Detection of the singularities and computation of a card of size of elements

Summarized:

One proposes here a method which aims at improving the processing of the singularities in the strategies of mesh adaptation with the software HOMARD (in the case of refinement) or with software GMSH (case of mending of meshes). This mechanism allows, on the one hand to detect the finite elements connected to singular zones and on the other hand to obtain, for a given total error, the size of the finite elements of the new mesh in the event of mending of meshes.
This functionality is accessible in command `CALC_ERREUR` by computation options `SING_ELEM` (constant field by element) or `SING_ELNO` (field at nodes by element). This option is valid only in mechanics. It is necessary to have calculated beforehand an estimator of error in mechanics and strain energy on each element. In any rigor, this method is valid only in elasticity.

Contents

1 Introduction

the purpose option suggested here is to improve the processing of the singularities in the strategies of mesh adaptation suggested in the Code_Aster . Indeed, the presence of singularities (present in practice in any real structural analysis by finite elements) implies two kinds of difficulties which one will describe here as "theoretical" and of "practices".

The "theoretical" difficulties come owing to the fact that the contribution to the error in energy of the elements touching a singularity is form $C h^{\alpha}$ (C a constant, h size of the element and α the order of the singularity) while the contribution of the elements except singularity is form $C h^{q}$ (q depending only on the degree of interpolation of the shape functions of the element). The adaptation of the mesh must take into account this difference to be most effective possible. For example, to divide the contribution of the error by 4, it will be necessary in the case of to take elements 16 times smaller a crack ($\alpha=1/2$) and elements 2 times smaller in the case except crack with quadratic elements ($p=2$).

The "practical" difficulties come owing to the fact that, in zones of singularities, the contributions to the error in energy are important. If one aims at obtaining an error in weak energy, these zones thus should be very strongly refined. However, one can wonder about the influence of these errors in energy on physical quantities which interest the engineer (displacement in such point, maximum stress in such sensitive area, etc…). In other words, it is not because the zones of singularities cause important errors on the energy which they have a great influence on computation apart from these zones. **In practice, the estimators of error quickly indicate the only zones of singularities as being refining: the zones of singularities mask the other errors, for example a zone with strong gradient which one would wish to refine.**

The Laboratory of Mechanics and CAO of Saint-Quentin developed a method making it possible, on the one hand, to detect the singularities, and on the other hand, to determine, for a given total error, the size of the finite elements of the new mesh in the event of mending of meshes.

The use of this two information can be under consideration under two angles:

- The finite elements considered as "singular" by the method can be excluded from the process of cutting,
- the new size of the finite elements is given to a remaillor so that this one builds the new mesh by as well as possible respecting this new card of size. Currently, the software HOMARD cuts out the element once (for example into 2D, a triangle is divided into 4 but not more). To continue cutting, it is necessary to call on HOMARD again. An evolution is thus to envisage so that one can divide several times an element and thus as well as possible respect the card of size of the new mesh. It is however possible to use the free mesh generator GMSH which directly takes a card of size in entry.

Note:

> *This document shows for the group the note resulting from a CRECO between the LMCAO and the department AMA whose reference is quoted in bibliography ([bib1]).*

2 Detection of the singularities

2.1 Principle of the method

When the exact solution of the studied problem present of the singularities, the order of convergence of the solution finite elements is modified and thus also that of the estimator of error. Let us consider, for example, a problem of plane elasticity discretized with triangular elements of degree p .
If the exact solution U_{ex} is regular, it is known that ([bib2], [bib3]):

$$\|u-u_h\|_\Omega=\|e\|_\Omega\leq C\,h^p \qquad \text{éq 2.1-1}$$

Where $\|e\|_\Omega\leq C\,h^p$ is the contribution to the error in energy, is:

$$\|e\|_\Omega\leq\frac{1}{2}\int_\Omega \varepsilon\left(e_h\right)\ K\ \varepsilon\left(e_h\right)d\,\Omega \qquad \text{éq 2.1-2}$$

On the other hand, if the exact solution presents a singularity, for example if, locally in the vicinity of a point M_0 , the field of displacement is form (with r and θ polar coordinates in the vicinity of the point M_0):

$$U_{ex}=r^\alpha V\left(\theta\right)+W\quad avec\quad 0<\alpha<1 \qquad \text{éq 2.1-3}$$

Then, one shows that [Strang & Fix, 1976]:

$$\|e_h\|_\Omega\leq C\,h^\alpha \qquad \text{éq 2.1-4}$$

It results from it that the rate of convergence of the total error in energy becomes independent of the degree p of the finite elements used and it is the same of that of the measurement of the error (for example, so $p=1\,\mathrm{ou}\,2$ then $\alpha=1/2$ for a crack).

