# Homogenization of a network of beams bathing in a Summarized

**fluid:**

This note describes a model obtained by a method of homogenization to characterize the vibratory behavior of a periodic network of tubes bathed by an incompressible fluid. Then the development of a finite element associated with this homogenized model is presented.

The tubes are modelled by beams of Eulerian and the fluid by a model with potential.

This modelization is accessible in the command AFFE_MODELE by choosing modelization `3D_FAISCEAU`.

# Contents

# 1 Introduction

Into nuclear industry, certain structures consist of networks quasi-periodicals of tubes bathed by fluids: the “combustible” assemblies, the steam generators,… to determine the vibratory behavior such structures, the classical approach (each tube is modelled, the volume occupied by the fluid is with a grid) is expensive and tiresome even impracticable (in particular, development of a complicated mesh containing a large number of nodes). The studied structures presenting a character quasi-periodical, it seems interesting to use methods of homogenization.

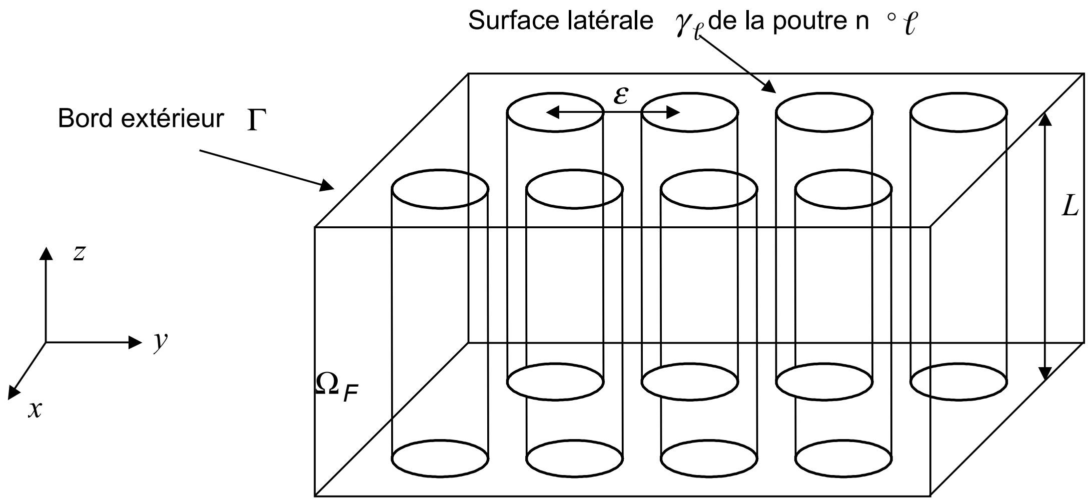
Techniques of homogenization applied to a network of tubes bathed by a fluid were with various already elaborate recoveries [bib1], [bib5], [bib4]. The models obtained differ by the assumptions carried out on the fluid (compressibility, initial velocity of flow, viscosity). According to the allowed assumptions, the action of the fluid on the network of tubes corresponds to an added mass (lowers frequencies of vibration compared to those given in absence of fluid), with a damping even added to an added stiffness [bib5].

At the beginning, of the finite elements associated with two-dimensional models (network of runners bathed by a fluid) were elaborate [bib2]. To study the three-dimensional problems (network of tubes), a solution to consist in projecting motion on the first mode of bending of the beams [bib4]. Later on, of the finite elements three-dimensional were developed [bib3], [bib8].

# 2 Initial physical problem

## 2.1 Description of the problem

One considers a set of identical beams, of axis $z$ , laid out periodically (either $\varepsilon$ the period of space). These beams are located inside an enclosure filled with fluid (see [fig 2.1-a]). One wishes to characterize the vibratory behavior of such a medium, by considering for time only the effect of added mass of the fluid which is dominating [bib6].




**Appear 2.1-a**

## 2.2 Assumptions of modelization

One considers that the fluid is a true fluid initially at rest, incompressible. As the assumption of small displacements around the equilibrium position was carried out (fluid initially at rest), the field of displacement of the fluid particles is irrotational so that there exists a potential of displacement of the noted fluid $\Phi$ . There is no fluid flux through external surface $\Gamma$ .

It is considered that the beams are homogeneous and with constant section according to $z\in]0,L[$ . To model the beams, the model Eulerian is used and the motions of bending are only taken into account. The section of beam is rigid and the displacement of any point of the section is noted:

$\mathbf{s}^l$ the bending of the beam n° $l$ $\left(\mathbf{s}^l(z)=\left(\mathbf{s}_x^l(z),\mathbf{s}_y^l(z)\right)\right)$ .

The beams are embedded at their two ends.

The variational form of the vibro-acoustic problem fluid-structure (conservation of the mass, dynamic equation of each tube) is written:

$$\int_{\Omega_F}\nabla\Phi.\nabla\Phi^*=\sum_l\int_{\gamma_l}\left(\mathbf{s}^l\cdot\mathbf{n}\right)\Phi^* \qquad \forall\Phi^*\in V_\Phi \qquad \textbf{éq 2.2-1}$$

$$\int_0^L\rho_S S.\frac{\partial^2\mathbf{s}^l}{\partial t^2}\cdot\mathbf{s}^{l*}+\int_0^L E\,\mathbf{I}\cdot\frac{\partial^2\mathbf{s}^l}{\partial z^2}\cdot\frac{\partial^2\mathbf{s}^{l*}}{\partial z^2}=-\int_0^L\left(\int_{\gamma_l}\rho_F.\frac{\partial^2\Phi}{\partial t^2}\cdot\mathbf{n}\right)\mathbf{s}^{l*} \qquad \forall\mathbf{s}^{l*}\in V_s \qquad \textbf{éq 2.2-2}$$

with:

$$V_s=\left(H_0^2(]0,L[)\right)^2 \text{ and } V_\Phi=H^1\left(\Omega_F\right)$$

where:

- $\mathbf{n}$ is the norm entering to the beam n° $l$ ,
- $\rho_F$ is the constant density of the fluid in all the field,
- $\rho_S$ is the density of the material constituting the beam,
- $S$ is the section of the beam,
- $E$ is the Young modulus,
- $\mathbf{I}$ is the tensor of inertia of the section of the beam.

