Error indicator in residue for modelizations HM

Résumé

In this document, one presents the error indicators a posteriori developed for modelizations HM. The estimate a posteriori concerned is of standard explicit residue. two types of problems are treated: the permanent version and the transitory version of saturated modelization HM. One gives initially a frame of work for the study a posteriori of these problems. One introduces then the families of error indicators for the two types of problem and one states the theoretical results of reliability and optimality which guarantee the validity of the indicators. The evidence of the announced results will not be given, the interested reader will be able for that to consult [1].

Contents

1 Frame of work

In this paragraph, one points out the equations which constitute the versions permanent and transitory problem HM. One proposes some then adimensionnement who will be useful a posteriori for the analysis of error. One advises with the reader as a preliminary to have consulted the documents [2,3,4] for more details on the establishment of the equations given in this section and on their resolution in Code_Aster.

1.1 The continuous model problem

the frame of work is that of the linear poroelasticity. One considers a porous environment Ω , of border Γ , saturated by liquid water, with a linear elastic structural mechanics behavior. The behavior of the fluid influences that of the squelette and reciprocally. Spatial dimension is noted $\dim$. One solves a mechanical problem of equilibrium (assessment of linear momentum for the squelette and the fluid) and a problem of evolution in hydraulics (fluid weight breakdown). It is pointed out that the processing of these two problems is completely coupled in Code_Aster. The unknowns of the problem are displacement u and the water pressure p .
Being given a time of simulation T , the mechanical equilibrium is formulated

$$-\nabla\cdot\sigma'(u)+b\nabla p=f \text{ in } [0,\mathrm{T}]\times\Omega \qquad \textbf{1.1-1}$$

where one posed $f=\rho_{ref}F^m$ and

σ'	Tensor of the effective stresses
b	Coefficient of Biot
ρ_{ref}	homogenized Density
F^m	Force of gravity

In the frame of isotropic linear elasticity, the tensor of the effective stresses admits like statement

$$\sigma'(u)=\lambda_1(\nabla\cdot u)Id+\lambda_2(\nabla u+\nabla u^t) \qquad \textbf{éq 1.1-2}$$

where λ_1 and λ_2 are the coefficients of Lamé. The hydraulic equation comes as for it from the writing from the conservation from the fluid mass in the course of time and the model from Darcy. The model of Darcy provides a relation of proportionality between hydraulic flux M_{lq} and the gradient of pressure. More precisely, she is written

$$\frac{M_{lq}}{\rho}=\kappa(-\nabla p+\rho F^m) \qquad \textbf{éq 1.1-3}$$

the hydraulic equation is written

$$\partial_t\left(\frac{1}{M}p+b\nabla\cdot u\right)-\nabla\cdot(\kappa\nabla p)=g \text{ in } s[0,\mathrm{T}]\times\Omega \qquad \textbf{1.1-4}$$

where $1/M=(b-\varphi)/K_s$ and $g=-\rho\nabla\cdot(\kappa F^m)$.

φ	Lagrangian porosity
ρ	Density hydraulic
κ	hydraulic Conductivity

K_s	Compressibility of the solid matter constituents

One introduces the permanent version of the equation [1.1-4]

$$-\nabla\cdot(\kappa\nabla p)=g \text{ into } \Omega \qquad \text{éq 1.1-5}$$

the equation [1.1-1] coupled with [1.1-4] constitutes the transitory **version** of problem HM.

The equation [1.1-1] coupled with [1.1-5] constitutes the permanent **version** of problem HM.

The boundary conditions are of mixed type Dirichlet/Neumann on displacement and the pressure. One considers a partition of the border in the form $\Gamma=\Gamma_D^H\cup\Gamma_N^H=\Gamma_D^M\cup\Gamma_N^M$.

On Γ_D^H , one imposes boundary conditions of Dirichlet in hydraulics: $p=p_D$.
On Γ_N^H , one imposes boundary conditions of Neumann in hydraulics: $M_{lq}\cdot n=M_{lq,nor}$.

