrac{\mathbf{X}}{\mathbf{s}}\right)$ *.*

The operators describing structure become: $\mathbf{K}=\begin{bmatrix}\mathbf{k}_{xx} & \mathbf{k}_{xs}\\ \mathbf{k}_{sx} & \mathbf{k}_{ss}\end{bmatrix}$ $\mathbf{C}=\begin{bmatrix}\mathbf{c}_{xx} & \mathbf{c}_{xs}\\ \mathbf{c}_{sx} & \mathbf{c}_{ss}\end{bmatrix}$

$\mathbf{M}=\begin{bmatrix}\mathbf{m}_{xx} & \mathbf{m}_{xs}\\ \mathbf{m}_{sx} & \mathbf{m}_{ss}\end{bmatrix}$

The problem moving relative motion of structure compared to the bearings with decomposition Absolute motion = Relative motion + Motion of training led to introduce the change of variable $\mathbf{U}=\mathbf{u}+\mathbf{E}$.

Assumption

One supposes that no force of excitation is applied to the degrees of freedom of structure, which reduces the second member $\mathbf{F}(t)$ *, with the same partition with* $\mathbf{F}=\begin{pmatrix}\mathbf{0}_x\\ \mathbf{r}_s\end{pmatrix}$

equation 2.1-1 becomes then:

$$\begin{bmatrix}\mathbf{m}_{xx} & \mathbf{m}_{xs}\\ \mathbf{m}_{sx} & \mathbf{m}_{ss}\end{bmatrix}\begin{bmatrix}\ddot{\mathbf{X}}\\ \ddot{\mathbf{s}}\end{bmatrix}+\begin{bmatrix}\mathbf{c}_{xx} & \mathbf{c}_{xs}\\ \mathbf{c}_{sx} & \mathbf{c}_{ss}\end{bmatrix}\begin{bmatrix}\dot{\mathbf{X}}\\ \dot{\mathbf{s}}\end{bmatrix}+\begin{bmatrix}\mathbf{k}_{xx} & \mathbf{k}_{xs}\\ \mathbf{k}_{sx} & \mathbf{k}_{ss}\end{bmatrix}\begin{bmatrix}\mathbf{X}\\ \mathbf{s}\end{bmatrix}=\begin{bmatrix}\mathbf{0}_x\\ \mathbf{r}_s\end{bmatrix} \quad \textbf{éq. 2.1-2}$$

the first system of equations extracts from [éq. 2.1-2]:

$$\mathbf{m}_{xx}\ddot{\mathbf{X}}+\mathbf{c}_{xx}\dot{\mathbf{X}}+\mathbf{k}_{xx}\mathbf{X}=-\mathbf{m}_{xs}\ddot{\mathbf{s}}-\mathbf{c}_{xs}\dot{\mathbf{s}}-\mathbf{k}_{xs}\mathbf{s} \quad \textbf{éq. 2.1-3}$$

allows the determination of the response of the internal degrees of freedom of structure, while the second:

$$\mathbf{m}_{sx}\ddot{\mathbf{X}}+\mathbf{c}_{sx}\dot{\mathbf{X}}+\mathbf{k}_{sx}\mathbf{X}+\mathbf{m}_{ss}\ddot{\mathbf{s}}+\mathbf{c}_{ss}\dot{\mathbf{s}}+\mathbf{k}_{ss}\mathbf{s}=r$$

the determination of the reaction between $r(t)$ structure and its supports allows.

Notations moving relative motion

One breaks up the vector of motion $\mathbf{U}=\begin{pmatrix}\mathbf{X}\\ \mathbf{s}\end{pmatrix}$ into the sum of two vectors $\mathbf{u}$ and $\mathbf{E}$ with:

$\mathbf{E}=\begin{pmatrix}\mathbf{e}_{xs}\mathbf{s}_x\\ \mathbf{s}_x\end{pmatrix}$ the static motion of strain of structure under the effect of the displacements imposed on the supports, which one calls motion of training,

and $\mathbf{u}=\begin{pmatrix}\mathbf{x}_s\\ \mathbf{0}_s\end{pmatrix}$ the motion of residual strain of structure compared to the preceding strain to obtain the absolute strain $\mathbf{U}=\begin{pmatrix}\mathbf{X}\\ \mathbf{s}\end{pmatrix}$.

The vector $\mathbf{e}_{xs}\mathbf{s}_s$ is obtained by carrying out the static raising, on the internal degrees of freedom of structure, of the displacements imposed on the bearings is (by means of the first line of equation 2.1-2 and by eliminating the dynamic components): $\mathbf{e}_{xs}\mathbf{s}_s=-\mathbf{k}_{xx}^{-1}\mathbf{k}_{xs}\mathbf{s}_s$, that is to say $\mathbf{e}_{xx}=-\mathbf{k}_{xx}^{-1}\mathbf{k}_{xs}$.

The transition of absolute motion to relative motion can be also written by introducing the operator of transition $\boldsymbol{\Psi}$:

$$\mathbf{U}=\begin{pmatrix}\mathbf{X}\\ \mathbf{s}\end{pmatrix}=\mathbf{u}+\mathbf{E}=\begin{pmatrix}\mathbf{x}_s\\ \mathbf{0}_s\end{pmatrix}+\begin{pmatrix}\mathbf{e}_{xs}\mathbf{s}_s\\ \mathbf{s}_s\end{pmatrix}=\boldsymbol{\Psi}\begin{pmatrix}\mathbf{x}_s\\ \mathbf{s}_s\end{pmatrix} \text{ with } \boldsymbol{\Psi}=\begin{bmatrix}\mathbf{I}_{xx} & \mathbf{e}_{xs}\\ \mathbf{0}_{sx} & \mathbf{I}_{ss}\end{bmatrix}$$

the system [éq. 2.2.1-1] takes the general shape then:

$$\mathbf{M}\boldsymbol{\Psi}\begin{pmatrix}\ddot{\mathbf{x}}_x\\ \ddot{\mathbf{s}}_s\end{pmatrix}+C\boldsymbol{\Psi}\begin{pmatrix}\dot{\mathbf{x}}_x\\ \dot{\mathbf{s}}_s\end{pmatrix}+\mathbf{K}\boldsymbol{\Psi}\begin{pmatrix}\mathbf{x}_x\\ \mathbf{s}_s\end{pmatrix}=\begin{pmatrix}\mathbf{0}_x\\ \mathbf{r}_s\end{pmatrix} \quad \text{éq 2.1-4}$$

2.1.1 multiple imposed Motion: multi-bearing

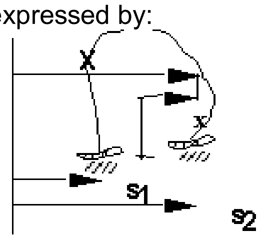
This situation corresponds to a discrete number of points of connection of structure in bearings subjected to different imposed displacements.

We interest initially in the quasi-static response $\mathbf{X}^{\mathbf{qs}}$ of the degrees of freedom of structure. Then $\mathbf{e}_{\mathbf{xs}}=\boldsymbol{\varphi}_{\mathbf{s}}$, where the matrix $\boldsymbol{\varphi}_{\mathbf{s}}$ indicates the matrix of the static modes reduced to the degrees of freedom of structure.

One thus obtains: $\mathbf{X}^{\mathbf{qs}}=\boldsymbol{\varphi}_{\mathbf{s}}\mathbf{s}$

The matrix $\boldsymbol{\varphi}_{\mathbf{s}}$ gathers $6\,n_{appuis}$ static modes for the models of structures and 3 times the number of bearings for the models of continuums. Each static mode $\boldsymbol{\varphi}_{\mathbf{s}}=-\mathbf{k}_{\mathbf{xx}}^{\mathbf{-1}}\mathbf{k}_{\mathbf{xs}_j}$ is an attach mode, corresponding to a unit displacement imposed on a component of bearing, the other components being null, and produces MODE_STATIQUE [28] by the operator.

The change of reference can then be expressed by:



$$\mathbf{U}=\left(\frac{\mathbf{X}}{\mathbf{s}}\right)=\left(\frac{\mathbf{x}_{\mathbf{s}}}{\mathbf{0}_{\mathbf{s}}}\right)+\left(\frac{\boldsymbol{\varphi}_{\mathbf{s}}\mathbf{s}_{\mathbf{s}}}{\mathbf{s}_{\mathbf{s}}}\right)$$

The absolute response is written then in the form: $\mathbf{U}=\begin{bmatrix}\mathbf{I}_{\mathbf{xx}} & \boldsymbol{\varphi}_{\mathbf{s}}\\ \mathbf{0}_{\mathbf{sx}} & \mathbf{I}_{\mathbf{ss}}\end{bmatrix}\begin{bmatrix}\mathbf{x}_{\mathbf{x}}\\ \mathbf{s}_{\mathbf{s}}\end{bmatrix}$

where $\mathbf{I}$ indicates the matrix identity, $\mathbf{x}_{\mathbf{x}}$ the vector of relative displacements of structure compared to the supports, and $\begin{bmatrix}\mathbf{I}_{\mathbf{xx}} & \boldsymbol{\varphi}_{\mathbf{s}}\\ \mathbf{0}_{\mathbf{sx}} & \mathbf{I}_{\mathbf{ss}}\end{bmatrix}=\boldsymbol{\Psi}$ is the transition matrix of absolute motion to relative motion.