The second member of the equation [éq 2.2-2] represents the forces exerted on the beam by the fluid. The pressure $p$ of the fluid is related to the potential of displacement by: $p=-\rho_F\frac{\partial^2\Phi}{\partial t^2}$ . In the same way, the second member of the equation [éq 2.2-1] represents fluid flux induced by motions of the beams. At the border of each beam $l$ one a: $\mathbf{s}^l\cdot\mathbf{n}=\nabla\Phi\cdot\mathbf{n}$ .

This formulation leads to an asymmetric system matric, which is not very convenient, in particular at the time of the search for modes of vibration.

# 3 Problem homogenized

## 3.1 homogenized Problem obtained

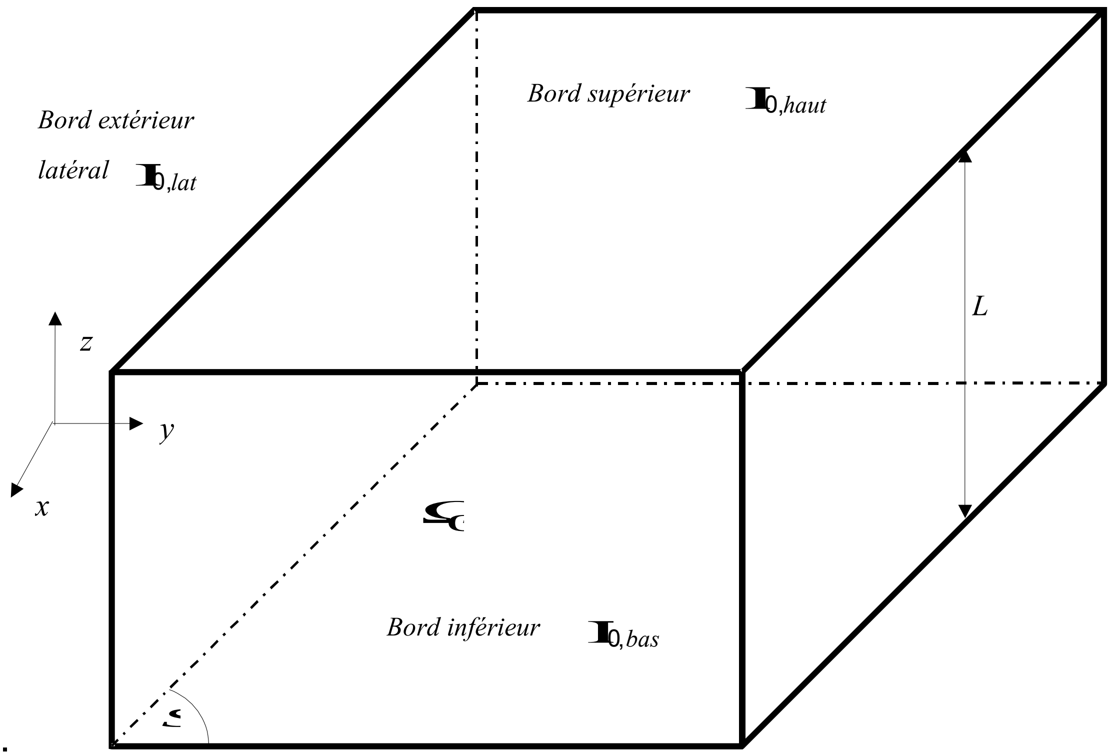
to take account of the periodic character of the studied medium, one uses a method of homogenization based in this precise case on an asymptotic development of the variables intervening in the physical starting problem. With regard to the operational approach, one returns the reader to the following references [bib2], [bib4], [bib5], [bib6]. One will be satisfied here to state the got results.

In the homogenized medium $\Omega_0$ (see [fig 3.1-a]), the two following homogenized variables are considered: $\mathbf{s}_0$ (displacement of the beams) and $\Phi_0$ (potential of displacements of the fluid). In variational form, these variables are connected by the equations of natural vibrations :

$$\begin{cases} \int_{\Omega_0} \mathbf{A} . \nabla \Phi_0 . \nabla \phi^* = - \int_{\Omega_0} \mathbf{D} . \mathbf{s}_0 . \phi^* & \forall \phi^* \in V_\Phi^{\text{hom}} \\ \int_{\Omega_0} \mathbf{M} . \frac{\partial^2 \mathbf{s}_0}{\partial t^2} \mathbf{s}^* + \int_{\Omega_0} \mathbf{K} . \frac{\partial^2 \mathbf{s}_0}{\partial z^2} \frac{\partial^2 \mathbf{s}^*}{\partial z^2} = \rho_F \int_{\Omega_0} \mathbf{D} . \nabla \frac{\partial^2 \Phi_0}{\partial t^2} . \mathbf{s}^* & \forall \mathbf{s}^* \in V_s^{\text{hom}} \end{cases}$$ **éq 3.1-1**

where:

$$V_s^{\text{hom}} = L^2(\Omega_0)^2 \times H_0^2(\Omega_0)^2$$
$$\text{où} \quad \Omega_0 = S \times ]0, L[$$
$$H_0^2(\Omega_0) = \{ v ; \ \forall (x, y) \in S \quad z \to v(x, y, z) \ \in \ H_0^2(]0, L[) \}$$




**Appear 3.1-a**

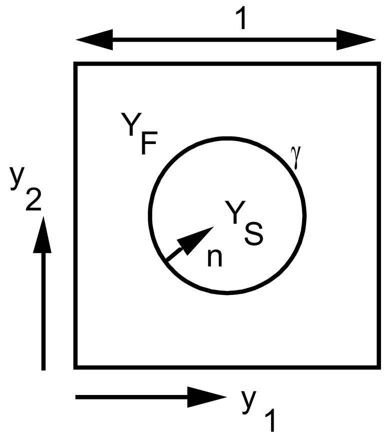



**Figure 3.1-b**

the various tensors which intervene in [éq 3.1-1] are defined using two functions in the following way $\chi_\alpha(\alpha=1,2)$ :

