Fluid-structure coupling for tubular structures and the coaxial shells

Abstract:

This document fluid-structure coupling describes the various models from available from operator CALC_FLUI_STRU. These models make it possible to simulate the forces of coupling fluid-elastic in the following configurations:

- tube bundles under transverse flow (primarily for the tubes of GV),
- transition rod of command/plate of housing (exclusively for the control rods),
- coaxial cylindrical shells under annular flow (space ferments/envelope of heart,…),
- tube bundles under axial flow (fuel assemblies,…).

For each configuration, the model of forces fluid-elastics is initially presented. The resolution of the modal problem is then described. The methods of resolution employed integrate specificities of the various models of forces fluid-elastics.

1 General presentation

1.1 Recalls

the dynamic fluid forces being exerted on a structure moving can be classified in two categories:

- forces **independent** of the motion of structure, at least in the range of small displacements; they are mainly random forces generated by the turbulence or the diphasic nature of flow,
- the fluid forces **dependant** on the motion of structure, called "**forces fluid-elastics**", persons in charge of fluid-structure coupling.

In this document, one is interested in the four models of forces **fluid-elastics** integrated in operator `CALC_FLUI_STRU`. The data-processing aspects related to the integration of these models were the object of notes of specifications [feeding-bottle. 1], [feeding-bottle. 2].

1.2 Modelization

the dependence of the forces fluid-elastics with respect to the motion of structure is translated, for the low amplitudes, by **a matrix of transfer between the force fluid-elastic and the vector displacement**. The projection of the equation of the motion of the system coupled fluid-structure on modal base of structure alone is written, in **the field of Laplace** :

$$\left\{[\mathbf{M}_{ii}]s^2+[\mathbf{C}_{ii}]s+[\mathbf{K}_{ii}]-[\mathbf{B}_{ij}(U,s)]\right\}(\mathbf{q})=(\mathbf{Q}_t)$$

éq. 1.2 - 1

where $[\mathbf{M}_{ii}],[\mathbf{C}_{ii}]\text{et}[\mathbf{K}_{ii}]$ the diagonal matrixes of mass, damping and stiffness structural in air indicate respectively;
$(\mathbf{q})$ indicate the vector of the displacements generalized in air;
$(\mathbf{Q}_t)$ is the vector of the generalized random excitations (independent forces);
and $[\mathbf{B}_{ij}(U,s)]$ the matrix of transfer of the forces fluid-elastics represents, projected on the basis of the structure modal base alone. This matrix depends in particular on U, velocity characteristic of flow, as well as frequency of motion via the variable of Laplace s.

A priori, $[\mathbf{B}_{ij}(U,s)]$ is an unspecified matrix whose diagonal terms, if they are not null, introduce a coupling between modes. In addition, the terms of $[\mathbf{B}_{ij}(U,s)]$ evolve in a nonlinear way with the complex frequency s.

A each model of force fluid-elastic is associated a specific matrix of transfer. In all the cases, the formulation of the modal problem under flow can be characterized by the relation [éq. 1.2-1].

For the various types of configurations being able to be simulated using the operator `CALC_FLUI_STRU`, the representations of the matrixes of transfer of the forces fluid-elastics are clarified in the continuation of this document.

2 Excitation fluid-elastic acting on the tube bundles under transverse flow (primarily for the tubes of GV)

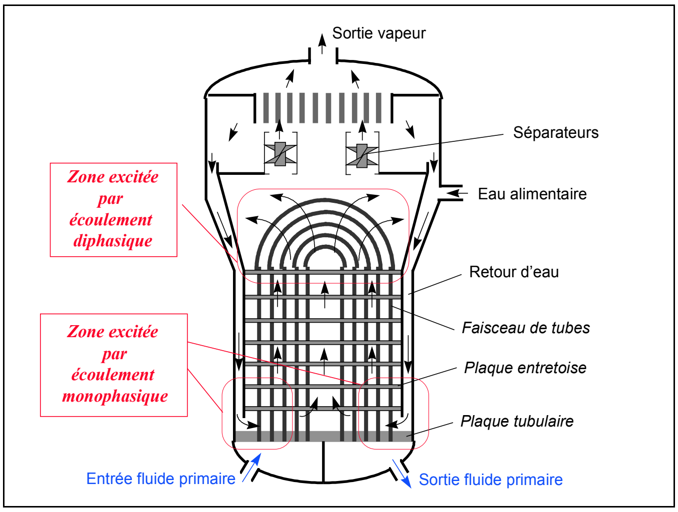
Two methods of simulation of the excitation fluid-elastic are available in *Code_Aster*.

The first goes up at the end of the Seventies. It is very widespread in the scientific community, within which it is known under the denomination of “method of Connors”. The results provided by this method depend mainly on the value which one allots to the one of his principal parameters of entry: the “constant of Connors”. Conservative values thus have being determined for this constant on the basis of test many carried out in the world. The method of Connors is well adapted to the design of the tube banks against the vibratory risk at the stage of the design. It is described hereafter in the paragraph § 2.5.

The second integrates more physics that the method of Connors. However, the complete modelization of the phenomena being too complex compared to current knowledge, this second method remains based on a set of experimental correlations, known as correlations fluid-elastics. The integration of this second model of excitation fluid-elastic in *the Code_Aster* was approached in the note of specifications [bib.1]. The note of principle of software FLUSTRU [feeding-bottle. 3] constitutes the theoretical documentation of reference. She is recalled in her broad outlines in the paragraphs § 2.1 to 2.4 hereafter.

2.1 Description of the studied configuration

One considers a tube bundle excited by a transverse external flow. Transverse external flows tend to destabilize the mechanical system when the rate of flow increases. An industrial case is that of vibrations of the tubes of GV. On this component, transverse flows are observed as starter tube bundle (monophasic flow liquidates), and the curved part of the tubes (diphasic flow) [Figure 2.1-a].



Appear 2.1-a: Diagram of steam generator

From the point of view of the coupling fluid-elastic, the study of the dynamic behavior of the various tubes of a beam subjected to a transverse flow is brought back under investigation equivalent tube; the definition of the equivalent tube depends on the environment of the tube to treating.

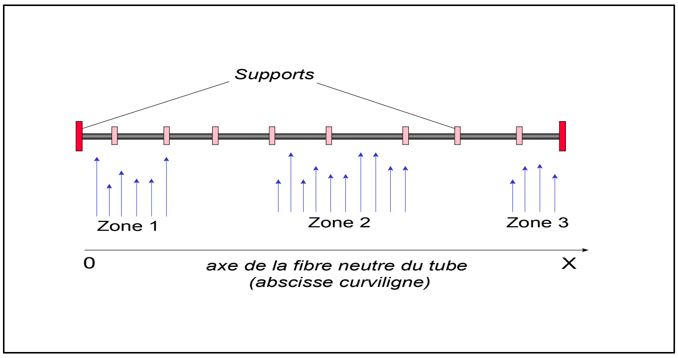
When the tube considered has vibratory characteristics appreciably different from those of its neighbors, this tube can be comparable to only one tube, vibrating in the middle of a rigid tube bundle.

In the contrary case, the problem is more complex because one must consider a mechanical system with coupling between tubes of the beam and thus comprising a large number of degrees of freedom.

To treat this kind of configuration, a model was developed at Department TTA, "the model total" [feeding-bottle. 7]; this model allows the definition of a system equivalent to a degree of freedom, which represents the complete coupled system.

The approach adopted to lead computations can be summarized in the following way [Figure 2.1-b]:

- Taking into account the telegraphic nature of structures studied, the computation of the fluid coupling - elastic in the tube bundle is carried out by describing the tube by its curvilinear abscisse.
- In computation, the fluid environment of the tube is characterized, at the same time by the physical properties of the fluid circulating inside the tube (fluid primary education), and by those of the fluid circulating outside the tube (fluid exiting secondary). These physical properties, such as the density, can vary along the tube, according to the curvilinear abscisse.
- The rate of flow taken into account for the computation of coupling fluid-elastic is the component, norm with the tube in the plan of the tube, the velocity of the secondary fluid. This velocity can vary along the tube.
- In order to be able to take into account the various possible types of excitation, several excitation zones can be defined along structure. In the case of the steam generator, for example, one may find it beneficial to distinguish, on the one hand the zones where the excitation is exerted by a fluid in a monophasic state, which are in foot of tube, and on the other hand, the zone where the excitation is diphasic withstrong rate of vacuum, localised in the curved part of the tube.
- The computation of coupling is realized starting from the mechanical characteristics of fluid structure "at rest". The forces fluid-elastics of coupling are estimated from adimensional correlations which are obtained on analytical experiments in similarity. On each excitation zone, one can thus apply the adequate correlations; the excitation zones must be disjoined.



Appear 2.1-b: representation of the configuration studied

For this configuration of coupling fluid-elastic, the following notations will be used:

L	Overall length of the tube

L_k	Length of the zone k
d_e	Diameter external of the tube
d_i	Internal diameter of the warp-tube
ϕ_i	modal of mode I
$\rho_e(x)$	Density of the external fluid with the curvilinear abscisse x
$\rho_i(x)$	Density of the internal fluid with the curvilinear abscisse x
ρ_t	Density of the tube (structure alone)
$\rho_{eq}(x)$	Density equivalent to the curvilinear abscisse x
U	Velocity of the external fluid specified by the user in operator `DEFI_FLUI_STRU`
$V(x)$	Velocity of the external fluid to the curvilinear abscisse x
$V_k(x)$	Velocity of the external fluid to the curvilinear abscisse x (excitation zone k) defined by the product of U and a profile velocity specified by the user in operator `DEFI_FLUI_STRU`
U_k	Mean velocity of the external fluid calculated from $V_k(x)$ for the Average θ
$\overline{U}$	excitation zone velocities U_k on all the zones ofexcitation

2.2 Stages of computation

- the first stage of computation consists in calculating the structural features in “fluid at rest”. One proceeds by considering an equivalent mass of the tube; this equivalent mass gathers, on the one hand the mass of the tube alone, and on the other hand the added masses by the fluids internal and external.

  An equivalent density is thus defined along the tube according to the curvilinear abscisse x by the statement:

$$\rho_{eq}(x)=\frac{1}{\left(d_e^2-d_i^2\right)}\left[\rho_i(x).d_i^2+\rho_t.\left(d_e^2-d_i^2\right)+\rho_e(x).d_{eq}^2\right]$$ **éq. 2.2 - 1**

  with

$$d_{eq}^2=\frac{2.C_m.d_e^2}{\pi}$$ **éq. 2.2 - 2**

  In the equation [**éq. 2.2-1**], the term $\rho_e(x).d_{eq}^2$ represents the added mass by the external fluid. This term depends, via the parameter C_m, of the arrangement of the tube bundle (not square or triangular), and of the containment of the beam (not reduced). For computations of coupling fluid-elastic of the tube bundles subjected to a transverse flow, one usually uses, to estimate the coefficient C_m, of the analytical statements determined from experimental results. All the data necessary to the estimate of the coefficient C_m are collected by the operator `DEFI_FLUI_STRU`.

- Knowing the equivalent density of the tube, the elementary matrixes of water mass and stiffness at rest are then calculated by means of the profile of equivalent density, by the operator

CALC_MATR_ELEM ; one uses options MASS_FLUI_STRU and RIGI_FLUI_STRU. Operator MODE_ITER_SIMULT allows, after assembly of the elementary matrixes, to directly calculate the modes out of water at rest of studied structure.

- The forces fluid-elastics of coupling are calculated by the operator CALC_FLUI_STRU starting from the adimensional correlations established on analytical models in similarity. These forces of coupling $\left[\mathbf{B}_{ij}(U,s)\right]$, dependant on the motion of structure are then taken into account in the general equation of motion [**éq. 1.2-1**] to compute: characteristics of the system coupled flow-structure for a given velocity of flow.

