# Generation of seismic signals

**Summarized:**

In the seismic studies “best-estimate”, one is often brought to carry out transitory dynamic analyzes. The seismic signal is then modelled by a stochastic process. This process expresses acceleration on the ground according to time.

One calls trajectories the temporal signals (accelerograms) which are achievements of the stochastic process. These signals can come from data bases (accelerations on the ground measured during seisme) or be obtained by computational simulation (cf also [bib17]*)*. The simulation of artificial seismic signals, as realized by the operator GENE_ACCE_SEISME, is described in this document.

# Contents

# 1 Modelization of the seismic excitation by a stochastic process

the gaussian stochastic processes are entirely characterized by their power spectral density (DSP). If the process is steady, then one can very easily simulate trajectories using the theorem of spectral representation as described in the §2. This theorem also applies to the nonsteady gaussian processes under the condition which the process can be expressed in the form of a DSP evolutionary with low oscillation (i.e. the properties change slowly with time).

In a simplified seismic analysis, one models only the strong phase of the seismic signal. This last is then supposed to be steady during this period. It is in particular the case in the frame of the linear seismic analyses by modal combination (CQC, SRSS).

Nevertheless, of the studies and statistical analyzes showed that the seismic signal is nonsteady in amplitude and frequential contents. As for the frequential contents, its evolution can be estimated from natural signals. In most of the time one observes a decrease of the center frequency with time [bib9]. This fall of the center frequency of the seismic excitation can go hand in hand with the fall of the eigenfrequencies of structure subjected to the seisme. This coincidence can have worsening effects which it is advisable to take into account in the nonlinear modelizations of structures.

## 1.1 Modelization of the seisme by an evolutionary process with DSP evolutionary

One models seismic motion by achievements of a centered gaussian process described by its evolutionary power spectral density [bib8]:

$$S_X(\omega, t) = q(t) S_Y(\omega, t) q(t) \quad \in \mathbb{R} \tag{1}$$

where $S_Y(\omega, t)$ is the nonseparable part of the spectral concentration which models the evolution of the frequential contents of the seismic signal in the course of time. The function $q(t)$ east is a deterministic function of modulation which determines the amplitude of the signal (for example the function of modulation of Jennings & Housner [bib4] or the function Gamma). If one has $S_Y(\omega, t) \equiv S_Y(\omega)$, i.e. the DSP does not depend on time, $Y$ is a steady process and $X(t) = q(t) Y(t),\ t \in T$ a quasi steady process whose only amplitude varies with time. One speaks then about separable DSP: $S_X(\omega, t) = q^2(t) S_Y(\omega)$.

The model established in *Code_Aster* the DSP of Kanai-Tajimi [ bib13] leans on which expresses a filtered white vibration and models seismic acceleration in free field. The two parameters of the original model of Kanai-Tajimi are thus the own pulsation of the filter as well as the damping of this last. This model knew evolutions since, in particular with regard to the filtering of the contents low frequencies [bib2] and the introduction of the evolution of the frequential contents of the seismic signals (*Ahmadi & Fan* [bib1]*)*. The filtering of the low frequencies makes it possible to obtain signals in acceleration which can be integrated without drifts in displacement and of velocity (it is so necessary to resort to the “baseline correction” of the constants of integration must be eliminated). Lastly, a function of modulation is applied in order to obtain the variation of the amplitude. The modelization retained is summarized in what follows:

- The process $Y$ is characterized by the DSP of Kanai-Tajimi (KT): this DSP models the response in absolute acceleration of a mass subjected to an excitation by white vibration (seismic motion with the rock). In other words, the DSP of Kanai-Tajimi describes a white vibration filtered by an oscillator of pulsation clean $\omega_0$ and reduced damping $\xi_0$.

- The fundamental pulsation $\omega_0$ of the DSP of KT evolutionary is a function of time:

$$S_{KT}(\omega, t)=\frac{(\omega_0^4+4\xi_0^2\omega_0(t)^2\omega^2)}{((\omega_0(t)^2-\omega^2)^2+4\xi_0^2\omega_0(t)^2\omega^2)}S_0$$

One supposes a linear evolution of the own pulsation compared to time during the strong phase of the seisme:

$$\omega_0(t)=\begin{cases}\omega_1 & si\ \ t<t_1\\ \omega_1-\omega'(t-t_1) & si\ \ t_1<t<t_2\\ \omega_2 & si\ \ t>t_2\end{cases}$$

where $t_1$ and $t_2$ the beginning and the end of the strong phase indicate respectively (cf §1.2 for a definition of these quantities). For $t<t_1$ the own pulsation is supposed to be constant is equal to $\omega_1$ .

For $t>t_2$ , the own pulsation is supposed to be constant is equal to $\omega_2=\omega_1-\omega'(t_2-t_1)$ . The user must take care that the own pulsation given by this relation remains positive in the course of time.