On Γ_D^M , one imposes boundary conditions of Dirichlet in mechanics: $u=u_D$.
On Γ_N^M , one imposes boundary conditions of Neumann in mechanics: $(\sigma'(u)-bp\chi_S Id)\cdot n=\sigma_{nor}$,
where one noted $\Gamma_S=\Gamma_N^M\cap\Gamma_N^H$, of characteristic function χ_S , the portion of border where the stresses and the hydraulic flux are imposed.

1.2 Adimensionnement

the model problem utilizes two unknowns, displacements and the pressure of the fluid from which the orders of magnitude can be very different. One thus proposes a setting at the level of the equations constituting the model problem, which will be used as starting point with the analysis of error a posteriori. One draws attention to the fact that this scaling is not used in the numerical stage of resolution. The estimates of error a posteriori of section 2 will thus be obtained on the adimensionné problem, then redimensionnées for their use in Code_Aster.

For any variable a , a^* the corresponding adimensionnée variable and A the corresponding characteristic quantity are noted. For example for the water pressure, there is the relation

$$p=Pp^* ,$$

where P indicates a characteristic pressure. One uses same symbolism for the mathematical operators. Thus, one a: $\nabla^*\cdot u^*=\frac{L}{U}\nabla\cdot u$ $\varepsilon^*(u^*)=\frac{L}{U}\varepsilon(u)$ $\nabla^* p^*=\frac{L}{P}\nabla p$.

1.2.1 Transitory case

the mechanical equation rewrites

$$-\nabla^*\cdot\sigma^*(u^*)+b\nabla^* p^*=f^* \qquad \text{éq 1.2-1}$$

with $f^*=\frac{L}{P}f$. By noting ν the Poisson's ratio, $\sigma^*(u^*)$ of the tensor of the stresses given by formula éq 1.2-2 One

$$\sigma^*(u^*)=\frac{\nu}{(1-2\nu)(1+2\nu)}(\nabla^*\cdot u^*)Id+\frac{1}{1+\nu}\varepsilon^*(u^*) \qquad \textbf{a adimensionnée}$$

version proposes the strategy of scaling following. The user chooses two quantities: the scale length L and the scale of pressure P. The scale length L is determined by the geometry of the model. The scale of pressure is determined by the limiting conditions of Dirichlet imposed on the pressure or possibly by the initial conditions. For the mechanical part, one imposes

$$U=\frac{LP}{E} \quad \textbf{éq 1.2-3}$$

In addition, the hydraulic equation is written, by taking account of [1.2-3],

$$\partial_t^*\left(\frac{E}{M}p^*+b\nabla^*\cdot u^*\right)-\frac{E}{M}\Delta^* p^*=g^* \quad \textbf{éq 1.2-4}$$

with $g^*=\frac{E}{M}\frac{L^2}{\kappa P}g$.

The adimensionnée version of transitory problem HM is thus:

$$\begin{cases} -\nabla^*\cdot\sigma^*(u^*)+b\nabla^* p^*=f^* & \text{dans } [0,T^*]\times\Omega^* \\ \partial_t^*\left(\frac{E}{M}p^*+b\nabla^*\cdot u^*\right)-\frac{E}{M}\Delta^* p^*=g^* & \text{dans } [0,T^*]\times\Omega^* \end{cases} \quad \textbf{éq 1.2-5}$$

1.2.2 Cases permanent

One proceeds like above: the user chooses L and P then evaluates U according to [1.2-3]. The mechanical equation is treated as in the preceding paragraph. One thus has the equation [1.2-1]. The hydraulic equation is written

$$-\frac{E}{M}\Delta^* p^*=g^* \quad \textbf{éq 1.2-6}$$

where g^* is defined like above. The multiplication of the hydraulic equation by the factor E/M can seem a little artificial because the parameter M does not intervene in a permanent modelization. It is introduced so that the system [1.2-1] - [1.2-6] provides well the permanent version of [1.2-1] - [1.2-5].

