Interaction soil-structure with spatial variability (operator DYNA_ISS_VARI)

Summarized:

This document is a theoretical note describing the methods developed in operator DYNA_ISS_VARI. This operator allows to deal with problems of interaction (ISS) soil-structure in seismic analysis where one wishes to take account of the spatial variability of the incidental seismic field. In the frame of a standard seismic study with interaction soil-structure using the sequence *Code_Aster* /ProMISS3D, one supposes that the seismic excitation does not vary spatially. However, spatial variability can have considerable effects on the responses of structures subjected to a seisme. One observes in particular a reduction of the response in translation what can make it possible to release from the margins in computations of ISS.

DYNA_ISS_VARI makes it possible to calculate the response of a structure subjected to a variable seismic motion in space from a function of coherence, matrix of impedance and seismic force. These last can be calculated by the ProMISS3D software. More precisely, one builds the spectral vectors of modal response (exits of a spectral decomposition of the matrix of coherence) via a harmonic computation out of generalized components. In output, one obtains the spectral concentration of the modal response (for a unit excitation) or the temporal response (in acceleration).

Contents

1 Introduction

Into seismic computations with interaction soil-structure, the common practice consists in considering a motion of uniform free field in any point of the surface of the soil. The recent observations of strong motions on seismographic networks however revealed spatial variations on rather weak scales. The taking into account of the spatial variability of the incidental field led to a filtering of the translatory movements for the high frequencies. The introduction of spatial variability can thus lead to reduced floor spectrums.

The computation of the response is done in three stages:

- Computation of the matrixes generalized of clamped structure on the interface.
- Determination of the matrixes of impedance of interface and the seismic force by ProMISS3D: The ProMISS3D software rests on a method of substructuring. The field of study is broken up into subdomains - in our case soil and the structure - coupled between them by interfaces. One applies a method of resolution multi-fields and only the interfaces between fields require to be with a grid by of the finite elements of border. This makes it possible to model the structure (building) as well as the loadings which are applied to him entirely by *Code_Aster*. The ProMISS3D code, as for him, makes it possible to determine the impedances of interface between soil and structure as well as the seismic force exerted by the incidental field on the level of the interface. The resolution of the problem of dynamics and the post-processing are carried out again with *Code_Aster*.
- Resolution of the problem of dynamics on reduced basis (resulting from the substructuring) in the field of frequencies (`DYNA_ISS_VARI`) and postprocessing.

2 Description of the command DYNA_ISS_VARI

operator `DYNA_ISS_VARI` [U4.53.31] makes it possible to calculate the response of a structure subjected to a variable seismic motion in space from a function of coherence, matrix of impedance of interface and the seismic force. These last can be calculated using the sequence *Code_Aster*/ProMISS3D, cf [U2.06.07]. In output of `DYNA_ISS_VARI`, one obtains, in generalized coordinates, the spectral concentration of response or a temporal response with a temporal excitation.

More precisely, one builds the spectral vectors of modal response (exits of a spectral decomposition of the matrix of coherence) via a harmonic computation out of generalized components. Then, one determines power spectral density (DSP) of the modal response or the temporal response out of generalized components.

The results which one can get by means of command `DYNA_ISS_VARI` (for the cases with or without spatial variability) are the following:

- Computation of transfer transfer functions between the seismic excitation and the response of structure (the transfer transfer functions are obtained by choosing an excitation by white vibration).
- Computation of the spectral concentration of response for the case where the seismic excitation is given by one spectral concentration (the spectrum of Kanai-Tajimi is generally used to describe the seismic excitation, to also see [R4.05.02]).
- Computation of the temporal response out of generalized components. The simulation of a realization of temporal response then makes it possible to determine floor spectrums.

2.1 Seismic analysis with ISS in the field of the frequencies

the ProMISS3D software is founded on a frequential resolution side soil; it makes it possible to determine the matrixes of impedances as well as the seismic force with the interface. The problem of interaction soil-structure amounts solving on the interface the equation of harmonic dynamics:

$$\left[\underbrace{K_b+i\omega C_b-\omega^2 M_b}_{\text{Equation d'équilibre du bâtiment}}+\underbrace{K_s(\omega)}_{\text{Impédance d ' interface sol}}\right]q(\omega)-\underbrace{f_s(\omega)}_{\text{Force sismique}}=0 \quad (3\text{-}1)$$

In the equation [éq 3], $q(\omega)\in\mathbb{C}^m$ is the vector of the generalized unknowns describing displacement.