$$\mathbf{B}=(b_{ij})=\frac{1}{|Y|}\begin{pmatrix}\int_{Y_F}\frac{\partial\chi_1}{\partial y_1} & \int_{Y_F}\frac{\partial\chi_1}{\partial y_2} & 0\\ \int_{Y_F}\frac{\partial\chi_2}{\partial y_1} & \int_{Y_F}\frac{\partial\chi_2}{\partial y_2} & 0\\ 0 & 0 & 0\end{pmatrix}\qquad \mathbf{A}=(a_{ij})=\left(\frac{|Y_F|}{|Y|}\delta_{ij}-b_{ij}\right)$$ **éq 3.1-2**

$$\mathbf{D}=(d_{ij})=\mathbf{B}+\frac{1}{|Y|}\begin{pmatrix}Y_S & 0 & 0\\ 0 & Y_S & 0\\ 0 & 0 & 0\end{pmatrix}\qquad \mathbf{M}=(m_{ij})=\rho_F\mathbf{B}+\frac{\mu^2}{|Y|}\begin{pmatrix}\rho_S S & 0 & 0\\ 0 & \rho_S S & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$\mathbf{K}=(k_{ij})=\frac{E\mu^2}{|Y|}\begin{pmatrix}I_{xx} & I_{xy} & 0\\ I_{xy} & I_{yy} & 0\\ 0 & 0 & 0\end{pmatrix}$$ **éq 3.1-3**

where $|Y_F|$ and $|Y_S|$ respectively represent the areas of the fluid fields and structure of the basic cell of reference (cf [fig 3.1-b]). $|Y|$ represent the sum of the two preceding areas. The basic cell of reference is homothetic of ratio $\mu$ to the real cell of periodicity of the heterogeneous medium.

The two functions $\chi_\alpha(\alpha=1,2)$ are solutions of a two-dimensional problem, called cellular problem. On the basic cell of reference, the functions $\chi_\alpha(\alpha=1,2)$ are defined by:

$$\int_{Y_F}\nabla\chi_\alpha.\nabla v=\int_\gamma n_\alpha.v\qquad\forall v\in V$$
$$\int_{Y_F}\chi_\alpha=0\quad(\text{pour avoir une solution unique})$$ **éq 3.1-4**

where:

$$V=\{\, v\in H_1(Y_F),\quad v(y)\ \text{ périodique en } y \text{ de période } 1\,\}$$

**Note:**

*It is shown that the two-dimensional part of $\mathbf{B}$ is symmetric and definite positive [bib5].*

**Note:**

*In the matrix $\mathbf{M}$, the term $\rho_F \mathbf{B}$ plays the part of a specific mass matrix added to each beam in its cell.*

**Note:**

*For the various tensors, one can put in factor the multiplicative end $\frac{1}{|Y|}$. It was added in order to obtain the "good one masses" tubes in absence of fluid. One has then $\int_{\Omega_0} \mathbf{M}\, dV$ = mass of the tubes component $\Omega_0$.*

## 3.2 Matricial problems

By discretizing the problem [éq 3.1-1] by finite elements, one lead (with obvious notations) to the following problem:

$$\begin{cases} \hat{\mathbf{A}}\,\Phi_0=-\hat{\mathbf{D}}\,\mathbf{s}_0 \\ \hat{\mathbf{M}}\dfrac{\partial^2 \mathbf{s}_0}{\partial t^2}+\hat{\mathbf{K}}\,\mathbf{s}_0=\rho_F\,\hat{\mathbf{D}}^{\mathbf{T}}\dfrac{\partial^2 \Phi_0}{\partial t^2} \end{cases} \quad \text{éq 3.2-1}$$

what can be put in the form (one pre - multiplies the first equation by $\rho_F$ ):

$$\tilde{\mathbf{M}}\begin{pmatrix} \dfrac{\partial^2 \mathbf{s}_0}{\partial t^2} \\ \dfrac{\partial^2 \Phi_0}{\partial t^2} \end{pmatrix}+\tilde{\mathbf{K}}\begin{pmatrix} \mathbf{s}_0 \\ \Phi_0 \end{pmatrix}=\begin{pmatrix} \hat{\mathbf{M}} & -\rho_F\,\hat{\mathbf{D}}^{\mathbf{T}} \\ -\rho_F\,\hat{\mathbf{D}} & -\rho_F\,\hat{\mathbf{A}} \end{pmatrix}\begin{pmatrix} \dfrac{\partial^2 \mathbf{s}_0}{\partial t^2} \\ \dfrac{\partial^2 \Phi_0}{\partial t^2} \end{pmatrix}+\begin{pmatrix} \hat{\mathbf{K}} & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \mathbf{s}_0 \\ \Phi_0 \end{pmatrix}=0 \quad \text{éq 3.2-2}$$

**Note::**

*The problem obtained is symmetric. So instead of choosing the potential of displacement to represent the fluid, one had chosen the potential velocity, one would have obtained asymmetric matricial problems also revealing a damping matrix.*

**Note:**

*It is necessary to have $\rho_F>0$ so the mass matrix $\tilde{\mathbf{M}}$ is invertible. If one wishes to do a calculation in "air", cf §16, it is necessary to block the degrees of freedom in $\Phi_0$.*

# 4 Resolution of the cellular problem

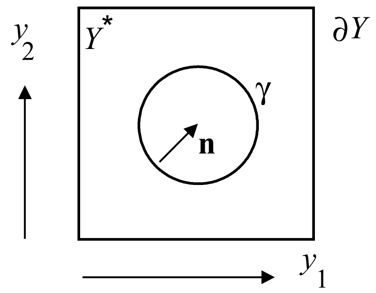
## 4.1 Problem to be solved

On the two-dimensional basic cell (see [fig 4.1-a]), one seeks to calculate the functions $\chi_\alpha(\alpha=1,2)$ checking:

$$\begin{aligned}&\int_{Y^*} \nabla \chi_\alpha . \nabla v = \int_\gamma n_\alpha v \qquad \forall v \in V\\&\int_{Y^*} \chi_\alpha = 0 \quad (\text{pour avoir une solution unique})\end{aligned} \qquad \textbf{éq 4.1-1}$$

where:

$$V = \{ v \in H_1(Y^*), \quad v(y) \ \text{ périodique en } y \text{ de période } 1 \}$$