2.3 Form of the matrix of transfer of the forces fluid-elastics

In the case as of tube bundles excited by a transverse flow, the forces fluid-elastics of coupling are distributed forces along structure. They are characterized by linear adimensional coefficients of damping and stiffness added, named respectively Cd and Ck . The statement of the coefficients of the matrix of transfer of the forces fluid-elastics projected on the basis of the fluid structure modal base "at rest" is then the following one:

$$\mathbf{B}_{ij}(U,s)=\begin{bmatrix}\left\{\int_L \frac{1}{2}\rho_e(x)V(x)d_e Cd(x,s_r)\phi_i^2(x)dx\right\}s\\ +\left\{\int_L \frac{1}{2}\rho_e(x)V^2(x)Ck(x,s_r)\phi_i^2(x)dx\right\}\end{bmatrix}\cdot\delta_{ij}$$

éq. 2.3 - 1

the dependence of the coefficients Cd and Ck with respect to the motion of structure and rate of flow of the fluid is translated by their evolution according to the reduced frequency complexes s_r , definite by:

$$s_r=\frac{sD}{U}$$

éq. 2.3 - 2

the statement [**éq. 2.3-1**] watch which one retains a diagonal matrix of transfer. That implies:

- the various eigen modes of structure are rather distant from/to each other so that one can suppose that there is not coupling between modes.
- the modal deformed shapes of fluid structure "at rest" are not disturbed by the flow setting of the fluid.

These two assumptions could be checked in experiments on the tube bundles subjected to a transverse flow.

In practice, taking into account the various excitation zones taken into account along structure, the diagonal coefficients of the matrix of forces fluid-elastics projected on modal base are written:

$$\mathbf{B}_{ii}(U,s)=\sum_k\begin{bmatrix}\left\{\int_{L_k}\frac{1}{2}\rho_e(x)V_k(x)d_e Cd_k\left(\frac{sd_e\overline{U}}{UU_k}\right)\phi_i^2(x)dx\right\}s\\ +\left\{\int_{L_k}\frac{1}{2}\rho_e(x)V_k^2(x)Ck_k\left(\frac{sd_e\overline{U}}{UU_k}\right)\phi_i^2(x)dx\right\}\end{bmatrix}$$

éq. 2.3 - 3

where Cd_k and Ck_k respectively indicate the adimensional coefficients of coupling, damping and stiffness, retained for the excitation zone k. The fluid velocity $\frac{UU_k}{\overline{U}}$ intervening in the reduced frequency complex in argument of the coefficients of coupling corresponds to the mean velocity on the excitation zone k , after renormalization of the profile $V_k(x)$, so that its average on all the excitation zones is worth U .

It is in addition very important to note that each modal deformed shape taken into account in equations 2.3-1, 2.3-3, etc is actually only via its component in translation according to the direction of the bearing pressure. This is with the fact that the damping coefficients and of stiffness added which appear in these equations were given (in experiments) only for the direction of the bearing pressure. This remark applies to all the méthodes de calcul of instabilities fluid-elastics of tubes of GV presented in this document, including with the method of Connors introduced to the paragraphs § 2.5.1 and § 2.5.2. It results from it in particular that the generalized matrixes of mass, damping and stiffness which appear in the equations associated with computations with instability fluid-elastic of the tubes of GV (as for example equation 2.4-1) are not matrixes generalized with the usual meaning of the term, i.e. leaning on the three components in translation and on the three components in rotation, but of the generalized matrixes which one can describe as "directed according to a privileged direction" insofar as they all are calculated on the basis of component only in translation of the modal deformed shapes according to the direction of the bearing pressure. This remark applies only to the application "vibrations of the tubes of GV" and, inside this application, with the computation of instabilities fluid-elastics.

2.4 Resolution of the modal problem under flow

In the configuration "Tube bundle subjected to a transverse flow", the problem is solved on modal base characterizing fluid structure "at rest".

Generally, the characteristics of the system coupled flow-structure are obtained by seeking the solutions of the equation:

$$\left\{[\mathbf{M}_{ii}]s^2+[\mathbf{C}_{ii}]s+[\mathbf{K}_{ii}]-[\mathbf{B}_{ii}(U,s)]\right\}(\mathbf{q})=(0)$$

éq. 2.4 - 1

where $[\mathbf{M}_{ii}],[\mathbf{C}_{ii}]\mathrm{et}[\mathbf{K}_{ii}]$ indicate respectively the diagonal matrixes of mass, damping and stiffness structural features in "fluid at rest";

$(\mathbf{q})$ indicate the vector of the displacements generalized in "fluid at rest".

As the matrix of forces fluid-elastics retained is diagonal, and that the modal deformed shapes are supposed not to be modified under flow, the problem of coupling fluid-elastic is reduced to the resolution of N scalar problems, N indicating the number of modes taken into account in modal base.

For each mode i and each rate of flow U, the problem to be solved is written:

$$\mathbf{M}_{ii}s^2+\left[\mathbf{C}_{ii}-\left(\sum_k\left\{\int_{L_k}\frac{1}{2}\rho_e(x)V_k(x)d_eCd_k\left(\frac{sd_e\overline{U}}{UU_k}\right)\phi_i^2(x)dx\right\}\right)\right]s$$
$$+\left[\mathbf{K}_{ii}-\left(\sum_k\left\{\int_{L_k}\frac{1}{2}\rho_e(x)V_k^2(x)Ck_k\left(\frac{sd_e\overline{U}}{UU_k}\right)\phi_i^2(x)dx\right\}\right)\right]=0$$

éq. 2.4 - 2

It will be noted that the equation [**éq. 2.4-2**] is nonlinear in s ; its solutions are obtained using an iterative method of Broyden type.

For each mode i, one obtains a solution s_i of the equation [**éq. 2.4-2**]. One then deduces from s_i, for this mode, the pulsation ω_i and the damping ξ_i of the system coupled flow-structure, by means of the relation:

$$s_i=-\xi_i\omega_i+J\omega_i\sqrt{1-\xi_i^2} \qquad \text{with} \qquad J^2=-1$$

éq. 2.4-33

the coupled system dynamically becomes unstable when one of the damping coefficients ξ_i becomes negative or is cancelled.

2.5 Method of Connors

2.5.1 Case of a single zone of fluid excitation

In 1978, H.J. Connors proposes to determine the critical engine failure speed V_{cn} associated with the mode with order N with a tube with Steam generator (GV) according to the relation [10]:

$$\frac{V_{cn}}{f_n D_e} = K\sqrt{\frac{\overline{m}\,\delta_n}{\overline{\rho}_s D_e^2}}$$

In this relation:

V_{cn} indicate the critical velocity inter-tubes of instability for mode N, f_n indicates the eigenfrequency of order N of the tube [1], D_e the diameter external of the tube indicates, K indicates the constant of Connors, $\overline{m}$ indicates the linear density of reference of the tube including the effects of added mass, δ_n indicates the decrement logarithmic curve of mode N out of fluid at rest, i.e. including the damping of structure and that brought by the external fluid at rest, and $\overline{\rho}_s$ indicates the density of reference of the secondary fluid.

It is pointed out that the decrement logarithmic curve δ_n is defined as:

$$\delta_n = \frac{2\pi\,\xi_n}{\sqrt{1-\xi_n^2}}$$

Where ξ_n the reduced modal damping of the n.étant mode about ξ_n the percent indicates, it is legitimate to pose $\sqrt{1-\xi_n^2} = 1$, and thus the approximation:

$$\frac{V_{cn}}{f_n D_e} = K\sqrt{\frac{2\pi\overline{m}\,\xi_n}{\overline{\rho}_s D_e^2}}$$

The constant of instability *K* is in experiments given from test results of instability. In the studies of vibratory design of the tube bundles of GV, the values usually adopted for this constant are:

- *K = 4* in the event of diphasic transverse flow on the level of the chignon,
- *K = 2,9* in the event of monophasic transverse flow with the top of the tubular plate.

By regarding as linear density of reference of the tube $\overline{m}$ his average linear density, one can determine $\overline{m}$ in the form:

$$\overline{m} = \frac{\pi}{4}\frac{\left(D_e^2 - D_i^2\right)}{L_{tube}}\int_{tube} \rho_{eq}(s)\,ds$$

1 any rigor, one should consider the value of f_n under flow. However, the method of Connors does not envisage the calculus of the variation of frequency ascribable to flow. At first approximation, one thus considers for f_n the value of the frequency out of fluid at rest.

Where D_i the internal diameter of the tube indicates, L_{tube} indicates its length, S indicates the curvilinear abscisse along the tube and $\rho_{eq}(s)$ indicates the density equivalent of the tube to the X-coordinate S:

$$\rho_{eq}(s) = \rho_t + \frac{D_i^2}{D_e^2 - D_i^2}\rho_p(s) + \frac{2C}{\pi}\frac{D_e^2}{D_e^2 - D_i^2}\rho_s(s)$$

Where ρ_t indicates the density of the tube presumedly independent of the curvilinear abscisse, $\rho_p(s)$ and $\rho_s(s)$ respectively indicate the density of the primary education fluid and the secondary fluid to the curvilinear abscisse s, and C is defined by:

$C = \frac{\pi}{2}\frac{(\Delta/D_e)^2 + 1}{(\Delta/D_e)^2 - 1}$, where Δ an equivalent diameter given by the relations indicates:

1) $\Delta/D_e = \left(1.07 + 0.56\frac{P}{D_e}\right)\frac{P}{D_e}$ for a square step (*C* is worth approximately *2,0* for GV French)
2) $\Delta/D_e = \left(0.96 + 0.50\frac{P}{D_e}\right)\frac{P}{D_e}$ for a triangular step (*C* is worth approximately *2,2* for GV French)

In the same way, by regarding as density of reference of the secondary fluid $\bar{\rho}_s$ his average density, one can determine $\bar{\rho}_s$ in the form:

$$\bar{\rho}_s = \frac{1}{L_{tube}}\int_{tube}\rho_s(s)ds$$

Mode N unstable if the critical velocity V_{cn} is lower at the effective speed V_{en} associated with mode N, thus is defined:

$$V_{en} = \sqrt{\frac{\int_{tube}\frac{\rho_s(s)}{\bar{\rho}_s}V^2(s)\varphi_n^2(s)ds}{\int_{tube}\frac{m(s)}{\bar{m}}\varphi_n^2(s)ds}}$$

Where $\varphi_n(s)$ the modal deformed shape of the mode indicates n, $V(s)$ indicates the rate of flow under operation (m/s), $m(s)$ indicate the linear density of the tube including the effects of added mass (kg/m) supposed to vary along the tube, obtained like:

$$m(s) = \frac{\pi}{4}(D_e^2 - D_i^2)\rho_{eq}(s)$$

One defines the ratio of instability for mode N within the meaning of Connors as being the ratio:

$$R_{Cn} = \frac{V_{en}}{V_{cn}}$$

2.5.2 Case of several excitation zones fluid

the approach of application of the method of Connors deserves to be specified if the tube is subjected to a multiform excitation on behalf of the fluid, in particular, if the latter is monophasic in bottom of beam and diphasic in the chignon. It is pointed out that such a situation is taken into account in software GEVIBUS [11].

To extrapolate the method of Connors to this general case, one proceeds by generalizing the establishment of the approach suggested by Connors [10].

That is to say W_n the energy added by flow during a cycle to a vibrating tube in its mode N:

$$W_n = \sum_{i=1}^{Nex} C_i \int_{Lex_i} \rho_s(s)\, V^2(s)\, \varphi_n^2(s) ds$$

Where, compared to the talk of Connors, the dependence of the constant C_i in the excitation zone i is added, Nex indicates the nombre total of excitation zones, and Lex_i indicates the length of i the -ème excitation zone.