In computations of ISS with ProMISS3D, one must provide an accelerogram in free field. The transform of Fourier of this signal and the matrix of impedance calculated by ProMISS3D make it possible to determine the seismic force in the field of the frequencies $f_s(\omega)$.

For what follows, one defines the complex transfer transfer function in displacement $H(\omega)\in Mat_{\mathbb{C}}(m,m)$ like:

$$H(\omega)f_s(\omega)=q(\omega)$$

If the excitation is supposed to be a gaussian steady stochastic process, then the response is also a gaussian steady stochastic process. This is true because the transfer transfer function is a linear filter. Thus, one can write the relation between the spectral concentrations of the seismic force and the response:

$$H(\omega)S_f(\omega)H^*(\omega)=S_q(\omega)$$

where H^* indicates the combined complex transposed of H and S_f is the matrix of spectral concentration of the seismic force which can be evaluated from L achievements of the temporal seismic force $f_s^l(t)$:

$$S_f(\omega)=\frac{1}{2\pi}\frac{1}{L}\sum_{l=1}^{L} f_s^l(\omega)f_s^l{}^*(\omega)$$
$$\text{où}\quad f_s^l(\omega)=\frac{1}{\sqrt{T}}\int_0^T f_s^l(t)e^{-i\omega t}dt \qquad (3)$$

In this statement, T the time interval indicates and $W(t)=1/\sqrt{(T)}$ is the natural window on $[0,T]$ (cf [bib10]). It is about an estimator not skewed of the spectral concentration [bib10].

Note:

> *One can also write:*
>
> $$S_f(\omega)=\lim_{T\to\infty}\frac{1}{2\pi T}E\left(f_s(\omega)f_s{}^*(\omega)\right)$$
>
> *where* $E(\cdot)$ *the operator expectation indicates.*

2.2 Taken into account of spatial variability

DYNA_ISS_VARI is founded on a probabilistic description of the incidental seismic field by its power spectral density (DSP). The latter is generally built using a specific spectrum and of a function of spatial coherence.

Thus, the crossed spectral concentration of the motion of the soil in free field is written:

$$S_u(x,x',\omega)=\gamma(x,x',\omega)S_0(\omega) \qquad (4\text{-}1)$$

where $\gamma(x,x',\omega)$ is the function of coherence of the seismic signal between two points x and x' $S_0\in\mathbb{R}$ is specific spectral concentration of seismic motion in free field. It is supposed here that the spectral concentration is defined on the interval $\Omega=\left[-\omega_s,\ +\omega_s\right]$ and that it is null apart from this frequency range. If one discretizes compared to the variable spatial, x and x' , one obtains the matrix of following spectral concentration:

$$S_u(\omega)=\boldsymbol{\gamma}(\omega)S_0(\omega) \quad \in \mathrm{Mat}_{\mathbb{C}}(m,m) \qquad (4)$$

In this statement, $\boldsymbol{\gamma}\in\mathrm{Mat}(m,m)$ indicates the matrix of coherence and one notes his components $\boldsymbol{\gamma}_{ij}(\omega)=\gamma(x_i,x_j,\omega)$. The elements of $S_u(\omega)=[S_{ij}(\omega)]\in Mat(m,m)$ are the crossed spectral concentrations $S_{ij}(\omega)=S(x_i,x_j,\omega)$, m being the number of points of spatial discretization. In general, it is supposed that the incidental seismic field is homogeneous, i.e. the stochastic description of the field depends only on the distance $d=|x-x'|$, but is independent of the spatial position.