**Appear 4.1-a**

After having determined the functions $\chi_\alpha(\alpha=1,2)$, one calculates the coefficients homogenized definite by $b_{\alpha\beta}(\alpha=1,2 ; \beta=1,2)$ the formula:

$$b_{\alpha\beta} = \int_{Y^*} \frac{\partial \chi_\alpha}{\partial y_B} \qquad \textbf{éq 4.1-2}$$

By means of the formula of Green and the periodic character of $\chi_\alpha$, one shows that the coefficients $b_{\alpha\beta}$ can be written differently:

$$b_{\alpha\beta} = \int_\gamma \chi_\alpha n_\beta \qquad \textbf{éq 4.1-3}$$

to estimate this quantity, one needs a discretization by finite elements during, to determine for each element the outgoing norm, which can be tiresome. Another way then is operated; while taking in the equation [éq 4.1-1] $v=\chi_\beta$, one obtains:

$$b_{\alpha\beta} = \int_{Y^*} \nabla \chi_\alpha \nabla \chi_\beta \qquad \textbf{éq 4.1-4}$$

From the function potential energy defined by the classical formula:

$$W\ (v)=-\frac{1}{2}\int_{Y^*}\nabla v\,.\,\nabla v \qquad \textbf{éq 4.1-5}$$

one can rewrite the coefficients homogenized in the form:

$$b_{\alpha\beta}=-(W\ (\chi_\alpha+\chi_\beta)-W\ (\chi_\alpha)-W\ (\chi_\beta)) \qquad \textbf{éq 4.1-6}$$

In the two-dimensional general case, one must calculate three coefficients of the homogenized problem (it $b_{11,}b_{12}=b_{21,}b_{22}$ is known that the matrix $\mathbf{B}=(b_{\alpha\beta})$ is symmetric). One must solve the two following problems:

$$\begin{cases}\text{Calculer }\chi_1\in V\ \ /\ \int_{Y^*}\nabla\chi_1\nabla v=\int_{Y^*}n_1 v\\ \text{Calculer }\chi_2\in V\ \ /\ \int_{Y^*}\nabla\chi_2\nabla v=\int_{Y^*}n_2 v\\ \text{Calculer }\chi^*\in V\ \ /\ \chi^*=\chi_1+\chi_2\end{cases} \qquad \textbf{éq 4.1-7}$$

One has then:

$$\begin{cases}b_{11}=-2W\ (\chi_1)\\ b_{22}=-2W\ (\chi_2)\\ b_{12}=b_{21}=-(W\ (\chi^*)-W\ (\chi_1)-W\ (\chi_2))\end{cases} \qquad \textbf{éq 44.1-84.1-8}$$

**Note::**

*If the basic cell has symmetries, that makes it possible to solve the problem on part of the cell with quite suitable boundary conditions and to calculate only certain coefficients of the homogenized problem. For example for the cell of the figure n°4.1 - there is a:* $b_{11}=b_{22,}\ \ b_{12}=b_{21}=0$ .

## 4.2 Equivalent problem to define $\chi_\alpha$

In the equation [éq 4.1-1], the computation of the second member requires the determination of the norm to edge. To avoid a determination of the norm, one can write an equivalent problem, checked by the functions $\chi_\alpha$ .

Are the vectors $G_1=\begin{pmatrix}1\\0\end{pmatrix},G_2=\begin{pmatrix}0\\1\end{pmatrix}$ et $G^*=\begin{pmatrix}1\\1\end{pmatrix}$ , one seeks the functions $\chi^*,\ \chi_{1,}\chi_2\in V$ such as:

$$\begin{cases}\int_{Y^*}\nabla\chi_1\,.\,\nabla v=\int_{Y^*}G_1\,.\,v\quad\forall\, v\in V\\ \int_{Y^*}\nabla\chi_2\,.\,\nabla v=\int_{Y^*}G_2\,.\,v\quad\forall\, v\in V\\ \int_{Y^*}\nabla\chi^*.\,\nabla v=\int_{Y^*}G^*.\,v\quad\forall\, v\in V\end{cases} \qquad \textbf{éq 4.2-1}$$

By means of the formula of Green and the anti-periodicity of the norm $\mathbf{n}$ , one shows that the problems [éq 4.1-1] and [éq 4.2-1] are equivalent.

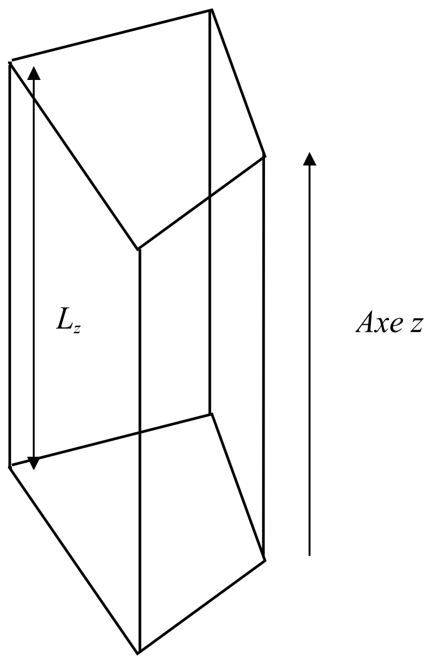
## 4.3 Practical application in *the Code_Aster*

In *the Code_Aster*, to solve the problem [éq 4.2-1], the thermal analogy by defining a material having a coefficient $c_p$ equal to zero and one coefficient $\lambda$ equal to is used. To impose the computation of the second member utilizing the term in $G_\alpha$, key word `PRE_GRAD_TEMP` in command `AFFE_CHAR_THER` is selected. The thermal problem is solved by means of command `THER_LINEAIRE`. The computation of potential energy $\bar{W}$ by the command POST_ELEM with option ENER_POT is provided. In the general case, three computations are carried out to determine the values $\bar{W}\ (\chi_1), \bar{W}\ (\chi_2), \bar{W}\ (\chi^*)$ and then, the values of the coefficients of the homogenized problem are deduced from it manually. To impose the periodic character of the space in which the solution is sought, key word `LIAISON_GROUP` in command `AFFE_CHAR_THER` is used.

# 5 Choice of the finite element for the problem homogenized

## 5.1 Choice of the finite elements

In the model previously presented, the axis $z$ plays a paramount role as a principal axis of the beams. The developed finite elements check this characteristic. Meshes are of the cylindrical type: the quadrangular bases are contained in planes $z=Cte$ and the axis of the cylinder is parallel to the axis $z$ (see [fig 5.1-a]).