That is to say E_n the energy dissipated during a cycle by the vibrating tube in its mode N:

$$E_n = C_2 f_n^2 \delta_n \int_{tube} m(s)\, \varphi_n^2(s) ds$$

While equalizing W_n and E_n, i.e. while placing themselves at instability, by introducing like Connors the variables of reference $\bar{\rho}_s$ and $\bar{m}$ (although they do not appear essential), while posing $\frac{C_i}{C_2} = \frac{1}{K_i{}^2}$, where K_i the constant of Connors associated with - i ème excitation zone indicates, and while seeking to reveal the effective velocity V_{en} such as Connors defines it, one obtains all done calculations the statement:

$$\frac{\int_{tube} \frac{\rho_s(s)}{\bar{\rho}_s} V^2(s)\, \varphi_n^2(s) ds}{\int_{tube} \frac{m(s)}{\bar{m}} \varphi_n^2(s) ds} \sum_{i=1}^{Nex} \frac{1}{K_i{}^2} \frac{\int_{Lex_i} \rho_s(s)\, V^2(s)\, \varphi_n^2(s) ds}{\int_{tube} \rho_s(s)\, V^2(s)\, \varphi_n^2(s) ds} \left(\frac{1}{f_n D_e}\right)^2 = \delta_n \frac{\bar{m}}{\bar{\rho}_s D_e^2}$$

From where:

$$\frac{\int_{tube} \frac{\rho_s(s)}{\bar{\rho}_s} V^2(s)\, \varphi_n^2(s) ds}{\int_{tube} \frac{m(s)}{\bar{m}} \varphi_n^2(s) ds} = (V_{en})^2 = \frac{\int_{tube} \rho_s(s)\, V^2(s)\, \varphi_n^2(s) ds}{\sum_{i=1}^{Nex} \frac{1}{K_i{}^2} \int_{Lex_i} \rho_s(s)\, V^2(s)\, \varphi_n^2(s) ds} (f_n D_e)^2 \delta_n \frac{\bar{m}}{\bar{\rho}_s D_e^2}$$

One from of deduced in the case of a multiform excitation the form of the critical velocity V_{cn} associated with the mode with order n :

$$\frac{V_{cn}}{f_n D_e} = \sqrt{\frac{\int_{tube} \rho_s(s) V^2(s) \varphi_n^2(s) ds}{\sum_{i=1}^{Nex} \frac{1}{K_i^2} \int_{Lex_i} \rho_s(s) V^2(s) \varphi_n^2(s) ds}} \sqrt{\frac{\overline{m}\,\delta_n}{\overline{\rho}_s D_e^2}}$$

It is checked that, when the excitation is of comparable nature on all the excited zones $K_{i(i=1,Nex)} = K$, the relation above finds the form suggested by Connors:

$$\frac{V_{cn}}{f_n D_e} = K \sqrt{\frac{\overline{m}\,\delta_n}{\overline{\rho}_s D_e^2}}$$

2.5.3 Alternative of the method

an alternative of the method presented to the paragraphs § 2.5.1 and § 2.5.2 consists in calculating the ratio of instability R_n of the mode " n " (in the case of a single zone of excitation) in the form:

$$R_n = \frac{V_{moy}}{f_i D_e K \cdot \left[\frac{2\Pi \xi_n M_n}{\overline{\rho} D_e{}^2 \int_0^L r(s) u^2(s) \varphi_n^2(s) ds} \right]^{1/2}}$$

Where V_{moy} the average velocity indicates on the excitation zone considered, M_n indicates the generalized mass (not directed according to a direction privileged and fascinating thus in account at the same time three components in translation and the three components in rotation) of the mode n, and φ_n indicates the modal deformed shape of the mode n.

By φ_n^2 one understands here the sum of the squares of the three components in translation of φ_n. The three components in rotation are not taken into account in the computation of φ_n^2. The computation ratio according to this alternative is systematically carried out by Code_Aster when one asks for the implementation of the method of Connors.

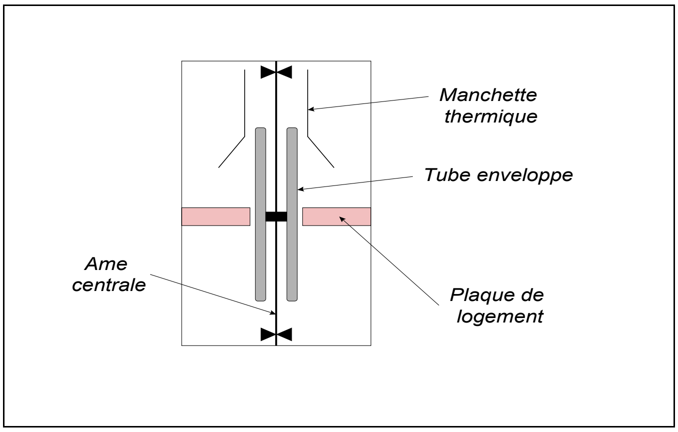
The ratio of instability calculated according to this alternative is provided beside the ratio of instability calculated according to the method specified in the paragraphs § 2.5.1 and § 2.5.2. Most of the time, the two results are identical. If there exists a variation, the reason of this variation must be required in the contribution of the components in rotation of the mode considered, for example in the contribution of rotations of the right parts of the tubes around their axis. One will adopt then result more penalizing of both.

3 Excitation fluid-elastic operating the rod of command on the level of the plate of housing (exclusively for the control rods)

the forces fluid-elastics acting on this kind of configuration were identified on model GRAPPE2 of department TTA. The theoretical aspects of the identification of these sources are developed in reference [feeding-bottle. 4]. The integration of model GRAPPE2 in *Code_Aster* is approached in the note of specifications [feeding-bottle. 2].

3.1 Description of the studied configuration

model GRAPPE2 represents the rod of command, the upper part of the guide of cluster, and the thermal cuff of an engine of the type 900 or 1300 MWe [Figure 3.1-a].



Appear 3.1-a: General diagram of model GRAPPE 2

This model primarily consists of a hollow cylindrical tube of low thickness, fixed on a full cylindrical central heart. The hollow tube is entirely immersed in water with room temperature. A plate, representing the plate of housing, makes it possible to reproduce annular containment. Flow through the plate can be ascending or descendant. The rod of command can be centered or offset (50% of average clearance) on the level of the plate of housing.

Four experimental configurations are thus possible, according to the meaning of flow and the centering or not of the rod of command. The coefficients of forces fluid-elastics were identified for each one of these configurations and are available in *the Code_Aster*.

Model GRAPPE2 was dimensioned in geometrical, hydraulic similarity and of frequency reduced compared to the configuration engine. The only data of the diameter of the rod of command thus makes it possible, in particular, to deduce all from the other geometrical magnitudes.

3.2 Stages of computation

- the first stage of computation consists in calculating the modal base of water structure at rest, the effects of added mass induced locally on the level of the containment of the plate of housing being neglected. This stage is carried out by the operator `MODE_ITER_SIMULT`.

With this intention, a homogeneous equivalent density is assigned to the group of structure, in order to take into account the apparent mass added by the fluid, except for that induced by the effects of containment on the level of annular space. This equivalent density is defined by:

$$\rho_{eq} = \alpha \frac{\pi R^2}{S} \rho_f + \rho_{tube}$$ **éq. 3.2 - 1**

where:

α indicate an adimensional coefficient of containment depend on the studied configuration; $\alpha=1$ is the value used for computations of control rods. It corresponds to a vibrating roller in an unlimited fluid field.

R indicate the radius external of the tube,

S indicates the area of the cross-section of the tube,

ρ_{tube} indicates the density of the material constituting the vibrating tube.

- The second stage is the taking into account of the coupling with fluid flow. It is carried out using operator `CALC_FLUI_STRU`.

3.3 Representation of the excitation fluid-elastic

Is x the direction of neutral fiber of the tube. The excitation fluid-elastic identified on model GRAPPE2 is represented by a resulting force and a moment, applied in the same point of X-coordinate x_o, corresponding to the central zone of the transition of the rod of command through the plate of housing. The excitation is thus defined, in physical base, by the relation:

$$\hat{\mathbf{f}}_c(x,s)=\mathbf{F}_c(s)\delta(x-x_0)-\mathbf{M}_c(s)\delta'(x-x_0) \quad \textbf{éq. 3.3 - 1}$$

where δ' the derivative compared to formula x the distribution of Dirac indicates δ.

The resultant force $\mathbf{F}_c$, acts thus under the effect of transverse displacements of the rod of command; and the resulting moment $\mathbf{M}_c$, acts under the effect of the rotation of the latter.

One notes $\mathbf{X}_T(s)$ the vector of transverse displacements and $\Theta(s)$ the vector of associated rotations, defined by:

$$\mathbf{X}_T(s)=\begin{pmatrix}0\\u_y(x_0,s)\\u_z(x_0,s)\end{pmatrix} \quad \textbf{éq. 3.3 - 2}$$

$$\Theta(s)=\begin{pmatrix}0\\ \dfrac{\partial u_y}{\partial x}(x_0,s)\\ \dfrac{\partial u_z}{\partial x}(x_0,s)\end{pmatrix} \quad \textbf{éq. 3.3 - 3}$$

following relations are used to compute: the forces and the moments resulting fluid-elastics starting from the added masses Cm_1, Cm_2, of added depreciation $Cd_1(V_r), Cd_2(V_r)$ and the added stiffness $Ck_1(V_r), Ck_2(V_r)$, adimensional coefficients identified on model GRAPPE2:

$$\mathbf{F}c(s)=\left\{-\frac{1}{2}\rho_f D^2 L_p Cm_1 s^2+\frac{1}{2}\rho_f DUL_p Cd_1(V_r)s+\frac{1}{2}\rho_f U^2 L_p Ck_1(V_r)\right\}\mathbf{X}_T(s) \quad \textbf{éq. 3.3 - 4}$$

$$\mathbf{M}c(s)=\left\{-\frac{1}{2}\rho_f D^2 L_p{}^3 Cm_2 s^2+\frac{1}{2}\rho_f DUL_p{}^3 Cd_2(V_r)s+\frac{1}{2}\rho_f U^2 L_p{}^3 Ck_2(V_r)\right\}\Theta(s)$$ **éq. 3.3 - 5**

In order to simplify the writing of the equations, one notes thereafter:

$$\mathbf{F}c(s)=H_1(s)\mathbf{X}_T(s) \text{ et } \mathbf{M}c(s)=H_2(s)\Theta(s)$$

The adimensional fallback speed V_r is defined here using the relation $V_r=\frac{U}{sD}$, where s the variable of Laplace indicates.

The statements [**éq. 3.3-4**] and [**éq. 3.3-5**] utilize the thickness L_p of the plate of housing. This thickness results from the value of the diameter of the rod of command D, because of geometrical similarity with the configuration engine. The force fluid-elastic $\hat{\mathbf{f}}_c(x,s)$ is thus completely characterized by the data of the following quantities:

ρ_f	Density of the fluid,
U	Rate of average flow in annular space between rod of command and plate of housing,
D	Diameter of the rod of command,
Cm_1	Coefficient of added mass associated with the translatory movement,
$Cd_1(V_r)$	added Damping coefficient associated with the translatory movement,
$Ck_1(V_r)$	Coefficient of added stiffness associated with the translatory movement,
Cm_2	Coefficient of added mass associated with the rotation movement,
$Cd_2(V_r)$	added Damping coefficient associated with the rotation movement,
$Ck_2(V_r)$	Coefficient of added stiffness associated with the rotation movement.

The adimensional coefficients of added mass, Cm_1 and Cm_2, allow the taking into account of the inertial effects induced by local containment of the rod of command the level of the plate of housing. These effects are estimated as follows.

That is to say H the thickness of annular flow on the level of containment, deduced from D geometrical similarity compared to the configuration engine; α indicate the adimensional coefficient of containment introduced by the relation [**éq. 3.2-1**]. One obtains [feeding-bottle then. 4]:

$$\frac{1}{2}\rho_f D^2 L_p Cm_1=\left\{\rho_f\frac{\pi D^3}{8H}-\alpha\rho_f\frac{\pi D^2}{4}\right\}L_p=\rho_f\frac{\pi D^2}{4}\left\{\frac{D}{2H}-\alpha\right\}L_p$$

$$\frac{1}{2}\rho_f D^2 L_p{}^3 Cm_2\theta=\rho_f\frac{\pi D^2}{4}\left\{\frac{D}{2H}-\alpha\right\}\int_{L_p}\theta(x-x_o)^2dx=\rho_f\frac{\pi D^2}{4}\left\{\frac{D}{2H}-\alpha\right\}\theta\frac{L_p{}^3}{3}$$

One from of deduced the values from Cm_1 and Cm_2 by:

$$Cm_1=\frac{\pi}{2}\left\{\frac{D}{2H}-\alpha\right\}$$ **éq. 3.3 - 6**

$$Cm_2=\frac{Cm_1}{3}=\frac{\pi}{6}\left\{\frac{D}{2H}-\alpha\right\}$$ **éq. 3.3 - 7**

coefficients Cd_1,Ck_1,Cd_2 and Ck_2 are directly deducted from measurement and are expressed in the form of adimensional correlations.

3.4 Projection on the basis of modal base and statement terms of the matrix of transfer of force fluid-elastic

Decomposition of motion on modal base

One notes $\Phi_{\mathbf{j}}(x)$ the modal deformed shape of j the ème mode of structure. The decomposition of the vector of displacements in modal base is expressed in the form:

$$\mathbf{u}(x,s)=\sum_{j=1}^{N}\Phi_j(x)q_j(s)=\sum_{j=1}^{N}\begin{pmatrix}DX_j(x)\\DY_j(x)\\DZ_j(x)\end{pmatrix}q_j(s)\qquad\text{éq. 3.4 - 1}$$

Where DX_j, DY_j and DZ_j correspond to the three components of translation characterizing the modal deformed shapes calculated using *Code_Aster*.

