Computation of the thermal strain

Summarized

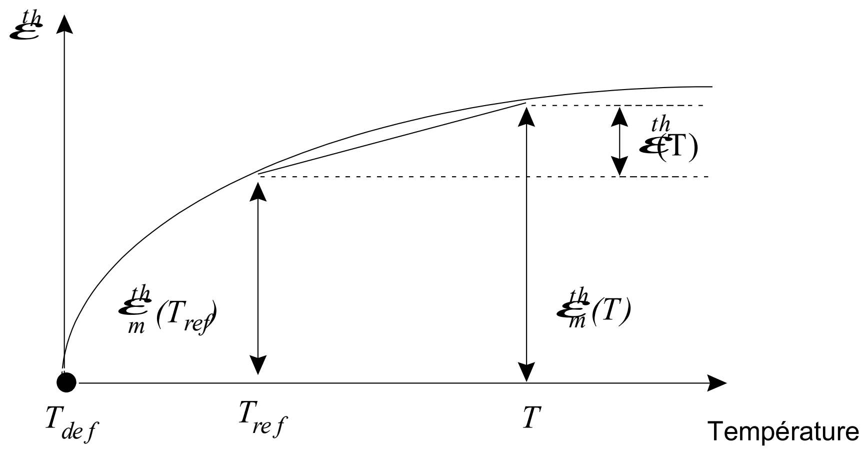
This document is devoted to the presentation of the computation of the thermal strain. One indicates the necessary information to it to the computation of the thermal strain and the various possibilities of definition of this information by the user.

Contents

1 Introduction

the values of the coefficients of thermal expansion are determined by tests of dilatometry which take place from the room temperature ($0°C$ or more generally $20°C$). So one in general has the values of the coefficient of thermal expansion defined compared to $20°C$ (temperature to which one supposes the thermal strain null).

Certain studies require to take a reference temperature different from the room temperature (thermal strain null for another temperature that the room temperature). It is then necessary to carry out a change of reference in the computation of the thermal strain (equation [éq 1-1] and appears below).



$$\varepsilon^{th}(T)=\varepsilon^{th}_m(T)-\varepsilon^{th}_m(T_{ref})$$ éq 1-1

where ε^{th}_m is the measured thermal strain (definite compared to the room temperature)

ε^{th} is the thermal strain calculated (definite compared to a reference temperature)

In *Code_Aster*, the thermal strain is calculated by the statement $\varepsilon^{th}(T)=\hat{\alpha}(T)(T-T_{ref})$ where $\hat{\alpha}(T)$ is the average coefficient of thermal expansion (with meaning RCC_M) with the temperature T determined compared to the temperature T_{ref} (T_{ref} being the temperature to which one considers that $\varepsilon^{th}(T_{ref})=0$).

2 Known thermal coefficient of thermal expansion compared to Tréf

the values of the thermal coefficient of thermal expansion were determined by tests of dilatometry carried out from the temperature T_{ref}.

In this case, key word `TEMP_DEF_ALPHA` should not be specified in command `DEFI_MATERIAU` [U4.23.01].

The equation [éq 1-1] is simplified, since $\varepsilon_m^{th}(T_{ref})=0$.

From where:

$$\varepsilon^{th}(T)=\hat{\alpha}(T)(T-T_{ref}) \qquad \text{éq 2-1}$$

and $\varepsilon^{th}(T_{ref})=0$

where the values of the coefficient of thermal expansion $\hat{\alpha}(T)$ are indicated under key word `ALPHA` (or `ALPHA_*`) in `DEFI_MATERIAU`.

3 Coefficient of thermal expansion known compared to a temperature $T_{def}\neq T_{ref}$

the values of the thermal coefficient of thermal expansion were determined by tests of dilatometry which took place from a temperature T_{def} different from the reference temperature T_{ref}.

Indeed, in general one has the values of the coefficient of thermal expansion defined compared to the room temperature, $0°C$ or more generally $20°C$, and certain studies require to take a reference temperature different from the room temperature.

It is then necessary to carry out a change of reference [éq 1-1].

In this case, the user informs under key word `TEMP_DEF_ALPHA` of the command `DEFI_MATERIAU`, the value of the temperature T_{def} , and under key word `ALPHA` (or `ALPHA_*`) the values of the coefficient of thermal expansion $\alpha(T)$ (definite compared to the temperature T_{def}). In command `AFFE_MATERIAU` under key word `TEMP_REF`, it indicates the value of the reference temperature T_{ref}.