In order to obtain a good prediction of the optimized meshes, the preceding observations lead us to use a rate of convergence q_E per element such as the estimator of error ε_E checks:

$$\varepsilon_E=O\left(h^{q_E}\right) \qquad \text{éq 2.1-5}$$

a way simple to define these local coefficients consists in taking:

- $q_E=\alpha$ if the element E is connected to a singularity of order α ;
- $q_E=q$ for all the other finite elements where q depends only on the type of finite elements used.

The method presented thereafter thus comprises three phases:

- detection of the singular zones, in fact singular nodes of the mesh;
- numerical evaluating of the coefficient q_E for the elements connected to the nodes considered as singular (for the other elements, one fixes then $q_E=q$);
- computation of the coefficient of modification of size r_E .

2.2 Detection of the singular nodes

the idea is to use the site errors. Indeed, the experiments show that these site errors present a peak in the vicinity of a singularity. For each node i of the mesh, one thus compares the average error $\bar{m}^i$

of the elements connected to the node i with the average error $\bar{M}$ on the group of structure. The node i is regarded as singular if:

$$\bar{m}^i \geq \beta \bar{M} \qquad \text{éq 2.2.1-1}$$

with

$$\bar{m}^i = \sqrt{\frac{\sum_{\text{E connecté à i}} \varepsilon_E^2}{\sum_{\text{E connecté à i}} mes(E)}} \text{ and } \bar{M} = \sqrt{\frac{\sum_{E \in \text{structure}} \varepsilon_E^2}{\sum_{E \in \text{structure}} mes(E)}} \qquad \text{éq 2.2.1-2}$$

where β is a coefficient larger than 1 and $mes(E)$ surface it in 2D or volume in 3D of the element E . The numerical experiments showed that the singular nodes are well detected while fixing $\beta=2$ in dimension 2, $\beta=3$ in dimension 3 for of the finite elements linear and $\beta=2$ in dimension 3 for of the finite elements quadratic.

Notice 1:

From a numerical point of view, the detection of the singular nodes differs between the cases 2D and 3D. The conditions given thereafter for this detection are not based on a particular theory but rather on the experience gained in this field by the Laboratory of Mechanics and CAO of Saint-Quentin.

In 2D: a node I am regarded as singular if he meets the 3 following conditions:

$$\bar{m}_1^i \geq \beta \bar{M}$$
$$\bar{m}_1^i \geq \bar{m}_2^i \qquad \text{éq 2.2.1-3}$$
$$\bar{m}_1^i \geq 3\,\mathrm{Min}\left(\bar{m}_2^i, \bar{m}_3^i\right)$$

Where $\bar{m}_1^i$, $\bar{m}_2^i$ *and* $\bar{m}_3^i$ *is the averages of the error for the elements belonging to layers 1,2 and 3, respectively, compared to the node* i *considered.*

The layers are defined as follows:

Lay down 1: elements which have node I to test,
Layer 2: elements in contact (face, edge or node) with an element of layer 1,
Layer 3: elements in contact (face, edge or node) with an element of layer 2.

In 3D: the node i *is regarded as singular if he meets the condition* $\bar{m}_1^i \geq \beta \bar{M}$ *and if one of the nodes connected to the node* i *considered meets the condition* $\bar{m}_i^{\text{Noeud connecté à i}} \geq \beta \bar{M}$ *. Contrary to the case 2D, the node* i *is singular only if one of its neighbors is also (one forgets the singular nodes isolated to keep only the singular edges).*

Notice 2:

In 2D, only the nodes tops are examined. In 3D, only the nodes tops located on an edge of structure are examined (only for reasons of time computation; this condition could thus be modified).

