Indicators of discharge and loss of proportionality of the loading in elastoplasticity

Summarized

One presents a set of scalar parameters called indicators, allowing to appreciate a discharge or a loss of proportionality of a loading during his history. Two types of indicators are proposed: indicators being appeared as scalar fields allowing to detect the zones of structure undergoing of the discharges or the nonradial loadings, and the total indicators integrated on a zone of structure chosen by the user. The latter are more especially intended for the evaluating of the validity of the rate of refund of energy in nonlinear fracture mechanics.

The indicators described in this document are available:

- for the local indicators under the command CALC_CHAMP, options DERA_ELGA and DERA_ELNO ;
- for the total indicators, under the command POST_ELEM , options INDIC_SEUIL and INDIC_ENER.

Contents

1 Introduction

1.1 Definition of a loading proportional

Considering a structure subjected to a thermomechanical loading into the time interval $[0,t]$ one will say that this loading is proportional (or radial) at the material point p if the stress field represented in this point by the tensor $\boldsymbol{\sigma}$ is proportional to a tensor independent of time considered, the proportionality factor being a monotonous function of time. Formally, that will be expressed by:

$$\forall \tau [0,t], \boldsymbol{\sigma}(P,\tau)=\alpha(\tau)\boldsymbol{\sigma}_0(P), \alpha(\tau)>0 \text{ monotonous function in } [0,t].$$

This definition implies, in particular, that the principal directions of the stresses remain constant, at the point considered, throughout the way of loading (these directions can be of course variable of a point with another).

1.2 Importance of the loading proportional and utility of indicators

For the plastic materials, the mechanical fields depend on all the history run out during the way of loading. The flow models are thus incremental and their integration depends on each case of loading. A notable exception precisely relates to the loading proportional for which the flow model can be integrated once and for all. For example, the model of plasticity of Prandtl-Reuss based on the criterion of Von Mises can be replaced by a nonlinear elastic model (called model of Hencky-Put). The cases of loading strictly proportional are rather rare. Indeed, it is necessary to meet a large number of conditions to carry out such a case [bib1] and these last are seldom checked for industrial structures. One can even say that for structures presenting of the geometrical defaults such as cracks, these conditions never are strictly checked.

When the loadings are multiaxial, cyclic, or thermomechanical transients, certain sections of the way of loading can be strongly nonproportional. It is then useful to locate these sections and to evaluate the importance of the loss of proportionality, so for example adjusting the discretization in time of the elastoplastic problem for the section considered, or measuring the validity of certain postprocessings (in fracture mechanics for example).

1.3 Various types of indicators of loss of proportionality

It seems difficult to define a single and simple quantity which could detect at the same time spatial zones of loss of proportionality and sections of loading path (temporal zones) in a material point. This is why we propose scalar quantities having each one their specificity: two, defined by fields measuring in each point the discharge and the deviation of the stresses between two time step (indicating buildings), two others of more total nature, characterizing in a given zone of structure a load history nonproportional.

Note:

> *These indicators are closely related to the discretization in time of the problem. In particular, if this discretization is too coarse, one can not detect very well the discharge or the loss of radiality occurring during the increment of time.*

1.4 indicators of elastoplastic discharge

Another application of the indicators of discharge, consists in alerting the user, if, in the event of important discharge, the choice of a kinematic hardening would provide a solution very different from isotropic hardening used (cf CR-AMA-11.035 9).

This has a practical interest: in several studies, the behavior chosen very often “by default” (because one has very often only one curve of tension) is `VMIS_ISOT_TRAC`. However so of the local discharges are possible, this can lead typically to over-estimate the stresses (with imposed strain) or to underestimate the strains (with imposed stress). It thus seems relevant to inform the user if, when it used models of Von Mises with isotropic hardening, it is likely to get false results when the discharge becomes too important (thus does not remain in the initial field of elasticity). That can indicate to him that it is necessary to use constitutive laws with kinematic hardening (VMIS_CINE_LINE, VMIS_ECMI*, VMIS_CIN1_CHAB, etc.).