The adimensionnée version of permanent problem HM is thus:

$$\begin{cases} -\nabla^*\cdot\sigma'^*(u^*)+b\nabla^* p^*=f^* & \text{dans } \Omega^* \\ -\frac{E}{M}\Delta^* p^*=g^* & \text{dans } \Omega^* \end{cases}$$

1.3 Discretization finite elements and notations

problem HM is discretized by a method finite elements which leans on a mesh $(T_h)_{h>0}$. It is pointed out that displacements are discretized by polynomials of degree 2 and the pressure by polynomials of degree 1. Discrete displacements and u_h the discrete pressure are noted p_h .
One indicates by F_h^i all the interior sides of the mesh and by F_h^∂ all the sides located on edge of the field Ω . For a given $K\in T_h$ mesh, one notes F_K all the sides of K and one installation

$$F_K^i = F_h^i \cap F_K, F_K^\partial = F_h^\partial \cap F_K$$

Is $F \in F_h^i$, i.e. such as there exist two meshes K_1 and K_2 in T_h with $F = K_1 \cap K_2$. One indicates by n_{K_1} and the n_{K_2} normal external with K_1 and K_2 , respectively (see figure 1). One notes, for almost all $x \in F$,

$$\left[\Phi\left(u_h\right) \cdot n\right](x) = \left(\Phi\left(u_h\right)\right)_{|K_1}(x) \cdot n_{K_1} + \left(\Phi\left(u_h\right)\right)_{|K_2}(x) \cdot n_{K_2}$$

the jump of the normal component of $\Phi\left(u_h\right)$ through F .

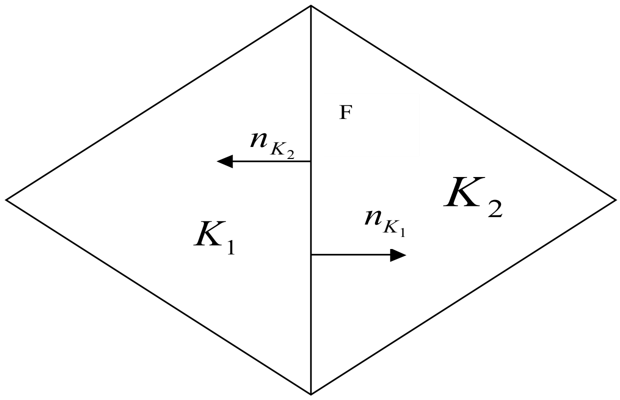

Figure 1: Example of interior face of mesh.

One observes that like the vectors n_{K_1} and n_{K_2} are opposed, the quantity above defines this jump well.
For any mesh K , one indicates by Δ_K the whole of meshes dividing an edge with K , including the mesh K , as illustrated on figure 2.

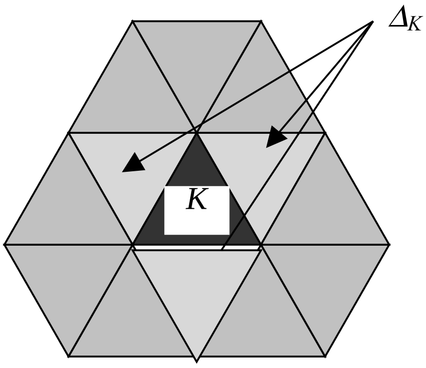

Figure 2: Example of mesh K and macro-element Δ_K

For the temporal discretization of the transitory problem, one considers an implicit diagram of Eulerian and one notes $\left\{t_i\right\}_{i=1}^N$ a discretization of the interval $[0, T]$. One notes $u_{h\tau}$ (respectively $p_{h\tau}$) the function continues and closely connected per pieces in time such as for all $n \in \{0, \ldots, N\}$, $u_{h\tau}\left(t_n\right) = u_h^n$ (respectively $p_{h\tau}\left(t_n\right) = p_h^n$). One also needs to consider constant functions per pieces in

time, namely $\pi^0 p_{h\tau}$ equal to p_h^n about $\left[t_{n-1}, t_n\right]$. For all $n\in\{1,\ldots,N\}$, one poses $\tau_n = t_n - t_{n-1}$ and $I_n = \left[t_{n-1}, t_n\right]$.
One also notes discrete "derivatives" of displacements and of the pressure

$$\delta_t u_h^n = \tau_n^{-1}\left(u_h^n - u_h^{n-1}\right)$$
$$\delta_t p_h^n = \tau_n^{-1}\left(p_h^n - p_h^{n-1}\right)$$

the notation $x<y$ means that there exists a constant $c>0$ independent of the mesh such as $x\leq cy$. The notation $\|\cdot\|_V$ indicates a norm on a space V. One introduces his version $\|\cdot\|_{V,K}$, localised to the mesh K, par. $\|\cdot\|_V^2 = \sum_{K\in T_h} \|\cdot\|_{V,K}^2$