2.3 Evaluating about the singularity

For each detected singular i node, the order of the singularity, i.e. the value of q_E which will be used for the elements connected to the node i , is given by identifying the value of the density of energy of the solution finite elements in the vicinity of the node i with the theoretical value in the vicinity of a singular point.

2.3.1 Case of dimension 2

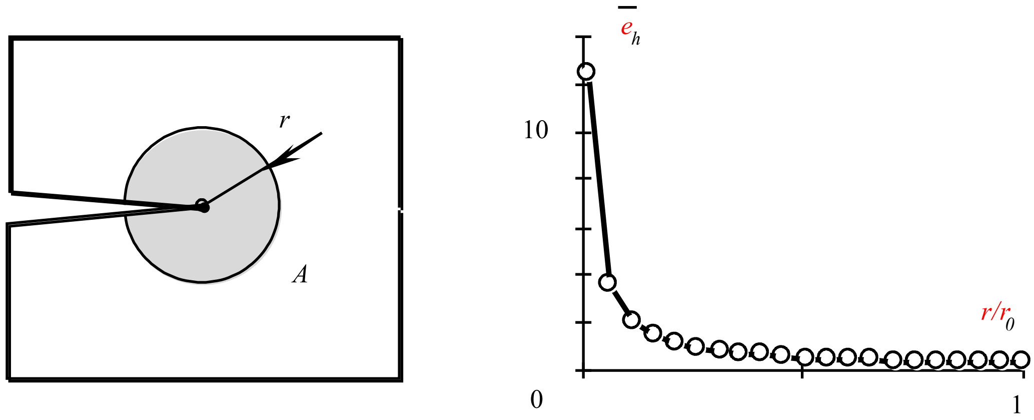
In this case, one calculates energy finite elements average, in discs A of center i and r :

$$\bar{e}_h(r)=\frac{1}{2\,mes(A)}\int_A \varepsilon(u_h)\;K\;\varepsilon(u_h)\,dA$$ éq 2.3.1-1

While identifying, by a method of least squares, this average energy with the theoretical value in the vicinity of a singularity of order α :

$$e(r)=k\,r^{2(\alpha-1)}+c$$ éq 2.3.1-2

one numerically obtains a value $\bar{\alpha}$ close to α .

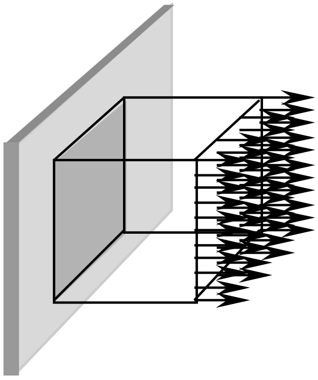


Appear 2.3.1-a: Numerical evaluating of α

In practice the numerical experiments show that it is enough to carry out the identification in a zone corresponding to 3 layers of elements around the singular point and to evaluate $\bar{e}^h(r)$ for 5 to 8 values of r regularly distributed in this zone (we took 10 values).

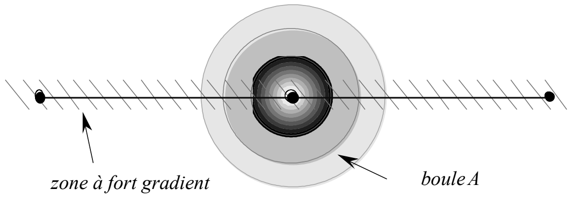
2.3.2 Case of dimension 3

In 3D the situation is more complex. The point singular, generally, are not isolated and it is thus frequent to be in the presence of singular edges. Let us consider, for example, the case of a cube embedded on a face and subjected to tractive efforts: all the points of the edges of the clamped face are singular [Figure 2.3.2-b].