The computation of the seismic response the spectral decomposition of the matrix of coherence leans on $\boldsymbol{\gamma}(\omega)$. Let us specify that it is about a spectral decomposition compared to the variable spatial and not compared to the variable time. Thus, one a:

$$S_u(\omega)=\Phi(\omega)\Lambda(\omega)\Phi^*(\omega)S_0(\omega) \qquad (4)$$

where Φ is a matrix containing the eigenvectors φ_k of the matrix of coherence $\boldsymbol{\gamma}$ and Λ is the diagonal matrix containing the eigenvalues $\Lambda=diag(\lambda_k),\ k=1,\ldots m$. Thereafter, one will speak about modes POD (Orthogonal Proper Decomposition) to indicate them φ_k. This makes it possible to distinguish them from the mechanical modes. From the statement [éq 4], one defines:

$$s_u^k(\omega)=\varphi_k(\omega)\sqrt{\lambda_k(\omega)}\sqrt{S_0(\omega)},\ \ \forall\,\omega\in\Omega \qquad (4)$$

knowing that $S_u(\omega)=\sum_{k=1}^{m} s_u^k(\omega)s_u^k{}^*(\omega)$. The spectral concentration of the seismic force is obtained starting from motion with the interface of the soil by the matric transfer transfer function $G(\omega)$:

$$S_f(\omega)=G(\omega)S_u(\omega)G^*(\omega) \qquad (4)$$

and as follows:

$$s_f^k(\omega)=G(\omega)\varphi_k(\omega)\sqrt{\lambda_k(\omega)}\sqrt{S_0(\omega)},\ \ \forall\,\omega\in\Omega \qquad (4\text{-}7)$$

In the studies of ISS with Code_Aster, the transfer transfer function $G(\omega)$ is calculated by ProMISS3D, this is described more in detail in section 2.2.1.
The statement [éq 4] is the entry for the traditional seismic analysis, to see equation [éq 4].
The model is “reduced” if one can truncate statement [éq 4] with $K\leq m$ modes POD.
As one has just seen it, the computation of the seismic forces with spatial variability of the incidental field implies a spectral decomposition of the matrix of coherence $\boldsymbol{\gamma}$. For the continuation of computations, one retains only one reduced number of modes POD, namely $K\leq m$ modes. The parameter accuracy gives the share of “the energy” of the matrix which one preserves by retaining only one reduced number of vectors and eigenvalues of $\boldsymbol{\gamma}$. If one indicates by K the number of modes POD selected (the K greatest eigenvalues are retained), one a:

$$\mathrm{precision}=\frac{\sum_{i=1}^{K}\lambda_i^2}{\sum_{i=1}^{M}\lambda_i^2}$$

a value of 0.999 is the value by default for the accuracy.

2.2.1 Functions of coherence

the function of coherence depends on the distance from separation d between two points x and x' and of the frequency. In general, one expresses it by a term of modulus and a term of phase:

$$\gamma(d,\omega)=|\gamma(d,\omega)|\,exp(-i\,\theta(\omega,d))$$

The term $exp(-i\,\theta(\omega,d))$ represents the phase shift of at various times of arrivals of the waves. The term of amplitude corresponds to "the pure inconsistency". It can be evaluated starting from the DSP (autospectrums and interspectrums) at the points x and x' :

$$|\gamma(d,\omega)|^2=\frac{S(\omega,x,x')^2}{S(\omega,x)\,S(\omega,x')}$$

The currently available functions of coherence in `DYNA_ISS_VARI` are the function of coherence of Mita&Luco and the function of coherence of Abrahamson for rock. One does not introduce a term of phase.

The function of coherence of Became moth-eaten & Luco [biberon5, bib6] is written, for $f=\frac{\omega}{2\pi}$:

$$\gamma(d,f)=\exp\left[-\left(\frac{\alpha\,f\,d}{v_s}\right)^2\right]$$

In this statement, v_s is the apparent velocity of propagation on the surface of the wave SH (typically 200-1000 *m/s*) and the parameter α can vary from 0.1 with 0.5 according to the case but is generally taken equal to 0.5. If one chooses $\alpha=0.0$, then one carries out a computation without spatial variability.
One sees that correlation length for the function of coherence of Became moth-eaten and Luco [bib5, bib6], [éq 2,2-3], is characterized by the statement $(f\,\alpha/v_s)^{-1}$.

The function of coherence of Abrahamson [biberon9] for a rock site is written:

$$\gamma(d,f)=\left[1+\left(\frac{f\,tanh(a_3 d)}{f_c\,a_1}\right)^{n1}\right]^{-0.5}\left[1+\left(\frac{f\,tanh(a_3 d)}{f_c\,a_2}\right)^{n2}\right]^{-0.5}$$

with the values of parameters for horizontal motion following:

$$f_c=-1.886+2.221\,ln(4000/(d+1)+1.5)\ ,\ n_1=7.02,\ n_2=5.1-0.51\,ln(d+10)$$

and $a_1=1.647,\ a_2=1.01\ a_3=0.4$.