The system [éq. 2.1-3] becomes as follows:

$$\begin{bmatrix}\mathbf{m}_{\mathbf{xx}} & \mathbf{m}_{\mathbf{xs}}\\ \mathbf{m}_{\mathbf{sx}} & \mathbf{m}_{\mathbf{ss}}\end{bmatrix}\begin{bmatrix}\mathbf{I}_{\mathbf{xx}} & \boldsymbol{\varphi}_{\mathbf{s}}\\ \mathbf{0}_{\mathbf{sx}} & \mathbf{I}_{\mathbf{ss}}\end{bmatrix}\begin{bmatrix}\ddot{\mathbf{x}}_{\mathbf{x}}\\ \ddot{\mathbf{s}}_{\mathbf{s}}\end{bmatrix}+\begin{bmatrix}\mathbf{c}_{\mathbf{xx}} & \mathbf{c}_{\mathbf{xs}}\\ \mathbf{c}_{\mathbf{sx}} & \mathbf{c}_{\mathbf{ss}}\end{bmatrix}\begin{bmatrix}\mathbf{I}_{\mathbf{xx}} & \boldsymbol{\varphi}_{\mathbf{s}}\\ \mathbf{0}_{\mathbf{sx}} & \mathbf{I}_{\mathbf{ss}}\end{bmatrix}\begin{bmatrix}\dot{\mathbf{x}}_{\mathbf{x}}\\ \dot{\mathbf{s}}_{\mathbf{s}}\end{bmatrix}+\begin{bmatrix}\mathbf{k}_{\mathbf{xx}} & \mathbf{k}_{\mathbf{xs}}\\ \mathbf{k}_{\mathbf{sx}} & \mathbf{k}_{\mathbf{ss}}\end{bmatrix}\begin{bmatrix}\mathbf{I}_{\mathbf{xx}} & \boldsymbol{\varphi}_{\mathbf{s}}\\ \mathbf{0}_{\mathbf{sx}} & \mathbf{I}_{\mathbf{ss}}\end{bmatrix}\begin{bmatrix}\mathbf{x}_{\mathbf{x}}\\ \mathbf{s}_{\mathbf{s}}\end{bmatrix}=\begin{bmatrix}\mathbf{0}_{\mathbf{x}}\\ \mathbf{r}_{\mathbf{s}}\end{bmatrix} \quad \text{éq 2.1.1-1}$$

the first equation of this system is written:

$$\mathbf{m}_{\mathbf{xx}}\ddot{\mathbf{x}}_{\mathbf{x}}+\mathbf{c}_{\mathbf{xx}}\dot{\mathbf{x}}_{\mathbf{x}}+\mathbf{k}_{\mathbf{xx}}\mathbf{x}_{\mathbf{x}}=-\left(\mathbf{m}_{\mathbf{xx}}\boldsymbol{\varphi}_{\mathbf{s}}+\mathbf{m}_{\mathbf{xs}}\right)\ddot{\mathbf{s}}_{\mathbf{s}}-\left(\mathbf{c}_{\mathbf{xx}}\boldsymbol{\varphi}_{\mathbf{s}}+\mathbf{c}_{\mathbf{xs}}\right)\dot{\mathbf{s}}_{\mathbf{s}}-\left(\mathbf{k}_{\mathbf{xx}}\boldsymbol{\varphi}_{\mathbf{s}}+\mathbf{k}_{\mathbf{xs}}\right)\mathbf{s}_{\mathbf{s}}$$

The diagonalization of the term of stiffness is acquired:

$$\begin{bmatrix}\mathbf{k}_{\mathbf{xx}} & \mathbf{k}_{\mathbf{xs}}\\ \mathbf{k}_{\mathbf{sx}} & \mathbf{k}_{\mathbf{ss}}\end{bmatrix}\begin{bmatrix}\boldsymbol{\varphi}_{\mathbf{s}}\\ \mathbf{I}_{\mathbf{ss}}\end{bmatrix}=\begin{bmatrix}\mathbf{0}_{\mathbf{x}}\\ \mathbf{r}_{\mathbf{s}}\end{bmatrix}$$

Concerning the terms of damping, decoupling is acquired only if damping is proportional to the stiffness, usually allowed assumption.

This makes it possible well to uncouple the system [éq 2.1.1-1]

$$\mathbf{m}_{\mathbf{xx}}\ddot{\mathbf{x}}_{\mathbf{x}}+\mathbf{c}_{\mathbf{xx}}\dot{\mathbf{x}}_{\mathbf{x}}+\mathbf{k}_{\mathbf{xx}}\mathbf{x}_{\mathbf{x}}=\mathbf{g}_{\mathbf{x}}(t) \quad \text{éq 2.1.1-2}$$

with $\mathbf{g}_\mathbf{x}(t) = -\mathbf{m}_\mathbf{xx}\left(\boldsymbol{\varphi}_\mathbf{s} + \mathbf{m}_\mathbf{xx}^{-1}\mathbf{m}_\mathbf{xs}\right)\ddot{\mathbf{s}}_s = -\mathbf{m}_\mathbf{xx}\ddot{\mathbf{x}}^{qs} - \mathbf{m}_\mathbf{xs}\ddot{\mathbf{s}}_s$

the equivalent loading $\mathbf{g}_\mathbf{x}(t)$ is due contrary to the sum of the acceleration of the supports and relative acceleration to the static modes.
This formulation must be interpreted like the decomposition of the motion of structure in a motion of training corresponding to an instantaneous static deformed shape (differential displacement of the bearings) and a relative motion corresponding to the inertial effects around this new static deformed shape.

This interpretation is in conformity with the ranking of the requests defined by the rules of construction (ASME, RCC-M):

- the stresses induced by relative motion are, as for the statical stresses, of the primary stresses (effects of inertia),
- the stresses induced by differential displacements of the bearings which are they classified in secondary stresses.

The response of the degrees of freedom of structure being thus determined, the second equation of the system [éq. 2.1.1. - 1] makes it possible to obtain the reaction $\mathbf{s}(t)$.

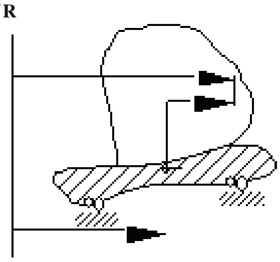
2.1.2 Single imposed motion: mono-bearing

the motion of training corresponds then to a noted rigid body motion $\begin{bmatrix}\boldsymbol{\varphi}_\mathbf{R}\\ \mathbf{s}_\mathbf{R}\end{bmatrix}$. The matrix $\boldsymbol{\varphi}_\mathbf{R}$ indicates the matrix gathering the modes of rigid bodies of reduced structure to the degrees of freedom of structure.
The structure is subjected to this total motion with an acceleration $\gamma(t)$, on which one superimposes the relative motion of the degrees of freedom of structure: absolute acceleration can thus be written in the form:

$$\ddot{\mathbf{U}} = \begin{bmatrix}\ddot{\mathbf{X}}\\ \ddot{\mathbf{s}}\end{bmatrix} = \begin{bmatrix}\ddot{\mathbf{x}}_\mathbf{x}\\ \mathbf{0}_\mathbf{s}\end{bmatrix} + \begin{bmatrix}\boldsymbol{\varphi}_\mathbf{R}\\ \mathbf{s}_\mathbf{R}\end{bmatrix}\gamma(t)$$

The rigid part of displacement checks the relation: $\begin{bmatrix}\mathbf{k}_\mathbf{xx} & \mathbf{k}_\mathbf{xs}\\ \mathbf{k}_\mathbf{sx} & \mathbf{k}_\mathbf{ss}\end{bmatrix}\begin{bmatrix}\boldsymbol{\varphi}_\mathbf{R}\\ \mathbf{s}_\mathbf{R}\end{bmatrix} = \mathbf{0}$

One from of deduced that: $\boldsymbol{\varphi}_\mathbf{R} = \boldsymbol{\varphi}_\mathbf{S}\mathbf{s}_\mathbf{R}$



$\mathbf{U} = \begin{pmatrix}\mathbf{X}\\ \hline \mathbf{s}\end{pmatrix} = \begin{pmatrix}\mathbf{x}_\mathbf{s}\\ \hline \mathbf{0}_\mathbf{s}\end{pmatrix} + \begin{pmatrix}\boldsymbol{\varphi}_\mathbf{R}\mathbf{s}_\mathbf{x}\\ \hline \mathbf{s}_\mathbf{s}\end{pmatrix}$ and $\boldsymbol{\Psi} = \begin{bmatrix}\mathbf{I}_\mathbf{xx} & \boldsymbol{\varphi}_\mathbf{R}\\ \mathbf{0}_\mathbf{sx} & \mathbf{I}_\mathbf{ss}\end{bmatrix}$

the second member of the equation [éq. 2.1.1-2] can then be written in the form:

$$\mathbf{g}_\mathbf{x}(t) = -\mathbf{m}_\mathbf{xx}\left(\boldsymbol{\varphi}_\mathbf{s} + \mathbf{m}_\mathbf{xx}^{-1}\mathbf{m}_\mathbf{xs}\right)\ddot{\mathbf{s}}_{xx} = -\left(\mathbf{m}_\mathbf{xx}\boldsymbol{\varphi}_\mathbf{R} + \mathbf{m}_\mathbf{xs}\mathbf{s}_R\right)\gamma(t) \qquad \textbf{éq. 2.1.2-1}$$

This writing highlights well which the seismic loading depends only on overall acceleration and inertia associated with the modes with rigid bodies.