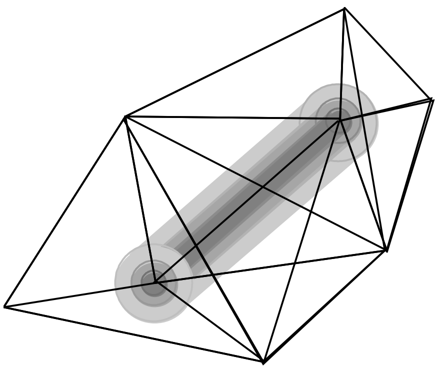
Appear 2.3.2-b: Cubic embedded in tension

In this situation, the evaluating of average energy in balls A of radius growing and centered on a singular node does not make it possible to identify q_E . Indeed, as the radius increases the extent of the singular zone contained in the ball A increases and one does not obtain a fast decrease of $\overline{e}_h$ [Figure 2.3.2-c].



Appear 2.3.2-c: Energy in concentric balls

When the singular points are not isolated, it is necessary to identify the coefficient q_E by calculating the density of energy in coaxial cylinders built on the edges whose ends were regarded as singular [Figure 2.3.2-d] and cf notices [§ 2.2].



Appear 2.3.2-d: Energy in coaxial cylinders

2.4 Extension to the zones of stress concentration

In practice, we noted that the preceding method, clarification on cases presenting of the singularities also makes it possible to take into account correctly the zones with strong gradients of stresses even if mathematically these zones do not correspond to singularities.

3 Construction of an optimal mesh

3.1 General information

the purpose of a procedure of adaptation is to guarantee to the user a level of accuracy on the total error while minimizing the costs of computation. To evaluate the errors of discretization, one and the uses a relative total measurement of ε the error local contributions associated ε_E with:

$$\varepsilon^2=\sum_E \varepsilon_E{}^2 \quad \text{éq 3.1-1}$$

the idea is and to use the results of this first analysis finite elements estimators of errors to determine an optimal mesh T^* i.e. a mesh which makes it possible to respect desired accuracy the while minimizing the costs of computation. One builds then the mesh T^* using an automatic mesh generator and one carries out one second analysis finite elements.

3.2 Definition of optimality

For a given total error ε_0, a mesh T^* is optimal compared to a measurement of error ε if:

$$\begin{array}{ll} \varepsilon^*=\varepsilon_0 & \text{précision demandée} \\ N^* & \text{nombre d'éléments de } T^* \text{ est minimum} \end{array} \quad \text{éq 3.2-1}$$

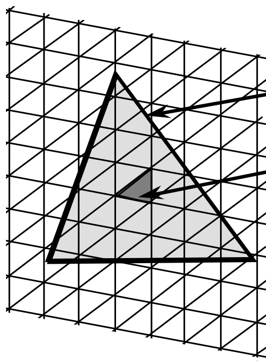
This criterion of optimization naturally results in minimizing the costs of computation.

3.3 Determination of an optimal mesh

to determine the characteristics of the optimal mesh T^*, the method consists in calculating on each element E of the mesh T a coefficient of modification of sizes:

$$r_E=\frac{h_E^*}{h_E} \quad \text{éq 3.3-1}$$

Where h_E is the current size of the element E and h_E^* the size (unknown) which it is necessary to force on the elements of T^* in the zone E to ensure optimality [Figure 3.3-a]. A possible choice to define the size of an element h_E is to take the size of largest with dimensions of this element. The determination of the optimal mesh is thus brought back to the determination, on the initial mesh T, of a card of coefficients of modification of size.



Appear 3.3-a: Definition of the sizes

The computation of the coefficients r_E is based on the rate of convergence of the error:

$$\varepsilon = O\left(h^q\right) \quad \text{éq 3.3-2}$$

where q depends on the type of finite element used but also on the regularity of the exact solution of with the dealt problem. For the "classical" estimators of error, one supposes that the rate of convergence of the estimator of error is equal to the order of convergence of the solution finite elements. For the estimators in quantity of interest, this rate of convergence is equal to the double about convergence of the solution finite elements ([bib4]).

Thereafter, to compute: the coefficient of modification of size r_E, one distinguishes the case from the regular solution (q depends only on p, degree of interpolation of the shape functions of the element) of the case of the singular solution (q depends only on α, order of the singularity of the field of displacement).

