Identification of forces on a Summarized modal

model:

Macro-command CALC_ESSAI gathers the features of Code_Aster for the correlation CALCUL-tests. One describes in the frame of this documentation the functionality "identification of forces", which must make it possible to identify the forces to apply to a structure in the form of an inter-spectrum, from the data of the inter-spectrum of measurements under operation, of a modal model, identified or calculated, and choice of the localization of the points of application of the forces.

Contents

1 Introduction, assumptions design

This documentation describe the methods used in the macro `CALC_ESSAI`.

1.1 Position of the problem

One considers a structure, which constitutes a dynamic system that one supposes **linear (assumption H1).**

It is supposed that this structure is subjected to a presumedly random excitation (typical example: turbulent fluid forces), but that one can describe perfectly using his DSP (power spectral density, to see the document "Modelization of the turbulent excitations" R4.07.02, section 2.1). This vibration is measured in a certain number of points of X-coordinates $x_k(1 \le k \le nmes)$. The forces applied on the structure are not, as for them, measurable. One thus wishes to find the forces from measurement. It is about inverse problems, for the resolution of which it is necessary to take many precautions in order to obtain result which, if it is not exact, is nevertheless relevant.

1.2 Assumption of modal behavior

the structure is supposed to be linear (H1), one can thus describe his structural mechanics behavior on modal base. Each mode I of structure, is defined by the following parameters:

- ω_i : own pulsation, f_i associated Eigen frequency
- ξ_i : reduced damping
- m_i : modal mass
- $\varphi_i(\underline{x})$: modal deformed shape.

Flow exerts a force on the structure $f(\underline{x}, t)$. It is supposed that these forces are applied in a direction. One will treat the two directions of measurement separately, by supposing that those are not coupled.

Displacement on modal base: it is supposed that the motion of structure is rather well described by its N first modes:

$$u(\underline{x}, t) \approx \sum_{i=1}^{N} q_i(t) . \varphi_i(\underline{x})$$

The generalized coordinates check the decoupled equations then:

$$m_i\left(\ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_{i^2} q_i\right) = \int_0^L \varphi_i(\underline{x}) f(\underline{x}, t)\, dx = Q_i(t) \Leftrightarrow m_i\left(-\omega^2 q_i + 2\xi_i\omega_i\omega q_i + \omega_i^2 q_i\right) = Q_i(\omega),\ 1 \le i \le N$$

2 Modelization and computation of the forces

2.1 Modelization on modal base, observability and commandability

the supposed linearity of the behavior of structure allows the decomposition of the model on modal base:

$$\underline{y} = \underline{\underline{C\,\Phi}} . \underline{\underline{Z}}^{-1} . \underline{\underline{\Phi^T B}} . \underline{f}$$

By means of the notion of inter-spectrum, privileged in the macro1La[1] :

$$\underline{\underline{S_{yy}}}=\underline{\underline{C\Phi}}.\underline{\underline{Z^{-1}}}.\underline{\underline{\Phi^T B}}.\underline{\underline{S_{ff}}}.\underline{\underline{\Phi^T B^H}}.\underline{\underline{Z^{-1}}}.\underline{\underline{C\Phi^H}} \quad (1)$$

$\underline{\underline{C}}$ and $\underline{\underline{B}}$ are the matrixes of observability and command. $\underline{\underline{C}}$ allows to project a field defined on a model, a more restricted field of sensors. $\underline{\underline{B}}$ project this same field on the points of application of the forces. The matrixes $\underline{\underline{C\Phi}}$ and $\underline{\underline{\Phi^T B}}$ are the matrixes of modal deformed shapes definite on a mesh sensor and a mesh of command. They can be obtained with the operator `PROJ_CHAMP` or the operator `OBSERVATION`, who allows, besides the precedent, to define for each node or groups node of the not measured directions, or to define local coordinate systems.

$\underline{\underline{Z}}$ is the matrix of impedance $diag\left(-\omega^2+2\mathrm{j}\,\xi_i\,\omega_i\omega+\omega_i^2\right)_{1\leq i\leq N}$ associated with the base with the modal deformed shapes $\underline{\underline{\Phi}}$. This base is in general defined on a model of good quality. A base of good quality must indeed have modal parameters close to reality. The deformed shapes must also be close to reality, and being sufficiently regular. Thus, an experimental base of deformed shapes is defined on a reduced number of sensors, and is thus not very regular (one is likely to have a resolution of the problematic inverse problems). One will prefer to him the base of deformed shapes of a readjusted digital model, or, better still, an experimental base extended on digital model. One clarifies this point below.