- Filtering of the DSP of KT by a clean filter of pulsation $\omega_f=0.5\pi\, rad/s$ and a reduced damping $\xi_f=1.0$ according to Clough & Penzien (CP). This filtering makes it possible to very remove the contents in low frequencies which lead to non-zero drifts (displacements and non-zero velocities at the end of the seisme). The seismologists speak on this subject about “Frequency Corner”, namely the minimal frequency below which the spectrum must tend towards 0. This leads us to the DSP corrected $S_{CP}(\omega)$ :

$S_{CP}(\omega, t)=|h(\omega)|^2 S_{KT}(\omega, t)$ , where the filter is written $h(\omega)=\dfrac{\omega^2}{(\omega_f^2-\omega^2+2\xi_f\omega_f\omega)}$

- One proposes two functions of temporal modulation of the amplitude $q(t)$ : the function gamma and the function of Jennings & Housner [bib4]. The function gamma is written:

$$q(t)=\alpha_1 t^{(\alpha_2-1)}\exp(-\alpha_3 t)$$

The parameters $\alpha_2$ and $\alpha_3$ respectively describe the form and the period of the strong phase of the signal. The parameter $\alpha_1$ determines the energy of the signal and can be given from the data of the intensity of Arias. The function of Jennings & Housner [bib4] is written:

$$q(t)=\begin{cases}t/t_1 & si\ \ 0\le t<t_1\\ 1 & si\ \ t_1\le t\le t_2\\ \exp(-\alpha(t-t_2)^\beta) & si\ \ t_2<t\le T\end{cases}$$

The parameters $\alpha$ and $\beta$ determine the pace of the slope afterwards $t_2$ .

Figure 1 shows the DSP of Kanai-Tajimi filtered (DSP of Clough & Penzien) for a given eigenfrequency. For regular functions of DSP, the eigenfrequency of the filter is close to the fundamental frequency of the DSP. In figure 1, one also visualizes the bandwidth of the DSP, noted $\delta$ . The bandwidth of the DSP is related to the damping of the filter as clarified in the §1.4.

**Note:**

*A model very similar to that described above is proposed in the reference* (Rezaeian & Der Kiureghian, [bib9, biberon10]) *. The difference between the two approaches lies mainly in the writing of the problem in the time field by* Rezaeian & Der Kiureghian *and not in the field of the frequencies through the spectral concentration as proposed here. However, the writing in the time* *field has a certain number of disadvantages, like the need for evaluating the convolution integral and for working with the process of Wiener (white vibration) which is not a process of the second order. The formulation of the problem by a filtered white vibration described by its DSP evolutionary as proposed here is, on the other hand, very easy and numerically much more effective.*

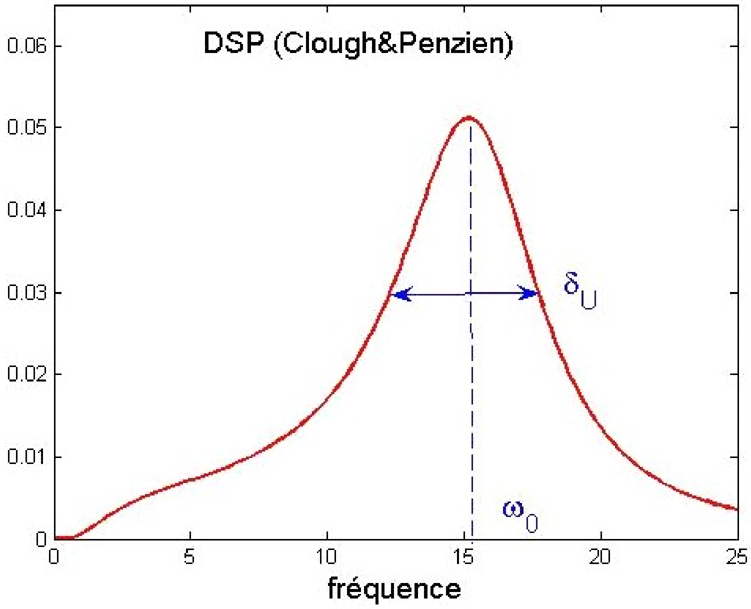
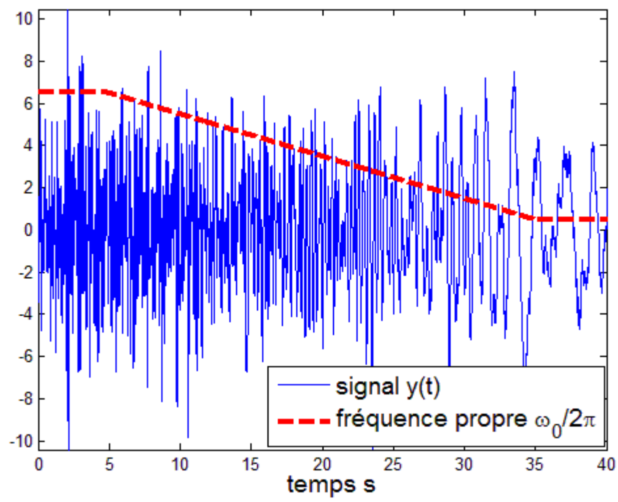