**Appear 5.1-a**

According to the equations [éq 3.1-1], of second derivative according to the coordinate $z$ intervene in the model, which requires of the finite elements $C^1$ in the direction $z$ . Shape functions of the type Hermit to represent the variations of $s$ following the axis $z$ are thus used. At the points of discretization, displacements $s_x, s_y$ but also the derivatives $\frac{\partial s_y}{\partial z}, \frac{\partial s_x}{\partial z}$ which are related to the degrees of freedom of rotation $\theta_x, \theta_y$ by the formulas $\theta_x = \frac{\partial s_y}{\partial z}$, $\theta_y = -\frac{\partial s_x}{\partial z}$ must be known. With regard to the variations according to $x, y$, one limits oneself for time to shape functions $Q_1$.

For the degree of freedom of potential, shape functions $Q_1$ or $Q_2$ according to the three directions $x, y, z$ of space are used.

The finite element thus has as unknowns the following degrees of freedom: $s_x, s_y, \theta_x, \theta_y, \Phi$.

**Note:**

*The order of the nodes of meshes the support is very important. Indeed, the edges parallel with the axis $z$ are in the same way represented only the edges contained in the planes $z = Cte$. The nodes of meshes are thus arranged in a quite precise order: list nodes of the lower base, then list of with respect to the higher base (or vice versa).*

With regard to the geometry, the shape functions making it possible to pass from the element of reference to the element running are $Q_1$. The finite element is thus under-parametric.

Two finite elements were developed:

- a associate with a mesh HEXA8. In each node of the mesh, the unknowns are $s_x, s_y, \theta_x, \theta_y, \Phi$. The shape functions associated with the potential $\Phi$ are $Q_1$.
- another associate with a mesh HEXA20. In each top node of the mesh, the unknowns are $s_x, s_y, \theta_x, \theta_y, \Phi$. In each medium node of the edges, the unknown is $\Phi$. The shape functions associated with the potential $\Phi$ are $Q_2$.

## 5.2 Finite elements of reference

### 5.2.1 Nets HEXA8

On the finite element of reference HEXA8 (see [fig 5.2-a]), the following shape functions are defined:

$$N^L_{\pm 1, \pm 1, \pm 1}(\xi) = P_{\pm 1}(\xi_1) P_{\pm 1}(\xi_2) P^L_{\pm 1}(\xi_3) \quad \text{avec } L = \Phi \text{ ou } D \text{ ou } R \qquad \textbf{éq 5.2.1-1}$$

indices $\pm 1$ represent the coordinates of the nodes of the mesh support of reference.

The functions which make it possible to define the shape functions write:

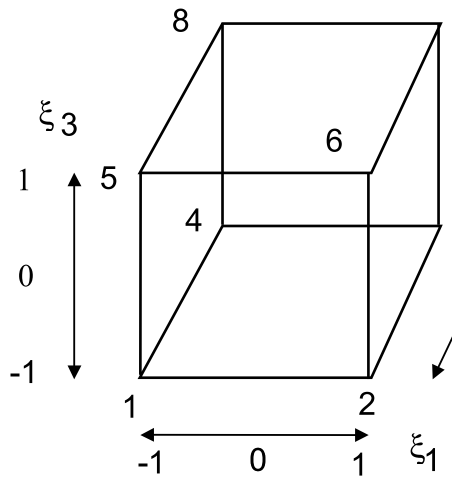
$$\begin{array}{ll} P_{(-1)}(\varsigma) = \frac{1-\varsigma}{2} & P_{(+1)}(\varsigma) = \frac{1+\varsigma}{2} \\ P^{\Phi}_{(-1)}(\varsigma) = \frac{1-\varsigma}{2} & P^{\Phi}_{(+1)}(\varsigma) = \frac{1+\varsigma}{2} \\ P^D_{(-1)}(\varsigma) = \frac{1}{2}(1 - \frac{3}{2}\varsigma + \frac{1}{2}\varsigma^3) & P^D_{(+1)}(\varsigma) = \frac{1}{2}(1 + \frac{3}{2}\varsigma - \frac{1}{2}\varsigma^3) \\ P^R_{(-1)}(\varsigma) = \frac{1}{4}(1 - \varsigma - \varsigma^2 + \varsigma^3) & P^R_{(+1)}(\varsigma) = \frac{1}{4}(-1 - \varsigma + \varsigma^2 + \varsigma^3) \end{array} \quad \varsigma \in [-1,1] \qquad \textbf{éq 5.2.1-2}$$

functions $P^D, P^R$ are the functions of Hermit.

The unknowns of the homogenized problem, on a mesh, break up in the following way:

$$\begin{cases} s_x(\xi)=\sum_{i=1}^{8} DX_i N_i^D(\xi)+\frac{L_S}{2}\sum_{i=1}^{8} DRX_i N_i^R(\xi) \\ s_y(\xi)=\sum_{i=1}^{8} DY_i N_i^D(\xi)+\frac{L_S}{2}\sum_{i=1}^{8} DRY_i N_i^R(\xi) \quad \xi=(\xi_{1,}\xi_{2,}\xi_3) \\ \Phi(\xi)=\sum_{i=1}^{8} \Phi_j N_j^{\Phi}(\xi) \end{cases}$$ **éq 5.2.1-3**

where $DX_i, DY_i, DRX_i, DRY_i, \Phi_i$ are the values of displacement according to $x$, of displacement according to $y$, of rotation around the axis $x$, rotation around the axis $y$ and the potential of displacement at the top $i$ of the mesh. In *the Code_Aster*, for each node, the degrees of freedom are arranged in the order quoted previously.




**Be reproduced 5.2.1-a**

### 5.2.2 Mesh HEXA20

On the finite element of reference HEXA20 (see [fig 5.2-b]), the following shape functions are defined:

$$N^L_{\pm 1,\pm 1,\pm 1}(\xi)=P_{\pm 1}(\xi_1)P_{\pm 1}(\xi_2)P^L_{\pm 1}(\xi_3) \quad \text{avec } L=\Phi \text{ ou } D \text{ ou } R$$ **éq 5.2.2-1**

$$N_j^{\Phi}(\xi)=Q_j(\xi_3) \quad j=1,\ldots,20$$ **éq 5.2.2-2**

indices $\pm 1$ represent the coordinates of the nodes tops of the mesh support of reference.