2 Study of the permanent problem

In this paragraph, one introduces 2 families of error indicators put in work in Code_Aster for the permanent problem. The first family is optimal for the estimate of error on the oscillations of pressure. Second is optimal for the estimate of error in energy on displacements.

One points out the statement of the problem in which one is interested. To find $\left(u^*, p^*\right)$ such as

$$\begin{cases} -\nabla^*\cdot\sigma'^*(u^*) + b\nabla^* p^* = f^* & \text{dans } \Omega^* \\ -\dfrac{E}{M}\Delta^* p^* = g^* & \text{dans } \Omega^* \end{cases}$$

2.1 Estimate for the pressure

One defines the estimators of error in displacement

$$\begin{aligned} E_u &= \sum_{K\in T_h} E_{u,K} \\ &= \sum_{K\in T_h} \Big(h_K^2 \frac{1}{P^2 L^{\dim}} \|f + \nabla\cdot\sigma'(u_h) - b\nabla p_h\|_{0,K}^2 + h_K \frac{1}{P^2 L^{\dim}} \sum_{F\in F_K^i} \|\left[\sigma'(u_h)\cdot n\right]\|_{0,F}^2 \\ &\quad + h_K \frac{1}{P^2 L^{\dim}} \sum_{F\in F_K^\partial\cap\Gamma_N^M} \|\sigma_{nor} - \left(\sigma'(u_h)\cdot n - bp_h n\right)\|_{0,F}^2 \Big) \end{aligned}$$

One defines the estimators in pressure

$$\begin{aligned} E_{p,0} &= \sum_{K\in T_h} E_{p,0,K} \\ &= \sum_{K\in T_h} \Big(h_K \frac{E^2}{P^2 L^{\dim-2} M^2 \kappa^2 \rho^2} \sum_{F\in F_K^i} \|\left[M_{lq,h}\cdot n\right]\|_{0,F}^2 \\ &\quad + h_K \frac{E^2}{P^2 L^{\dim-2} M^2 \kappa^2 \rho^2} \sum_{F\in F_K^\partial\cap\Gamma_N^H} \|M_{lq,nor} - M_{lq,h}\cdot n\|_{0,F}^2 \Big) \end{aligned}$$

One has result of total increase in space of the error:

Theorem 1 (Reliability)

$$\|u-u_h\|_a^2+\|p-p_h\|_d^2 < E_u+E_{p,0}$$

where for all $(v,q)\in\left[H^1(\Omega)\right]^{\dim}\times H^1(\Omega)$, one posed:

$$\begin{cases}-\nabla^*\cdot\sigma'^*(u^*)+b\nabla^* p^*=f^* & \text{dans } [0,T^*]\times\Omega^*\\ \partial_t^*\left(\frac{E}{M}p^*+b\nabla^*\cdot u^*\right)-\frac{E}{M}\Delta^* p^*=g^* & \text{dans } [0,T^*]\times\Omega^*\end{cases}$$

$$\|v\|_a^2=\frac{E^2}{P^2 L^{\dim}}\left(\int_\Omega \lambda_1(\nabla\cdot v)^2+2\int_\Omega \lambda_2\varepsilon(v)^2\right)$$
$$\|q\|_d^2=\frac{1}{P^2 L^{\dim-2}}\frac{E}{M}\int_\Omega(\nabla q)^2$$

There are the following results of local decrease in space of the error

Theorem 2 (Optimality)