**Figure 1. DSP of Kanai-Tajimi after filtering of the low frequencies according to Clough & Penzien [bib2].**




**Appear 1b. Illustration of an evolution of the frequential contents of a signal without modulation (DSP of Clough & Penzien evolutionary with $\xi_0 = 0.2$ ).**

## 1.2 Parameter setting of the functions of modulation

the parameters which characterize the function of modulation are the period of strong phase $T_{SM}$ , the time of the beginning of the strong phase $t_{ini}$ and the intensity of Arias $I_a$ .

The average intensity of Arias $\bar{I}_a$ of a process $Y(t)\in\mathbb{R}$ modulated by a function $q(t)$, described by the equation (1), is expressed like:

$$\bar{I}_a = \boldsymbol{E}\left(\frac{\pi}{2g}\int_0^\infty X^2(t)\,dt\right) = \frac{\pi}{2g}\int_0^\infty q^2(t)\,\boldsymbol{E}(Y^2(t))\,dt$$

knowing that $X(t)=q(t)Y(t)$. The operator $\boldsymbol{E}$ in the statement above indicates the expectation and $g=9.81\mathrm{m}/s^2$. If one standardizes the process $Y(t)$ of kind so that $\sigma_Y(t)=1$, one can write:

$$\bar{I}_a = \frac{\pi}{2g}\int_0^\infty q^2(t)\,dt \qquad (2)$$

In practice integration is done until the time of end $T_f$ of the seismic signals. The period of the average strong phase $\bar{T}_{SM}$ is defined from the intensity of Arias which expresses energy contained in the seismic signal. Thus, $\bar{T}_{SM}$ is defined like the period of time between the time of time $t_{0.05}$ and $t_{0.95}$ where respectively 5% and 95% of the intensity of Arias are carried out. Time $t_{0.05}$ indicates the beginning of the average strong phase consequently $\bar{t}_{ini}$, such as:

$$\bar{t}_{ini}:\ \frac{\pi}{2g\bar{I}_a}\int_0^{t_{ini}} q^2(t)\,dt = 0.05$$

and $\bar{t}_{ini}+\bar{T}_{SM}$ end of the strong phase:

$$\bar{t}_{ini}+\bar{T}_{SM}:\ \frac{\pi}{2g\bar{I}_a}\int_0^{\bar{t}_{ini}+\bar{T}_{SM}} q^2(t)\,dt = 0.95$$

Being given the period of the strong phase, the average intensity of Arias $\bar{I}_a$ makes it possible to determine the standard deviation of the process $X$.

For the parameter setting of the functions of modulation, one supposes given the averages of these three parameters: period of average strong phase $\bar{T}_{SM}$, time of the beginning of the average strong phase $\bar{t}_{ini}$ and the average intensity of Arias $\bar{I}_a$.
In addition, standardized processes $X(t)$ are considered so that the standard deviation is unit at every moment of time.
The amplitude of the signals to be generated is determined by the data of the intensity of Arias (energy), the standard deviation $\sigma_X$ over the period of the strong phase or the median maximum $a_m$ over the period of the strong phase.

In practice, the median maximum $a_m$ is associated with the data of the PGA (Peak Ground Acceleration). Thus, if the PGA is given, then it is considered that this one corresponds to the median of maximum $a_m = Médiane(max_{t\in T}(|X(t)|))$ of the trajectories of the process (during the strong phase $T_{SM}$ ). This enables us to determine the standard deviation $\sigma_X$ corresponding to this maximum using the factor of peak $\eta_{T_{SM}}$ :

$$a_m \approx \eta_{T_{SM}}\sigma_X$$

The factor of peak is given by the formula

$$\eta^2_{T_{SM},p} = 2\ln(2\mathrm{N}_\eta[1-\exp(-\delta^{1.2}\sqrt{\pi\ln(2\mathrm{N}_\eta)})])$$

where $N_\eta = 1{,}4427\,T_{SM}\,\nu_0^+$. The variables $\nu_0^+$ (center frequency) and $\delta$ (bandwidth) are calculated as from the moments of the DSP according to the equations (7) and (8) of the §1.3.