2.1.3 Abstract

the equations [éq 2.1.1-2] and [éq 2.1.2-1] lead to the general form (written without index for more clearness):

$$\mathbf{m}\ddot{\mathbf{z}}+\mathbf{c}\dot{\mathbf{z}}+\mathbf{k}\mathbf{z}=-\mathbf{m}\left(\boldsymbol{\varphi}_{\mathbf{X}}+\mathbf{m}^{-1}\mathbf{m}_{\mathbf{xs}}\right)\ddot{\mathbf{s}}=-\mathbf{m}\mathbf{O}\ddot{\mathbf{s}}$$ éq the 2.1.3-1

terms $\mathbf{m}_{\mathbf{xs}}$ correspond under the terms of coupling of the mass matrix with the degrees of freedom of bearing: this fraction of the total mass is very weak and it is justified to neglect it. Let us recall that this term is indeed null for the structure models whose mass matrix is diagonal: models masses - spring, models with elements of the type "lumped farmhouse".

In this case, the simplified formulas are obtained:

mono-bearing: $\mathbf{O}=\boldsymbol{\varphi}_{\mathbf{R}}$ where $\boldsymbol{\varphi}_{\mathbf{R}}$ are the six multi-bearing

solid state modes: $\mathbf{O}=\boldsymbol{\varphi}_{\mathbf{s}}$ where $\boldsymbol{\varphi}_{\mathbf{s}}$ are $6n$ the bearings attach modes

the second member $-\mathbf{m}\mathbf{O}$ is built by the operator CALC_CHAR_SEISME [U4.63.01].

2.2 Response in modal base

2.2.1 temporal Response of a modal oscillator

If the studied structure is represented by its spectrum of low frequency real eigen modes φ in embedded base, solution of $\left(K-M\omega^2\right)\varphi=0$ or of $\left(k-m\omega^2\right)\varphi=0$ one can introduce a new transformation $\mathbf{x}=\boldsymbol{\varphi}\,\mathbf{q}$ and the system of equations [éq 2.1.3-1] is written, by means of the matrix of modal participation factors $\mathbf{P}$:

$$\ddot{\mathbf{q}}+\frac{\boldsymbol{\varphi}^{\mathbf{T}}\mathbf{c}\boldsymbol{\varphi}}{\boldsymbol{\varphi}^{\mathbf{T}}\mathbf{m}\boldsymbol{\varphi}}\dot{\mathbf{q}}+\omega^2\mathbf{q}=\frac{\boldsymbol{\varphi}^{\mathbf{T}}\mathbf{m}\mathbf{O}}{\boldsymbol{\varphi}^{\mathbf{T}}\mathbf{m}\boldsymbol{\varphi}}\ddot{\mathbf{s}}=-\mathbf{P}\ddot{\mathbf{s}}$$ éq 2.2.1-1

Assumption:

For industrial studies concerned with the seismic analysis by spectral method, one limits oneself to the case of proportional damping, known as of RAYLEIGH, for which one can diagonalizing the term $\frac{\boldsymbol{\varphi}^{\mathbf{T}}\mathbf{c}\boldsymbol{\varphi}}{\boldsymbol{\varphi}^{\mathbf{T}}\mathbf{m}\boldsymbol{\varphi}}=2\xi\boldsymbol{\omega}$ *. The damping is then represented by a modal damping ,* ξ_i *possibly different for each eigen mode [R4.05.01].*

Each eigen mode, characterized by the parameters $\left(\omega_i,\xi_i\right)$ is compared to a simple oscillator whose behavior is represented in the general case by:

$$\ddot{q}_i+2\xi_i\omega_i\dot{q}_i+\omega^2 q_i=-(\mathbf{P}\ddot{\mathbf{s}})_i$$ éq 2.2.1-2

Let us recall that are $\ddot{\mathbf{s}}$ to them driving accelerations.

2.2.2 Modal participation factor out of mono-bearing

When the motion of training is single, [éq 2.2.1-2] becomes:

$$\ddot{q}_i+2\xi_i\omega_i\dot{q}_i+\omega^2 q_i=-p_i\ddot{s}$$ éq 2.2.2-1

with

$$p_i=\frac{\boldsymbol{\varphi}_{\mathbf{i}}^{\mathbf{T}}\mathbf{m}\,\mathbf{O}}{\boldsymbol{\varphi}_{\mathbf{i}}^{\mathbf{T}}\mathbf{m}\boldsymbol{\varphi}_{\mathbf{i}}}=\frac{\boldsymbol{\varphi}_{\mathbf{i}}^{\mathbf{T}}\mathbf{m}\boldsymbol{\varphi}_{\mathbf{R}}}{\mu_i}$$ éq 2.2.2-2

where μ_i is the generalized modal mass, which depends on the standardization of the eigen mode. Let us state some properties of the participation factors modal p_i in the case as of rigid of translation, but extensible modes with the modes of rotation.

A mode φ_{RX}, that we will note δ_X, to recall that the components in the direction X are unit, belongs to the space of dimension N degrees of freedom whose N eigen modes constitute a base in which $\delta_X=\sum_i \alpha_i \varphi_i$.

From the properties of orthogonality of the eigen modes $\boldsymbol{\varphi}_{\mathbf{i}}^{\mathbf{T}} \mathbf{m} \boldsymbol{\varphi}_{\mathbf{i}}=\mu_i \delta_{ij}$, one identifies the coefficients α_i with the participation factors modal p_{iX} in the direction X and

$$\delta_X=\sum_i p_{iX} \varphi_i \quad \text{éq 2.2.2-3}$$

Moreover $\delta_x^T \mathbf{m} \delta_X=m_T$ total mass of structure, which leads to:

$$\boldsymbol{\delta}_{\mathbf{x}}^{\mathbf{T}} \mathbf{m} \boldsymbol{\delta}_{\mathbf{X}}=\sum_{ij} p_{iX} p_{jX} \boldsymbol{\varphi}_{\mathbf{j}}^{\mathbf{T}} \mathbf{m} \boldsymbol{\varphi}_{\mathbf{i}}=\sum_i p_{iX}^2 \mu_i \text{ and } m_T=\sum_i p_{iX}^2 \mu_i \text{ or } \frac{\sum_i p_{iX}^2 \mu_i}{m_T}=1 \quad \text{éq 2.2.2-4}$$

the modal parameter p_{iX} depends on the norm on the eigen mode and is accessible, for each eigen mode, in the result concept of the mode_meca `type` under name `FACT_PARTICI_DX` ; in the same way $p_{iX}^2 \mu_i$, independent of the norm, is accessible under the name of `MASS_EFFE_UN_DX`.

2.2.3 Modal participation factor out of multi-bearing

For a multiple imposed motion, [éq 2.2.1-2] becomes:

$$\ddot{q}_i+2\xi_i \omega_i \dot{q}_i+\omega^2 q_i=-\sum_j p_{ij} \ddot{s}_j \quad \text{éq 2.2.3-1}$$

with

$$p_{ij}=\frac{\boldsymbol{\varphi}_{\mathbf{i}}^{\mathbf{T}} \mathbf{m} \mathbf{O}}{\boldsymbol{\varphi}_{\mathbf{i}}^{\mathbf{T}} \mathbf{m} \boldsymbol{\varphi}_{\mathbf{i}}}=\frac{\boldsymbol{\varphi}_{\mathbf{i}}^{\mathbf{T}} \mathbf{m} \boldsymbol{\varphi}_{\mathbf{s_j}}}{\mu_i} \quad \text{éq 2.2.3-2}$$

where μ_i is the generalized modal mass, which depends on the standardization of the eigen mode and the p_{ij} can be regarded as participation factors relating to the mode i and a direction j of motion imposed of a bearing.
As previously, one can establish [bib4] the two properties:

$$\boldsymbol{\varphi}_{\mathbf{sj}}=\sum_i p_{ij} \boldsymbol{\varphi}_{\mathbf{i}} \text{ etéq } \boldsymbol{\varphi}_{\mathbf{sj}}^{\mathbf{T}} \mathbf{m} \boldsymbol{\varphi}_{\mathbf{sj}}=\sum_i p_{ij}^2 \mu_i \quad 2.2.3\text{-}3$$

One does not make, at this stage, any assumption of dependence between the various terms p_{ij}. Let us recall that the components $\ddot{s}_j$ express the driving acceleration applied to a direction of bearing j. The participation factors p_{ij} are not built independently and seem only intermediate variables in command `COMB_SISM_MODAL` [U4.84.01].

3 Seismic response by spectral method

the spectral method is an approximate technique of evaluating of the maximum of the response of structure starting from the maxima of response of each modal oscillator read on the oscillator spectrum of the excitation.

3.1 Spectral response of a modal oscillator out of mono-bearing

the maximum response in relative displacement of a modal oscillator (ω_i, ξ_i) for a direction X is given by reading on an oscillator spectrum of absolute pseudo-acceleration confer [§1.4.2] the value of absolute pseudo-acceleration $a_{iX} = Sro\,\ddot{x}_X(A, \xi_i, \omega_i)$ and while dividing by ω_i^2, from where:

$$q_{iXmax} = \frac{Sro\,\ddot{x}_X(A, \xi_i, \omega_i)}{\omega_i^2} = \frac{a_{iX}}{\omega_i^2} \quad \text{éq 3.1-1}$$

the contribution of this oscillator to the relative displacement of structure for the component x^k depends on the participation factor and the component φ_i^k in physical space:

$$x_{iXmax}^k = \varphi_i^k\, p_{iX}\, q_{iXmax} = \varphi_i^k\, p_{iX} \frac{a_{iX}}{\omega_i^2} \quad \text{éq 3.1-2}$$

and the contribution to the pseudonym absolute acceleration $\ddot{x}^k$ is the same $\ddot{x}_{iXmax}^k = \varphi_i^k\, p_{iX}\, a_{iX}$.