For all $K\in T_h$, one has

$$E_{u,K}<\sum_{K'\in\Delta_K}\left(h_K^2\|f-f_h\|_{0,K'}^2+\|u-u_h\|_{a,K'}^2+\|p-p_h\|_{0,K'}^2\right)$$
$$E_{p,0,K}<\sum_{K'\in\Delta_K}\left(h_K^2\|g-g_h\|_{0,K'}^2+\|p-p_h\|_{d,K'}^2\right)$$

the estimates obtained are optimal for the estimate of error on the pressure. Indeed, analysis of error a priori watch that normalizes of it H^1 displacements convergent with order 2 and the pressure with order 1. By combining the estimates of theorem 1 with those of theorem 2 and by means of this result of analysis of error a priori, one has that the estimators a posteriori E_u and $E_{p,0}$ converge overall with order 1, which is optimal for the estimate of error on the pressure but not for the estimate on displacements. One "is bridled" by the convergence of $E_{p,0}$, which is only with order 1. That justifies the following paragraph.

2.2 Estimate for displacements

One defines a new estimator in pressure, which is a version only slightly modified of the preceding estimator (there is an additional order of convergence)

$$E_{p,1}=\sum_{K\in T_h}E_{p,1,K}$$
$$=\sum_{K\in T_h}\Big(h_K^3\frac{E^2}{P^2L^{\dim}M^2\kappa^2\rho^2}\sum_{F\in F_K^i}\|[M_{lq,h}\cdot n]\|_{0,F}^2$$
$$+h_K^3\frac{E^2}{P^2L^{\dim}M^2\kappa^2\rho^2}\sum_{F\in F_K^\partial\cap\Gamma_N^H}\|M_{lq,nor}-M_{lq,h}\cdot n\|_{0,F}^2\Big)$$

One has the following properties:

Theorem 1 (Reliability)

$$\|u-u_h\|_a^2 < E_u + E_{p,1}$$

Theorem 2 (Optimality)
For all $K\in T_h$, one has

$$E_{p,1,K} < \sum_{K'\in\Delta_K} \left(h_K^4 \|g-g_h\|_{0,K'}^2 + h_K^2 \|p-p_h\|_{d,K'}^2 \right)$$

the estimates obtained are optimal for the estimate of error on displacement because the convergence of the estimators E_u eta $E_{p,1}$ place to order 2.

3 Study of the transitory problem

One points out the statement of the continuous problem in which one is interested. To find $\left(u^*, p^*\right)$ such as

$$\begin{cases} -\nabla^*\cdot\sigma'^*(u^*)+b\,\nabla^* p^*=f^* & \text{dans } [0,T^*]\times\Omega^* \\ \partial_t^*\left(\dfrac{E}{M}p^*+b\,\nabla^*\cdot u^*\right)-\dfrac{E}{M}\Delta^* p^*=g^* & \text{dans } [0,T^*]\times\Omega^* \end{cases}$$

éq 3.1-1

In the thesis [1], 2 families of error indicators for this problem were proposed. Only was restored in Code_Aster, allowing to effectively evaluate the error on the pressure.

The estimators of error in space are defined:

- Estimator for the hydraulic equation: for all $m\in[1,N]$,

$$\tau_m E_{p,0}^m = \sum_{K\in T_h} \tau_m E_{p,0,K}^m$$

$$= \sum_{K\in T_h} \Big(\tau_m h_K^2 \frac{E^2}{P^2 L^{\dim}\kappa M} \left\| \frac{1}{M}\delta_t p_h^m + b\nabla\cdot\left(\delta_t u_h^m\right)\right\|_{0,K}^2 + \tau_m h_K \frac{E^2}{P^2 L^{\dim}\kappa\rho^2 M} \sum_{F\in F_K^i} \left\| \left[M_{lq,h}^m\cdot n \right] \right\|_{0,F}^2$$

$$+\tau_m h_K \frac{E^2}{P^2 L^{\dim}\kappa\rho^2 M} \sum_{F\in F_K^\partial\cap\Gamma_N^H} \left\| M_{lq,nor}^m - M_{lq,h}^m\cdot n \right\|_{0,F}^2 \Big)$$

