Computation of mass matrix ajoutéesur modal base

Summarized:

This document presents an aspect of the fluid coupling/structure: when a vibrating structure is immersed in a fluid that one supposes at rest, incompressible and nonviscous, it feels compressive forces whose resultant is proportional to the acceleration of structure in the fluid: the proportionality factor is homogeneous with a mass: it is called **added mass**. One specifies here the layer to consider a mass matrix added for one (or of) structure (S) at several degrees of freedom on modal base of (of) the structure (S) in the vacuum.

Contents

1 Notations

p	:	fluctuating pressure in the fluid
γ_l	:	contour of structure indexed by l
$\ddot{x}_{s_l}$	:	the field of displacements in structure l
ρf , ρs	:	density of the fluid, structure
$\boldsymbol{X}_{il}$	:	eigen mode of order I of structure l in air
a_{il} , $\dot{a}_{il}$ $\ddot{a}_{il}$	:	coordinates, velocities, generalized accelerations relating to mode I of structure l in air
$\bar{\bar{\sigma}}$	:	the tensor of the stresses in structure
$\boldsymbol{\Phi}$	:	the fluid flux vector
$\mathbf{H}$	:	the stiffness matrix of the fluid
$\mathbf{v}$	:	the field fluid velocities
$\mathbf{n}$	:	the interior norm of the fluid.

2 Introduction

Of many industrial components are in contact with fluid environments, which more is often out of flow. These surrounding fluid environments disturb the vibratory characteristics of structures, in particular their modal characteristics. This action of the fluid on the structure results in effects of fluid coupling/structure.

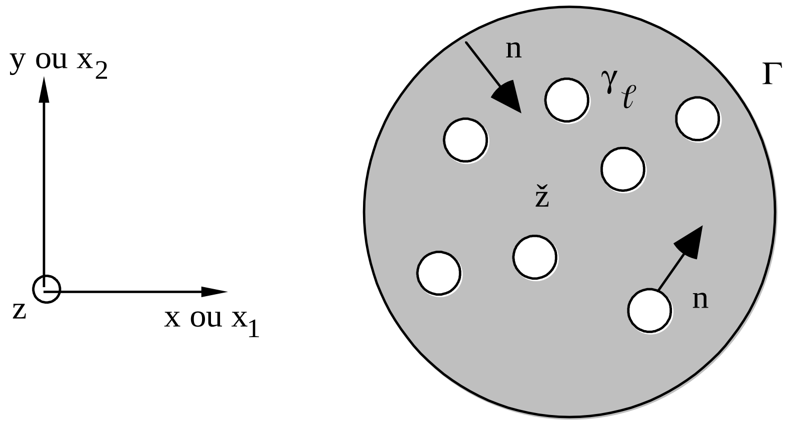
One supposes the incompressible, perfect fluid environment here surrounding and at rest. One will show that then, a structure which vibrates with a small amplitude in this fluid modifies the field of pressure in the fluid at rest, and thus feels a compressive force, proportional to his acceleration. The proportionality factor is a mass. It describes the inertial effect of the fluid on the structure: this is why one on the structure names this **mass** added mass of the fluid.

When several structures are in contact of the same fluid, when one of structures starts to vibrate, not only it feels the inertia of the fluid, but it modifies the field of pressure around the interfaces with the fluid of all other structures. The forces that each one feels are proportional to the acceleration of vibrating structure: there still the proportionality factors are masses called **added masses of coupling**.