3.4 "Classical" estimators of error

One designates by "classical" estimators of error the estimators who provide a norm (norm - *L2*, norm - *H1*, norm in energy) of the error in solution.

3.4.1 Case of the regular solution

Initially, we suppose that the exact solution is sufficiently regular so that the value of q depends only on the type of finite elements used and is equal to the degree of interpolation used p (p is worth 1 for of the finite elements linear and 2 for of the finite elements quadratic). In this case, to predict the optimal sizes, it is written that the ratio of the sizes is related to the ratio of the contributions to the error by:

$$\frac{\varepsilon_E^*}{\varepsilon_E} = \left[\frac{h_E^*}{h_E}\right] = r_E^p \quad \text{éq 3.4.1-1}$$

where ε_E^* represents the contribution of the elements of T^* located in the zone E, i.e.:

$$\varepsilon_E^* = \left[\sum_{E^* \subset E} \varepsilon_{E^*}^2\right]^{1/2} \quad \text{éq 3.4.1-2}$$

ε_E^* is the error of the element E^* calculated on the mesh T.

The square of the error on the mesh T^* can thus be evaluated by:

$$\sum_E \left(\varepsilon_E^*\right)^2 = \sum_E r_E^{2p} \varepsilon_E^2$$ éq 3.4.1-3

and the number of elements from T^* :

$$N^* = \sum_E \frac{1}{r_E^d}$$ éq 3.4.1-4

Where d is the dimension of space (in practice, $d=2$ or 3).

Indeed, $r_E = \frac{h_E^*}{h_E} = \left(\frac{V}{N_E{}^*}\right)^{1/d} \left(\frac{N}{V}\right)^{1/d}$ with N the number of element of T in E (thus 1), N_E^* the number of element of T^* in the zone of E . One thus has $N_E^* = \frac{1}{r_E^d}$, that is to say $N^* = \sum_E N_E^* = \sum_E \frac{1}{r_E^d}$ the nombre total of elements of T^* .

The problem to be solved is thus:

$$\text{Minimiser} \quad N^* = \sum_E \frac{1}{r_E^d} \quad \text{avec} \quad \sum_E r_E^{2p} \varepsilon_E^2 = \varepsilon_0^2$$ éq 3.4.1-5

It acts of a problem of optimization with a stress on the variables of optimization.
Introducing a multiplier of Lagrange, noted A , the problem [éq 3.4.1-5] reverts returning extremum the Lagrangian one:

$$L\left(\left\{r_E\right\}_{E \in T} \;;\; A\right) = \sum_E \frac{1}{r_E^d} + A\left(\sum_E r_E^{2p} \varepsilon_E^2 - \varepsilon_0^2\right)$$ éq the 3.4.1-6

conditions of extremality give:

$$\frac{\partial L}{\partial r_E} = \frac{-d}{r_E^{d+1}} + 2\,A\,p\,\varepsilon_E^2 r_E^{2p-1} = 0 \quad \forall E \in T$$ éq 3.4.1-7

From where:

$$r_E = \left[\frac{d}{2\,A\,p\,\varepsilon_E^2}\right]^{1/(2p+d)}$$ éq 3.4.1-8

While deferring in the second equation of [éq 3.4.1-5], one from of deduced A :

$$A = \frac{d}{2\,p}\left[\frac{\sum_E \varepsilon_E^{2d/(2p+d)}}{\varepsilon_0^2}\right]^{(2p+d)/2p}$$ éq 3.4.1-9

One replaces the statement of A thus obtained in the equation [éq 3.4.1-8] to obtain r_E :

$$r_E = \frac{\varepsilon_0^{1/p}}{\varepsilon_E^{2/(2p+d)} \left[\sum_E \varepsilon_E^{2d/(2p+d)} \right]^{1/2p}}$$ éq 3.4.1-10