- Estimators for the mechanical equation: for all $m\in[1,N]$,

$$E_u^m = \sum_{K\in T_h} E_{u,K}^m$$

$$= \sum_{K\in T_h} \Big(h_K^2 \frac{1}{P^2 L^{\dim}} \left\| f^m + \nabla\cdot\sigma'\left(u_h^m\right)\right\|_{0,K}^2 + h_K \frac{1}{P^2 L^{\dim}} \sum_{F\in F_K^i} \left\| \left[\sigma'\left(u_h^m\right)\cdot n \right] \right\|_{0,F}^2$$

$$+ h_K \frac{1}{P^2 L^{\dim}} \sum_{F\in F_K^\partial\cap\Gamma_N^M} \left\| \sigma_{nor}^m - \left(\sigma'\left(u_h^m\right)\cdot n - b p_h^m n \right) \right\|_{0,F}^2 \Big)$$

$$E_u^m(\delta_t)=\sum_{K\in T_h} E_{u,K}^m(\delta_t)$$

$$\sum_{K\in T_h}\Big(h_K^2\frac{1}{P^2 L^{\dim}}\|f^m-f^{m-1}+\nabla\cdot\sigma'\left(u_h^m-u_h^{m-1}\right)-b\nabla\left(p_h^m-p_h^{m-1}\right)\|_{0,K}^2$$

$$+h_K\frac{1}{P^2L^{\dim}}\sum_{F\in F_K^i}\|\left[\sigma'\left(u_h^m-u_h^{m-1}\right)\cdot n\right]\|_{0,F}^2$$

$$+h_K\frac{1}{P^2L^{\dim}}\sum_{F\in F_K^{\partial}\cap\Gamma_N^M}\|\sigma_{nor}^m-\sigma_{nor}^{m-1}-\left(\sigma'\left(u_h^m-u_h^{m-1}\right)\cdot n-b\left(p_h^m-p_h^{m-1}\right)n\right)\|_{0,F}^2$$

One defines the estimator in time

$$E_{tim}^m=\tau_m\frac{E}{P^2L^{\dim}\rho^2\kappa}\|M_{lq,h}^m-M_{lq,h}^{m-1}\|_{0,\Omega}^2$$

One has the following properties:

Theorem 1 (Reliability) For all $n\in[1,N]$,

$$\int_0^{t_n}\|\left(p-p_{h\tau}\right)(s)\|_d^2+\int_0^{t_n}\|\left(p-\pi^0 p_{h\tau}\right)(s)\|_d^2<\sum_{m=1}^N\tau_m E_{p,0}^m+\sup_{0\le m\le N}E_u^m+\left(\sum_{m=1}^N\left(E_u^m(\delta_t)\right)^{1/2}\right)^2+\sum_{m=1}^N E_{tim}^m$$

the operator π^0 appoints the operator of projection on the constant functions per pieces in time, with savoirégales $\pi^0 p_{h\tau}$ with p_h^n on I_n for all $n\in\{1,\cdots,N\}$.

Theorem 2 (Optimality of the indicator in time)

One has the following estimate

$$E_{tim}=\sum_{m=1}^N E_{tim}^m<\int_0^T\|\left(p-p_{h\tau}\right)(s)\|_d^2 ds+\int_0^T\|\left(p-\pi^0 p_{h\tau}\right)(s)\|_d^2 ds$$

Theorem 3 (Optimality of the indicators in space) For all $K\in T_h$, one has

$$E_{u,K}^m<\sum_{K'\in\Delta_K}\left[h_K^2\|f^m-f_h^m\|_{0,K'}+\|u^m-u_h^m\|_{a,K'}^2+\|p^m-p_h^m\|_{0,T'}^2\right]$$

$$E_{u,K}^m(\delta_t)<\sum_{K'\in\Delta_K}\tau_m^2\left[h_K^2\|\delta_t f^m-\delta_t f_h^m\|_{0,K'}+\|\delta_t u^m-\delta_t u_h^m\|_{a,K'}^2+\|\delta_t p^m-\delta_t p_h^m\|_{0,T'}^2\right]$$

$$\tau_m E_{p,0,K}^m<\sum_{K'\in\Delta_K}h_K^2\int_{I_m}[\|\left(g-\pi^0 g_{h\tau}\right)(s)\|_{0,K'}^2 ds+\tau_m^2\|\delta_t u^m-\delta_t u_h^m\|_{a,K'}$$

$$+\tau_m^2\|\delta_t p^m-\delta_t p_h^m\|_{0,K'}^2+h_T^{-2}\|\left(p-\pi^0 p_{h\tau}\right)(s)\|_{d,K'}^2]ds$$