3 Recalls of the equations of the problem

3.1 Equations in the fluid

It is supposed that K vibrating structures are immersed in a true fluid (nonviscous), incompressible and at rest. One neglects the effect of gravity. One can thus write the equations of Eulerian associated with the fluid at rest:



"

- conservation of the mass:

$$\frac{\partial \rho_f}{\partial t} + div(\rho_f \mathbf{v}) = 0 \quad \text{éq 3.1-1}$$

- conservation of the linear momentum:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\boldsymbol{grad})\mathbf{v} + \frac{1}{\rho_f}\boldsymbol{grad}\ p = 0 \quad \text{éq 3.1-2}$$

Because of incompressibility of the fluid, the equation [éq 3.1-1] becomes:

$$\mathrm{div}\ \mathbf{v} = 0 \quad \text{éq 3.1-3}$$

In the volume Ω of the fluid, one neglects the convection induced by the motion of low amplitude of structure. The equation [éq 3.1-2] thus becomes:

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\rho_f}\boldsymbol{grad}\ p = 0 \quad \text{éq 3.1-4}$$

While deriving [éq 3.1-3] compared to time and by deferring the statement of $\frac{\partial \mathbf{v}}{\partial t}$ according to the pressure in this equation, one obtains:

$$\mathrm{div}\ \boldsymbol{grad}\ p = 0$$

that is to say:

$$\Delta p = 0 \text{ in } \Omega$$

which is the equation of Laplace in a fluid at rest.

With the fluid interface/structure, one can write that the normal acceleration of the wall of structure is equal to the normal acceleration of the fluid (continuity of normal accelerations - condition of impermeability of structure). One uses here following convention for the norm: it is the norm **external** with structure, **directed structure towards the fluid.**

$$\frac{\partial \mathbf{v}}{\partial t}.\boldsymbol{n}=\ddot{\mathbf{x}}_{S_l}.\boldsymbol{n}$$

With the equation [éq 3.1-4], one obtains:

$$\mathbf{grad}\, p.\boldsymbol{n}=-\rho_f \frac{\partial \mathbf{v}}{\partial t}.\boldsymbol{n}=-\rho_f \ddot{\mathbf{x}}_{S_l}.\boldsymbol{n}$$

That is to say:

$(\frac{\partial p}{\partial n})_{\gamma_l}=-\rho_f \ddot{\mathbf{x}}_{S_l}.\boldsymbol{n}$ on γ_l, fluid interface/structure of indexed structure par. l

In short, the fluid problem consists in solving an equation of Laplace with boundary conditions of the type von Neumann:

$$\begin{cases} \Delta p=0 \quad \text{dans} \quad \Omega \\ (\frac{\partial p}{\partial n})_{\Gamma_1}=-\rho_f \ddot{\mathbf{x}}_s.\mathbf{n} \quad \text{sur} \quad \Gamma_1, \quad \Gamma_1=\bigcup_{l=1,K} \gamma_l \\ (\frac{\partial p}{\partial n})_{\Gamma_2}=0 \quad \text{sur} \quad \Gamma_2, \quad \Gamma_2=\partial\Omega-\Gamma_1 \end{cases} \qquad \text{éq 3.1-5}$$

3.2 Equations in structures

Let us consider K elastic structures divings in a fluid environment. The equation of their motion in the presence of fluid is written:

$$\begin{cases} \forall l \quad \text{indice de structure}, l\in\{0,\dots,K\}, \quad \mathbf{M_l}\ddot{\mathbf{X}}_l+\mathbf{K_l}\mathbf{X_l}=0 \quad \text{dans} \quad \Omega_{S_l}, \text{volume de la structure} \quad l \\ \forall l, \overline{\overline{\boldsymbol{\sigma}}}\boldsymbol{n}=-p\boldsymbol{n} \quad \text{sur}\, \gamma_l, \text{contour de la structure} \quad l \end{cases}$$

$\mathbf{M_l}$ is the mass matrix of structure, $\mathbf{K_l}$ its stiffness matrix. The boundary condition on the contour of structures translates the continuity of the normal stress to the fluid interface/structure (the tensor of the fluid stresses being tiny room to its nondeviatoric part, fluid being perfect). By integrating on the contour of each structure this normal stress, one obtains a resultant force $\mathbf{F_l}$ of the compressive forces of the fluid to the fluid interface/structure. This force is the integral of the field of pressure on the contour γ_l of each structure:

$$\forall l \quad \text{indice de structure}, l\in\{0,\dots,K\}, \quad \mathbf{F_l}=-\int_{\gamma_l} p\,\boldsymbol{n}\,d\Gamma$$

The field of pressure checks the problem [éq 3.1-5].