**Note:**

*The parameters of the functions of modulation given from the phase the strong average and average intensity of Arias, like are described above. The strong, maximum phases (PGA) and intensities of Arias of the seismic signals generated with this modelization will consequently display a certain variability around the averages.*

### 1.2.1 Function of modulation Gamma

One identifies, by least squares, the parameters $\alpha_2$ and $\alpha_3$ such as time $\bar{t}_1$ corresponds to the beginning of the strong phase and $\bar{t}_{ini}+\bar{T}_{SM}$ the strong phase. The parameter $\alpha_1$ is a constant of standardization which is selected such as the equation (2) is checked if the intensity of Arias is given like parameter of entry.
If a standard deviation or the median maximum is given, one standardizes the function Gamma of kind so that energy during the strong phase corresponds to the energy of a constant unit modulation: $\int_{t_1}^{t_2} q(t)^2\,dt=\bar{T}_{SM}$.

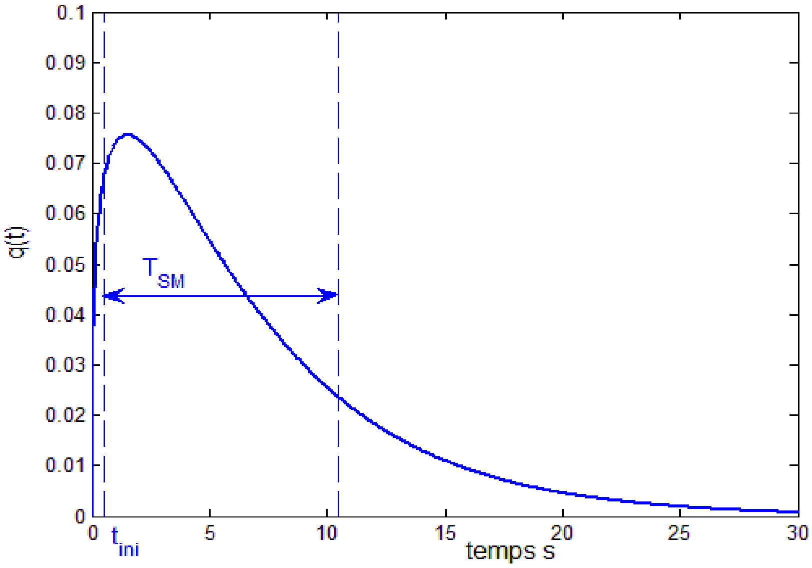



**Figure 2. Function of modulation Gamma for tini=0.5s and TSM=10s, Ia=0.5.**

### 1.2.2 Function of modulation of Jennings & Housner

time $t_1$ is taken equal to $\bar{t}_{ini}$ whereas the period of the strong phase gives the time $t_2$ which corresponds to the end you plate. One associates here the length of the plate with the period of the strong phase $\bar{T}_{SM}$. It should nevertheless be stressed that the length of the plate of the function of Jennings & Housner does not correspond in general (exactly) to the notion of period of above definite strong phase from the intensity of Arias.

The parameters $\alpha$ and $\beta$ of the model of Jennings & Housner are additional parameters which determine the pace of the function of modulation beyond the strong phase. They are with being informed by the user. The function is then multiplied by a constant in order to respect the average intensity of Arias $\bar{I}_a$, a standard deviation or the median maximum.

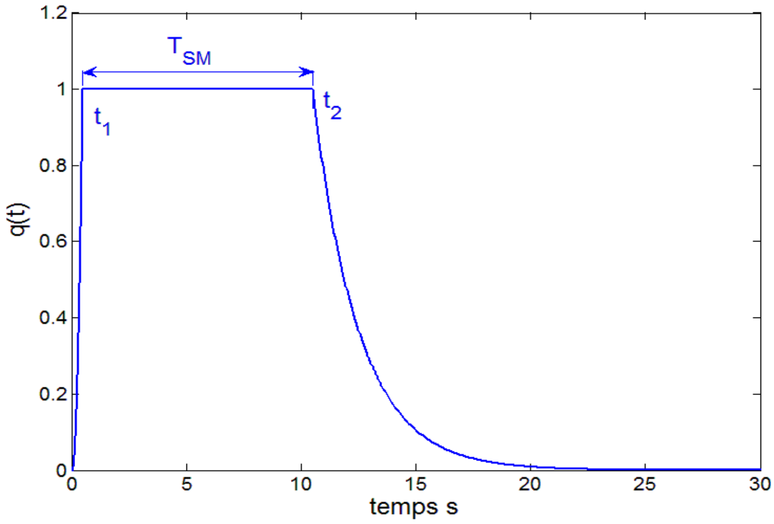



**Figure 3. Example of function of temporal modulation of Jennings & Housner for** $t_{ini}=0.5\mathrm{s}$ and $T_{SM}=10\mathrm{s}$ **.**

## 1.3 Signals compatible with a SRO

If the seisme is modelled by a gaussian steady process during phase strong *TSM,* it is possible to establish a relation (approximate) between the spectral concentration $S_Y(\omega)$ of the process and its response sprectrum of oscillator (SRO) $S_a(\omega,\xi)$ . This last indeed often is given by the seismologists or prescribed by codes and regulations.