Computation of the generalized excitation associated with mode I

the generalized excitation $\mathbf{Q}_i(s)$ associated with the mode i is defined by the relation:

$$\mathbf{Q}_i(s)=\int_0^L\hat{\mathbf{f}}_c(x,s).\Phi_i(x)dx\qquad\text{éq. 3.4 - 2}$$

where L the length of the structure indicates on which one wants to impose excitations GRAPPE2.

Transfer transfer functions $H_1(s)$ and $H_2(s)$ being defined starting from the relations [**éq. 3.3-4**] and [**éq. 3.3-5**], one from of deduced, taking into account the statements [**éq. 3.3-1**], [**éq. 3.3-4**] and [**éq. 3.3-5**]:

$$\begin{aligned}\mathbf{Q}_i(s)\;&=\sum_{j=1}^{N}\int_0^L H_1(s)\begin{pmatrix}0\\DY_j(x_o)\\DZ_j(x_o)\end{pmatrix}q_j(s)\delta(x-x_o).\begin{pmatrix}0\\DY_i(x)\\DZ_i(x)\end{pmatrix}dx\\&-\sum_{j=1}^{N}\int_0^L H_2(s)\begin{pmatrix}0\\DY'_j(x_o)\\DZ'_j(x_o)\end{pmatrix}q_j(s)\delta'(x-x_o).\begin{pmatrix}0\\DY_i(x)\\DZ_i(x)\end{pmatrix}dx\end{aligned}\qquad\text{éq. 3.4 - 3}$$

3D' where, after integration:

$$\begin{aligned}\mathbf{Q}_i(s)\;&=\sum_{j=1}^{N}\Big\{H_1(s)\big[DY_i(x_o).DY_j(x_o)+DZ_i(x_o).DZ_j(x_o)\big]\\&+H_2(s)\big[DY'_i(x_o).DY'_j(x_o)+DZ'_i(x_o).DZ'_i(x_o)\big]\Big\}\quad q_j(s)\\&=\sum_{j=1}^{N}B_{ij}(s)q_j(s)\end{aligned}\qquad\text{éq. 3.4 - 4}$$

Note::

$DY'_i(x_o)=DRZ_i(x_o)$ and $DZ'_i(x_o)=-DRY_i(x_o)$

3.5 Resolution of the modal problem under flow

the modal problem is solved by supposing, at first approximation, that the diagonal terms of the matrix of transfer of the forces fluid-elastics $[\mathbf{B}(s)]$ are dominating compared to the extra-diagonal terms.

The matrix $[\mathbf{B}(s)]$ being thus reduced to its diagonal, the modal deformed shapes are not disturbed by the taking into account of the coupling fluid-elastic; the only modified parameters are the eigenfrequencies and the reduced dampings modal.

The modal problem under flow breaks up then into N independent scalar problems, solved by a method of the Broyden type:

$$\left(\mathbf{M}_{ii}+\mathbf{M}^{aj}{}_{ii}\right)s^2+\left(\mathbf{C}_{ii}+\mathbf{C}^{aj}{}_{ii}(s)\right)s+\left(\mathbf{K}_{ii}+\mathbf{K}^{aj}{}_{ii}(s)\right)=0$$ **éq. 3.5 - 1**

where $\mathbf{M}^{aj}{}_{ii}$ indicates the generalized mass added by the fluid,

$\mathbf{C}^{aj}{}_{ii}(s)$ indicates the generalized damping added by the fluid,

$\mathbf{K}^{aj}{}_{ii}(s)$ indicates the generalized stiffness added by the fluid.

$\mathbf{M}^{aj}{}_{ii}$, $\mathbf{C}^{aj}{}_{ii}(s)$ and $\mathbf{K}^{aj}{}_{ii}(s)$ are calculated using the relations:

$$\mathbf{M}^{aj}{}_{ii}=+\frac{1}{2}\rho_f D^2 L_p\left[Cm_1\left\{DY_1^2(x_o)+DZ_i^2(x_o)\right\}+L_p{}^2 Cm_2\left\{DY'^2_i(x_o)+DZ'^2_i(x_o)\right\}\right]$$ **éq. 3.5 - 2**

$$\mathbf{C}^{aj}{}_{ii}(s)=-\frac{1}{2}\rho_f DUL_p\left[Cd_1(V_r)\left\{DY_1^2(x_o)+DZ_i^2(x_o)\right\}+L_p{}^2 Cd_2(V_r)\left\{DY'^2_i(x_o)+DZ'^2_i(x_o)\right\}\right]$$ **éq. 3.5-33**

$$\mathbf{K}^{aj}{}_{ii}(s)=-\frac{1}{2}\rho_f U^2 L_p\left[Ck_1(V_r)\left\{DY_1^2(x_o)+DZ_i^2(x_o)\right\}+L_p{}^2 Ck_2(V_r)\left\{DY'^2_i(x_o)+DZ'^2_i(x_o)\right\}\right]$$ **éq. 3.5-44**

$\mathbf{C}^{aj}{}_{ii}$ and $\mathbf{K}^{aj}{}_{ii}$ depend implicitly from s the intermediary on the fallback speed $V_r=\frac{U}{sD}$.

The three quantities necessary to dimension these terms are thus only ρ_f, D and U, L_p being deduced from D thanks to the geometrical property of similarity.

As that was indicated previously, the adimensional coefficients $Cd_1(V_r)$, $Ck_1(V_r)$, $Cd_2(V_r)$ and $Ck_2(V_r)$ result from the empirical correlations identified in experiments on model GRAPPE2.

4 Excitation fluid-elastic acting on two coaxial cylindrical shells under annular flow (example: space tank/envelope of heart)

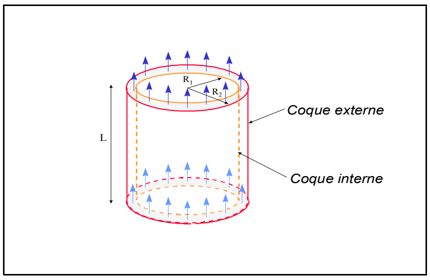
the integration of this model of excitation fluid-elastic in *Code_Aster* was approached in the note of specifications [feeding-bottle. 2]. The note of principle of model MOCCA_COQUE [feeding-bottle. 5] constitutes the theoretical documentation of reference.

4.1 Description of the studied configuration

the studied hardware configuration is made up of two coaxial cylindrical shells, separated by an annular space in which runs out a viscous incompressible monophasic fluid [Figure 4.1 - has]. Flow is done in the direction of the axis of revolution of the cylinders; to fix the notations, one supposes in the continuation of the document that it is the axis $\mathbf{x}$.

One notes:

L	the common length of the two cylindrical shells,
$R_1(\theta,x,t)$	the interior radius of annular space,
$R_2(\theta,x,t)$	the radius external of annular space,
$R(\theta,x,t)$	the average radius $\left(R(\theta,x,t)=\dfrac{R_1(\theta,x,t)+R_2(\theta,x,t)}{2}\right)$,
$H(\theta,x,t)$	annular clearance $\left(H(\theta,x,t)=R_2(\theta,x,t)-R_1(\theta,x,t)\right)$,
$\mathbf{e}_r,\mathbf{e}_\theta,\mathbf{e}_x$	vectors of the base of cylindrical coordinates.



Appear 4.1-a: general diagram coaxial shells

4.2 Stages of computation

- the first stage of computation consists in determining modal base in air of structure. This operation is carried out by the operator `MODE_ITER_SIMULT` . This computation is necessary because the decomposition of the matrix of transfer of the forces fluid-elastics $[\mathbf{B}(s)]$ is expressed in this base.
- The second stage relates to the taking into account of the forces fluid-elastics. It intervenes in operator `CALC_FLUI_STRU` . This stage breaks up into eight sub-tasks:

4.2.1 Preprocessings

1°/ Determination of the characteristic geometrical magnitudes, from the topology of mesh: common length of the two shells, average radius, average annular clearance.
2°/ Characterization of the modal deformed shapes in air: determination of the orders of shell, the principal planes, the wave numbers and the coefficients of deformed shapes of beam associated with each mode of structure, both for the inner shell the outer shell.

4.2.2 Resolution of the modal water problem at rest

3°/ Computation of the mass matrix added by the fluid $\left[\mathbf{M}_{aj}\right]$ in the modal base of structure in air
4°/ Computation of the modal characteristics of water structure at rest while solving:
$\left\{\left(\left[\mathbf{M}_i\right]+\left[\mathbf{M}_{aj}\right]\right)s^2+\left[\mathbf{K}_i\right]\right\}\left(\mathbf{q}\right)=0$

One obtains the new structural features out of water at rest $\mathbf{M}_i^{\ e},\mathbf{K}_i^{\ e},f_i^{\ e}$ (generalized mass and stiffness, eigenfrequency of the mode i) as well as the modal deformed shapes $\psi_i^{\ e}$, expressed in the base in air.
5°/ Computation of the water deformed shapes at rest in physical base, by basic change: $\left[\phi_i^{\ e}\right]=\left[\phi_i^{\ a}\right]\left[\psi_i^{\ e}\right]$

4.2.3 Resolution of the modal problem under flow

For each rate of flow:

6°/ Computation of $\left[B(s)\right]$ in modal base in air.
This computation is carried out by solving the non stationary fluid problem according to the method specified in the paragraph § 4.3.1.
7°/ Computation of the forces fluid-elastics induced by the effects of added damping and stiffness, in water modal base at rest.

$$\left[\mathbf{B}^e(s)\right]={}^t\left[\psi_i^{\ e}\right]\left\{\left[\mathbf{B}(s)\right]-\left[\mathbf{M}_{aj}\right]s^2\right\}\left[\psi_i^{\ e}\right]$$

8°/ Resolution of the modal problem by neglecting the extra-diagonal terms of this last matrix, **by the method of Broyden** (buckles on the sub-tasks 6° and 7°).

$$\mathbf{M}_i^{\ e}s^2+\mathbf{C}_i^{\ e}s+\mathbf{K}_i^{\ e}-\mathbf{B}_{ii}^{\ e}(s)=0$$

Modal characteristics of structure: $\mathbf{M}_i^{\ ec},f_i^{\ ec},\xi_i^{\ ec}$ (generalized mass, eigenfrequency and damping of the mode i , under flow) are given. The modal deformed shapes are supposed to be identical to those out of water at rest.

End of loop on the rates of flow

Note:

- *The computation terms of the matrix of transfer of the forces fluid-elastics requires the resolution of the non stationary fluid problem (sub-task 6°). This resolution is it - even possible only if one beforehand determined certain geometrical magnitudes characteristic of the configuration, as well as the coefficients of the analytical forms of the modal deformed shapes of the structures (preprocessings 1° and 2°).*
- *If the user chooses to carry out the first stage (computation of modal base by the operator `MODE_ITER_SIMULT`) by taking directly into account the effects of added mass, those should not be taken into account by the operator CALC_FLUI_STRU `any more` . For that, key word `MASS_AJOU` of the command `DEFI_FLUI_STRU` must be indicated by "`NON`" . Under - tasks 3° with 7° become then:*

3°/ Computation of the effects of added mass by the fluid, in the modal base of water structure, in order to be able to cut off these effects of the force total fluid-elastic, since the terms of added mass are already taken into account.
4°/ removed Sub-task.
5°/ removed Sub-task.

For each rate of flow

6°/ Computation of the matrix $[\mathbf{B}(s)]$ in water modal base.
7°/ Computation of the forces fluid-elastics induced by the effects of damping and stiffness added in water modal base:

$$\left[\mathbf{B}^e(s)\right]=[\mathbf{B}(s)]-\left[\mathbf{M}_{aj}\right]s^2$$

The sub-tasks 1°, 2° and 8° are not modified.

For each rate of flow

6°/ Computation of the matrix $[\mathbf{B}(s)]$ in water modal base.
7°/ Computation of the forces fluid-elastics induced by the effects of damping and stiffness added in water modal base:

$$\left[\mathbf{B}^e(s)\right]=[\mathbf{B}(s)]-\left[\mathbf{M}_{aj}\right]s^2$$

The sub-tasks 1°, 2° and 8° are not modified.

4.3 Resolution of the non stationary fluid problem

4.3.1 simplifying Assumptions

Some assumptions on the nature of flow make it possible to simplify the non stationary Navier-Stokes equations, at the base of the problem fluid-structure.