3.3 Equations of the coupled problem - description of the mass matrix added

Ultimately, the fluid coupled problem/structure is written:

$$\begin{cases} \Delta p=0 \text{ dans } \Omega \\ \forall l\in\{0,\dots,K\},\left(\dfrac{\partial p}{\partial n}\right)_{\gamma_l}=-\rho_f\,\ddot{\mathbf{x}}_{S_l}.\boldsymbol{n} \text{ sur } \gamma_l \\ \forall l\in\{0,\dots,K\},\ \mathbf{M_l}\ddot{\mathbf{X}}_l+\mathbf{K_l}\mathbf{X_l}=0 \text{ dans } \Omega_{S_l} \\ \forall l\in\{0,\dots,K\},\ F_l=-\int_{g_l} p\,\boldsymbol{n}\,d\Gamma \text{ sur } \gamma_l \end{cases}$$

éq 3.3-1

One will show from now on that the force that feel the immersed structures is proportional to their acceleration. A good layer to show that is to place itself in the modal base of structures in the vacuum. One can thus break up acceleration on this basis (which is in fact the meeting of modal bases of each structure). As follows:

$$\mathbf{x}_{\mathbf{S_l}}(r,t)=\sum_{i=1}^{\infty}a_{il}(t)\mathbf{X}_{il}(\mathbf{r})$$

By deferring this statement in the second equation of the system [éq 3.3-1], one is brought to search the field of pressure in the form:

$$p=\sum_{l=1,\dots,K}\ \sum_{i=1,\dots,\infty}\ddot{a}_{il}(t)\,p_{il}(\mathbf{r})$$

By deferring in the problem [éq 3.3-1] these statements, one has to as many solve in the fluid problems of Laplace than one chose modes for each structure. This results in:

$$\forall l\in\{1,\dots,K\},\ \ \forall i\,\{1,\dots,\infty\},\quad \begin{cases} \Delta p_{il}=0 \text{ dans } \Omega \\ \left(\dfrac{\partial p_{il}}{\partial n}\right)_{\gamma_l}=-\rho_f\,\mathbf{X_{il}}.\boldsymbol{n} \text{ sur } \gamma_l \\ [\mathbf{m_{il}}](\ddot{\mathbf{a}}_l)+[\mathbf{k_{il}}](\mathbf{a_l})=(\mathbf{f_{il}}) \text{ dans } \Omega_l \end{cases}$$

The “matrixes” of mass and stiffness written in these bases are diagonal.

Each one of the components of the resulting force of pressure project on modal base is written:

$$\forall i\,\{1,\dots,\infty\},\forall l\,\{1,\dots,K\},\ (f_{il})=-\sum_{k=1}^{K}\sum_{j=1}^{\infty}\ddot{a}_{jk}\int_{\gamma_l}p_{jk}\boldsymbol{X_{il}}.\boldsymbol{n}\,N_j\,d\Gamma$$

One can then write the vector of the force generalized of pressure on a structure immersed in matric form:

$$(\mathbf{f}_{il})=-[\boldsymbol{m_{il\,jk}}]\ddot{\boldsymbol{a}}_{jk} \text{ avec } m_{il\,jk}=\int_{\gamma_l}p_{jk}\boldsymbol{X_{il}}.\boldsymbol{n}\,d\Gamma$$

Here, l is built-in: the matrix $\left[\boldsymbol{m}_{il\ jk}\right]$ is called on the structure **added** mass matrix of the fluid of contour γ_l . When one considers the modal base of all K structures, one generalizes the notation of **the mass matrix added** $\left[\boldsymbol{m}_{il\ jk}\right]$ on modal base in the vacuum, l varying from 1 with K . This matrix is in general not diagonal.

3.4 Some definitions

3.4.1 Definition 1

When $l=k$ (even structure) and $i=j$ (even order of mode), the coefficient m_{ilil} is **l'auto - added mass** of the mode i of structure l . It is additional inertia due to the fluid moved by the mode of order i of structure, taking into account the geometrical containments induced in the fluid by the presence of other presumedly fixed structures.