Notice on the modal expansion

One has an experimental $\left(f_i, \xi_i, m_i, \underline{\varphi_{\mathrm{exp}i}}\right)$ modal base. The modal parameters are supposed to be identified with a good approximation, but the deformed shapes, of good quality, are defined only on one restricted number of sensors. One thus tries to extend this base of deformed shapes on a digital model which, is rather representative of structure (but inevitably not readjusted). The base of expansion can be the base of the modes of the digital model (simulated with `MODE_ITER_SIMULT` for example). To carry out a modal expansion thus means to find the vector of generalized parameters η minimizing:

$$\|\underline{\varphi_{\mathrm{exp}i}}-\underline{\underline{\Phi_{num}}}.\underline{\eta}\|^2$$

The mitre "correlation" of `CALC_ESSAI`, makes it possible to carry out this expansion, thanks to the use of `PROJ_MESU_MODAL`. For more details on the principle of the expansion, one will refer to the documentation of `PROJ_MESU_MODAL` (U4.73.01).

2.1.1 Resolution of the inverse problems

the equation (1) of the direct problem, can, under certain conditions, being reversed:

$$\underline{\underline{S_{ff}}}=\underline{\underline{\Phi^T B}}^{\oplus}.\underline{\underline{Z}}.\underline{\underline{C\Phi}}^{\oplus}.\underline{\underline{S_{yy}}}.\underline{\underline{C\Phi}}^{H^{\oplus}}.\underline{\underline{Z}}.\underline{\underline{\Phi^T B}}^{H^{\oplus}}$$

The matrixes $\underline{\underline{C\Phi}}$ and $\underline{\underline{\Phi^T B}}$ being rectangular, the sign $\oplus$ indicates their pseudo-opposite of Moore Penrose. This opposite pseudonym can be obtained by the use of an algorithm of SVD (Singular Been worth Decomposition, cf Doc. R6.03.01). In `CALC_ESSAI`, two possibilities of regularization are available:

1notion of inter-spectrum, or DSP, makes it possible to describe random phenomena. For a deterministic phenomenon $\underline{\underline{S_{yy}}}=\underline{y}.\underline{y}^H$, where H indicates the hermitian operator. For a random signal, $\underline{\underline{S_{yy}}}$ contains, on its diagonal, the spectral powers (auto--spectrums) and on its extra-diagonal terms, the correlations between the signals. For example, in the case of a fluid force applied to a telegraphic structure, one imagines easily that the vortexes created at the base tend to be propagated along structure. The extra-diagonal terms thus describe this propagation with the phase (see R4.07.02 for more details).

1) truncation of the small singular values: one notes $\sigma_{\max}$, the greatest value singular. The SVD consists in fixing for all the singular values lower than $\varepsilon\sigma_{\max}$, for ε given, value 0. They are thus not taken into account in opposite computation. Truncation eliminates from information on the matrixes to reverse, but improves conditioning;
2) regularization of Tikhonov: the reverse of the matrix of the singular values is not worth $diag\left(\frac{1}{s_i}\right)_{1\leq i\leq N}$ but $diag\left(\frac{s_i}{s_{i^2}+\alpha}\right)_{1\leq i\leq N}$. The parameter α is called parameter of Tikhonov, and makes it possible to limit the divergence of the opposite solution.

In CALC_ESSAI, one calculates successively the inter-spectrums:

- generalized displacements (inversion of $\underline{\underline{C\,\Phi}}$),
- reconstituted physical displacements (for checking of the quality of the inversion),
- of the generalized forces (multiplication by the matrix of impedance),
- of the physical efforts (inversion of $\underline{\underline{\Phi^T B}}$),
- the generalized forces reconstituted (for checking)
- of physical displacements by synthesis on modal base.

A good quality standard of the results of the inversion can be the comparison between measured displacements, and those reconstituted on the same modal base which was used for the inversion (matrix of impédence, and matrixes of modal deformed shapes).

3 References

[1]L. Perotin, R. Nhili, *Software MEIDEE version 2.1: note principle* . Note EDF/R & D HT-32/92/014/B.

[2]S. Granger, *Software MEIDEE Version 2.1: Data-processing documentation* . Note EDF/R & D HT-32/92-15/ A.C.

[3]Raynaud, *Identification of the fluid excitation spectrums turbulent on the model SNIPE from* the tests pencil modes for configurations AFAG2G – 4 grids and ENUSA Westinghouse V5H – 4grills. Note EDF/R & D HI-86/03/030/A.C.

[4]Bodel, *Identification of fluid forces applied to a tube of control rod, model EPR. Methodology and results* . Note EDF/R & D H-T61-2007-02808-FR.

[5]A. Adobes, *Modelization of the turbulent excitations*. Documentation R4.07.02 Code_Aster.

4 History of the versions of the document

Index Doc.	Version Aster	Author (S) or contributor (S), organization	Description of the modifications
A	9.4	C.BODEL EDF/R & D /MMN	initial Text