3.4.2 Cases of the singular zones

to predict the optimal sizes, one uses a rate of convergence q_E defined by element:

$$\frac{\varepsilon_E^*}{\varepsilon_E} = \left[\frac{h_E^*}{h_E} \right]^{q_E} = r_E^{q_E}$$ éq 3.4.2-1

where ε_E^* represents the contribution of the elements of T^* located in the zone E , i.e.:

$$\varepsilon_E^* = \left[\sum_{E^* \subset E} \varepsilon_{E^*}^2 \right]^{1/2}$$ éq 3.4.2-2

the square of the error on the mesh T^* can thus be evaluated by:

$$\sum_E \left(\varepsilon_E^* \right)^2 = \sum_E r_E^{2 q_E} \varepsilon_E^2$$ éq 3.4.2-3

and the number of elements of T^* is always evaluated by:

$$N^* = \sum_E \frac{1}{r_E^d}$$ éq 3.4.2-4

the new problem to be solved is thus:

$$\text{Minimiser} \quad N* = \sum_E \frac{1}{r_E^d} \quad \text{avec} \quad \sum_E r_E^{2 q_E} \varepsilon_E^2 = \varepsilon_0^2$$ éq 3.4.2-5

which is a problem of optimization with a stress on the variables of optimization.
Introducing a multiplier of Lagrange, noted A , the problem reverts returning extremum the Lagrangian one:

$$L\left(\left\{ r_E \right\}_{E \in T} \; ; \; A \right) = \sum_E \frac{1}{r_E^d} + A \left(\sum_E r_E^{2 q_E} \varepsilon_E^2 - \varepsilon_0^2 \right)$$ éq the 3.4.2-6

conditions of extremality give:

$$\frac{\partial L}{\partial r_E} = -\frac{d}{r_E^{d+1}} + 2 A q_E \varepsilon_E^2 r_E^{2 q_E - 1} = 0 \quad \forall E \in T$$ éq 3.4.2-7

From where:

$$r_E = \left[\frac{d}{2 A q_E} \varepsilon_E^2 \right]^{1/(2 q_E + d)}$$ éq 3.4.2-8

While deferring in the second equation of [éq 3.4.2-5], one obtains a nonlinear equation in A (because q_E depends on the elements):

$$\sum_E \left[\left[\frac{d}{2 A q_E} \right]^{2 q_E/(2 q_E + d)} \varepsilon_E^{2d/(2 q_E + d)} \right] - \varepsilon_0^2 = 0 \qquad \text{éq 3.4.2-9}$$

It is solved by the method of Newton (the multiplier of Lagrange is initialized by taking the multiplier of Lagrange of the regular solution i.e. the statement [éq 3.4.2-9] with $q_E = p$). Once A calculated, one from of deduced r_E by the equation [éq 3.4.2-8].

3.5 Estimators of error in quantities of interest

One designates by estimators of error in quantities of interest the estimators who provide an error on a precise physical quantity (quantity of interest) on a selected zone.

3.5.1 Case of the regular solution

In the case of the estimators in quantity of interest, the value of q is worth $2p$ [bib4] (p is worth 1 for of the finite elements linear and 2 for of the finite elements quadratic). To predict the optimal sizes, it is written that the ratio of the sizes is related to the ratio of the contributions to the error by:

$$\frac{\varepsilon_E^*}{\varepsilon_E} = \left[\frac{h_E^*}{h_E} \right]^{2p} = r_E^{2p} \qquad \text{éq 3.5.1-1}$$

where ε_E^* represents the contribution of the elements of T^* located in the zone E , i.e.:

$$\varepsilon_E^* = \sum_{E^* \subset E} \varepsilon_{E^*} \qquad \text{éq 3.5.1-2}$$

ε_E^* is the error of the element E^* calculated on the mesh T .

The error on the mesh T^* can thus be evaluated by:

$$\sum_E \varepsilon_E * = \sum_E r_E^{2p} \varepsilon_E \qquad \text{éq 3.5.1-3}$$

and the number of elements from T^* :

$$N^* = \sum_E \frac{1}{r_E^d} \qquad \text{éq 3.5.1-4}$$

Where d is the dimension of space (in practice, $d = 2$ or 3).

The problem to be solved is thus:

$$\text{Minimiser} \quad N^* = \sum_E \frac{1}{r_E^d} \quad \text{avec} \quad \sum_E r_E^{2p} \varepsilon_E = \varepsilon_0 \qquad \text{éq 3.5.1-5}$$

There still, it acts of a problem of optimization with a stress on the variables of optimization.