3.4.2 Definition 2

When $l=k$ (even structure) and $i\neq j$ (different orders of mode), the coefficient m_{iljl} is **the added mass of coupling** between the modes of order i and j structure l . In air, these extra-diagonal terms of mass are null, because the modes are orthogonal between them. Taking into account the general statement of the coefficient $m_{il\ jk}$, the modes i and j can be coupled out of mass, because the field of pressure p_{jl} created by the mode j of structure l is not necessarily orthogonal with the mode of order i of this same structure. It is enough that this structure is immersed in an environment not comprising geometrical symmetry so that this coefficient is non-zero. In a symmetric environment, on the other hand, the orthogonality of the field of pressure with the mode is observed.

3.4.3 Definition 3

When $l\neq k$ (different structures) and $i\neq j$ (different orders of mode), the coefficient $m_{il\ jk}$ is **the added mass of coupling** between the modes of order i and j respectively of structures l and k . This coefficient translates the inertial force which makes undergo the structure k vibrating on its mode of order J to structure l vibrating on its mode i .

3.5 Properties of the added mass matrix

3.5.1 Theorem 1: the added mass matrix is symmetric

to simplify the demonstration, we will consider a single structure immersed in a true, incompressible and nonviscous fluid. We break up the motion of structure on its modal base (truncated with n modes), but result can be just as easily shown in "physical" base (*i.e* the base of the nodal interpolation functions). Lastly, result spreads with the case of structures K immersed in the same fluid. One

must show that: (respectively $m_{ij}=\int_{\Gamma} p_i \boldsymbol{X}_j . \boldsymbol{n}\, d\Gamma = m_{ji}=\int_{\Gamma} p_j \boldsymbol{X}_i . \boldsymbol{n}\, d\Gamma$

- p_i) p_j represents the field of pressure created in the fluid and with the interface with structure by the mode of order (respectively i of order) j of structure, (respectively
- $\boldsymbol{X_j}$) $\boldsymbol{X_i}$ the modal deformed shape of the mode of order represents (respectively j of order). i However

: and

$$\begin{cases} \Delta p_i = 0 \quad \text{dans} \quad \Omega \quad \text{volume fluide} \\ \dfrac{\partial p_i}{\partial n} = -\rho_f X_i . \boldsymbol{n} \quad \text{sur} \quad \Gamma \end{cases} \quad \text{formulates} \quad \begin{cases} \Delta p_j = 0 \quad \text{dans} \quad \Omega \, \text{volume fluide} \\ \dfrac{\partial p_j}{\partial n} = -\rho_f \mathbf{X_j} . \boldsymbol{n} \quad \text{sur} \quad \Gamma \end{cases}$$

, by means of the formula of Green with a norm directed of structure towards the fluid and harmonicity of and p_i of: p_j Theorem

$$\begin{aligned} m_{ij} &= \int_\Gamma p_i \boldsymbol{X}_j . \boldsymbol{n} \, d\Gamma = -\frac{1}{\rho_f} \int_\Gamma p_i \frac{\partial p_j}{\partial n} d\Gamma \\ &= -\frac{1}{\rho_f} \Big(\underbrace{\int_\Omega p_i \Delta p_j \, d\Omega}_{0} - \int_\Omega \boldsymbol{grad}\, p_i . \boldsymbol{grad}\, p_j \, d\Omega \Big) \\ &= -\frac{1}{\rho_f} \Big(\underbrace{\int_\Omega p_j \Delta p_i \, d\Omega}_{0} - \int_\Omega \boldsymbol{grad}\, p_j . \boldsymbol{grad}\, p_i \, d\Omega \Big) \\ &= -\frac{1}{\rho_f} \int_\Gamma p_j \frac{\partial p_i}{\partial n} d\Gamma = \int_\Gamma p_j \boldsymbol{X_i} . \boldsymbol{n} \, d\Gamma \\ &= m_{ji} \end{aligned}$$