3.2 Spectral response of a modal oscillator out of multi-bearing

One proceeds of the same way to determine, from the value read $\ddot{S}_{jX}$ on the oscillator spectrum of absolute pseudo-acceleration associated with $\ddot{s}_j$, the contribution of the bearing j in the direction X:

$$q_{iXmax\,j} = \frac{Sro\,\ddot{s}_j(A, \xi_i, \omega_i)}{\omega_i^2} = \frac{\ddot{S}_{jXX}}{\omega_i^2} \quad \text{éq 3.2-1}$$

the statement of the contribution of this oscillator to the relative displacement of structure for the component x^k in physical space and an imposed motion j becomes:

$$x_{iXmax\,j}^k = \varphi_i^k\, p_{ijX}\, q_{iXmax\,j} = \varphi_i^k\, p_{ijX} \frac{\ddot{S}_{jX}}{\omega_i^2} \quad \text{éq 3.2-2}$$

3.3 Generalization with other quantities

Note:

The spectral method of analysis is strictly restricted with the quantities depending linearly on displacements in linear elasticity: generalized strains, stresses, forces, nodal forces, reactions of bearings.

In particular it cannot apply to equivalent quantities of strain or stresses (Von Mises).

For each quantity R^k, component of a field by elements, it is possible to calculate the component modal r_i^k associated with the eigen mode φ_i what leads to:

$$R_{iXmax}^k = r_i^k\, p_{iX}\, q_{iXmax} = r_i^k\, p_{iX} \frac{a_{iX}}{\omega_i^2} \quad \text{éq 3.3-1}$$

or

$$R_{iXmax\,j}^k = r_i^k\, p_{ijX}\, q_{iXmax\,j} = r_i^k\, p_{ijX} \frac{\ddot{S}_{jX}}{\omega_i^2} \quad \text{éq 3.3-2}$$

4 Combination rules of the modal responses

to evaluate one raising of the response R of structure, one must now combine the previously definite R^{k}_{imax} modal responses. Several levels of combination are necessary:

combination of the eigen modes selected,
static correction by pseudo-mode,
effect of the excitations different applied to groups from bearings,
combination according to the directions of excitation seisme.

4.1 Direction of the seisme and directional response

Various considerations result in separately studying the seismic behavior according to each direction of space:

- for the study of a building on a soil, the accelerogram of vertically imposed motion is different from that describing horizontal motion, itself different following two orthogonal directions from space;
- for the study of equipment, the floor spectrums differ significantly according to the three directions from space, since they integrate the participations of various modes of the building (bending of bottoms, bending or torsion of the framework.).

This resulted in drawing up a directional modal response R_x from oscillator spectrums different and of participation factors modal established in each direction X representing one of the directions of reference GLOBAL of definition of the mesh (X, Y, Z) or an explicitly definite particular direction by the user.

4.2 Choice of the eigen modes to combine

to represent correctly the modes of strain likely to be excited by imposed motion, it would be necessary to know all the eigen modes of frequency lower than the cut-off frequency of the spectrum, beyond which there is no significant dynamic amplification. This condition can prove to be difficult to fill for complex structures having a large number of eigen modes.

The size of modal base necessary must thus be evaluated to make sure that no mode having an important contribution in the internal forces and the forced was omitted in each studied direction.

4.2.1 Statement of modal strain energy

strain energy associated with each eigen mode $U_i=\frac{1}{2}\mathbf{x}^{\mathbf{T}}_{\mathbf{i\,max}}\mathbf{k}\,\mathbf{x}_{\mathbf{i\,max}}$ can be expressed for a particular direction

$$U_{iX}=\frac{1}{2}\left(p_i\frac{a_{iX}}{\omega_i^2}\right)^2\boldsymbol{\varphi}_{\mathbf{i}}^{\mathbf{T}}\mathbf{k}\,\boldsymbol{\varphi}_{\mathbf{i}}=\frac{1}{2}\left(p_{iX}\frac{a_{iX}}{\omega_i^2}\right)^2\omega_i^2\,\mu_i=\frac{1}{2}\frac{a_{iX}^2}{\omega_i^2}\,p_{iX}^2\,\mu_i \qquad \text{éq 4.2.1-1}$$

This statement corresponds to an excitation mono-bearing and can extend to the case from the multi-bearing.

The ranking of the modes with decreasing strain energies makes it possible not to retain systematically, for a general study of structure, modes which do not produce significant strains. On the other hand, for the study of the effect of the requests in a particular zone of structure, it will be necessary to use the “local” modes which can be detected by an analysis of the distribution of strain energy on groups of mesh.

Let us note that one does not have an estimate of total strain energy to quantify the mistake made by being unaware of certain modes.

4.2.2 Statement of modal kinetic energy

kinetic energy associated with each eigen mode is written $V_i=\frac{1}{2}\dot{\mathbf{x}}_{\mathbf{i\,max}}^{\mathbf{T}}\mathbf{m}\,\dot{\mathbf{x}}_{\mathbf{i\,max}}$ which gives

$$V_{iX}=\frac{1}{2}\left(p_{iX}\frac{a_{iX}}{\omega_i}\right)^2\boldsymbol{\varphi}_{\mathbf{i}}^{\mathbf{T}}\mathbf{m}\,\boldsymbol{\varphi}_{\mathbf{i}}=\frac{1}{2}\frac{a_{iX}^2}{\omega_i^2}p_{iX}^2\mu_i \qquad \text{éq 4.2.2-1}$$

the statement [éq 4.2.2-1] utilized the definite mass modal $p_{iX}^2\mu_i$ effective with [§2.2], which makes it possible to state the criterion of office plurality of the unit effective modal masses [éq 2.2.2-4].

Criterion of office plurality of the effective modal masses

the quality of a modal base, from the point of view of the representation of the inertial properties of structure, is evaluated by cumulating, for this direction, the unit effective modal masses of the modes available. A threshold of admissibility of 95% of the total mass is usually allowed. The same criterion can apply partially in the case of an excitation multi-bearing with n modes while comparing $\boldsymbol{\varphi}_{\mathbf{sj}}^{\mathbf{T}}\mathbf{m}\boldsymbol{\varphi}_{\mathbf{si}}$ and $\sum^{n} p_{ij}^2\mu_i$.

The sum of the effective modal masses is worth in fact the total mass which works on selected modal base. In other words, this working total mass is worth the total mass minus the contributions out of mass which are carried by clamped degrees of freedom (which thus do not work on modal base). Thus, for example, on a system with 1 mass-spring degree of freedom with a mass $M1$ at the top and another mass $M2$ at the level to erase it, then the working mass will be worth $M1$ and the total mass $M1+M2$. Consequently, the unit effective modal mass for the only mode of the system will be worth $M1/(M1+M2)$. The total office plurality will thus have the same value and, according to the ratio in $M1$ and $M2$, one will not be able thus inevitably to reach 90% of the total mass $(M1+M2)$, even by considering all the modes (there is only one only mode on this example). In practice, the model with the finite elements will be so and realistic, more the difference between the working mass and the total mass will be weak.

Estimate of the mistake made with an incomplete modal base

the criterion of office plurality of the effective modal masses cannot always be satisfied. Indeed one limits oneself in general to a modal base of n eigen modes with n degrees of freedom $\ll N$ modes. For rigid foundations, the spectrum of the eigenfrequencies necessary usually exceeds the cut-off frequency of the oscillator spectrum.

From the statement [éq 4.2.2-1], one can write total kinetic energy in the form:

$$V_X=\sum_1^n V_{iX}+\sum_{n+1}^N V_{iX}$$

who allows to express the absolute error from [éq 3.1-1]:

$$2\Delta V_X=\sum_{n+1}^N V_{iX}=\sum_{n+1}^N\frac{a_{iX}^2}{\omega_i^2}p_{iX}^2\mu_i\le\frac{a_{(n+1)X}^2}{\omega_i^2}\sum_{n+1}^N p_{iX}^2\mu_i$$

by noting $a_{(n+1)X}=Sro\,\ddot{x}_X(A,\xi_{min},\omega_{n+1})$ the value read on the spectrum of absolute pseudo-acceleration for $\omega_n\le\omega_{n+1}$ and the weakest modal damping ξ_{min} likely to give the raising amplitude. If the maximum frequency of the base f_n exceeds the cut-off frequency, then $a_{(n+1)X}=a_{nX}=|A(t)_{max}|$. This gives one raising of the absolute error:

$$\Delta V_X=\frac{1}{2}\frac{a_{(n+1)X}^2}{\omega_i^2}\sum_{n+1}^N p_{iX}^2\mu_i=\frac{1}{2}\frac{a_{(n+1)X}^2}{\omega_i^2}\left(m_T-\sum_1^n p_{iX}^2\mu_i\right) \qquad \text{éq 4.2.2-2}$$

4.2.3 Conclusion

the quantities allowing for choice of the modes necessary to each analysis are available in *Code_Aster* (operator `POST_ELEM` with the options `MASS_INER`, `ENER_POT` and `ENER_CIN` and parameters modal `FACT_PARTICI_DX` and `MASS_EFFE_UN_DX` in the result concept of the mode_meca `type`).

No criterion of automatic admissibility is programmed currently and the quantities $\varphi_{Sj}^{T}\mathbf{m}\,\varphi_{Sj}$ and $\sum_{1}^{n} p_{ij}^{2}\mu_{i}$, necessary to the checking of the criterion for an excitation multi-bearing, are not printed.