The problem of the first transition makes it possible to bind the SRO (for a given $\omega_n$ pulsation and $\xi_0$ a reduced damping) to the standard deviation of the process via the factor of peak $\eta$ :

$$S_a(\omega_n,\xi_0)\approx\omega_n^2\eta_{T_{SM},p}\sigma_n \tag{3}$$

where $\sigma_n$ is the standard deviation of the process response

$$\sigma_n^2=\int\left|h_{\omega_n,\xi_0}(\omega)\right|^2 S_Y(\omega)\,d\omega$$

and:

$$h_{\omega_n,\xi_0}(\omega)=\frac{1}{\omega_n^2-\omega^2+2\xi_0\omega_n\omega}$$

is the transfer transfer function (filter) of own pulsation $\omega_n$ and reduced damping $\xi_0$ .

The factor of peak makes it possible to estimate *the p-fractiles* of the distribution of maximum of a gaussian process from the standard deviation. The factor of peak, due to Vanmarcke [bib16], is written:

$$\eta_{T_{SM},p}^2=2\ln(2\mathrm{N}_\eta[1-\exp(-\delta^{1.2}\sqrt{\pi\ln(2\mathrm{N}_\eta)})]) \,. \tag{4}$$

In this statement, $\delta$ is the bandwidth of the process (cf Figure 1) and the

$$N_\eta=\frac{T_{SM}}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_0}}(-\ln p)^{-1}$$

$\lambda_i$ are the spectral moments of the DSP of the process response:

$$\lambda_i = \int_{-\infty}^{+\infty} |\omega|^i \left| h_{\omega_n, \xi_0}(\omega) \right|^2 S_Y(\omega)\, d\omega \,, \tag{5}$$

with in particular $\sqrt{\lambda_0}$ the standard deviation of the process. In order to determine the DSP compatible with the SRO, it is necessary to reverse the statement (3). One can show that one has, at first approximation for an excitation by white vibration ( $S_Y(\omega)=const$ ) and for a filter with eigenfrequency $\omega_n$ :

$$\lambda_0 = \int_{-\infty}^{+\infty} \left| h_{\omega_n, \xi_0}(\omega) \right|^2 S_Y(\omega)\, d\omega \approx S_Y(\omega_n) \int_{-\infty}^{+\infty} \left| h_{\omega_n, \xi_0}(\omega) \right|^2 d\omega = \frac{\pi S_Y(\omega_n)}{2\xi\omega_n^3}$$

This formula was improved thereafter by Vanmarcke [bib15, bib16] to take account of the contribution of $S_Y(\omega)$ in the range of pulsations $[-\omega_n, \omega_n]$ where $h_{\omega_n}(\omega) \approx 1/\omega_n^2$ :

$$\lambda_0 \approx \frac{\pi S_Y(\omega_n)}{2\xi\omega_n^3} + \frac{2}{\omega_n^4} \int_0^{\omega_n} S_Y(\omega_n)\, d\omega - \frac{2}{\omega_n^3} S_Y(\omega_n)$$

This leads us to the formula of Vanmarcke to evaluate a DSP $G^S(\omega)$ from a SRO $S_a(\omega, \xi)$ (for a given $\xi_0$ damping):

$$G^S(\omega_n) = \frac{1}{\omega_n \left( \frac{\pi}{2\xi_0} - 2 \right)} \left[ \frac{S_a^2(\omega_n, \xi_0)}{\eta_{T_{SM}, p}^2} - 2 \int_0^{\omega_n} G^S(\omega)\, d\omega \right], \quad \omega_n > 0 \tag{6}$$

under condition that these two functions are sufficiently smooth. The original formula of Vanmarcke was then used in more or less close forms by many authors [bib7, biberon3]. It is established in its original version (equation (6)) in `INFO_FONCTION`. This DSP makes it possible to generate trajectories of the steady or quasi steady process (variation of the amplitude by multiplication by a function of modulation) by calling upon operator `GENE_FONC_ALEA` (cf §1.2).

Alternatively, one can estimate the parameters of the DSP of Kanai-Tajimi, namely the eigenfrequency $\omega_0$ and reduced damping $\xi_0$, as from the moments of the DSP compatible with the spectrum:

$$\lambda_i = \int_{-\infty}^{+\infty} |\omega|^i G^S(\omega)\, d\omega$$

The standard deviation of the process is given by the moment of order 0:: $\sigma^2 = \lambda_0 = \int_{-\infty}^{+\infty} G^S(\omega)\, d\omega$ . Then, it is necessary to identify the parameter $\omega_0$ and $\xi_0$ so that the moments of order 1 and 2 of the DSP of Kanai-Tajimi and DSP $G^S(\omega)$, compatible with the SRO, are close or coincide. This makes it possible to build a nonsteady model of seisme (nonsteady in amplitude and frequential contents) by introducing the evolution of the center frequency.