3.5.2 2: the added mass matrix is definite positive One

returns to the reference [bib1] for the complete demonstration. Theorem

3.5.3 3 Let us suppose

that one has structures K having properties of linear elasticity identical and who are immersed in the same fluid. Moreover, these structures admit two degrees of freedom of displacement in the plane (cf Oxy diagram). Each one of these structures admits the same frequency spectrum $f_1, \ldots, f_n, \ldots$ clean in the vacuum. For

any eigenfrequency, f_n there exist eigenfrequencies $2\mathrm{K}$ of $\{\omega_1, \ldots, \omega_{2\mathrm{K}}\}$ the fluid coupled system/structure checking One $\forall i \in [1, \ldots, 2K], \ \omega_i \leq f_n$

returns to the reference [bib1] for the complete demonstration. Other

3.5.4 properties

- the coefficients of added auto--mass are always positive One

always supposes that one has only one structure immersed in a true fluid, incompressible and at rest. The demonstration spreads without difficulty with immersed K structures. One

must show that: However

$$\forall i \text{ indice de mode} \in [1, \ldots, n], m_{ii} = \int_\Gamma p_i \boldsymbol{X_i} . \boldsymbol{n} \, d\Gamma \geq 0$$

: let us suppose

$$\begin{aligned} m_{ii} &= \int_\Gamma p_i \boldsymbol{X_i} . \boldsymbol{n} \, d\Gamma = -\frac{1}{\rho_f} \int_\Gamma p_i \frac{\partial p_i}{\partial n} d\Gamma \\ &= -\frac{1}{\rho_f} (\underbrace{\int_\Omega p_i \Delta p_i d\Omega}_{0} - \int_\Omega \boldsymbol{grad}\, p_i . \boldsymbol{grad}\, p_i d\Omega) \\ &= \frac{1}{\rho_f} \int_\Omega (\boldsymbol{grad}\, p_i)^2 d\Omega \\ &\geq 0 \end{aligned}$$

- that one has structures K immersed in the same fluid. It is supposed that they have only one degree of freedom of translation according to. Ox Then the sum of all the coefficients of added mass of this matrix gives the auto--mass added on all structures K moving same sinusoidal rectilinear motion very. One

returns to the reference [bib2] for the complete demonstration. Put

4 in numeric work Resolution

4.1 of the equation of Laplace by finite elements of volume Let us take again

the fluid problem of Laplace with boundary conditions of the type von Neumann: Let us write

$$\begin{cases} \Delta p = 0 \quad \text{dans}\,\Omega \\ (\frac{\partial p}{\partial n})_{\Gamma_1} = -\rho_f \ddot{\mathbf{x}}_s . \boldsymbol{n} \quad \text{sur}\,\Gamma_1, \quad \Gamma_1 = \bigcup_{l=1,K} \gamma_l \\ (\frac{\partial p}{\partial n})_{\Gamma_2} = 0 \quad \text{sur}\,\Gamma_2, \quad \Gamma_2 = \partial\Omega - \Gamma_1 \end{cases}$$

a variational formulation of this problem: By means of

$$\int_\Omega \mathbf{v} \Delta p \, d\Omega = 0$$

the formula of Green with a norm which one supposes directed of structure towards the fluid (thus interior with fluid volume) and while posing: $\Gamma=\Gamma_1 \cup \Gamma_2$ That is to say

$$\int_{\Omega} \textbf{grad}\, v \,.\, \textbf{grad}\, p\, d\Omega + \int_{\Gamma} v \frac{\partial p}{\partial n} d\Gamma = 0$$

: éq

$$\int_{\Omega} \textbf{grad}\, v \,.\, \textbf{grad}\, p\, d\Omega = \rho_f \int_{\Gamma_1} v\, \ddot{x}_n\, d\Gamma$$ 4.1-1 One

considers a partition of volume in Ω a finished number of elements. On this discretization of the field, one can write the approximate shape of the hydrodynamic field of pressure: represent

$$p=\sum_{i=1}^{N} N_i(\mathbf{r})\, p_i$$

N_i the nodal interpolation functions definite on the elements: they are worth 1 with the node number, i and on all the 0 others. Then