4.3 Static correction by pseudo-mode

4.3.1 Mono-bearing

the evaluating of one raising of the response to a seismic excitation requires, as suggests it the preceding analysis, a correction by a term representing the static contribution of the neglected eigen modes.

If one subjects structure to a quasi-static constant acceleration in the direction X , the response φ_{aX} is solution of $\mathbf{k}\,\varphi_{aX}=\mathbf{m}\,\delta_{X}\,\mathbf{I}$, without dynamic amplification. The field of displacements φ_{aX} of the nodes of structure subjected to a constant acceleration in each direction is produced by the operator `MODE_STATIQUE` [U4.52.14] with key word `PSEUDO_MODE`.

By breaking up this deformed shape on the basis as of eigen modes, one obtains (cf [§2.2.2]):

$$\mathbf{k}\,\varphi_{aX}=\mathbf{m}\sum_{i}^{k} p_{IX}\,\varphi_{i} \text{ from where } \phi_{aX}=\mathbf{k}^{\mathbf{-1}}\,\mathbf{m}\sum_{1}^{N} p_{iX}\,\varphi_{i}=\sum_{1}^{N}\frac{p_{iX}}{\omega_{i}^{2}}\varphi_{i}$$

This makes it possible to introduce a pseudo-mode φ_{cX} , for each direction, by withdrawing from the quasi-static mode φ_{aX} the static contributions of the modes used φ_{i} :

$$\varphi_{cX}=\varphi_{aX}-\sum_{1}^{n}\frac{p_{iX}}{\omega_{i}^{2}}\varphi_{i} \qquad \text{éq 4.3.1-1}$$

the statement [éq 4.3.1-1] is homologous with the term $\left(m_{T}-\sum_{i}^{n} p_{iX}^{2}\mu_{i}\right)$ of [the éq 4.2.2-2] and the pseudo-mode makes it possible to introduce a correction of the static effects of the neglected modes. The contribution of the pseudo-mode is the value read on the spectrum of absolute pseudo-acceleration $a_{(n+1)X}=Sro\,\ddot{x}_{X}(A\,,\xi_{min}\,,\omega_{n+1})$ for $\omega_{n}\leq\omega_{n+1}$ and the weakest modal damping ξ_{min} .

The correction to be brought to relative displacements and the quantities which result some (generalized forces, stresses, reactions of bearings) in excitation mono-bearing is then $x_{cX}^{k}=\varphi_{cX}^{k}\,a_{(n+1)X}$ in accordance with the conditions of estimate of the error cf [§ 4.2.2].

For the evaluating of the correction of absolute acceleration, one obtains:

$$\ddot{x}_{cX}^{k}=\left(\delta_{X}-\sum_{i}^{n} p_{iX}\,\varphi_{i}\right)^{k} a_{(n+1)X}$$

4.3.2 Multi-bearing

In excitation multi-bearing, the formulation of the pseudo-mode and its contribution take again the preceding principle.

The field of displacements $\boldsymbol{\varphi}_{\mathbf{ajX}}=\mathbf{k}^{\mathbf{-1}}\mathbf{m}\boldsymbol{\varphi}_{\mathbf{SjX}}$ of the nodes of structure subjected to a unit acceleration of the bearing J in the direction X is produced by the operator `MODE_STATIQUE` [U4.52.14] with the key word `PSEUDO_MODE`.
The correction to be brought to relative displacements and the quantities which result some writes then, for the bearing j in the direction X :

$$x_{cjX}=\varphi_{cjX}\,a_{(n+1)\,jX} \text{ with } \varphi_{cjX}=\varphi_{ajX}-\sum_{1}^{n}\frac{P_{ijX}\,\varphi_i}{\omega_i^2}$$

For absolute acceleration, the correction is written:

$$\ddot{x}_{cjX}=\left(\varphi_{SjX}-\sum_{1}^{n}\varphi_i\,P_{ijX}\right)a_{(n+1)\,jX}$$

4.4 General information on the combination rules

the combination rules or of office plurality of the various components, modal or directional, is multiple and more or less complex to implement.

One presents the "natural" methods from the point of view of their aptitude required one raising realistic requests induced in a structure represented by a base of real eigen modes resulting from a model in linear elasticity, raising estimated without transient analysis for a quantity of component G^k, which one will name G^k_{max}. For the continuation the suffix max indicates the estimate of the maximum value reached during the seismic excitation, by being unaware of time when it was reached and the index r applies to eigen modes, pseudo-modes, directions of bearings,…

Note::

Whatever the method of combination used, the value of a component obtained by combination cannot to compute: be used as data a new quantity: for example, the computation of one raising of a differential displacement between two points must be calculated mode by mode, then combined.

4.4.1 Arithmetic combination

$$G^k_{max}=\sum_r G^k_{r\max}$$

It is not usable for directional responses since the spectral method disregards times when the maximum values are reached in two directions or for two different modes. No relation of phase, and thus of sign, exists between the contributions to combine. It is thus available only in the case multi-bearing, for the office plurality of directional responses modal of bearing and for the office plurality of differential displacements.

4.4.2 Combination in absolute value

$$G^k_{max}=\sum_r |G^k_{r\max}|$$

In an obvious way, it can provide a higher limit, since it supposes that all the contributions reach their maximum at the same time with the same sign. Too much penalizing, it is available, but unusable industrially.

4.4.3 Simple quadratic combination

This method is also known under denomination SRSS (Public garden Root of Sum of Squares).

$$G^k_{max}=\sqrt{\sum_r (G^k_{r\max})^2}$$

Assumption:

***The assumption** which justifies this method of combination can be stated:*
the probable maximum of the energy stored in structure is the sum of the probable maxima of the energy stored on each mode and each one of the components directional of the seisme, i.e., with

respect to energy, the eigen modes and the component of the seisme are decoupled. It is similar to the rule of addition of the gaussian random variables and to average null.

The validity of this assumption, which will be discussed for each typical case of use of this method of combination, is not established and various proposals were presented to obtain a better approximation whenever it is put at fault cf [§ 3.4.1.2].

In addition, one will be able to refer to [bib3] for a criticism of this approach, in particular of his aptitude to consider a maximum probable of the strains and stresses, but the alternative approach that it evokes was the object of any development in *Code_Aster*.

4.5 Establishment of the directional response out of mono-bearing

the directional response, previously definite, is obtained by simple quadratic combination of two terms which we will discuss:

$$R_X = \sqrt{R_m^2 + \left(R_{qs} + R_c\right)^2}$$

R_m dynamic combined response of the modal oscillators

R_{qs} quasi-static combined response of the modal oscillators (=0 except for the combination of the Gupta type)

R_c contribution of the static correction of the neglected modes (pseudo-mode)

the assumptions justifying the method of quadratic combination simple, on this level, do not seem to have to be reconsiderations [bib1].
To simplify the notations, one notes R_m instead of R_{mX} ,…

4.5.1 Combined response of the modal oscillators

the response of structure R_m , in a direction of seisme, is obtained by one of the possible combinations of the contributions of each eigen mode taken into account for this direction. The number of possible methods proves simply the difficulty in releasing a sufficient justification to guarantee a conservative and realistic estimate. If the simple quadratic combination (SRSS or CQS) is evoked by all, one will retain [bib1] that it is often put at fault and one will prefer the quadratic combination to him supplements (CQC). The other methods are available for possible comparisons.

4.5.1.1 Absolute values This

combination Somme corresponds to an assumption of complete dependence of the oscillators associated with each eigen mode and conduit with a systematic overvaluation with the response:

$$R_m = \sum_i^n |R_i|$$

4.5.1.2 Quadratic combination simple (CQS)

By considering that the contribution of each modal oscillator is an independent random variable, an estimate of the maximum response, for the component of displacement $x_{\max}^k$, can be obtained by simple quadratic combination of the contributions of each mode from where, for an excitation mono-bearing:

$$x_{max}^k = \sqrt{\sum_i^n \left(x_{i\,max}^k\right)^2} = \sqrt{\sum_i^n \left(\varphi_i^k p_i q_{i\,max}\right)^2} \qquad \text{éq 4.5.1.2 - 1}$$

Generally, for any quantity R_i associated with a modal oscillator (ω_i, ξ_i) :

$$R_m = \sqrt{\sum_i^n R_i^2}$$

It constitutes a good approximation of reality when the oscillator spectrum defining the seisme is to broad waveband, and when the eigen modes of structure are quite separate from/to each other and are located inside or in the vicinity of this tape. She is in particular put at fault if eigen modes are with frequencies close or for modes far away from the peak to excitation [bib2]. The other methods of combination of the modal responses try to correct this point.