In practice, it is advisable to identify $\omega_0$ and $\xi_0$ such as the center frequency $\nu_0^+$ and the bandwidth $\delta$ of the DSP coincide. The center frequency, namely the frequency where the energy of the process is concentrated, can be calculated by the classic formulates of Rice:

$$\nu_0^+ = \frac{1}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_0}}\,, \tag{7}$$

This value corresponds to the rate of transitions by zero with positive slope. It can also be given from an accelerogram by counting of transitions positive by zero. The bandwidth $\delta$ of the DSP is expressed as follows [bib15] :

$$\delta = \sqrt{\left(1 - \frac{\lambda_1^2}{\lambda_0 \lambda_2}\right)}\,, \tag{8}$$

These two parameters are illustrated on figure 1. They can be calculated using `POST_DYNA_ALEA` and make it possible to estimate the parameters of the DSP of Kanai-Tajimi, namely the own pulsation $\omega_0$ and reduced damping $\xi_0$.

For the response in displacement of an oscillator with a white vibration, one knows analytical statements of the parameters $\omega_0$ and $\xi_0$ according to the moments and thus of the center frequency and bandwidth. These statements also apply to the DSP of Kanai-Tajimi, if damping is weak. The DSP of Kanai-Tajimi corresponds indeed to the response in acceleration (absolute) of an oscillator subjected to a white vibration and not the response in displacement.
Thus, if damping is weak, the eigenfrequency of the DSP of Kanai-Tajimi can be taken equal to the center frequency. But the approximation is not in general step very good. Studies showed that a better approximation can be obtained by taking the frequency corresponding to the center of gravity of the positive part of the DSP.
The bandwidth is expressed according to reduced damping $\xi$, always for the response of an oscillator with a white vibration, by the formula [bib15]:

$$\delta = \sqrt{1 - \frac{1}{1-\xi^2}\left(1 - \frac{1}{\pi}\arctan\left(\frac{2\xi\sqrt{1-\xi^2}}{1-2\xi^2}\right)\right)^2}$$

For low values of damping $\xi < 0.1$, one can estimate reduced damping by the asymptotic relation for $\xi \to 0$, following [bib15]:

$$\delta \approx \sqrt{\frac{4\xi}{\pi}} \tag{9}$$

a good approximation of reduced damping can be obtained while taking $\xi \approx \frac{\pi\delta}{4}^{2/1.2}$ (cf also (4)).

The own pulsation $\omega_0$ and reduced damping $\xi_0$ identified are then used to build seisme with GENE_ACCE_SEISME `the model`.

# 2 Simulation of trajectories of the process

the classical method for simulation of a steady process stocha stic gaussian, centered leans on the integral representation of the processes, cf for example [bib11] . The spectral representation as well as the algorithm of computational simulation are pointed out in what follows.
One treats the general case of a vectorial gaussian process $\boldsymbol{Y}(t) \in \mathbb{R}^M, t \in [0, T]$ characterized by his DSP matric $\boldsymbol{S}_{\boldsymbol{Y}}(\omega) \in Mat_{\mathbb{C}}(M, M)$ . This method can be wide with the case of the evolutionary processes with DSP evolutionary $\boldsymbol{S}'_Y(\omega, t) \in Mat_{\mathbb{C}}(M, M)$ if the process is with slow variation.

## 2.1 Discretization

computational simulation requires the discretization of the temporal field. One uses a temporal discretization with constant $\Delta t$ step. The cut-off frequency results directly from this value: $\Omega_C = \pi/(\Delta t)$ . For a discretization by N time step, one notes $t_j = j\,\Delta t,\ j=0,\dots N-1$ the points of temporal sampling. In addition, one defines the step of frequency by $\Delta\omega = 2\,\Omega_C/N$ and the period of the temporal signal is $T=\Delta t(N-1)$ .

The spectral concentrations are defined on the interval $F=[-\Omega_C, +\Omega_C]$ and the points of frequential discretization are selected like the mediums of the paving stones:: $\omega_j = -\Omega_C + (0.5+j)\Delta\omega,\quad j=0,\dots N-1$ .