This function of coherence is an empirical model which was readjusted starting from the 78 recorded seismes with Pinyon Flat (the USA) by Abrahamson. It is also used for other soil types for which it is regarded as conservative.

2.2.2 Modelization of the seismic forces with *Code_Aster*

Case of the rigid foundation

the modes of interface are the 6 modes of rigid body. The modal seismic force calculated by ProMISS3D is written then for a unit excitation (white vibration):

$$f_s(\omega)=K_s(\omega)x_0$$

where $K_s(\omega)$ is the matrix of modal impedance and $x_0=(1.,0.,0.,0.,0.,0.)$ for a seismic excitation in direction x, $x_0=(0.,1.,0.,0.,0.,0.)$ for a seismic excitation in y and $x_0=(0.,0.,1.0.,0.,0.,0.)$ for a vertical seisme. The seismic force is non-zero only for the modal component in the direction of the seisme (x, y, z). In the same way, the function of coherence is built only for the degrees of freedom of translation in the direction of the seisme. The other degrees of freedom are not affected.
To take account of spatial variability, one determines the modal participation for each mode POD characterizing spatial variability:

$$f_s^k(\omega)=K_s(\omega)\,\Theta^T s_u^k$$

where Θ is the matrix containing the modes (mechanical) of interface.

Cases of the flexible foundation

the modes of interface are $d\times 6$ the unit modes relating to d the nodes of the interface. The modal seismic force calculated by ProMISS3D is written then:

$$f_s(\omega)=K_s(\omega)x_0$$

x_0 is the vector of modal participation comprising of 1 for degrees of freedom relating to the direction of the seisme and the zeros for the other directions. In the same way, the seismic force and the function of coherence are non-zero only for the degrees of freedom in the direction of the seisme. One builds the matrix of coherence and then the vectors s_u^k for the degrees of freedom in the direction of the seisme :

$$f_s^k(\omega)=K_s(\omega)\,s_u^k$$

Case of an “unspecified” foundation

In these cases, corresponding either with an inserted foundation, or with a case of interaction soil-fluid-structure, or with a case where the modes of interface are unspecified modes different of unit modes relating to the nodes of the interface, the vector of modal participation x_0 is not any more itself unit, nor independent of the frequency and there is not any more identity between the physical and modal coordinates of the interface.
The modal seismic force calculated by ProMISS3D is written then:

$$f_s(\omega)=K_s(\omega)x_0(\omega)$$

$x_0(\omega)$ the vector of modal participation is thus obtained by inversion of the seismic force $f_s(\omega)$ compared to the impedance of soil $K_s(\omega)$; the physical vector on the interface corresponding $X_0(\omega)$ is written then: $X_0(\omega)=\Theta x_0(\omega)$ where Θ is the matrix containing the modes (mechanical) of interface.

To take account of spatial variability, one determines the physical contribution on the interface for each mode POD characterizing spatial variability:

$$X_u^k(\omega)=X_0(\omega)s_u^k$$

the physical vector on the interface $X_u^k(\omega)$ corresponds to a vector of modal participation $S_u^k(\omega)$ such as: $X_u^k(\omega)=\Theta S_u^k(\omega)$ where Θ is the matrix containing the modes (mechanical) of interface. Then the corresponding seismic force will be expressed as follows :

$$f_s^k(\omega)=K_s(\omega)S_u^k(\omega)$$

2.2.3 Computation of transfer functions transfer and DSP of response

By linear filtering, one obtains:

$$s_q^k(\omega)=H(\omega)s_f^k(\omega) \quad (8\text{-}1)$$

What makes it possible to rebuild the matrix of spectral concentration of response like:

$$S_q(\omega)=\sum_{k=1}^{K\leq m} s_q^k\,(s_q^k)^* \quad (8\text{-}2)$$

The model is reduced if one can truncate statement [éq 8] with $K\leq m$ modes POD.
For an excitation by white vibration, one directly obtains the DSP of the transfer transfer function in output.