Introducing a multiplier of Lagrange, noted A, the problem [éq 3.5.1-5] reverts returning extremum the Lagrangian one:

$$L\left(\left\{ r_E \right\}_{E\in T};\mathrm{A}\right)=\sum_E \frac{1}{r_E^d}+A\left(\sum_E r_E^{2p}\varepsilon_E-\varepsilon_0\right)$$ éq the 3.5.1-6

conditions of extremality give:

$$\frac{\partial L}{\partial r_E}=-\frac{d}{r_E^{d+1}}+2\,A\,p\,\varepsilon_E r_E^{2p-1}=0 \quad \forall E\in T$$ éq 3.5.1-7

From where:

$$r_E=\left[\frac{d}{2\,A\,p\,\varepsilon_E}\right]^{1/(2p+d)}$$ éq 3.5.1-8

While deferring in the second equation of [éq 3.5.1-5], one from of deduced A :

$$A=\frac{d}{2p}\left[\frac{\sum_E \varepsilon_E^{d/(2p+d)}}{\varepsilon_0}\right]^{(2p+d)/2p}$$ éq 3.5.1-9

One replaces the statement of A thus obtained in the equation [éq 3.5.1-8] to obtain r_E :

$$r_E=\frac{\varepsilon_0^{1/2p}}{\varepsilon_E^{1/(2p+d)}\left[\sum_E \varepsilon_E^{d/(2p+d)}\right]^{1/2p}}$$ éq 3.5.1-10

3.5.2 Cases of the singular zones

to predict the optimal sizes, one imposes now:

$$\frac{\varepsilon_E^*}{\varepsilon_E}=\left[\frac{h_E^*}{h_E}\right]^{2q_E}=r_E^{2q_E}$$ éq 3.5.2-1

where ε_E^* represents the contribution of the elements of T^* located in the zone E , i.e.:

$$\varepsilon_E^*=\sum_{E^*\subset E}\varepsilon_{E^*}$$ éq 3.5.2-2

the square of the error on the mesh T^* can thus be evaluated by:

$$\sum_E \varepsilon_E^*=\sum_E r_E^{2q_E}\varepsilon_E$$ éq 3.5.2-3

and the number of elements of T^* is always evaluated by:

$$N^*=\sum_E \frac{1}{r_E^d}$$ éq 3.5.2-4

the new problem to be solved is thus:

$$\text{Minimiser} \quad N^*=\sum_E \frac{1}{r_E^d} \quad \text{avec} \quad \sum_E r_E^{2q_E}\varepsilon_E=\varepsilon_0$$ éq 3.5.2-5

which is a problem of optimization with a stress on the variables of optimization.
Introducing a multiplier of Lagrange, noted A, the problem reverts returning extremum the Lagrangian one:

$$L\left(\{r_E\}_{E\in T}\ ;\ A\right)=\sum_E \frac{1}{r_E^d}+A\left(\sum_E r_E^{2q_E}\varepsilon_E-\varepsilon_0\right)$$ éq the 3.5.2-6

conditions of extremality give:

$$\frac{\partial L}{\partial r_E}=-\frac{d}{r_E^{d+1}}+2Aq_E\varepsilon_E r_E^{2q_E}-1=0 \quad \forall E\in T$$ éq 3.5.2-7

From where:

$$r_E=\left[\frac{d}{2Aq_E}\varepsilon_E\right]^{1/(2q_E+d)}$$ éq 3.5.2-8

While deferring in the second equation of [éq 3.5.2-5], one obtains a nonlinear equation in A :

$$\sum_E\left[\left[\frac{d}{2Aq_E}\right]^{2q_E/(2q_E+d)}\varepsilon_E^{d/(2q_E+d)}\right]-\varepsilon_0=0$$ éq 3.5.2-9

It is solved by the method of Newton (the multiplier of Lagrange is initialized by taking the multiplier of Lagrange of the regular solution i.e. the statement [éq 3.5.2-9] with $q_E=p$). Once A calculated, one from of deduced r_E by the equation [éq 3.5.2-8].