H1 It is supposed that flow is the superposition of an average flow steady, obtained when the structures are fixed, and from a non stationary flow induced by the motion of the walls.

H2 It is supposed that structure vibrations are of low amplitude with respect to the thickness of average annular flow.

H3 One supposes that the disturbances velocity induced by vibratory motions are, on average on a radius, primarily directed in the directions θ and x : one supposes thus that vibratory motion induced a helicoid motion of fluid around structures rather than a radial motion compared to these last. These disturbances velocity define order 1.

H4 One supposes finally that the velocity field and of pressure is uniform, with order 1, in the radial direction.

These simplifying assumptions make it possible to solve the fluid problem analytically. The matrix of transfer of the forces fluid-elastics $[\mathbf{B}(s)]$ is deduced from non stationary flow resulting from this resolution.

4.3.2 Analyzes in disturbances

With the help of the assumptions stated previously, the analysis in disturbances of the fluid problem led to search non stationary flow in the form:

$$U_r = 0 + 0 + \text{ordre 2} \quad \textbf{éq. 4.3.2-1}$$

$$U_\theta = 0 + \tilde{u}_\theta(\theta, x, t) + \text{ordre 2} \quad \textbf{éq. 4.3.2-2}$$

$$U_x = \overline{U}(x) + \tilde{u}_x(\theta, x, t) + \text{ordre 2} \quad \textbf{éq. 4.3.2-3}$$

$$P = \overline{P}(x) + \tilde{p}(\theta, x, t) + \text{ordre 2} \quad \textbf{éq. 4.3.2-4}$$

with:

$$R_1 = \overline{R}_1 + \tilde{r}_1(\theta, x, t) \quad \textbf{éq. 4.3.2-5}$$

$$R_2 = \overline{R}_2 + \tilde{r}_2(\theta, x, t) \quad \textbf{éq. 4.3.2-6}$$

One defines the variables $\tilde{h}$ and $\tilde{R}$ as: $\tilde{h} = \tilde{r}_2 - \tilde{r}_1$ and $\tilde{R} = \dfrac{\tilde{r}_2 + \tilde{r}_1}{2}$.

By limiting the development of the Navier-Stokes equations to the first order, one obtains two systems of equations characterizing the steady part and the disturbed part of flow, the second system being a linear system.

The resolution of the steady fluid problem leads thus to:

$$\overline{U}(x) = \overline{U} \text{ constant and } \frac{\partial \overline{P}}{\partial x} = -\frac{1}{\overline{\overline{H}}}\rho \overline{C}_f \overline{U}^2 \quad \textbf{éq. 4.3.2-7}$$

In the equation [**éq. 4.3.2-7**], ρ indicates the density of the fluid and $\overline{C}_f$ the steady part of the coefficient of kinetic friction to the wall. The incompressible fluid being supposed, its density is not broken up partly steady and fluctuating part. C_f is deduced from the model of Nikuradzé characterizing flows in control:

$$C_f = C_{fo}(R_e,\varepsilon) R_e^{m(R_e,\varepsilon)} \text{ with } R_e = \frac{2\overline{H}U_x}{\nu}$$ **éq. 4.3.2-8**

where m indicates the value of an exhibitor, ν indicates the kinematical viscosity of the fluid and ε the surface roughness.

It results from this:

$$\overline{C}_f = \overline{C}_{fo}(\overline{R}_e,\varepsilon)\overline{R}_e^{m(\overline{R}_e,\varepsilon)}$$

$$\widetilde{C}_f = C_f(\widetilde{R}_e) - \overline{C}_f(\widetilde{R}_e) \quad \text{with } \overline{R}_e = \frac{2\overline{H}\overline{U}}{\nu} \text{ and } \widetilde{R}_e = \frac{2\overline{H}\widetilde{u}_x}{\nu}$$

$$= (m+2)\,\overline{C}_f \frac{\widetilde{u}_x}{\overline{U}} + \text{ordre } 2$$

the linear differential connection of a nature 1 characterizing the non stationary part of flow induced by motions of walls is written in the field of Laplace:

$$\begin{cases} \dfrac{\partial \widetilde{u}_x}{\partial x} + \dfrac{1}{\overline{R}}\dfrac{\partial \widetilde{u}_\theta}{\partial \theta} = -\dfrac{\overline{U}}{\overline{H}}\left[\dfrac{\partial \widetilde{h}}{\partial x} + \dfrac{s}{\overline{U}}\widetilde{h}\right] - \dfrac{\overline{U}}{\overline{R}}\left[\dfrac{\partial \widetilde{R}}{\partial x} + \dfrac{s}{\overline{U}}\widetilde{R}\right] \\ \overline{U}\dfrac{\partial \widetilde{u}_\theta}{\partial x} + \left(s + \overline{C}_f \dfrac{\overline{U}}{\overline{H}}\right)\widetilde{u}_\theta + \dfrac{1}{\rho \overline{R}}\dfrac{\partial \widetilde{p}}{\partial \theta} = 0 \\ \overline{U}\dfrac{\partial \widetilde{u}_x}{\partial x} + \left(s + \overline{C}_f (m+2)\dfrac{\overline{U}}{\overline{H}}\right)\widetilde{u}_x + \dfrac{1}{\rho}\dfrac{\partial \widetilde{p}}{\partial x} = \overline{C}_f \left(\dfrac{\overline{U}}{\overline{H}}\right)^2 \widetilde{h} \end{cases}$$ **éq. 4.3.2-9**

Three boundary conditions of input-output make it possible to solve this system. The first of these conditions is obtained by supposing that flow is sufficiently regular upstream of annular space, so that the tangential component velocity of entry can be neglected:

$u_\theta = 0$ in $x = 0$ **éq. 4.3.2-10**

the two others are obtained by applying the conservation equation of kinetic energy, in its quasi-stationary form, between the infinite upstream and the entry of annular space, then between the output of annular space and the infinite downstream. One obtains then respectively, with the order of the disturbances:

$$\begin{cases} \displaystyle\int_{\overline{R_1}}^{\overline{R_2}} \left[\widetilde{p} + \rho\overline{U}\,\widetilde{u}_x\left(1 + \overline{C}_{d_e}\right) + \frac{1}{2}\rho\,\widetilde{C}_{d_e}\overline{U}^2\right]\overline{U}\, r dr = 0 \quad \text{en } x = 0 \\ \displaystyle\int_{\overline{R_1}}^{\overline{R_2}} \left[\widetilde{p} + \rho\overline{U}\,\widetilde{u}_x\left(1 - \overline{C}_{d_s}\right) - \frac{1}{2}\rho\,\widetilde{C}_{d_s}\overline{U}^2\right]\overline{U}\, r dr = 0 \quad \text{en } x = L \end{cases}$$ **éq. 4.3.2-11**

In these statements, $\overline{C}_{d_e}$ and $\overline{C}_{d_s}$ the steady parts of the singular loss ratios represent of load of entry and output. They take into account the dissipation of energy induced, when the walls are fixed, by possible abrupt evolutions of the geometry at the entry or the output of annular space. In most case, these coefficients can be estimated simply using data of the literature (Idel' cik for example). When the geometrical configuration of entry or output is very particular, these coefficients can also be given using a two-dimensional code of mechanics of the fluids adapted under investigation problems to fixed walls, of type N3S.

$\tilde{C}_{d_e}$ and $\tilde{C}_{d_s}$ are the non stationary parts of the singular loss ratios of load. These coefficients take into account the disturbances of the lines of separation induced by structure motions. They can be modelled thanks to a quasi-stationary approach of comparable nature that introduced for the estimate of the coefficient of kinetic friction of wall.

The system [**éq. 4.3.2-9**] is solved analytically, using the limiting conditions [**éq. 4.3.2-10**] and [**éq. 4.3.2-11**], by clarifying the functions $\tilde{h}$ et $\tilde{R}$ characterizing the second member.

The disturbances $\tilde{r}_1(\theta,x,s)$ and $\tilde{r}_2(\theta,x,s)$ defining the motion of the walls, the disturbed parts of annular clearance and the average radius are then defined, in the field of Laplace, by:

$$\tilde{h}(\theta,x,s)=\tilde{r}_2(\theta,x,s)-\tilde{r}_1(\theta,x,s) \quad \textbf{éq. 4.3.2-12}$$

$$\tilde{R}(\theta,x,s)=\frac{\tilde{r}_1(\theta,x,s)+\tilde{r}_2(\theta,x,s)}{2} \quad \textbf{éq. 4.3.2-13}$$

4.3.3 Decomposition on modal base

Is N the number of oscillatory modes of structure in the studied waveband. Decomposition on the basis of motion modal base of the walls is expressed in the following way:

$$\tilde{r}_1(\theta,x,s)=\sum_{i=1}^{N}\mathbf{cos}[k_{1i}(\theta-\theta_{1i})].r_{1i}^{*}(x).\alpha_i(s) \quad \textbf{éq. 4.3.3-1}$$

$$\tilde{r}_2(\theta,x,s)=\sum_{i=1}^{N}\mathbf{cos}[k_{2i}(\theta-\theta_{2i})].r_{2i}^{*}(x).\alpha_i(s) \quad \textbf{éq. 4.3.3-2}$$

where k_{1i} and k_{2i} represent the orders of shell of $^{i\text{ ème}}$ mode for respective motions of the shells internal and external,

θ_{1i} and θ_{2i} make it possible to characterize the principal planes of these modes,

$r_{1i}^{*}(x)$ and $r_{2i}^{*}(x)$ are deducted from the deformed shapes of beam of structures internal and external associated with the mode considered,

and $\alpha_i(s)$ represents generalized displacement.

Note:

The functions $r_{1i}^{*}(x)$ and $r_{2i}^{*}(x)$ are represented, in the frame of the analytical resolution, in the form of linear combinations of sine, cosine, hyperbolic sine and hyperbolic cosine:

$$r_{1i}^{*}(x)=A_{1i}\,\mathbf{cos}\left(\frac{\delta_{1i}}{L}x\right)+B_{1i}\,\mathbf{sin}\left(\frac{\delta_{1i}}{L}x\right)+C_{1i}ch\left(\frac{\delta_{1i}}{L}x\right)+D_{1i}sh\left(\frac{\delta_{1i}}{L}x\right) \quad \textbf{éq. 4.3.3-3}$$

$$r_{2i}^{*}(x)=A_{2i}\,\mathbf{cos}\left(\frac{\delta_{2i}}{L}x\right)+B_{2i}\,\mathbf{sin}\left(\frac{\delta_{2i}}{L}x\right)+C_{2i}ch\left(\frac{\delta_{2i}}{L}x\right)+D_{2i}sh\left(\frac{\delta_{2i}}{L}x\right) \quad \textbf{éq. 4.3.3-4}$$

with δ_{1i} and δ_{2i} wave numbers of $^{i\text{ ème}}$ mode for motions of the shells internal and external respectively.