4.5.1.3 Quadratic combination supplements (CQC)

the quadratic combination supplements (established by DER KIUREGHIAN [bib5]) makes a correction to the preceding rule by introducing coefficients of correlation depending on depreciation and the distances between close eigen modes:

$$R_m=\sqrt{\sum_{i_1}\sum_{i_2}\rho_{i_1 i_2}R_{i_1}R_{i_2}}$$

with the coefficient of correlation:

$$\rho_{ij}=\frac{8\sqrt{\xi_i\xi_j\omega_i\omega_j}(\xi_i\omega_i+\xi_j\omega_j)\omega_i\omega_j}{(\omega_i^2-\omega_j^2)^2+4\xi_i\xi_j\omega_i\omega_j(\omega_i^2+\omega_j^2)+4(\xi_i^2+\xi_j^2)\omega_i^2\omega_j^2}$$

or by introducing the ratio of pulsation or frequencies between two modes $\eta=\omega_j/\omega_i$:

$$\rho_{ij}=\frac{8\eta\sqrt{\xi_i\xi_j\eta}(\xi_i+\xi_j\eta)}{(1-\eta^2)^2+4\eta\xi_i\xi_j(1+\eta^2)+4\eta^2(\xi_i^2+\xi_j^2)}$$

and for ξ constant:

$$\rho_{ij}=\frac{8\eta\xi^2(1+\eta)\sqrt{\eta}}{(1-\eta^2)^2+4\eta\xi^2(1+\eta^2)+8\eta^2\xi^2}$$

4.5.1.4 Combination of ROSENBLUETH

This rule (proposed by E. ROSENBLUETH and J. ELORDY [bib6]) introduced a correlation between modes, different from that of method CQC. The responses of the oscillators are combined by double sum (Double Sum Combination):

$$R_m=\sqrt{\sum_{i_1}\sum_{i_2}\rho_{i_1 i_2}R_{i_1}R_{i_2}}$$

It requires an additional data, period s of the “strong” phase of the seisme. The coefficient of correlation is then:

$$\rho_{ij}=\left(1+\left(\frac{\omega'_i-\omega'_j}{\xi'_i\omega_i+\xi'_j\omega_j}\right)^2\right)^{-1} \text{ where } \omega'_i=\omega_i\sqrt{1-\xi'^2_i} \text{ and } \xi'^2_i=\xi_i+\frac{2}{s\omega_i}$$

4.5.1.5 Combination with rule of the 10%

the close modes (of which the frequencies different from less than 10%) are initially combined by summation of the absolute values. The values resulting from this first combination are then combined quadratically (simple quadratic combination). This method was proposed by American regulation U.S. Nuclear Regulatory Commission (Regulatory Guides 1.92 - February 1976) to attenuate the conservatism of the method of sum of the absolute values. It remains at fault for structures with a dense frequency spectrum clean and should not be used any more.

4.5.1.6 Combination according to Gupta

Gupta [NRC1.92], to take into account the correlations between modes due to the quasi-static part of the response, introduced the rigid factor of response, which varies from 0 to 1 the correlation between the modal responses of intermediate frequencies between f_1 and f_2, two frequencies to be determined (typically $2\ Hz$ with $20\,Hz$).

Gupta breaks up each modal response R_r in a dynamic part R_r^p a quasi-static part R_r^{qs} : $R_r^{qs}=\alpha_r R_r$ and $R_r^p=\sqrt{1-\alpha_r^2}R_r$

Thus, for each mode r , one affects the rigid factor of response α_r the modal response R_r :

$\alpha_r=0$ for $f\le f_1$ $\alpha_r=1$ for $f\ge f_2$

α_r estimated for the fréquenceformuleselon f_r the following formula:

$$\alpha_r=\frac{\ln f_r/f_1}{\ln f_2/f_1}$$

the dynamic combined response of the modal oscillators is carried out according to combination "CQC":

$$R_d=\sqrt{\sum_{r_1}\sum_{r_2}\rho_{r_1 r_2}R_{r_1}^p R_{r_2}^p}$$

the quasi-static combined response of the modal oscillators is carried out according to an algebraic combination:

$$R_{qs}=\sum_{r=1}^{nmod}R_r^{qs}$$

This combination according to GUPTA is available only in the case mono-bearing.

4.5.2 Contribution of the static correction of the neglected modes

the contribution of the pseudo-mode cf [§4.3.1] can be combined quadratically because independence with the modal contributions of vibration is not disputed.

4.6 Establishment of the directional response out of multi-bearing

4.6.1 Computation of the total response

the order of the combinations to be carried out differs according to whether the excitations of the bearings (or the groups of bearings) can be regarded as correlated or uncorrelated between them.

4.6.1.1 Case with groups of uncorrelated bearings

the sequence of the combinations can be stated as follows, in the order:

- for each direction X , each mode i , each group of bearings:
    1) computation of directional responses of bearings modal (combination intra-group on the bearings): considering that the bearings of the same group are correlated, one proposes a summation of the algebraic values;
- for each direction X , each group of bearings j :
- computation of the combined response of the modal oscillators (combination on the modes);
- computation of the pseudo-mode R_{Xj}^c (static correction of the neglected modes);
- computation of the motion of training R_{Xj}^e ;
- computation of the directional response of bearing: $R_{Xj}=\sqrt{R_{Xj}^{m\,2}+R_{Xj}^{c\,2}+R_{Xj}^{e\,2}}$
- for each direction (combination joint committee on the groups of bearings): computation of the directional response, according to a quadratic summation CQS because the groups are supposed to be uncorrelated between them;
- computation of the total response (combination on the directions).

4.6.1.2 Case with bearings correlated

If all the bearings are correlated between them (when one cannot display of groups of uncorrelated bearings), one proposes the following diagram, which has the advantage of being able to establish a parallel between the processing of the case mono-bearing treated like such and the case mono-bearing treated like a typical case of the case multi-bearing. The sequence of the combinations can be summarized as follows:

- for each direction X , each mode i :
- computation of directional responses modal (combination on the bearings): the bearings being supposed correlated between them, one proposes a summation of the algebraic values or absolute values;
- for each direction X :
- computation of the combined response R_X^m of the modal oscillators (combination on the modes);
- computation of the pseudo-mode R_X^c (static correction of the modes neglected, algebraic summation on the bearings of the pseudo-modes of bearing R_{Xj}^c);
- computation of the motion of training R_{Xj}^e (algebraic summation on the bearings of motions of training of bearing R_{Xj}^e);
- computation of the directional response: $R_X=\sqrt{R_X^{m2}+R_X^{c\,2}+R_X^{e\,2}}$,
- computation of the total response (combination on the directions).

4.6.2 Computation separated of the components primary and secondary from the response

Each component is the object of a similar separate processing. This approach is adapted to postprocessings RCC-M in force for the seismic analysis of the pipework [§ 4.9]:

4.6.2.1 Primary component RIX (inertial response)

the order of the combinations to be carried out differs according to whether the excitations of the bearings (or the groups of bearings) can be regarded as correlated or uncorrelated between them.

- groups of uncorrelated bearings

for each imposed motion $\ddot{s}_j$, computation of directional responses of bearing (office plurality on the modes):

$$R_{IjX}=\sqrt{R_j^{m2}+R_j^{c2}}$$

R_j^m response of bearing combined of the modal oscillators (office plurality on the modes)

R_j^c contribution of the static correction of the neglected modes (pseudo-mode of bearing)

combination of the responses R_{IjX} (office plurality on the bearings)

- bearings correlated

for each imposed motion $\ddot{s}_j$, computation of directional responses modal (office plurality on the bearings):

$$R_{IiX}=\sqrt{R_i^{m2}+R^{c2}}$$

R_i^m combined modal response of the modal oscillators (office plurality on the bearings)

R^c contribution of the static correction of the modes neglected (pseudo-mode)

combination of the responses R_{IiX} (office plurality on the modes)

4.6.2.2 Component secondary RII (quasi-static response)

combination of the responses R_{ej}

4.6.3 Office plurality on the modes

the selection criterion of the method of combination of the modal contributions is the same one as for an excitation mono-bearing and one will use method CQC preferentially.

4.6.4 Contribution of the pseudo-mode

the corrective term by pseudo-mode cf [§4.3.2] can be combined quadratically.

4.6.5 Contribution of motions of training

the motion of training of structure not being uniform, one can add a term with the computation of the directional response. This is not necessary if one chooses to regard this static contribution as a specific loading case inducing of the secondary stresses. This term is defined starting from the maximum relative displacement which cannot be known starting from the only spectrums of absolute pseudo-acceleration of the bearings.

$$R_{ej}=\varphi_{Sj}\delta_{jmax}$$

φ_{Sj} static mode for the bearing j

δ_{jmax} maximum relative displacement of the bearing j compared to a bearing of reference (for which $\delta_{jmax}=0$)

4.6.6 Office plurality on the bearings

This stage is compulsory, but the choice of the method of combination of directional responses remains very open. Indeed, the assumption of independence of $\ddot{s}_j$ strongly depends on the eigen modes of the structure support of the studied equipment. An analysis of the studied system is necessary to possibly gather the bearings by groups: for example, for a pipework connecting two buildings, either all the bearings are regarded as correlated between them, or one can display uncorrelated groups between them (the group of the bearings of the building 1, that of the building 2 and, finally, that of the intermediate stanchions), whose bearings inside each group are correlated.

Office plurality intra-group
the excitations with the bearings of the same group being supposed correlated between them, the office plurality is carried out algebraically according to the linear combination defined by: $R_X=\sum R_{jX}$
Office plurality joint committee
the groups of bearings being made up so that they are uncorrelated between them, the office plurality joint committee is carried out by simple quadratic combination.

4.7 Combination of directional responses

the Two combination rules of directional responses are available.

4.7.1 Quadratic combination

This combination corresponds to the assumption of strict independence of the responses in each direction cf [§ 3.3.3]. Let us recall that this combination rule does not have any geometrical meaning, although the three directions of analysis are orthogonal.

$$R=\sqrt{R_X^2+R_Y^2+R_Z^2}$$

The assumptions justifying the method of quadratic combination simple, on this level, do not seem to have to be reconsiderations [bib3], but this method is not used.