4 Use in *Code_Aster*

4.1 the commands

the order of the singularity and the card of modification of size are calculated by the command `CALC_ERREUR` by activating options "`SING_ELEM`" (constant field by element) or "`SING_ELNO`" (field at nodes by element).

Option "`SING_ELEM`" calculates, on each element, two components:

- "`DEGRE`" : the order of the singularity i.e. the value of the coefficient q_E (which is worth p if the element is not connected to a singular node and who is worth α if not);
- "`RATIO`" : the relationship between the current size h_E and the new size h_E^* of the finite element ($h_E/h_E^*=1/r_E$);
- "`TAILLE`" : new size h_E^* of the finite element ($h_E^*=r_E h_E$).

The computation of this option requires, as a preliminary, the computation of an estimator of error (it is the absolute component which is used and it is coded into tough in *Code_Aster*) and of total strain energy. If one of these options is not calculated, an alarm message is transmitted and option "SING_ELEM" is not calculated.

The user can inform optional key word "TYPE_ESTI" by indicating one of the options following:

- "ERME_ELEM" for the estimator based on the residues;
- "ERZ (1 or 2) _ELEM_SIGM" for the estimator based on the smoothed stresses (Zhu - Zienkiewicz version 1 or 2);
- "QIRE_ELEM" for the estimator in quantity of interest based on the residues;
- "QIZ (1 or 2) _ELEM_SIGM" for the estimator in quantity of interest based on the smoothed stresses (Zhu - Zienkiewicz version 1 or 2).

If this key word is not then indicated the estimator based on the residues "ERME_ELEM" is chosen by default (transmitted alarm message). If the two estimators of Zhu-Zienkiewicz are present, one chooses "ERZ1_ELEM".

For total strain energy, one uses:

- With STAT_NON_LINE: "ETOT_ELEM" which is total strain energy on a finite element (valid for an elastic behavior and an elastoplastic behavior "VMIS_ISOT_XXX").
- With MECA_STATIQUE : "EPOT_ELEM" which is the potential energy of elastic strain on a finite element and integrated starting from displacements and temperature (valid only for one elastic behavior).

The user must also inform the key word "PREC_ERR" which makes it possible to calculate the accuracy ε_0 of the equation [éq 2.2-1] in the following way: $\varepsilon_0 = \mathrm{PREC\_ERR} * Erreur_{totale}$. The value of "PREC_ERR" strictly lies between 0 and 1 (a fatal message is transmitted if this condition is not checked).

For the option "SING_ELNO", it acts of a recopy of the values of "SING_ELEM" to the nodes of the element. The computation preliminary of "SING_ELEM" is thus necessary. If "SING_ELEM" is absent, an alarm message is transmitted and option "SING_ELNO" is not calculated.

4.2 Perimeter of use

the perimeter of use is the same one (but more reduced) than that of the estimator of error chosen namely:

- For the estimator in residue: finite elements of the continuums in 2D (triangles and quadrangles) or 3D (only tetrahedrons) for an elastoplastic behavior,
- the estimator of Zhu-Zienkiewicz: finite elements of the continuums in 2D (triangles and quadrangles) for an elastic behavior.

In any rigor (cf [§2]), computation about the singularity is obtained from theoretical energy at a peak of crack [éq 2.3.1-2], valid equation only in elasticity. The use of this option in elastoplasticity is thus to handle with prudence.

5 Bibliography

[1] COOREVITS P.: Mechanism of detection of the singularities. First part. Note Laboratory of Mechanics and CAO (Saint-Quentin).

[2] CIARLET P. - G.: The finite element method for elliptic problems, North-Holland, 1978.

[3] STRANG & FIX: Year analysis of the finite element method, Prentice hall, 1976.

[4] Estimators of error in quantities of interest. [R4.10.06]

6 Description of the versions of the document

Index Doc.	Version Aster	Author (S) or contributor (S), organization	Description of the modifications
A	8.4	V.Cano EDF/R & D /AMA	initial Text
B	9.4	J.Delmas EDF/R & D /AMA	Recasting of the document + addition of the estimators of error in quantities of interest