## 2.2 Spectral representation of the steady processes

the theorem of the spectral representation (cf [bib11]) known as that there exists a stochastic process $d\,\boldsymbol{Z}(\omega) \in \mathbb{C}^M$ with orthogonal increments such as:

$$\boldsymbol{Y}(t) = \int_{\mathbb{R}^M} e^{i\omega t}\, d\,\boldsymbol{Z}(\omega) \qquad (10)$$

This process, called spectral process associated with $\boldsymbol{Y}$, checks:

$$\boldsymbol{E}(d\,\boldsymbol{Z}(\omega)\,d\,\boldsymbol{Z}(\omega')^*) = \begin{cases} 0 & si \quad \omega \neq \omega' \\ \boldsymbol{S}_{\boldsymbol{Y}}(\omega)\,d\,\omega & si \quad \omega = \omega' \end{cases}, \qquad (11)$$

where $\boldsymbol{E}$ is the operator of the expectation and $d\,\boldsymbol{Z}$ is a process with orthogonal increments.

The matrix of spectral concentration $\boldsymbol{S}_{\boldsymbol{Y}}(\omega)$ being, by construction, a positive hermitian matrix for all $\omega \in F$ , there exists a matrix $\boldsymbol{L}(\omega) \in \mathbb{C}^M$ such as:

$$\boldsymbol{S}_{\boldsymbol{Y}}(\omega) = \boldsymbol{L}(\omega)\,\boldsymbol{L}(\omega)^*$$

The matrix $\boldsymbol{L}(\omega)$ can be obtained by decomposition of Cholesky if the row of $\boldsymbol{S}_{\boldsymbol{Y}}(\omega)$ is maximum. $\boldsymbol{L}(\omega)$ is then a lower triangular matrix. One can then simulate trajectories of the process by the formula

$$\boldsymbol{Y}(t) = \sqrt{\Delta\omega}\,\Re e \sum_{j=0}^{N-1} \boldsymbol{L}(\omega_j)\,\chi_j\, e^{i\omega_j t} \qquad (12)$$

where are $\chi_j$ to them complex gaussian random vectors of which the components are independent.

The use of the IFFT makes it possible to reduce considerably the numerical cost of simulation. Indeed, one can write the statement (12) in the form

$$\boldsymbol{Y}(t_k) = \sqrt{\Delta\omega}\,\Re e\left(\boldsymbol{V}_{\boldsymbol{k}}\, e^{-i\pi k(1-1/N)}\right) \qquad (13)$$

where $V_k$ is computable by IFFT:

$$\boldsymbol{V}_{\boldsymbol{k}} = \sum_{j=0}^{N-1} \boldsymbol{L}(\omega_j)\,\chi_j\, e^{2i\pi k\, j N^{-1}}$$

This algorithm can be used for the steady and nonsteady gaussian processes with separable DSP. In these cases, one has $S_Y(\omega, t) \equiv S_Y(\omega)$ in the equation (1) and one can generate trajectories of the process

$Y \in \mathbb{R}$ on the interval $[0,T]$. One obtains the seismic signals modulated by applying the function of modulation of kind so that $X(t)=q(t)\,Y(t)$. This is carried out by the operator GENE_ACCE_SEISME.

## 2.3 Spectral representation of the evolutionary processes with DSP evolutionary

the spectral representation always applies to evolutionary processes described by their nonseparable DSP evolutionary:

$$\boldsymbol{Y}(t)=\int_{\mathbb{R}^M} \mathrm{e}^{i\omega t}\, d\,\boldsymbol{Z}(\omega,t) \qquad (14)$$

under condition which the DSP evolves slowly with time [bib8, biberon6]. There then exists a stochastic process $d\,\boldsymbol{Z}(\omega,t)\in\mathbb{C}^M$ with orthogonal increments such as

$$\boldsymbol{E}(d\,\boldsymbol{Z}(\omega,t)\,d\,\boldsymbol{Z}(\omega',t)^*)=\begin{cases} 0 & si \;\; \omega\neq\omega' \\ \boldsymbol{S}_{\boldsymbol{Y}}(\omega,t)\,d\omega & si \;\; \omega=\omega' \end{cases}$$

where $\boldsymbol{E}$ is the operator of the expectation and $d\,\boldsymbol{Z}$ is a process with orthogonal increments. One can then simulate trajectories of the process by the formula

$$\boldsymbol{Y}(t)=\sqrt{\Delta\omega}\,\Re e\sum_{j=0}^{N-1}\boldsymbol{L}(\omega_j,t)\,\chi_j\,\mathrm{e}^{i\omega_j t} \qquad (15)$$

where are $\chi_j$ to them complex gaussian random vectors of which the components are independent. The matrix $\boldsymbol{L}(\omega,t)$ can always be obtained by decomposition of Cholesky if the row of $\boldsymbol{S}_{\boldsymbol{Y}}(\omega,t)$ is maximum. On the other hand, it is not possible to call on the fast transform of Fourier as in the steady case (cf equation (13)).