4.7.2 Combination of NEWMARK

This empirical combination rule is most usually used and in general leads to estimates slightly stronger than the preceding one. It supposes that when one of directional responses is maximum, the others are with most equal to $4/10$ their respective maximum contributions. For each direction $i(X,Y,Z)$, one calculates the 8 values:

$$R_l=\pm R_X\pm 0{,}4\,R_Y\pm 0{,}4\,R_Z$$

what leads, by circular shift, 24 values and $R=\max(R_l)$

4.8 Warning on the combinations

Several remarks are forced to warn the user on the way of using the methods of combination and the quantities combined in a note of study.

Notice 1:

If one wishes to use arithmetic combinations (direction) and quadratic combinations (modes), the quadratic office pluralities must be always carried out in the last.

Notice 2:

Any quadratic combination applies only to the quantities for which, in instantaneous values, the office plurality has the meaning of a sum: combination of the components of displacement, or force generalized or of stress of each eigen mode.
The modal or directional quadratic combination cannot thus apply to intensities of stress (principal stresses, of Von Mises, Tresca).

Notice 3:

The results of a combination, whatever the rule of office plurality, should not be used as data to compute: of other quantities: for example, a differential displacement between two points (or a strain) can be calculated only starting from modal differential displacements that one combines then.
A fortiori the generalized forces and the forced can be calculated only mode by mode before any combination and not from inertia forces deduced from the fields of acceleration obtained by combination of modal accelerations.

4.9 Lawful practices

4.9.1 Partition of the components primary and secondary of the response

the various supports from line of pipework can be animated different motions. The same section of pipework can be distributed on different buildings, different levels or equipment. It thus undergoes a multiple excitation. This results in two types of loading [§ 2.1.2]:

- an excitation whose frequential contents vary from one support to another and who constitutes a primary education according to classification RCC-M, of
- Differential Displacements Seismic (DDS) inducing a stress state by displacements imposed on the bearings and classified loading like secondary.

The generalized moments resulting from these 2 loadings intervene separately in the inequations of design RCC-M and on several levels.
For a deepened postprocessing RCC-M, It is thus necessary to have of the components primary and secondary the seismic response.
In a more general way, the method of combination of the responses of bearings can differ according to whether one treats the case of the components inertial or differential. Moreover, the number of bearings concerned with these two summations can not be equal. One is often brought to impose overall differential motions even for bearings associated with spectrums different users. In addition, of the DDS formulated in rotation are sometimes to consider. They cannot be associated with an inertial loading (restricted with the translations).

Code_Aster thus proposes two processing:

- Determination of the total response:
- The contributions inertial and static of training are cumulated during the computation of directional responses of bearing [§4.6].
- Partition of the components primary and secondary of the total response:

The two preceding contributions are not any more cumulated during the computation of directional responses and are the object of 2 independent processing:

- The inertial component is obtained by removing the term of training R_{ej} in the computation of the total response [§ 4.6].
- The static component is given by combining the terms of training defined under key word DEPL_MULT_APPUI. The methods of combination of these loading cases DDS are indicated in key word COMB_DEPL_APPUI.

4.9.2 Method of the spectrum envelope

Even if the pipework are subjected to a multiple seismic excitation, the common practice is to be reduced to the computation of a structure mono-supported while preserving loading cases DDS. This simplified approach implies to define a single spectrum by direction for all the supports of the pipework. For each direction, one adopts a spectrum then “wraps” various spectrums with the supports. The spectrums retained for the horizontal directions X and Y are identical.
In almost the whole of the cases, this method is generating of “margin of design”.

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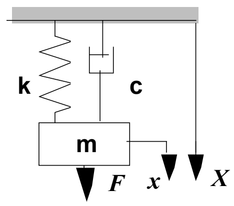
6 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of modifications 5.
5	*LEVESQUE, L. VIVAN, D. SELIGMANN, EDF-R&D/AMA* initial	Text 4/6/11
04/06/11	*LEVESQUE, L. VIVAN, Y. PONS EDF R & D /AMA* 9.4	
S.AUDEBE RT	EDF R & D /AMA 10 S	
10	EDF R & D /AMA Taken	into account of file REX 12005: modification of the office plurality §4.6.6 intra-group 11 S
11	EDF R & D /AMA Taken	into account of file REX 17054: introduction of the method of modal recombination according to Gupta Transient response

Annexe 1of an oscillator simple damped A1.1

Forced vibrations of a system with a degree of freedom in translation For

a simple oscillator of stiffness, k mass and m viscous damping, c the equation of motion is form: for



$$m\ddot{X} + c\dot{X} + kX = 0$$

which the traditional notations are: The total

la pulsation propre du système non amorti :	$\omega_0 = \sqrt{\frac{k}{m}}$
l'amortissement critique :	$c_{critique} = 2m\omega_0$
l' amortissement réduit : (exprimé en pourcentage de l' amortissement critique)	$\xi = \frac{c}{c_{critique}} = \frac{c}{2m\omega_0}$
la pulsation propre du système amorti :	$\omega'_0 = \omega_0 \sqrt{(1-\xi^2)}$
la déflexion statique pour une force F_0 :	$\delta_{st} = \frac{F_0}{k}$
la fréquence réduite :	$\eta = \frac{\omega}{\omega_0}$
équation réduite du mouvement :	$\ddot{X} + 2\xi\omega_0\dot{X} + \omega_0^2 X = 0$

response **with** a harmonic excitation of the form is $F(t) = F_0 \cos(\omega t)$ the sum: of

- a free response damped $X_1(t)$ oscillatory general solution oùetsont X_{l0} φ_0 determined by the initial conditions: of

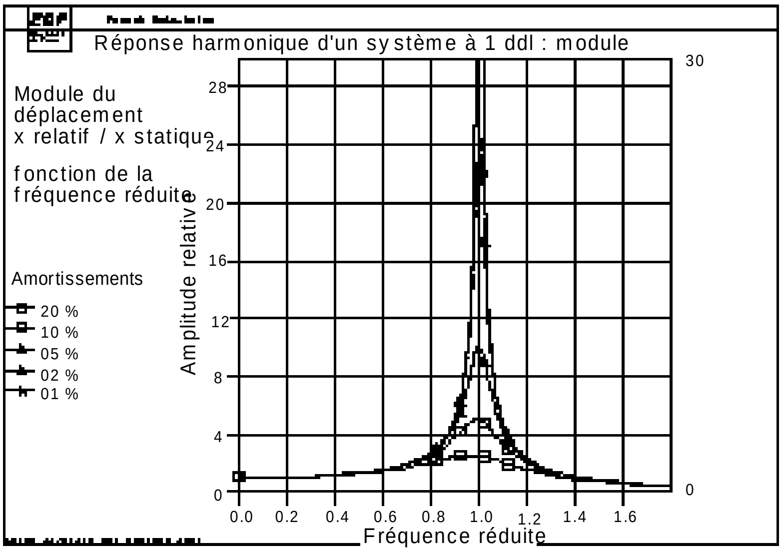
$$X_1(t) = X_0 e^{-\xi\omega_0 t} \cos(\omega'_0 t + \varphi_0)$$

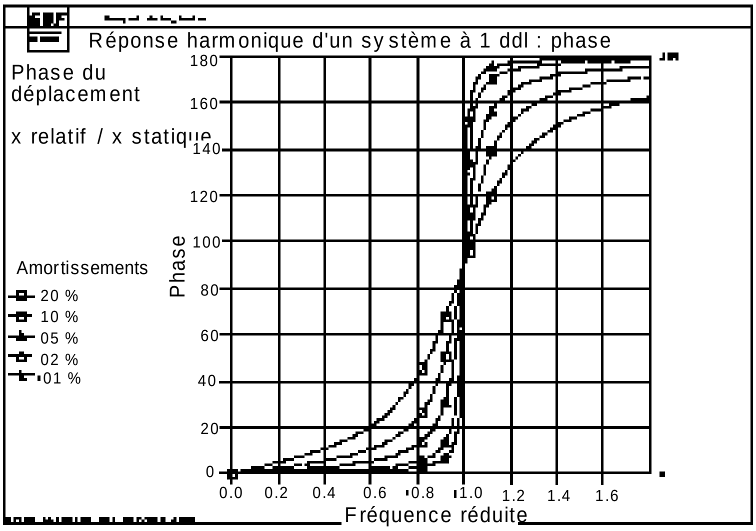
- a forced response permanent $X_f(t)$ particular solution éq $X_f(t) = X_{f0} \cos(\omega t - \varphi)$

$$X_{f0} = \frac{F_0}{\sqrt{(k-m\omega^2)^2 + (c\omega)^2}} \quad \varphi = arctg\left(\frac{c\omega}{k-m\omega^2}\right)$$ A1.1-1 which

is written in reduced form: éq

$$\frac{X_{f0}}{\delta_{st}} = \frac{k X_{f0}}{F_0} = \frac{1}{\sqrt{(1-\eta^2)^2 + (2\xi\eta)^2}} \quad \varphi = arctg\left(\frac{2\xi\eta}{1-\eta^2}\right)$$ A1.1-2 Appears





A1.1-a: Response of an oscillator in imposed force (modulus and phase) the response

with a harmonic excitation of the form $F(t)=F_0 e^{j\omega t}$ is written with a forced response permanent particular solution éq $X_f(t)=X_{f0} e^{(j\omega t-\varphi)}$

$$X_{f0}=\frac{F_0}{\sqrt{(k-m\omega^2)^2+(c\omega)^2}}\ \varphi=\text{arctg}\left(\frac{c\omega}{k-m\omega^2}\right)$$ A1.1-3 which

is written in reduced form: éq

$$\frac{kX_{f0}}{F_0}=\frac{1}{1-\eta^2+j2\xi\eta}\equiv H(j\omega)\varphi=\text{arctg}\left(\frac{2\xi\eta}{1-\eta^2}\right)$$ A1.1-4 where