, by taking as function-tests successively v the nodal interpolation functions, one obtains a system of equations N while deferring in [éq 4.1-1]: what

$$j=1,\ldots,N\,;\quad \int_{\Omega} \sum_{i=1}^{N} p_i\, \textbf{grad}\, N_i(\boldsymbol{r}) . \boldsymbol{\Delta}\, N_j(\boldsymbol{r})\, d\Omega = \rho_f \int_{\gamma_1} N_j\, \ddot{x}_n\, d\Gamma$$

can be written in the form: formulate

$\mathrm{HP}=\boldsymbol{\Phi}$ formula $\boldsymbol{\Phi}$ of components formulates $\Phi_j=\rho_f \int_{\Gamma} N_j\, \ddot{x}_n\, d\Gamma$

matrix $\boldsymbol{H}$ of coefficients formuleéq $H_{ij}=\int_{\Omega} \textbf{grad}\, N_i . \textbf{grad}\, N_j\, d\Omega$ 4.1-2 In

any rigor, this system is singular. He admits an infinity of solutions differing from a constant. It is thus necessary to impose a pressure (boundary condition of the Dirichlet type) in a point of the fluid to raise the indetermination on the solution. These

precautions taken, by reversing the system [éq 4.1-2], one obtains the field of pressure in all the volume of Ω fluid, including with the fluid interface/structure, where it interests us obviously. Computation

4.2 of the coefficients of the mass matrix added on modal base It

is necessary to estimate the value of the integral numerically: éq

$$m_{il\,jk}=\int_{\gamma_l} p_{jk}\, \boldsymbol{X}_{il} . \boldsymbol{n}\, d\Gamma$$ 4.2-1 from

a field at nodes of pressure represented by a vector column noted and $\boldsymbol{P}_{jk}$ of a field at nodes of displacement corresponding to structure a modal deformed shape in air and represented by the vector column. $\boldsymbol{X}_{il}$ However

, on the fluid interface/structure, the field of pressure approximate due p_{jk} to the discretization of the interface in edge N elements can be written: while

$$p_{jk}=\sum_{m=1}^{N} N_m(\boldsymbol{r})\, p_{jk_m}$$

the field of “modal” displacement is written on this same discretization: Thus

$$\mathbf{X}_{il}=\sum_{n=1}^{N} N_n(\mathbf{r})\, \boldsymbol{X}_{il_n}$$

, by deferring these two statements in the integral [éq 4.2-1], one obtains: One

$$m_{il\,jk} \simeq \int_{\gamma_l} (\sum_{m=1}^{N} N_m(\boldsymbol{r})\, p_{jk_m}) \left[\sum_{n=1}^{N} N_n(\boldsymbol{r})\, X_{ilx_n} A\, n_x + \sum_{n=1}^{N} N_n(\boldsymbol{r})\, X_{ily_n} A\, n_y\right] d\,\Gamma$$

$$m_{il\,jk} \simeq \sum_{m=1}^{N}\sum_{n=1}^{N} p_{jk_m} (\int_{\gamma_l} N_m(\boldsymbol{r})\, N_n(\boldsymbol{r}) n_x\, d\,\Gamma)\, X_{ilx_n} + \sum_{m=1}^{N}\sum_{n=1}^{N} p_{jk_m} (\int_{\gamma_l} N_m(\boldsymbol{r})\, N_n(\boldsymbol{r}) n_y\, d\,\Gamma)\, X_{ily_n}$$

supposes in the demonstration that the problem is two-dimensional. This

can be put in the shape of a scalar product, utilizing a product stamps vector: formulate

$m_{il\,jk}=\mathbf{P}_{jk}^{\mathbf{T}}\mathbf{A_x}\mathbf{X_{ilx}}+\mathbf{P}_{jk}^{\mathbf{T}}\mathbf{A_y}\mathbf{X_{ily}}$ formula $\boldsymbol{A}_x$ coefficients formulates $\int_{\gamma_l} N_i N_j n_x\, d\,\Gamma$

formulates $\boldsymbol{A}_y$ of coefficients formulates $\int_{\gamma_l} N_i N_j n_y\, d\,\Gamma$