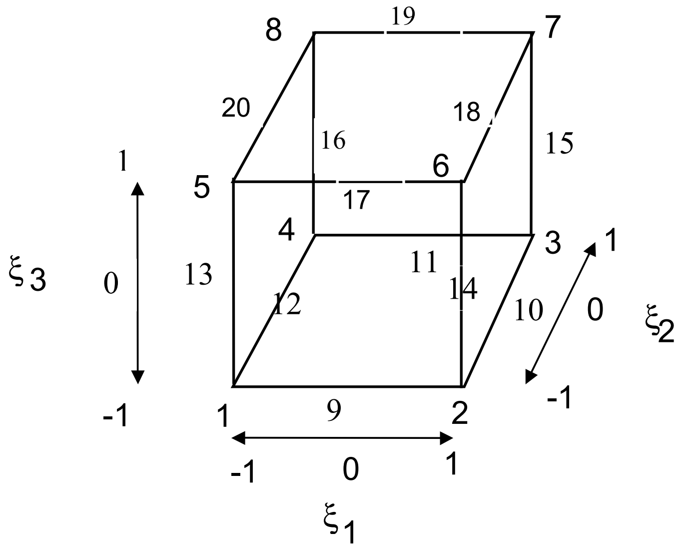
The functions $P_{\pm 1}, P_{\pm 1}^{L}$ were already defined in the paragraph [§5.2.1]. The functions $Q_i$ are defined by:

$$\begin{aligned}
&Q_i(\xi)=\frac{1}{8}\left(1+\xi_1\xi_1^i\right)\left(1+\xi_2\xi_2^i\right)\left(1+\xi_3\xi_3^i\right)\left(\xi_1\xi_1^i+\xi_2\xi_2^i+\xi_3\xi_3^i-2\right) \quad i=1,\ldots,8\\
&Q_i(\xi)=\frac{1}{4}\left(1-\left(\xi_1\xi_1^i\right)^2\right)\left(1+\xi_2\xi_2^i\right)\left(1+\xi_3\xi_3^i\right) \quad i=9, 11, 17, 19\\
&Q_i(\xi)=\frac{1}{4}\left(1-\left(\xi_2\xi_2^i\right)^2\right)\left(1+\xi_1\xi_1^i\right)\left(1+\xi_3\xi_3^i\right) \quad i=10, 12, 18, 20\\
&Q_i(\xi)=\frac{1}{4}\left(1-\left(\xi_3\xi_3^i\right)^2\right)\left(1+\xi_1\xi_1^i\right)\left(1+\xi_2\xi_2^i\right) \quad i=13, 14, 15, 16
\end{aligned} \qquad \textbf{éq 5.2.2-3}$$

where $(\xi_{1,}^i \xi_{2,}^i \xi_3^i)$ are the coordinates of the node $i$ of the mesh.
The unknowns of the homogenized problem, on a mesh, break up in the following way:

$$\begin{cases}
s_x(\xi)=\sum_{i=1}^{8} DX_i N_i^D(\xi)+\frac{L_S}{2}\sum_{i=1}^{8} DRX_i N_i^R(\xi)\\
s_y(\xi)=\sum_{i=1}^{8} DY_i N_i^D(\xi)+\frac{L_S}{2}\sum_{i=1}^{8} DRY_i N_i^R(\xi) \qquad \xi=(\xi_{1,}\xi_{2,}\xi_3)\\
\Phi(\xi)=\sum_{i=1}^{8} \Phi_j N_j^{\Phi}(\xi)
\end{cases} \qquad \textbf{éq 5.2.2-4}$$

where $DX_i, DY_i, DRX_i, DRY_i, \Phi_i$ are the values of fluid displacement according to $x$, of displacement according to $y$, of rotation around the axis $x$, rotation around the axis $y$ and the potential of displacement at the top $i$ of the mesh $(i=1,\ldots,8)$ and $\Phi_j$ fluid potential of displacement to the medium node of the edges $(j=9,\ldots,20)$.




**Gauss points 5.2.2-a**

## 5.3 5.2.2-a Choice of

the Each integral which intervenes in the forms of the elementary matrixes, is transformed into an integral on the element of reference (a change of variable is carried out) who is then calculated by means of a formula of squaring of the GAUSS type.

Gauss points are selected in order to integrate exactly the integrals on the element of reference. Families of different points of integration are used to compute: the mass matrixes and the stiffness matrixes (the degrees of the polynomials to be integrated are different). But here, to compute: the various contributions of the mass matrix, various families of Gauss points can still be used.

The element of reference being a HEXA8 or a HEXA20, the integral on volume can be separate in a product of three integrals which correspond each one to a direction of the space of reference. The number of points of integration necessary is determined by direction.

According to the mesh of reference, the number of points of integration by direction is the following:

| | Net HEXA8 | | Nets HEXA20 | |
|---|---|---|---|---|
| | **direction $x$ or $y$** | **direction $z$** | **direction $x$ or $y$** | **direction $z$** |
| of Gauss points $\hat{\mathbf{K}}$ | stamps | 2 | 2 | 2 |
| 2 $\hat{\mathbf{A}}$ | matrix | 2 | 2 | 3 |
| 3 $\hat{\mathbf{D}}$ | matrix | 2 | 3 | 2 |
| 3 $\hat{\mathbf{M}}$ | matrix | 2 | 4 | 2 |

4 Four families were defined. Each family corresponds to one of the matrixes of the problem to solve.

On the segment [- 1,1], the coordinates of the points of integration and their weights are the following [bib7]:

| **Number of points from integration** | **Coordinated** | **Weight** |
|---|---|---|
| the 2 | $\pm 1/\sqrt{3}$ | $1$ |
| 3 | $0$<br>$\pm\sqrt{3/5}$ | $8/9$<br>$5/9$ |
| 4 | $\pm\sqrt{\dfrac{3-2\sqrt{6/5}}{7}}$<br>$\pm\sqrt{\dfrac{3+2\sqrt{6/5}}{7}}$ | $\dfrac{1}{2}+\dfrac{1}{6\sqrt{6/5}}$<br>$\dfrac{1}{2}-\dfrac{1}{6\sqrt{6/5}}$ |

weights of a Gauss point in the three-dimensional element of reference is obtained by multiplying the three weights corresponding to each coordinate of the Gauss point.