Note:

Without spatial variability the seismic field and thus the seismic force are the same ones for all the nodes of the interface (the foundation):

$$S_u(x,x',\omega)=S_0(\omega)$$

Caution: In the actual position, `DYNA_ISS_VARI` can only treat the case of a shallow foundation. As for the modes of interface, one can choose between a modelization by the 6 modes of rigid body (foundation rigid) and a modelization by all the modes EF (flexible foundation). The choice of a reduced base of flexible modes of interface, as described in [U2.06.07], is not possible.

2.2.4 Computation of the temporal response to a seisme

In a general way, It is possible to simulate trajectories of a Gaussian steady process which one knows the DSP by means of the theorem of the spectral representation.

One could thus obtain a realization the temporal structural response (stochastic process characterized by its spectral concentration $S_q(\omega)$) by the formula (cf [bib4]):

$$q(t)=\sum_k \sum_{n=0}^{N_T} H(\omega_n)G(\omega_n)e^{i\omega_n t}\varphi_k(\omega_n)\sqrt{\lambda_k(\omega_n)}\sqrt{S_0(\omega_n)}\xi_n^k\sqrt{\Delta\omega} \quad (8)$$

where are ξ_n^k to them complex random variables independent of reduced centered normal model and where $\Delta\omega$ the step of frequency indicates (constant), $\omega_n,\ n=1,N_T$ are frequencies resulting from the discretization. Under the assumption that one can approach the specific spectral concentration using a number of accelerograms, one a:

$$S_0^L(\omega)=\frac{1}{2\pi}\frac{1}{L}\sum_{l=1}^{L}u_0^l(\omega).u_0^l*(\omega) \quad (8)$$

where $u_0(\omega)$ is obtained from accelerogram $u(t)$ in free field:

$$u_0(\omega)=\frac{1}{\sqrt{T}}\int_0^T u_0(t)\,e^{-i\omega t}\,dt$$

Here, one does not use this method of generation of signals from their DSP because one works with evolutionary processes whose one knows a realization, namely the accelerogram in entry of mechanical computation. Being given this context, one places oneself rather in a deterministic frame of filtering of signals. Thus, one introduces a deterministic filter modelling the effect of the spatial inconsistency. This filter is given by the matrix $\Phi(\omega)\Lambda(\omega)^{1/2}$. One can thus calculate the frequency response and obtain the temporal response by opposite FFT. The frequency response is written:

$$q(\omega)=\sum_k H(\omega)G(\omega)\varphi_k(\omega)\sqrt{\lambda_k(\omega)}\,u_0(\omega) \qquad (8)$$

By considering the equation [éq 2.2.3-3], one can check that the spectral concentration of the response q is that given by the equation [éq 8] . The model is "reduced" if one can truncate the statement [éq 8] with $K\leq m$ modes POD.

2.3 Results calculated by DYNA_ISS_VARI

In short, operator DYNA_ISS_VARI allows to get the following results:

- Computation of the spectral concentration of the modal response (TYPE_RESU=' SPEC') for a unit seismic excitation (in displacement). In order to obtain the response with a seismic excitation, one multiplies the DSP obtained for unit excitation by the spectrum modelling the excitation (Kanai-Tajimi or other).
- Computation of the transfer transfer functions (TYPE_RESU=' SPEC') for a unit seismic excitation (in displacement). The transfer transfer functions are given by the root of the modulus of the auto--spectrums of response, $\sqrt{\left|\left[S_v(\omega)\right]_{ii}\right|}$ (root of the absolute value of the auto--spectrum [bib9] in physical coordinates v.). By comparing the transfer transfer functions with and without spatial variability, one can determine margins related to the inconsistency of the seismic signal (cf also [bib8]: factor of margin "inconsistency" in the seismic EPS).
- Computation of the floor spectrums via transient response (TYPE_RESU=' TRANS'). DYNA_ISS_VARI makes it possible to calculate the transient response in acceleration. The response sprectrums of oscillator (SRO) for a bottom can be obtained in postprocessing.

3 Other software: SASSI and CLASSI

the two software most used to do calculations of ISS with spatial variability for the nuclear power are software CLASSI and SASSI. The method established in *Code_Aster* is close to that of SASSI [bib8, bib9].