5 in work in thermal *Code_Aster*

5.1 Analogy

to solve the problem of Laplace in pressure, one uses a thermal analogy: it is a question in hover of solving the equation of heat with a material of thermal conductivity equal to the unit. As follows: represent

$$\begin{cases} \Delta\, p=0 \,\text{dans}\, \Omega \\ (\dfrac{\partial p}{\partial n})_G=-\rho_f\, \ddot{\mathbf{x}}_s \cdot \mathbf{n} \,\text{dans}\, \Gamma \end{cases} \Leftrightarrow \begin{cases} div(\lambda\, grad\, T)=0 \quad \text{dans}\, \Omega \Leftarrow \Delta\, T=0 \,\text{si}\, \lambda=1 \\ (\dfrac{\partial T}{\partial n})_G=\phi_n \text{dans}\, \Gamma \end{cases}$$

T the temperature in the medium, it plays the part of the pressure in the fluid environment. is ϕ_n the normal heat flux with the wall, it plays the part of the term which $-\rho_f\, \ddot{\mathbf{x}}_s . \boldsymbol{n}$ is comparable to the variation in the course of the time of flux of mass (fluid) to the wall of structure. This quantity is $-\rho_f\, \ddot{\mathbf{x}}_s . \boldsymbol{n}$ indeed homogeneous with a mass divided by a surface and a time squared. Implemented

5.2 operator

CALC _MATR_AJOU [U 4.66.01] practice was developed to take into account the inertial coupling (added mass : OPTION = "MASS_AJOU") between structures bathed in the same true, incompressible fluid and at rest. The fluid is described by equivalent thermal characteristics (operator DEFI _MATERIAU [U 4.43.01]) and the part of the mesh representing is affected by thermal elements (operator AFFE_MODELE [U 4.41.01]). This operator CALC _MATR_AJOU also allows to calculate the stiffness or added damping. In order to facilitate its use in certain cases, there exists also macro-command MACRO _MATR_AJOU [U 4.66.11].

The operator uses five compulsory key words:

- key word MODELE_FLUIDE : it is on this model that one solves the problem of Laplace with boundary conditions of Von Neumann (or its thermal problem are equivalent),
- key word MODE_MECA (or CHAM_NO , or MODELE_GENE): this key word makes it possible to calculate the boundary conditions of type flux to the wall of structure,
- key word MODELE_INTERFACE : it is on this model which understands all the thermal elements of edge of the fluid interface/structure that one calculates the scalar product mentioned in the paragraph [§4.2],
- key word CHAM_MATER : it is the fluid material (described by equivalent thermal characteristics),
- key word CHARGE : it is a thermal load (temperature imposed in an unspecified node of the fluid mesh) which corresponds to the boundary condition of Dirichlet to raise the singularity of the problem of Laplace (see [§4.1]).

A generalized added mass matrix is thus obtained. This matrix having a full profile sky line but (operator NUME_DDL_GENE [U 4.65.03]) can be added with the generalized mass matrix of structure by means of operator COMB _MATR_ASSE [U 4.72.01]. This makes it possible to calculate the coupled modes fluid/structure of immersed structures ("wet" modes) (operator MODE_ITER_SIMULT or MODE _ITER_INV [U 4.52.03], [U4.52.04]). Bibliography

6 C.

1) CONCA, J. PLANCHARD, B. THOMAS, Mr. VANNINATHAN: "Mathematical Problems in fluid coupling/structure" _EYROLLES (1994). F.

2) BEAUD, G. ROUSSEAU: "Validation inter-software of the computation of added mass with the Code_Aster *and* the code CALIPH", HT-32/95/004/A Description

7 of the versions of the document Version

Aster Author	(S) Organization (S) Description	of modifications 3
3	***ROUSSEAU (EDF*** */EP/AMV) initial*	Text