## 5.4 Addition of the problems of tension and torsion

to supplement the problem of bending homogenized described previously, the problem of tension and the problem of torsion are added in a decoupled way (these problems do not utilize the fluid).

### 5.4.1 Problem of tension

the problem of tension homogenized is written in the following form:

$$\int_{\Omega} \frac{E\,S\,\mu^2}{|Y|}\frac{\partial s_z}{\partial z}\frac{\partial v}{\partial z}+\int_{\Omega}\frac{\mu^2\rho_S S}{|Y|}\frac{\partial^2 s_z}{\partial t^2}v=0 \quad \forall v\in V \text{ with } V=H^1([0,L])$$

$\mu$ being the ratio on side of the basic cell of reference, area $|Y|$, compared to the real cell of periodicity of the heterogeneous medium.
The finite element of reference is a HEXA8 having for unknown displacement $DZ$ in each node. The associated shape functions are $Q_1$ .

### 5.4.2 Problem of torsion

the problem of torsion homogenized is written in the following form:

$$\int_{\Omega} \frac{E\,J_z\,\mu^2}{2(1+\nu)|Y|}\frac{\partial \theta_z}{\partial z}\frac{\partial v}{\partial z}+\int_{\Omega}\frac{\mu^2\rho_S J_z}{|Y|}\frac{\partial^2 \theta_z}{\partial t^2}v=0 \quad \forall v\in V \text{ with } V=H^1([0,L])$$

where $J_z$ is the constant of torsion.

The finite element of reference is a HEXA8 having for unknown displacement $DRZ$ in each node. The associated shape functions are $Q_1$

## 5.5 Integration in *the Code_Aster* of this finite element

the finite element is developed in *Code_Aster* in 3D. A modelization was added in the catalog of the modelizations:

- "FAISCEAU_3D" for 3D.

In the catalog of the elements, the element can apply to two the meshes following ones:

| Net | Many nodes in displacement and rotation | Many nodes in fluid potential | Name of the element in catalog |
|---|---|---|---|
| HEXA8 | 8 | 8 | meca_poho_hexa8 |
| HEXA20 | 8 | 20 | meca_poho_hexa20 |

In the routines of initialization of this element, one defines:

- two families of shape functions respectively associated with displacements and rotation with the beams (shape function linear in $x, y$ and cubic in $z$ ) and under the terms with potential with the fluid (linear function in $x, y, z$ ),
- four families of Gauss points to compute: the stiffness matrix and the various parts of the mass matrix.

During the computation of the elementary terms, the derivatives first or seconds of the shape functions on the element running are calculated. In spite of the simplified geometry of the finite element (the axis of the mesh cylindrical is parallel with the axis $z$ and the sections lower and higher are in planes $z=Cte$ ), a general subroutine to compute: the second derivative was written [bib7]. In addition, two news subroutines was developed starting from the subroutines existing for the isoparametric elements to take account of the under-parametric character of the element.

# 6 Use in *Code_Aster*

## 6.1 the data necessary

the characteristics of the beams (section $S$, tensor of inertia $\mathbf{I}$, constant of torsion $J_z$) are directly indicated under the key word factor POUTRE of the command AFFE_CARA_ELEM.

The characteristics of the homogenized coefficients and the cell of reference are indicated under the key word factor POUTRE_FLUI of the command AFFE_CARA_ELEM. For the simple keywords, the correspondence is the following one:

B_T: $b_{11}$
B_N : $b_{22}$
B_TN : $b_{12}$
A_FLUI : $Y_F$
A_CELL : $Y=Y_F+Y_S$
COEF_ECHELLE : $\mu$

The characteristics of the materials are indicated in command DEFI_MATERIAU. For the tubes, the key word factor ELAS is used to indicate the modulus Young (E: $E$), the Poisson's ratio (NU: $\nu$) and density (RHO: $\rho_S$). For the fluid, the key word factor FLUIDE is used to indicate the density of the fluid (RHO: $\rho_F$).

## 6.2 Directional sense of the axes of the beams

the generators of meshes cylindrical are obligatorily parallel to the axis of the beams and the bases of meshes perpendicular to this same axis. During the development of the mesh, it should be made sure that the order of the nodes (local classification) of each cylindrical mesh is correct: nodes of the lower base then nodes of the higher base (or vice versa). The direction of the axis of the beams is indicated under the key word factor ORIENTATION of the command AFFE_CARA_ELEM.
The following assumption was carried out: the reference of reference is the same one as the principal reference of inertia of the characteristic tube representing the homogenized medium. That means that in the equations [éq 3.1-3], the term $I_{xy}$ is null.

## 6.3 Modal computation

the developed finite element makes it possible to characterize the vibratory behavior of a network of beams bathed by a fluid. It is interesting to determine the frequencies of vibration of such a network in air and water.

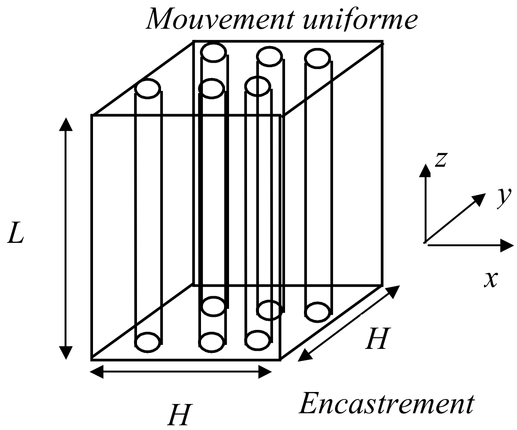
To carry out a modal computation in air ( $\rho_F=0$ ), all the degrees of freedom should be blocked corresponding to the fluid potential of displacement $\Phi$, if not the stiffness matrix (and even the shiftée matrix of the modal problem) are noninvertible [R5.01.01].

To carry out a water modal computation ( $\rho_F\neq 0$ ), it is necessary to use in command MODE_ITER_SIMULT, option CENTER of the key word factor CALC_FREQ. The shiftée matrix $(\tilde{\mathbf{K}}-\sigma\tilde{\mathbf{M}})$ is then invertible if $\sigma$ is not eigenvalue or if $\sigma$ is different from zero.