Just like DYNA_ISS_VARI in *Code_Aster*, SASSI makes it possible on the one hand to determine transfer functions transfer (excitation unit) and on the other hand to calculate temporal responses taking account of spatial variability.
The computation transfer functions transfer is done according to the same principle in two codes (*Code_Aster* and SASSI): one carries out a spectral decomposition (POD) matrix of coherence, one determines the response mode POD by mode POD in order to recompose of them the DSP of response as described in this note.
For the computation of temporal responses, there is a difference between methodology SASSI and the implementation in Code_Aster. more precisely, SASSI introduces a random phase for each mode POD [bib9]:

$$q(\omega)=\sum_k H(\omega)G(\omega)\varphi_k(\omega)\sqrt{\lambda_k(\omega)}e^{i\eta^k(\omega)}u_0(\omega) \qquad (9)$$

where are $\eta^k(\omega)$ to them uniform random variables on the interval $[-\pi, \pi]$. One determines then m responses and m SRO to retain the average of it. However, the introduction of the random phase is not useful if one regards the problem as a problem of filtering (deterministic) of a deterministic signal (the accelerogram) as described above. One can check moreover that the hope of the statement [éq 3-1] is equal to the statement [éq 8].

The method used by CLASSI is different, insofar as one remains in the frame of the classical stochastic dynamics: a linear problem of filtering is solved (as it is the case for `DYNA_ISS_VARI` when no seismic signal is given). The DSP of the seismic excitation is directly approximate using an algorithm making equivalence between the DSP and the SRO. The SRO of response is obtained same way from the DSP of response, namely by determining the SRO corresponding to the DSP of calculated response [bib9].

4 Bibliography

1) D. CLOUTEAU: M annual user of ProMISS3D - MISS2D, revision 6.3, by (LMSSMat Central School of Paris).

2) V. GUYONVARH - G. DEVESA: Méthodes de calcul of the seismic excitations to the works of the CP N4. HP-52/99/006/A, 1999.

3) V. GUYONVARH - G. DEVESA: Methods to consider the interaction soil-structure on nuclear small island EPR with the Code_Aster and MISS3D . HP-62/00/007/A, 2007.

4) I. ZENTNER: Proposal for a method to take into account the spatial variability of the seismic field incid ent. Note EDF R & D H-T61-2006-02 165,2007.

5) J.E. LUCO - H.L. WONG: Response of has rigid foundation to has spatially random ground motion. Earthquake Engrg. Struct. Dyn. 14,1986, pp.891-908.

6) J.E. LUCO - A. BECAME MOTH-EATEN: Response of has circular foundation to spatially random ground motion. J. Engrg. Mech. 113,1987, pp.1-15.

7) EPRI: Soil-structure interaction analysis incorporating spatial inconsistency of ground motion. Ratio TR-102631 2225, Palo Alto CA, 1997.

8) EPRI: Seismic Probabilistic Risk Assessment Implementation Guides, Final Carryforward 1002989,2003.

9) EPRI: Program one technology innovation: Validation of CLASSI and SASSI codes to treat seismic wave inconsistency in soil-structure interaction (SSI) analysis of nuclear power seedling structures. ratio EPRI 101511, Palo Alto, CA, 2007.

10) C. SOIZE - Methods mathematical in analysis of the signal . Masson, Paris 1993.

11) F. POIRION and C. SOIZE - Computational simulation of homogeneous and NON-homogeneous gaussian fields vector stochastic. The Aerospace Search 1,1989, pp.41-61.

12) A. PREUMONT - NON-separable The of generation artificial earthquake accelerograms for the design of nuclear power seedlings . Nuclear Engineering and Design 88,1985.

13) M.B. PRIESTLEY - Evolutionary will spectra and nonstationary processes . J. Royal Statistical Society 27(2), 1965, pp.204-237.

14) F. WANG - Effect of the spatial variation of seismic motion on the floor spectrums . Ratio techniqu E French atomic energy agency, SEMT/EMSI/RT /99-064, 2000.

5 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
10.2	I .ZENTNER EDF-R&D/AMA	initial Text
10.4	I .ZENTNER, F.VOLDOIRE EDF-R&D/AMA	Small corrections.