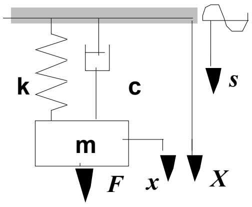
is $H(j\omega)$ the harmonic response complexes of a simple oscillator: Motion

$$H(j\omega)=\frac{1}{\sqrt{(1-\eta^2)^2+(2\xi\eta)^2}}$$

Annexe 2imposed of a system on a degree of freedom in translation A2.1

Absolute motion of a system to a degree of freedom For

a simple oscillator of stiffness, k mass and m viscous damping, c the equation of absolute motion **is** form: The forced



$$m\ddot{X} + c(\dot{X} - \dot{s}) + k(X - s) = 0$$
$$m\ddot{X} + c\dot{X} + kX = ks + c\dot{s}$$
$$\ddot{X} + 2\xi\omega_0\dot{X} + \omega_0^2 X = \omega_0^2 s + 2\xi\omega_0\dot{s}$$

response **with** a harmonic imposed motion of the form is $s(t)=s_0\cos(\omega t)$ form adds $X_m(t)=X_{m0}\cos(\omega t-\varphi_1-\varphi_2)$ of two terms of response, permanent particular solutions: term

induced by the excitation in term displacement $X_{d0}\cos(\omega t-\varphi_d)$

$$X_{d0}=\frac{k\,s_0}{\sqrt{(k-m\omega^2)^2+(c\,\omega)^2}} \qquad \varphi_d=\mathrm{arctg}\left(\frac{c\,\omega}{k-m\omega^2}\right)$$

induced by the excitation of velocity what $X_{v0}\cos(\omega t-\varphi_v)$

$$X_{v0}=\frac{\omega\,c\,s_0}{\sqrt{(k-m\omega^2)^2+(c\,\omega)^2}}\ \varphi_v=\mathrm{arctg}\left(\frac{c\,\omega}{k-m\omega^2}\right)$$

leads to a total forced response: from where

$$X_m(t)=X_m\cos(\omega t-\varphi_1-\varphi_2)\equiv s_0\sqrt{\frac{k^2+(c\,\omega)^2}{[(k-m\omega^2)^2+(c\,\omega)^2]}}\cos(\omega t-\varphi_1-\varphi_2)$$

the reduced form of the absolute amplitude: If

$$\frac{X_m}{s_0}=\sqrt{\frac{1+(2\xi\eta)^2}{[(1-\hat{e}ta^2)^2+(2\xi\eta)^2]}} \qquad \varphi_1=\mathrm{arctg}\left(\frac{2\xi\eta}{1-\eta^2}\right) \qquad \varphi_2=\mathrm{arctg}\left(\frac{1}{2\xi\eta}\right)$$

the motion imposed on the base is expressed in complex form, $s(t)=\Re(s_0 e^{j\omega t})$ the relative amplitude or transmissibility can be written from the harmonic response complexes of a simple oscillator éq $H(j\omega)$

$$\frac{X_m}{s_0}=\sqrt{1+(2\xi\eta)^2}\,|H(j\omega)|$$ A2.1-1 A2.2

Relative motion of a system to a degree of freedom the problem

of the response to an imposed motion can be treated in relative displacement of the mass compared to the bases by posing $x=X-s$

the equation of relative motion **for** a harmonic imposed motion of the form is $s(t)=s_0\cos(\omega t)$ then of the form or in $m\ddot{x}+c\dot{x}+kx=-m\ddot{s}$ reduced form: éq

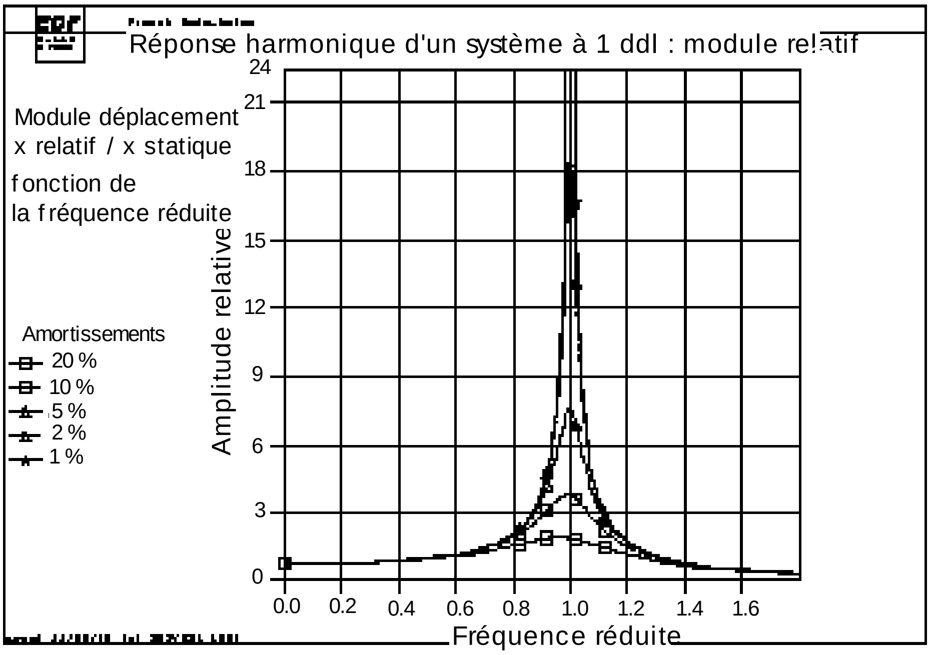
$$\ddot{x}+2\xi\omega_0\dot{x}+\omega_0^2 x=-\ddot{s}=\omega^2 s_0\cos(\omega t)$$ A2.2-1 the relative

forced **response** is then, for a permanent solution, éq $x_{m0}\cos(\omega t-\varphi)$

$$x_{m0}=\frac{m\omega^2 s_0}{\sqrt{(k-m\omega^2)^2+(c\omega)^2}} \qquad \varphi=\mathrm{arctg}\left(\frac{c\omega}{k-m\omega^2}\right)$$ A2.2-2 which

is written in reduced form: éq

$$\frac{x_{m0}}{s_0}=\frac{\eta^2}{\sqrt{(1-\eta^2)^2+(2\xi\eta)^2}}$$ A2.2-3 Appears



A2.2-a: Response of an oscillator moving imposed (modulus of relative displacement) Motion

Annexe 3imposed not periodical of a system on a degree of freedom with the problem

dealt previously was limited to a periodic imposed motion. For a nonperiodic excitation, of variable amplitude with time, being exerted for a finished length of time, one considers the response with a series of impulses. A3.1

Impulse response the

simplest form is the unit impulse force, which applied to a rest mass before the application of the impulse (for $x=\dot{x}=0$ or $t<0$) can $t=0^-$ be written:

$$\tilde{f}=\lim_{\Delta t\to 0}\int_t^{t+\Delta t} F\,dt=F\cdot dt=1=m\dot{X}(t=0)-m\dot{X}(t=0^-)=m\dot{X}_0$$

The initial conditions are then noted and $X(t=0)=X_0$ $\dot{X}(t=0)=\dot{X}_0=\frac{1}{m}$

the general equation of the response in free vibration of a system with a degree of freedom: then

$$X_l(t)=e^{-\xi\omega_0 t}\left(X_0\cos\omega'_0 t+\frac{\dot{X}_0+\xi\omega_0 x_0}{\omega'_0}\sin\omega'_0 t\right)$$

becomes the impulse response of $g(t)$ a system with a degree of freedom éq

$$X_l(t)=g(t)=\frac{e^{-\xi\omega_0 t}}{m\omega'_0}\sin\omega'_0 t$$ A3.1-1 For

a nonunit impulse, $\tilde{F}=F\cdot\Delta t$ the initial velocity is and $\dot{X}_0=\frac{F}{m}$ the response becomes: éq

$$X_l(t)=\frac{\tilde{F}e^{-\xi\omega_0 t}}{m\omega'_0}\sin\omega'_0 t=\tilde{F}g(t)$$ A3.1-2 If

the impulse force is applied to one unspecified time τ , the response is: A3.2

$$X_l(t)=\tilde{F}g(t-\tau)$$

Response in unspecified forced vibrations the force

of excitation can $F(t)$ be broken up into a series of impulses of variable amplitude applied $F(\tau)$ to time during τ a time. If τ , $\Delta\tau\to 0$ the response at one time t is obtained by: and while

$$X(t)=\int_0^t F(\tau)g(t-\tau)d\tau$$

replacing by the statement of the impulse response [éq A.3-2], one obtains the equation of convolution for a system at rest at time 0 of the form: éq

$$X(t)=\frac{1}{m\omega'_0}\int_0^t F(\tau)e^{-\xi\omega_0(t-\tau)}\sin\omega'_0(t-\tau)d\tau$$ A3.2-1 known

under the name of integral of DUHAMEL. A3.3

Response moving unspecified imposed For

an analysis moving relative motion represented by [éq A2.2-1]: the integral

$$\ddot{x}+2\xi\omega_0\dot{x}+\omega_0^2 x=-\ddot{s}=\omega^2 s_0\cos(\omega t)$$

of DUHAMEL becomes: éq

$$x(t)=\frac{1}{\omega'_0}\int_0^t \ddot{s}(\tau)e^{-\xi\omega_0(t-\tau)}\sin\omega'_0(t-\tau)d\tau \qquad \text{A3.3-1}$$