The solutions of the fluid problem $\tilde{p}, \tilde{u}_x$ et $\tilde{u}_\theta$ are required in the form of decompositions on modal base deduced from those of $\tilde{r}_1$ et $\tilde{r}_2$ clarified by the relations [**éq. 4.3.3-1**] and [éq 4.3.3-2]. One obtains thus, in the field of Laplace:

$$\tilde{p}(\theta,x,s)=\sum_{i=1}^{N}\left(\frac{p_{1i}^{*}(x,s)}{k_{1i}^{2}}\mathbf{cos}[k_{1i}(\theta-\theta_{1i})]+\frac{p_{2i}^{*}(x,s)}{k_{2i}^{2}}\mathbf{cos}[k_{2i}(\theta-\theta_{2i})]\right)\alpha_i(s) \qquad \textbf{éq. 4.3.3-5}$$

$$\tilde{u}_x(\theta,x,s)=\sum_{i=1}^{N}\left(u_{1i}^{*}(x,s)\,\mathbf{cos}[k_{1i}(\theta-\theta_{1i})]+u_{2i}^{*}(x,s)\,\mathbf{cos}[k_{2i}(\theta-\theta_{2i})]\right)\alpha_i(s) \qquad \textbf{éq. 4.3.3-6}$$

$$\tilde{u}_\theta(\theta,x,s)=\sum_{i=1}^{N}\left(\frac{v_{1i}^{*}(x,s)}{k_{1i}}\mathbf{sin}[k_{1i}(\theta-\theta_{1i})]+\frac{v_{2i}^{*}(x,s)}{k_{2i}}\mathbf{sin}[k_{2i}(\theta-\theta_{2i})]\right)\alpha_i(s) \qquad \textbf{éq. 4.3.3-7}$$

4.3.4 Statement of the terms of the fluid matrix of transfer of the elastic forces -

the surface force fluid-elastic $\mathbf{F}$, is the resultant of the field of pressure and the viscous and turbulent stresses exerted by flow on the walls of structure moving.

$$\mathbf{F}=-P\,\mathbf{n}+\tau_\theta\mathbf{t}_\theta+\tau_x\mathbf{t}_x \qquad \textbf{éq. 4.3.4-1}$$

the force generalized fluid-elastic associated with i the ème oscillatory mode of structure $\mathbf{Q}_i(s)$, is written as follows:

$$\mathbf{Q}_i(s)=\int_{S_i}\mathbf{F}.\mathbf{X}_i\,ds_i \qquad \textbf{éq. 4.3.4-2}$$

Where S_i the surface of the walls of structure indicates wet by flow, and the vector $\mathbf{X}_i$ represents I ème vector deformed modal in this statement. The representation of the velocity field and pressure and the representation in the form of a model of wall of the viscous and turbulent stresses exerted on the structure moving make it possible to express the force in the following way generalized $\mathbf{Q}_i(s)$ fluid-elastic:

$$\mathbf{Q}_i(s)=\sum_{j=1}^{N}\mathbf{B}_{ij}(s)\alpha_j(s) \qquad \textbf{éq. 4.3.4-3}$$

with $\mathbf{B}_{ij}(s)=\mathbf{B}_{1ij}(s)+\mathbf{B}_{2ij}(s)$

$\mathbf{B}_{1ij}(s)$ and $\mathbf{B}_{2ij}(s)$ indicate the contributions of the shells respectively interior and external. These contributions are defined by:

$$\begin{aligned}\mathbf{B}_{1ij}(s)= & -\pi\frac{\overline{R}_1}{k_{1i}^{2}}\mathbf{cos}[k_{1i}(\theta_{1i}-\theta_{1j})]\delta_{k_{1i}k_{1j}}\int_0^L\left(p_{1i}{}^{*}(x,s)+\frac{1}{2}\rho\overline{C}_f\overline{U}v_{1i}{}^{*}(x,s)\right)r_{1j}{}^{*}(x)dx\\ & -\pi\frac{\overline{R}_1}{k_{2i}^{2}}\mathbf{cos}[k_{2i}(\theta_{2i}-\theta_{1j})]\delta_{k_{2i}k_{1j}}\int_0^L\left(p_{2i}{}^{*}(x,s)+\frac{1}{2}\rho\overline{C}_f\overline{U}v_{2i}{}^{*}(x,s)\right)r_{1j}{}^{*}(x)dx\end{aligned}$$

éq. 4.3.4-4

and

$$\mathbf{B}_{2ij}(s) = -\pi \frac{\overline{R}_2}{k_{1i}^2} \mathbf{cos}\left[k_{1i}\left(\theta_{1i}-\theta_{2j}\right)\right]\delta_{k_{1i}k_{2j}} \int_0^L \left(p_{1i}{}^*(x,s) + \frac{1}{2}\rho \overline{C}_f \overline{U} v_{1i}{}^*(x,s) \right) r_{2j}{}^*(x) dx \\ -\pi \frac{\overline{R}_2}{k_{2i}^2} \mathbf{cos}\left[k_{2i}\left(\theta_{2i}-\theta_{2j}\right)\right]\delta_{k_{2i}k_{2j}} \int_0^L \left(p_{2i}{}^*(x,s) + \frac{1}{2}\rho \overline{C}_f \overline{U} v_{2i}{}^*(x,s) \right) r_{2j}{}^*(x) dx \quad \text{éq. 4.3.4-5}$$

4.4 Resolution of the modal problem under flow

As one explained in the paragraph [§ 4.2], one solves beforehand the modal problem out of water at rest, in order to take into account the inertial coupling between modes. One considers thus the mass matrix added by the fluid, while calculating $[\mathbf{B}(s)]$ for a mean velocity of flow null. The modal characteristics of the system under flow are then obtained by disturbing the water characteristics at rest. One does not take any more account but of damping and the stiffness added: the terms of added mass previously calculated are cut off from the matrix $[\mathbf{B}(s)]$. The coupling between modes is then neglected; consequently, the modal deformed shapes remain unchanged compared to those out of water at rest. Only parameters disturbed by the setting in the frequency and reduced modal damping. These parameters are calculated by solving N nonlinear equations mode by mode, by implementation of a method of the Broyden type.

5 Axial flow (example: fuel assemblies)

the integration of this model of excitation fluid-elastic in *Code_Aster* was approached in the note of specifications [feeding-bottle. 2]. The note of principle of model MEFISTEAU [feeding-bottle. 6] constitutes the theoretical documentation of reference.

5.1 Description of the studied configuration

One considers a beam of K circular cylinders mobile in bending and subjected to viscous an incompressible fluid flow, limited by a cylindrical rigid enclosure of circular or rectangular section [Figure 5.1-a].

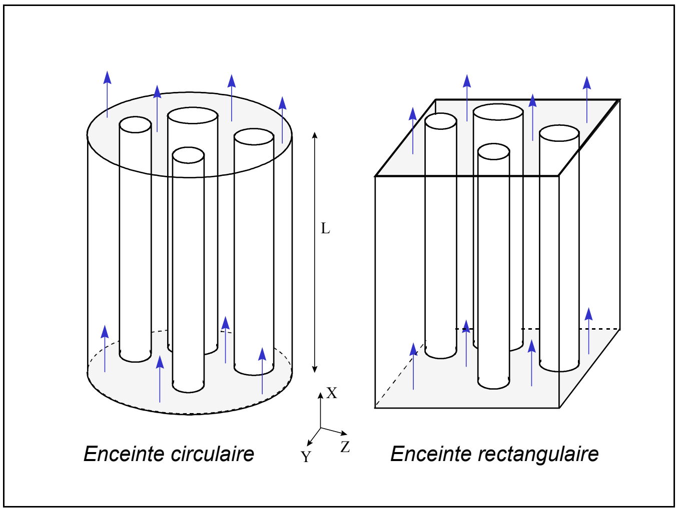


Figure 5.1-a : Beam under axial flow

the cylinders all parallel, are directed along the axis of the enclosure. They have a common length, noted L. To simplify the notations, it is supposed thereafter that $\mathbf{x}$ is the directing axis. The steady flow axial and is supposed to be uniform in each section. The density of the fluid which can be variable along the axis $\mathbf{x}$ (heat gradients), the rate of steady flow also depends on the variable x.

5.2 Stages of computation

- the first stage relates to the determination of modal base in air of the beam. This operation is carried out by the operator `MODE_ITER_SIMULT`. This stage is essential because the forces fluid-elastics are projected on this basis.
- The second phase by the operator relates to the taking into account of the forces fluid-elastics `CALC_FLUI_STRU`. This stage breaks up into 7 sub-tasks:

5.2.1 Preprocessings

1°/ By means of the topology of the mesh, deduction of the coordinates of the centers of the cylinders of the beam then checking of the good provision of the cylinders ones compared to the others (it is checked in particular that there is not overlapping between two cylinders) and compared to the rigid enclosure.

2°/ Determination the length of excitation of the fluid, commune to all the cylinders, as well as associated discretization along the directing axis.

3°/ Constitution of the tables giving the modal deformed shapes in air of each cylinder of the beam, for each mode taken into account for fluid-structure coupling. One interpolates for that the deformed shapes at the points of the discretization determined before.

5.2.2 Resolution of the modal problem under flow

4°/ Resolution of the disturbed fluid problem. The determination of the potential disturbed velocities requires the inversion of linear systems of high natures calling the implementation of the method of Crout.

For each rate of flow

5°/ Computation of the mass matrixes, damping and stiffness added giving the matrix of transfer of the forces fluid-elastics in modal base in air:

$$\left[\mathbf{B}_{ij}(s)\right] = -\left[\mathbf{M}_a\right]s^2 - \left[\mathbf{C}_a\right]s - \left[\mathbf{K}_a\right]$$

$\left[\mathbf{M}_a\right]$ full symmetric; $\left[\mathbf{C}_a\right]\text{et}\left[\mathbf{K}_a\right]$ a priori full and asymmetric.

6°/ Resolution of the modal problem under flow; one solves the complete problem with the vectors and with clean

$$\left\{\left[\mathbf{M}_{ij}\right]s^2 + \left[\mathbf{C}_{ij}\right]s + \left[\mathbf{K}_{ij}\right] - \left[\mathbf{B}_{ij}(s)\right]\right\}.(\mathbf{q}) = (0)$$

One does not neglect the extra-diagonal terms of $\left[\mathbf{B}_{ij}(s)\right]$. After reformulation, the resolution is carried out using algorithm QR: obtaining the masses, frequencies and modal dampings reduced under flow $\mathbf{M}_i^{ec}, f_i^{ec}, \xi_i^{ec}$, complex modal deformed shapes ψ_i^{ec} expressed in the base in air; of these last, one retains only the real part after minimization of the imaginary part (computation of a criterion on the imaginary part).

7°/ Restitution of the deformed shapes under flow in physical base.

$$\left[\varphi_i^{ec}\right] = \left[\varphi_i\right]\left[\psi_i^{ec}\right]$$

$\left[\varphi_i\right]$ is the matrix whose columns are the modal deformed shapes in air, expressed in physical base.

End of loop on the rates of flow

Note:

- *The knowledge of the coordinates of the centers of the cylinders (preprocessing 1°) is necessary to the resolution of the disturbed fluid problem (sub-task 4°). This resolution leads to the estimate of the terms of the matrix of transfer of the forces fluid-elastics (under - task 5°), which utilize the disturbances of pressure and velocity.*
- *The determination a common length of excitation and the creation of an associated discretization (pre - 2° processing) make it possible to define a field of integration on structures for projection of the forces fluid-elastics on modal base. The interpolation of the modal deformed shapes at the same points is thus necessary (preprocessing 3°).*
- *The dynamic behavior of the beam under flow can also be studied using a simplified representation of the beam (with equivalent tubes). The stages of computation for the taking into account of fluid-structure coupling are then identical to those described previously, the only differences appearing in the preprocessings. This second approach is described more precisely in the note [feeding-bottle. 2]. In the stage 1° of the pre - processing, the coordinates of the centers of the cylinders of the beam are then specified directly by the user, who also establishes the correspondence between the cylinders of the beam and the beams of the simplified representation given by the mesh. In the stage 3° of the preprocessings, this correspondence makes it possible to assign to the cylinders of the beam, at the points of discretization determined in the stage 2°, the modal deformed shapes of the beams of the simplified representation.*

5.3 Resolution of the fluid problem non stationary

5.3.1 simplifying Assumption

H1 the field non stationary fluid velocities is analytically given by supposing that disturbed flow $\tilde{\phi}$ is potential in all the fluid field, and that the steady flow is uniform transversely, but function of the axial position x :

$$\mathbf{u}=\overline{\mathbf{U}}+\tilde{\mathbf{u}}=\overline{U}(x)\,\mathbf{x}+\nabla(\tilde{\phi})$$ **éq. 5.3.1-1**

Such a velocity field admits a sliding on the walls of the cylinders which will make it possible to calculate the viscous stress by a friction law.

H2 the motion of the cylinders generates disturbances velocity $\tilde{\mathbf{u}}=\nabla(\tilde{\phi})$ only radially and orthoradialement (assumption of the slender bodies): $\tilde{\mathbf{u}}=\tilde{u}_y\,\mathbf{y}+\tilde{u}_z\,\mathbf{z}$

H3 the field of pressure is broken up into parts steady and disturbed according to $P=\overline{P}+\tilde{p}$
the steady field of pressure depends only on x and its gradient is worth:

$$\frac{d\overline{P}}{dx}(x)=-\rho\overline{U}\frac{d\overline{U}}{dx}(x)-2\rho\frac{C_{fl}}{D_H}\left|\overline{U}\right|\overline{U}+\rho\mathbf{g.x}$$ **éq. 5.3.1-2**

where D_H the hydraulic diamtere of the beam indicates,
C_{fl} indicates the local coefficient of kinetic friction for the steady velocity $\overline{U}$. It depends amongst Reynolds, calculated using the steady velocity $\overline{U}$, of the hydraulic diamtere of the beam and the surface roughness. This coefficient is deduced from the model of Nikuradzé characterizing flows in control;
$\mathbf{g}$ indicate the field of gravity. Its action on the steady field of pressure depends on the slope of the beam $(\mathbf{g.x})$.