# 7 Characterization of the spectrum of the homogenized model

## 7.1 Models heterogeneous

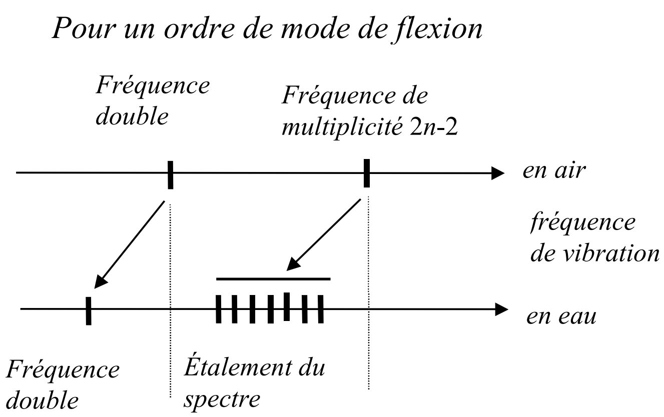
Is a network with square step of $n$ beams fixed in their low ends and whose higher ends move in the same way (uniform motion) (cf appears [fig 7.1-a]). Only motions of bending are considered.




**Appear 7.1-a**

spectrum of vibration in air of this network in the following form. For each order of mode of vibration of bending, the modal structure makes up of a double frequency corresponding to a mode in $x$ and with a mode in $y$ where all the upper part moves (all the beams have the same deformed shape) and of a frequency of multiplicity $(2n-2)$ corresponding to modes where all the upper part of the beams is motionless and where beams move in opposition of phase.

In the presence of fluid, the spectrum is modified. For each order of mode of vibration in bending, $2n$ the frequencies of vibration are lower than the frequencies of vibration obtained in air. The effect of the incompressible fluid is comparable to an added mass. There is always a double frequency corresponding to a mode in $x$ and with a mode in $y$ where all the upper part moves (all the beams have the same deformed shape). On the other hand, one obtains $(n-1)$ to couples different of frequency double (one in $x$ and one in $y$) corresponding with modes where all the upper part of the beams is motionless and where beams move in opposition of phase.




**Appear 7.1-b**

## 7.2 Models homogeneous

the heterogeneous medium was replaced by a homogeneous medium.

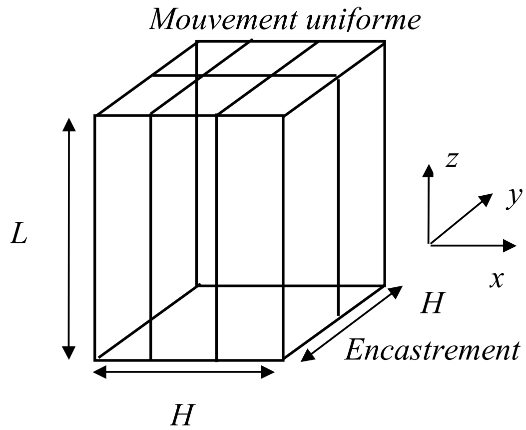
### 7.2.1 Continuous problem

Of recent works, concerning a problem of plane homogenization of a network of runners retained by springs and bathed by a fluid, show that the spectrum of the continuous homogeneous model consists of a continuous part and two frequencies of infinite multiplicity [bib10]. The spectrum of the eigenfrequencies of the water problem is also contained in a well defined interval limited exceptionally by the frequency of vibration in air of a runner [bib5].

These results are transposable for each order of bending of the network of tubes.

### 7.2.2 Discretized problem

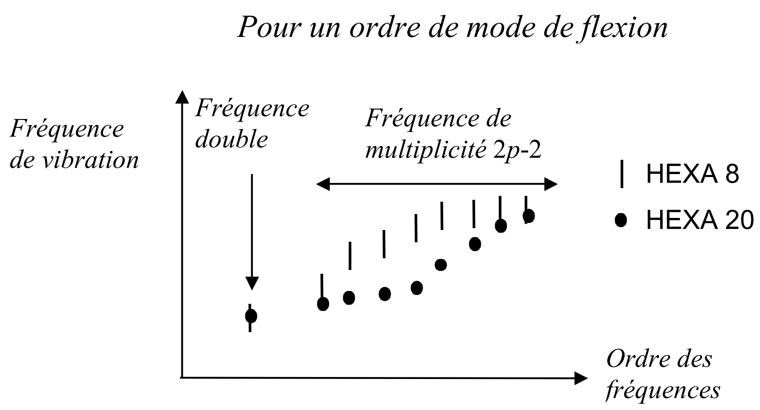
Is the homogeneous field with a grid by hexahedrons. That is to say $p$ the number of generators parallel with the axis $z$ of the network of beams.




**Appear 7.2.2-a**

One finds the model results similar to those obtained for heterogeneous. It is enough to replace $n$ par. $p$ For an order of bending of beam, the number of frequencies corresponding to modes where the beams do not vibrate all in the same meaning, depends on the discretization used in the transverse directions with the axis of the beams.

According to the finite element used (mesh HEXA8 or mesh HEXA20), the distribution of $(2p-2)$ the last frequencies is different. The first double frequency (that corresponding to the mode where the upper part moves) is the same one for the two finite elements.




**Appear 7.2.2-b**

All in all, the model homogeneous allows to obtain the frequencies of vibration easily corresponding to modes where all the beams vibrate in the same meaning. The other modes obtained provide only one vision partial of the spectrum. In the discretized spectrum, one can turn over one or the two frequencies of infinite multiplicity present in the spectrum of the continuous model.

# 8 Conclusion

the use of the finite elements developed associated with the homogenized model of a periodic tube bundle bathed by a fluid makes it possible to characterize overall vibratory motions (all the structure moves in the same meaning) of such a structure.

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# 10 Description of the versions of the document

| Version Aster | Author (S) Organization (S) | Description of the modifications |
|---|---|---|
| 4.0 | *B. QUINNEZ (EDF/IMA/MMN)* | initial Text |
| 10.2 | F.VOLDOIRE (EDF/AMA) | Corrections of working ; correction equation 3.2-2. |