The computation of $\varepsilon^{th}(T)$ is made the formula by means of:

$$\begin{aligned}\varepsilon^{th}(T) &= \alpha(T)(T-T_{def})-\alpha(T_{ref})(T_{ref}-T_{def})\\ &= \hat{\alpha}(T)(T-T_{ref}) \qquad \text{éq 3-1}\\ \text{et}\quad \varepsilon^{th}(T_{ref}) &= 0\end{aligned}$$

The computation of $\varepsilon^{th}(T)$ requires the preliminary computation of the values of the function $\hat{\alpha}(T)$.

The function $\hat{\alpha}(T)$ remains defined (or indicated) for the same values of T that $\alpha(T), i=1,N$ and keeps the same attributes (even standard of interpolation `("LIN", "LOG")` and even type of prolongation `("CONSTANT", "LINEAIRE", "EXCLUDED")`).

3.1 Computation of $\hat{\alpha}(T_i)$ into cubes temperatures different from T_{ref} (except for an accuracy)

One by means of obtains the statement $\hat{\alpha}(T_i)$ of the equation [éq 3-1].

$$\forall i \quad \text{tel que} \quad |T_i - T_{ref}| \geq Prec$$

$$\hat{\alpha}(T_i) = \frac{\alpha(T_i)(T_i - T_{def}) - \alpha(T_{ref})(T_{ref} - T_{def})}{T_i - T_{ref}} \qquad \text{éq 3.1-1}$$

the value of the accuracy is:

either specified by the user under the key word `accuracy` of the key word factor `ELAS_FO` (command `DEFI_MATERIAU` [U4.23.01]),
or equalizes to 1. : value by default.

3.2 Computation of $\hat{\alpha}(T_i)$ for temperatures close to T_{ref} (except for an accuracy)

One cannot use the equation [éq 3-1] directly. One derives the equation [éq 3-1] compared to the temperature and one takes the value of derivative to the temperature T_{ref}.

$$\varepsilon^{th}(T) = \alpha(T)(T - T_{def}) - \alpha(T_{ref})(T_{ref} - T_{def}) = \hat{\alpha}(T)(T - T_{ref})$$

from where $$\frac{\partial \varepsilon^{th}}{\partial T} = \alpha'(T)(T - T_{def}) + \alpha(T) = \hat{\alpha}'(T)(T - T_{ref}) + \hat{\alpha}(T)$$

and thus $$\hat{\alpha}(T_{ref}) = \alpha'(T_{ref})(T_{ref} - T_{def}) + \alpha(T_{ref}) \qquad \text{éq 3.2-1}$$

the equation [éq 3.2-1] gives the statement of $\hat{\alpha}(T_{ref})$.

It is considered that $\hat{\alpha}(T_i) = \hat{\alpha}(T_{ref}) \qquad \forall \text{i} \quad \text{tel que} |T_i - T_{ref}| < Prec$

the value of the accuracy is:

either specified by the user under the key word `accuracy` of the key word factor `ELAS_FO` (command `DEFI_MATERIAU` [U4.23.01]),
or equalizes to 1. : value by default.

Also, to compute: $\hat{\alpha}(T_i)$ it is necessary as a preliminary to calculate $\alpha'(T_{ref})$.

3.2.1 Computation of $\alpha'(T_{ref})$

1er cas : $\forall i \;\; \text{t . q .} \; |T_i - T_{ref}| < Prec \quad \text{et} \quad i \neq 1 \;\text{et}\; i \neq N$

$$\alpha'(T_{ref}) = \frac{1}{2}\left(\frac{\alpha(T_{i+1}) - \alpha(T_{ref})}{T_{i+1} - T_{ref}} + \frac{\alpha(T_{ref}) - \alpha(T_{i-1})}{T_{ref} - T_{i-1}}\right)$$

éq 3.2.1-1

2 ème cas : $\forall i \;\; t.q. \; |T_i - T_{ref}| < Prec \quad \text{et} \quad \text{si } i = \text{N}$

$$\alpha'(T_{ref}) = \frac{\alpha(T_{ref}) - \alpha(T_{i-1})}{T_{ref} - T_{i-1}}$$

éq 3.2.1-2

3 ème cas : $\forall i \;\; t.q. \; |T_i - T_{ref}| < \text{prec} \quad \text{et} \quad \text{si } i = 1$

$$\alpha'(T_{ref}) = \frac{\alpha(T_{i+1}) - \alpha(T_{ref})}{T_{i+1} - T_{ref}}$$

éq 3.2.1-3

4 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
04/01/00	***A.M. DONORE*** *(EDF/IMA/MMN)*	initial Text