5.3.2 Determination of the potential disturbed velocities

One seeks an analytical solution for $\tilde{\phi}\ (r,\theta,x,t)$ in the form of a superposition of elementary singularities which are written:

$$\sum_{n=1}^{N_{tronc}} \left\{ C_{nk}(x,t).r_k^{-n}.\cos(n\theta_k) + D_{nk}(x,t).r_k^{-n}.\sin(n\theta_k)\right\} \quad \textbf{éq. 5.3.2-1}$$

in the center of each cylinder k and:

$$\sum_{n=1}^{N_{tronc}} \left\{ A_n(x,t).r_o^n.\mathbf{cos}(n\theta_o) + B_n(x,t).r_o^n.\mathbf{sin}(n\theta_o)\right\} \quad \textbf{éq. 5.3.2-2}$$

in the center of the rigid enclosure when this one is circular where:

N_{tronc}	indicate the order of truncation of the series of Laurent $(N_{tronc}=3)$,
r_k,θ_k	indicate the polar coordinates in a plane perpendicular to the axis $\mathbf{x}$, centered in the center of the cylinder K,
r_o,θ_o	indicate the polar coordinates in a plane perpendicular to the axis $\mathbf{x}$, centered in the center of the circular rigid enclosure.

Coefficients $C_{nk}(x,t)$,$D_{nk}(x,t)$, $A_n(x,t)$ and $B_n(x,t)$ of the statements [**éq. 5.3.2-1**] and [**éq. 5.3.2 - 2**] are given by applying the boundary condition of nonpenetration:

- on the contour of each mobile cylinder k, this condition is written:

$$\forall\theta_k, \left(\frac{\partial\tilde{\phi}}{\partial r_k}\right)(r_k=R_k) = \frac{Dy_k}{Dt}(x,t)\,\mathbf{cos}(\theta_k) + \frac{Dz_k}{Dt}(x,t)\,\mathbf{sin}(\theta_k)$$

whe re
$y_k(x,t)$ and $z_k(x,t)$ indicate the components of the displacement of neutral fiber of the cylinder K to the X-coordinate x in the reference $(\mathbf{y},\mathbf{z})$,

r_k et θ_k indicate the polar coordinates in the reference $(\mathbf{y},\mathbf{z})$ whose origin is taken in the center of the cylinder k,

R_k indicates of the cylinder k

$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \overline{U}(x)\frac{\partial}{\partial x}$.

- on the contour of a circular rigid enclosure, she is written:

$\forall\theta_o, \left(\frac{\partial\tilde{\phi}}{\partial r_o}\right)(r_o=R_o)=0$ where R_o the radius of the enclosure indicates.

In the case of a rectangular rigid enclosure, this condition is taken into account by a method derived from the method of the "images" [feeding-bottle. 6]; the fluid problem confined by the rectangular enclosure is made equivalent to the problem in infinite medium by creating images of the mobile cylinders of the beam compared to the sides of the enclosure. This method results in introducing new singularities of the form [**éq. 5.3.2-1**], placed at the center of the cylinders "images", in the statement of $\tilde{\phi}$. She does not add however unknown to the problem since the coefficients for these new singularities are derived from those of the mobile cylinders of the beam by the clearance of the images.

Finally, the potential disturbed velocities is written:

$$\tilde{\phi}(r,\theta,x,t)=\sum_{k=1}^{K}\left\{f_k(r,\theta)\frac{Dy_k}{Dt}(x,t)+g_k(r,\theta)\frac{Dz_k}{Dt}(x,t)\right\}$$ **éq. 5.3.2-3**

Where K the number of mobile cylinders of the beam indicates. The functions $f_k(r,\theta)$ et $g_k(r,\theta)$ are linear combinations of $r^{-n}.\cos(n\theta), r^{-n}.\sin(n\theta)$, $r^n.\cos(n\theta)$ and $r^n.\sin(n\theta)$ whose coefficients are determined by the preceding boundary conditions. That requires the resolution of linear systems of high natures and with full matrixes. The inversions are carried out by implementing the method of Crout.

5.3.3 Modelization of the fluid forces

One retains initially the forces due to the disturbances of pressure $\tilde{p}$, connected to the potential velocities disturbed by:

$$\tilde{p}=-\rho\frac{D\tilde{\phi}}{Dt}$$ **éq. 5.3.3-1**

the resultant of the field of pressure disturbed around each mobile cylinder is a linear force $\mathbf{f}_{\tilde{p}}$ acting according to $\mathbf{y}$ et $\mathbf{z}$. This force depends linearly on $\frac{D^2 y_k}{Dt^2}$ and $\frac{D^2 z_k}{Dt^2}$, thus generating terms of mass, damping and stiffness added.

One then takes into account the forces related to the viscosity of the fluid.

In a quasi-static approach, one considers the action of the fluid velocity field $(\overline{\mathbf{U}}+\tilde{\mathbf{u}})$ around a cylinder at time t : in the reference related to the cylinder, flow, velocity $\overline{U}$ to order 0, presents an incidence compared to the cylinder which is function of the disturbances velocity and motion of the cylinder him - even. It results from it a force from drag and a force of bearing pressure. One shows that the components following $\mathbf{y}$ et $\mathbf{z}$ of the resulting linear force $\mathbf{f}_v$ are written, for the cylinder ℓ:

$$(\mathbf{f}_v^{\ell})_y=-\rho R_\ell|\overline{U}|\pi C_{fl}\left(\frac{\partial y_\ell}{\partial t}-\overline{\overline{u}}_y^{\ell}\right)-\rho R_\ell|\overline{U}|\pi C_p\left(\frac{D y_\ell}{Dt}-\overline{\overline{u}}_y^{\ell}\right)$$ **éq. 5.3.3-2**

$$(\mathbf{f}_v^{\ell})_z=-\rho R_\ell|\overline{U}|\pi C_{fl}\left(\frac{\partial z_\ell}{\partial t}-\overline{\overline{u}}_y^{\ell}\right)-\rho R_\ell|\overline{U}|\pi C_p\left(\frac{Dz_\ell}{Dt}-\overline{\overline{u}}_z^{\ell}\right)$$ **éq. 5.3.3-3**

where C_p indicates the slope with incidence null coefficient of bearing pressure around a cylinder very slightly tilted (C_p = 0,08).

$\overline{\overline{u}}_y$ and $\overline{\overline{u}}_z$ the averages of the disturbances velocity indicate along the axes $\mathbf{y}$ and $\mathbf{z}$ around the cylinders, which depend linearly on $\frac{Dy_k}{Dt}$ and $\frac{Dz_k}{Dt}$ (cf [**éq. 5.3.2-3**]).

These forces generate terms of added damping and stiffness.

One finally takes into account the action of the steady field of pressure on deformed mobile structures. One shows that the resulting linear force $\mathbf{f}_{\overline{p}}^{\ell}$ on the cylinder ℓ has as components, with order 1:

$$\left(\mathbf{f}_{\overline{p}}^{\ell}\right)_y = \pi R_\ell^2 \frac{\partial}{\partial x}\left(\overline{P}\frac{\partial y_\ell}{\partial x}\right) \quad \textbf{éq. 5.3.3-4}$$

$$\left(\mathbf{f}_{\overline{p}}^{\ell}\right)_z = \pi R_\ell^2 \frac{\partial}{\partial x}\left(\overline{P}\frac{\partial z_\ell}{\partial x}\right) \quad \textbf{éq. 5.3.3-5}$$

These forces generate only terms of added stiffness and any coupling between cylinders.

The statements [**éq. 5.3.3-1**], [**éq. 5.3.3-2**] and [**éq. 5.3.3-3**] highlight the need for solving the disturbed fluid problem before estimating the forces fluid-elastics.

5.3.4 Statement of the terms of the matrix of transfer of the elastic forces fluid -

Summary of the linear forces

For each cylinder ℓ, the forces fluid-elastics is written according to $\mathbf{y}$ and $\mathbf{z}$:

$$\mathbf{f}_\ell = \mathbf{f}_{\overline{p}}^{\ell} + \mathbf{f}_{v}^{\ell} + \mathbf{f}_{p}^{\ell} \quad \textbf{éq. 5.3.4-1}$$

and are linear combinations of:

$$\left(\frac{\partial y_k}{\partial t}, \frac{\partial^2 y_k}{\partial t^2}, \frac{\partial^2 y_k}{\partial t \partial x}, \frac{\partial^2 y_k}{\partial x^2}, \frac{\partial z_k}{\partial t}, \frac{\partial^2 z_k}{\partial t^2}, \frac{\partial^2 z_k}{\partial t \partial x}, \frac{\partial^2 z_k}{\partial x^2}\right)(k = 1, K)$$

Decomposition of motion on modal base

the motion of the beam of cylinders is broken up according to N modes of vibration into air. One notes φ_j^k ($1 \leq \mathrm{k} \leq \mathrm{K}$ and $1 \leq \mathrm{j} \leq \mathrm{N}$) the deformed shapes following $\mathbf{y}$ et $\mathbf{z}$ of the cylinder k corresponding to the J ème mode of the beam. The components of the displacement of neutral fiber of the cylinder k to the X-coordinate x can then be written:

$$y_k(t) = \sum_{j=1}^{N} \mathbf{q}_j(t) \varphi_j^k(x).\mathbf{y} \quad \textbf{éq. 5.3.4-2}$$

$$z_k(t) = \sum_{j=1}^{N} \mathbf{q}_j(t) \varphi_j^k(x).\mathbf{z} \quad \textbf{éq. 5.3.4-3}$$

where $(\mathbf{q}) = (\mathbf{q}_j)_{j=1,N}$ is the vector of generalized displacements.

Projection of the forces on modal base

- One notes $\mathbf{F}_i(t)$ the projection of the forces fluid-elastics according to I ème mode of the beam.

$$\mathbf{F}_i(t) = \sum_{k=1}^{K} \int_0^L \mathbf{f}_k(x,t).\varphi_i^k(x)dx \qquad \text{éq. 5.3.4-4}$$

$\mathbf{F}_i(t)$ is a linear combination of $(\mathbf{q}_j, \dot{\mathbf{q}}_j, \ddot{\mathbf{q}}_j)_{j=1,N}$

- One notes $\mathbf{F}(t)$ the vector of the modal forces fluid-elastics: $\mathbf{F}(t) = (\mathbf{F}_i(t))_{i=1,N}$ who is written:

$$\mathbf{F}(t) = -[\mathbf{M}_a](\ddot{\mathbf{q}}(t)) - [\mathbf{C}_a](\dot{\mathbf{q}}(t)) - [\mathbf{K}_a](\mathbf{q}(t)) \qquad \text{éq. 5.3.4-5}$$

Wher e $[\mathbf{M}_a]$ indicates the matrix of the terms of added mass by the fluid,

$[\mathbf{C}_a]$ indicates the matrix of the terms of damping added by the fluid,

$[\mathbf{K}_a]$ indicates the matrix of the terms of stiffness added by the fluid.

These matrixes are square real of order N and their terms are independent of the motion of structures. The matrix $[\mathbf{M}_a]$ is symmetric; the matrixes $[\mathbf{C}_a]$ and $[\mathbf{K}_a]$ are not it necessarily.

- The projection of the equations of motion on modal base provides:

$$([\mathbf{M}_{ii}]+[\mathbf{M}_a])(\ddot{\mathbf{q}}(t)) + ([\mathbf{C}_{ii}]+[\mathbf{C}_a])(\dot{\mathbf{q}}(t)) + ([\mathbf{K}_{ii}]+[\mathbf{K}_a])(\mathbf{q}(t)) = (0) \qquad \text{éq. 5.3.4-6}$$

where $[\mathbf{M}_{ii}],[\mathbf{C}_{ii}]\text{et}[\mathbf{K}_{ii}]$ the mass matrixes indicate, of depreciation and structure stiffness in air; these matrixes are of order N and diagonals.