One can then obtain the seismic signals modulated by applying the function of modulation of kind so that $X(t)=q(t)\,Y(t)$ . This is carried out in *Code_Aster* by the operator GENE_ACCE_SEISME.

# 3 Uncertainties and natural variability of the signals

the parameters of the model are:

- Average period of the strong phase $\overline{T}_{SM}$ and the time of beginning of the average strong phase $\overline{t}_{ini}$ (function of modulation),
- the own pulsation $\omega_1$ and the slope $\omega'$ of the DSP of Kanai-Tajimi (or $\omega_0$ if a constant value is considered),
- The damping reduced $\xi_0$ of the DSP of Kanai-Tajimi,
- the average intensity of Arias $\overline{I}_a$ (average energy contents in the seismic signal), the PGA ( maximum median $a_m$ ) or the standard deviation $\sigma_X$ .

In order to better represent the natural variability of the seismic signals and to take account of uncertainty on the parameters of the model, one can model the latter by random variables. Each simulated seismic signal corresponds then to a particular pulling of the parameters of the model. The method of pulling of Latin Hypercube constitutes an effective method which makes it possible to sweep well the field of definition of the parameters for a reduced number of pullings.

The mean values as well as the distributions of these parameters can be estimated from natural seismic signals corresponding to the required scenario. They are, for certain, also available in the literature. One finds of it a processing very exhaustive in the reference [bib9]. Averages, minimal and maximum values and distributions of the parameters, identified from accelerograms recorded on average soil with tough and $D>10\mathrm{km}$ (resulting from the base of seismes NGA [bib14]), are presented in the appendix of this document. These results are drawn from the ratio from Rezaeian & Der Kiureghian [bib9].

# 4 Ahmadi

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*Documentation of Code_Aster:*

[R4.05.02]: Stochastic approach for the seismic analysis.

[R7.10.01]: Examination of random responses.

[U4.84.04]: Operator POST_DYNA_ALEA

[U4.84.01]:: Operator COMB_SISM_MODAL

[U4.32.05]: Operator INFO_FONCTION

[U4.36.05]: Operator GENE_FONC_ALEA

[U4.36.04]: Operator GENE_ACCE_SEISME

# 5 Description of the versions of the document

| Version Aster | Author (S) Organization (S) | Description of the modifications |
|---|---|---|
| 11.2 | I.ZENTNER EDF/ R & D /AMA | initial Version of the document. |

# 6 Appendix

One reproduces below arrays of the PEER carryforward of Rezaeian & Der Kiureghian ([bib9], p.93-94) where mean values as well as the distributions of the parameters of the model of seisme were estimated from the base of seismes NGA for a selection of seismes recorded on average soil at tough and $D>10\,km$ .

**Caption:**

*$I_a$ : intensity Arias; $D_{5-95}$ : period of the strong phase (here $T_{SM}$ ); $\zeta_f$ : reduced damping of the filter (here $\xi_0$ ); $\omega_{mid}$ : eigenfrequency of the filter at time $t_{0.45}$ (one works here with the pulsation $\omega_0$ at time $t_{0.05}$ ); $\omega'$ : ratio of variation of the eigenfrequency of the filter (the slope of the function describing the evolution of the own pulsation of the filter).*

| Parameter | Minimum | Maximum | Sample Mean | Sample Standard Deviation | Coefficient of Variation |
|---|---|---|---|---|---|
| $I_a$ (s.g.) | 0.000275 | 2.07 | 0.0468 | 0.164 | 3.49 |
| $D_{5-95}$ (s) | 5.37 | 41.29 | 17.25 | 9.31 | 0.54 |
| $t_{mid}$ (s) | 0.93 | 35.15 | 12.38 | 7.44 | 0.60 |
| $\omega_{mid}/2\pi$ (Hz) | 1.31 | 21.6 | 5.87 | 3.11 | 0.53 |
| $\omega'/2\pi$ (Hz/s) | −1.502 | 0.406 | −0.089 | 0.185 | 2.07 |
| $\zeta_f$ (Ratio) | 0.027 | 0.767 | 0.213 | 0.143 | 0.67 |

| Parameter | Fitted Distribution[8] | Distribution Bounds |
|---|---|---|
| $\bar{I}_a$ (s.g.) | Lognormal | $(0,\infty)$ |
| $D_{5-95}$ (s) | Beta | [5,45] |
| $t_{mid}$ (s) | Beta | [0.5,40] |
| $\omega_{mid}/2\pi$ (Hz) | Gamma | $(0,\infty)$ |
| $\omega'/2\pi$ (Hz) | Two-sided Truncated Exponential | [−2,0.5] |
| $\zeta_f$ (Ratio) | Beta | [0.02,1] |

[8] Means and standard deviations of these distributions are according to columns 4 and 5 of Table 4.3.