4 Use in Code_Aster

The computation temporal indicators is started in `STAT_NON_LINE` by keyword `ERRE_TEMPS_THM='OUI'` in the keyword factor `CRIT_QUALITE`. It makes it possible to calculate quantities `ERRE_TPS_LOC` E_{tim}^m , and `ERRE_TPS_GLOB` $\sum_{m=1}^{N} E_{tim}^m$.

The computation indicators in space is started in `CALC_ERREUR` by option "`ERME_ELEM`" .

For the indicators in space into permanent, one has access to parameters `ERRE_MEC` , `ERRE_HYD_S` and `ERRE_HYD_D`. They are respectively the noted quantities E_u , $E_{p,0}$ and $E_{p,1}$ in this document.

Example of extract of command file:

```
RESU [K] =CALC_ERREUR (reuse =RESU [K],
             RESULTAT=RESU [K],
             LIST_INST=LINST,
             OPTION= ("ERME_ELEM", "ERME_ELNO",),);

    dictionary = RESU [K] .LISTE_PARA ()

    print dictionary ["ERRE_MEC"]
```

One thus recovers the values of the indicators E_u of list `LINST`.

For the indicators in space out of transient, one has access to the following parameters:

`ERRE_MEC_LOC`	E_u^m
`ERRE_MEC_LOC_D`	$E_u^m\left(\delta_t\right)$
`ERRE_MEC_GLOB`	$\sup_{0\leq m\leq N}\left(E_u^m\right)^{1/2}$
`ERRE_MEC_GLOB_D`	$\sum_{m=1}^{N}\left(E_u^m\left(\delta_t\right)\right)^{1/2}$
`ERRE_HYD_LOC`	$E_{p,0}^m$
`ERRE_HYD_GLOB`	$\left(\sum_{m=1}^{N}\tau_m E_{p,0}^m\right)^{1/2}$

5 Conclusion - Prospect

the estimators for modelizations HM come to supplement the consequent panoply estimators in existing space in Code_Aster. For the first time, indicators in time make it possible to quantify the error on the temporal discretization. The prospects for this work are several orders:

1) To extend the perimeter of use of estimators HM to the nonlinear modelizations;
2) To develop estimators for modelizations THHM in general;
3) To set up a procedure of recutting of time step starting from the estimators in time. For time, only the numerical values of the estimators in time are provided in `STAT_NON_LINE`, without being connected on an adaptation mechanism of the temporal discretization.

6 Bibliography

- S. MILLER. Analyzes error a posteriori for the couplings Hydro-mechanical and put in work in Code_Aster. Doctorate of the ENPC, 2007.
- S. MILLER. Modelization Thermo-Hydro-Mechanics in *Code_Aster* : Equations and discretization. Note HI- 5/23/018 /A.C.
- CHAVANT. Modelizations THHM. General information and algorithms. Document Aster R7.01.10-A
- S. GRANET. Models of behavior THHM. Document Aster R7.01.11-C
- Mr. AINSWORTH&J. T. ODEN. A posteriori error estimate in finite element analysis. Pure and Applied Mathematics. Wiley-Interscience, 2000.
- S. MILLER. Estimate of error *a posteriori* for the steady Hydro-mechanical problem. Note HI- 5/23/032 / A, 2005.
- S. MILLER. Modelization Thermo-Hydro-Mechanics in *Code_Aster* : Equations and discretization. Note HI- Historical 5/23/018

7 /A. versions of the document

Version Aster	Author (S) or contributor (S), organization	Description of the modifications
9.4	S.MEUNIER EDF-R&D/AMA	initial Text