In the field of Laplace, the relation [**éq. 5.3.4-6**] becomes:

$$([\mathbf{M}_{ii}]+[\mathbf{M}_a])s^2 + ([\mathbf{C}_{ii}]+[\mathbf{C}_a])s + ([\mathbf{K}_{ii}]+[\mathbf{K}_a])(\mathbf{q}(s)) = (0) \qquad \text{éq. 5.3.4-7}$$

- One introduces then the matrix of transfer of the forces fluid-elastics $[B(s)]$ defined by:

$$[\mathbf{B}(s)] = -[\mathbf{M}_a]s^2 - [\mathbf{C}_a]s - [\mathbf{K}_a] \qquad \text{éq. 5.3.4-8}$$

And one finds the relation [**éq. 1.2-1**] of the paragraph [§ 1.2]:

$$([\mathbf{M}_{ii}]s^2 + [\mathbf{C}_{ii}]s + [\mathbf{K}_{ii}] - [\mathbf{B}(s)])(\mathbf{q}(s)) = (0)$$

5.4 Resolution of the modal problem under flow

the modal problem under flow is formulated by the relation [**éq. 5.3.4-7**] of the preceding paragraph. This problem is solved after rewriting in the form of a standard problem to the vectors and the eigenvalues of the type $[\mathbf{A}](\mathbf{X}) = \lambda(\mathbf{X})$.

The new formulation is the following one:

$$\begin{bmatrix} [0] & [Id] \\ -([\mathbf{M}_{ii}]+[\mathbf{M}_a])^{-1}([\mathbf{K}_{ii}]+[\mathbf{K}_a]) & -([\mathbf{M}_{ii}]+[\mathbf{M}_a])^{-1}([\mathbf{C}_{ii}]+[\mathbf{C}_a]) \end{bmatrix} \begin{pmatrix} \mathbf{q} \\ s\mathbf{q} \end{pmatrix} = s \begin{pmatrix} \mathbf{q} \\ s\mathbf{q} \end{pmatrix}$$

éq. 5.4 - 1

Note:

1) *One doubles the dimension of the problem compared to that of the initial problem.*
2) *The properties of the matrixes* $[\mathbf{M}_{ii}]$ *and* $[\mathbf{M}_a]$ *allow the inversion.*

The resolution of this problem is done by means of algorithm QR. The moduli implemented by the operator CALC_FLUI_STRU are the same ones as those used by MODE_ITER_SIMULT.

The problem with the clean elements that one solves is a complex problem. One thus obtains an even number of combined complex eigenvalues two to two. One preserves only those of which the imaginary part is positive or null.

The eigenvectors are complex, defined except for constant a complex multiplicative. As one takes into account only real modes, it is initially a question of determining, for each eigenvector, the constant which minimizes the imaginary part of the vector compared to its real part, within the meaning of the euclidian norm. The eigenvectors are then redefined compared to this norm. Taking into account standardization used, it is then possible to preserve in the concept mode_meca only the real part of the eigenvectors. One restores however, in the message file , of the indicators on the relationship between imaginary part and real part of the eigenvectors thus normalized, so that the user can consider skew introduced by not taken into account of the imaginary part of the normalized vectors.

5.5 Taken into account of the presence of the grids of the tube bundle

The modelization described previously, of the forces induced by an axial flow on a beam of cylinders, does not take into account the presence of the grids of the beam (for example, grids of mixture and maintenance of the fuel assemblies). A comparison between this model and tests carried out on the model CHAIR (in the configuration of a beam of nine flexible tubes comprising a grid) is presented in a note of synthesis [feeding-bottle. 8]: it is noted that the coupling fluid-elastic between the grid and axial flow is not negligible and that it generates an increase in the reduced modal damping of the tubes. The object of this paragraph is the description of the additional effects due to the grids and their taking into account in the model MEFISTEAU.

5.5.1 Description of the configuration of the grids

One restricts here the study with two types of grids:

- the grids of maintenance which are located at the ends of the beam,
- the grids of mixture which are distributed between the grids of maintenance.

The grids all are positioned perpendicular to the beam of cylinders and are appeared as a prismatic network to square base on side d_g and height h_g (along the axis x of the cylinders). The grids of the same type are characterized by identical dimensions.

5.5.2 Additional stages of computation

the first additional stage by the operator relates to the specification of the type of configuration of grids DEFI_FLUI_STRU , then the checking of the good provision of the grids the ones compared to the others, and the ends of the beam.

The second phase relates to the resolution of the modal problem under flow. In the loop on the rates of flow, the matrix of transfer of the forces fluid-elastics in modal base in air is supplemented by the added computation of a damping matrix and an added stiffness matrix, been dependant on the grids.

5.5.3 Modelization of the fluid forces exerted on the grids

Computation of the jump of pressure

First of all, the presence of grids disturbs the steady field of pressure $\overline{P}(x)$; one regards each grid as a singularity involving a jump of pressure, whose statement is put in the form:

$$\Delta\overline{P}(x_g)=\frac{1}{2}\rho_g(x_g)\overline{U}_g^2(x_g)K_g(x_g)$$ **éq. 5.5.3-1**

where K_g indicates the loss ratio of load due to the grid,

$\overline{U}_g$ indicates the steady velocity of flow on the level of the grid,

ρ_g indicates the density of flow on the level of the grid,

x_g indicates the axial position of the medium of the grid along the beam.

The density $\rho_g(x_g)$ is calculated by linear interpolation of the profile of density $\rho(x)$ of flow in the absence of grid. The steady velocity $\overline{U}_g(x_g)$ is calculated pursuant to the conservation of the mass throughput, which results in the following equation:

$$\rho_g(x_g)\overline{U}_g(x_g)A_{Fg}=\rho_o\overline{U}_o A_F$$

Computation of the specific fluid forces exerted on each grid

According to the same quasi-static approach as that carried out in the paragraph [§5.3.3], one compared to the shows that the action of the fluid $\left(\overline{\mathbf{U}}+\tilde{\mathbf{u}}\right)$ velocity field around a grid implies a force of drag and a force of bearing pressure, according to the incidence of flow grid. The components **there** and **Z** of the resulting specific force **F** $_g$ are thus written, for each basic cell k of

where ρ_o and $\overline{U}_o$ indicate respectively the profile of density and steady velocity of flow in foot of beam,

A_F the fluid section of the beam in the absence of grid indicates,

A_{Fg} indicates the fluid section of the beam on the level of the grid: $A_{Fg}=A_F-A_g$ with A_g the solid section of the grid.

One from of deduced the statement:

$$\overline{U}_g(x_g)=\frac{1}{\left(1-\frac{A_g}{A_F}\right)}\frac{1}{\rho_g(x_g)}\rho_o\overline{U}_o$$

The loss ratio of load K_g is calculated from the statement of the total hydrodynamic force which applies to the grid, and we obtain:

$$K_g = \frac{1}{A_F}\left[A_g C_{dg}(x_g) + \left(1 - \frac{A_g}{A_F}\right)^2 h_g P_m C_{fl}(x_g)\right]$$ **éq. 5.5.3-2**

the 1st term (in $A_g C_{dg}$) comes from the drag load; $C_{dg}(x_g)$ is the drag coefficient of the grid. The 2nd term (in $P_m C_{fl}$) is an effort corrector term of friction applied to the beam alone to the altitude of the grid (P_m is the wet perimeter of the beam in the absence of grid).
By introducing the statement [**éq. 5.5.3-2**] in the relation [**éq. 5.5.3-1**], one thus obtains the statement of the jump of pressure $\Delta\overline{P}(x_g)$ for each grid of altitude x_g . This jump of pressure is taken into account on the level of the computation of the steady field of pressure $\overline{P}(x)$, in the following way:

$$\overline{P}(x_{i+1}) = \overline{P}(x_i) - \Delta\overline{P}(x_g) \qquad \forall x_g \in [x_{i+1}, x_i]$$

roast:

$$\left(\mathbf{f}_g\right)_y = -\tfrac{1}{2}\rho_g\left|\overline{U}_g\right|\frac{A_g}{K}\left(C_{dg}\left[\frac{\partial y_k}{\partial t} - \overline{\tilde{u}}_y^k\right] + C_{pg}\left[\frac{Dy_k}{Dt} - \overline{\tilde{u}}_y^k\right]\right)$$

$$\left(\mathbf{f}_g\right)_z = -\tfrac{1}{2}\rho_g\left|\overline{U}_g\right|\frac{A_g}{K}\left(C_{dg}\left[\frac{\partial z_k}{\partial t} - \overline{\tilde{u}}_z^k\right] + C_{pg}\left[\frac{Dz_k}{Dt} - \overline{\tilde{u}}_z^k\right]\right)$$

whe re C_{pg} indicates the slope with incidence null coefficient of bearing pressure around a grid very slightly tilted.

$\frac{A_g}{K}$ indicate the solid section of the basic cell k of the grid (which understands some K).

These forces will thus generate additional terms of added damping and stiffness, that one obtains after modal decomposition of the motion and projection of these forces on modal base.

5.6 Taken into depreciation account out of fluid at rest

Until now, the damping brought to a tube bundle by the presence of a fluid at rest was not taken into account in the modelization. One thus proposes here a model of damping out of fluid at rest, whose appendix 1 of the note of synthesis of the tests CHAIR [bib8] constitutes documentation of reference.

5.6.1 Modelization of the fluid force at rest exerted on a tube bundle

the method of calculating of fluid damping at rest which is put in work here, is a generalization of the method of CHEN [feeding-bottle. 9].

It is a question of calculating the resultant force on each tube of the stresses due to the shears in the boundary layer. It is a nonlinear problem because the fluid damping coefficient depends on the frequency One thus introduces following simplifications:

- the problem is written using the frequencies out of water at rest calculated without taking into account fluid damping,
- one neglects the coupling between modes.

The linear force f_i^k being exerted on the tube k subjected to a harmonic motion of the beam following the mode i to the frequency f_i is given by the following relation:

$$f_i^k = \rho \left| U_i^k \right| U_i^k R_k C_{Dki}$$ **éq. 5.6.1-1**

where U_i^k the velocity of sliding between the tube and k the fluid at rest indicates, on both sides of the boundary layer, defined by:

$$U_i^k = U_{im}^k \dot{\mathbf{q}}_i(t)$$ **éq. 5.6.1-2**

with $q_i(t) = \sin(2\pi f_i t)$ and

U_{im}^k depends on the averages $\overline{\overline{u}}_y$ and $\overline{\overline{u}}_z$ the disturbances velocity around the cylinders, calculated beforehand by the model.

C_{Dki} ad infinitum indicate the drag coefficient of a cylinder of R_k radius, subjected to a harmonic flow of amplitude $\left| U_i^k \right|_{\max} = \left| U_{im}^k \right| 2\pi f_i$, and is defined by:

$$C_{Dki} = \frac{f_i 2R_k}{\left| U_i^k \right|_{\mathbf{max}}} \frac{3\pi^3}{2} \sqrt{\frac{\nu}{\pi f_i (2R_k)^2}}$$ **éq. 5.6.1-3**

where ν the kinematical viscosity of the fluid indicates.

The relation obtained by replacing [**éq. 5.6.1-2**] and [**éq. 5.6.1-3**] in the equation [**5.6.1-1**] is linearized by a development in Fourier series (of the term $\left| \dot{\mathbf{q}}_i(t) \right| \dot{\mathbf{q}}_i(t)$) whose one retains only the first term; it comes:

$$f_i^k \approx 2\pi(2R_k)\rho U_{im}^k \sqrt{\pi f_i}\, \dot{\mathbf{q}}_i(t)$$

Projection on modal base

By projection on modal base and by neglecting the coupling between modes, one obtains the generalized force being exerted on the tube bundle following the mode i :

$$\mathbf{F}_i(t) = \sum_{k=1}^{K} \int_0^L \vec{f_i^k} . \vec{\varphi_i^k}(z) dz \cong \sum_{k=1}^{K} 2\pi(2R_k)\sqrt{\pi f_i} \left[\int_0^L \rho \sqrt{\nu}\, \vec{U_{im}^k} . \vec{\varphi_i^k}(z) dz \right] \dot{\mathbf{q}}_i(t)$$

$\mathbf{F}_i(t)$ is thus proportional to $\dot{\mathbf{q}}_i(t)$ and the associated force vector modal $\mathbf{F}(t)=(\mathbf{F}_i(t))_{i=1,N}$ is put in the form:

$$\mathbf{F}(t)=-[\mathbf{C}_a](\dot{\mathbf{q}}(t))$$

where $[\mathbf{C}_a]$ the damping matrix indicates added by the fluid at rest.

6 GAY

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7 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
8.4	T.Kestens EDF-R&D/MFEE	initial Text
9.2	A.Adobes, E. Longatte EDF-R&D/MFEE	Addition of precise details concerning the mode of computation of instability fluid-elastic by the method of Connors and the method of the correlations fluid-elastics.