2 Local indicators

the goal of these indicators is to determine the zones of structure where, at one particular time, occurs is a discharge or a loss of radiality of the stress field. They are produced in post - processing of a static or dynamic computation, 2D or 3D, using an elastic constitutive law or not. They are appeared as fields of scalars whose examination can be carried out by tracing their isovaleurs by a graphic post-processor.

2.1 Indicators of discharge

2.1.1 local Indicator of total discharge

This indicator measures at the point M and between time t and $t+\Delta t$, the relative variation of the norm of the stresses within the meaning of Von Mises. He is written formally: $I_1=\frac{\|\boldsymbol{\sigma}(M,t+\Delta t)\|-\|\boldsymbol{\sigma}(M,t)\|}{\|\boldsymbol{\sigma}(M,t+\Delta t)\|}$. This quantity is negative in the event of local discharge at the point Mr. the norm $\|\boldsymbol{\sigma}(M,t)\|$ can be written in four ways different according to the choice from the modelisator:

1) $\|\boldsymbol{\sigma}(M,t)\|=\sqrt{\frac{3}{2}\boldsymbol{\sigma}^D.\boldsymbol{\sigma}^D}$, where $\boldsymbol{\sigma}^D$ is the deviatoric part of the tensor of the stresses (this norm is useful in plasticity with isotropic hardening).
2) $\|\boldsymbol{\sigma}(M,t)\|=\sqrt{\frac{3}{2}\boldsymbol{\sigma}.\boldsymbol{\sigma}}$, where one considers the totality of the tensor of the stresses in order to detect for example the reductions in hydrostatic pressure.
3) $\|\boldsymbol{\sigma}(M,t)\|=\sqrt{\frac{3}{2}(\boldsymbol{\sigma}^D-\boldsymbol{X}).(\boldsymbol{\sigma}^D-\boldsymbol{X})}$, with $\boldsymbol{X}$ the tensor of the stress of recall in the case of an elastoplastic model with a kinematic hardening.
4) $\|\boldsymbol{\sigma}(M,t)\|=\sqrt{\frac{3}{2}(\boldsymbol{\sigma}-\boldsymbol{X}).(\boldsymbol{\sigma}-\boldsymbol{X})}$

2.1.2 Local indicator of elastoplastic discharge

This indicator makes it possible to know if the discharge remains elastic or if there would be a risk of plasticization if a pure kinematic hardening were used. It is an “extreme” indicator, knowing that, for metals, isotropic and kinematical hardenings are both present.

Option DERA_ELGA thus calculates (besides components DCHA_V and DCHA_T) the components:

I_decha=IND_DCHA:
- IND_DCHA=0 unconstrained initial value.
- IND_DCHA=1 if load elastic
- IND_DCHA=2 if plastic load
- IND_DCHA=-1 if licit elastic discharge (some is the type of hardening)
- IND_DCHA=-2 if abusive discharge (one would have plasticized with a kinematic hardening).

VAL_DCHA: indicate the proportion of output of the criterion (see further).

Operation is the following: for models VMIS_ISOT* only in each point of integration, at every moment t, starting from the tensor of the stresses $\boldsymbol{\sigma}(t)$, cumulated equivalent plastic strain $\boldsymbol{p}(t)$, and curve of isotropic hardening $\boldsymbol{R}(\boldsymbol{p}(t))$,

- Initialization: IND_DCHA=0, VAL_DCHA=0.
- as long as $p(t)=0$, IND_DCHA=1 (elasticity),
- if IND_DCHA=-2 (criterion of abusive discharge reached), one does not do anything any more
- if $\Delta p(t)>0$:
  - if one is in load, therefore if the angle between the tensor increase in forced stresses and the tensor total is "small": , $\frac{\tilde{\boldsymbol{\sigma}}(t-\Delta t)\cdot\Delta\tilde{\boldsymbol{\sigma}}}{\|\tilde{\boldsymbol{\sigma}}(t-\Delta t)\|\|\Delta\tilde{\boldsymbol{\sigma}}\|}>0$ IND_DCHA=2 ; (points enters A and B on the figure 2.1.2-a).
  - if one is in discharge, IND_DCHA=-2 (case rare: that means that in one time step, one crosses the surface of load in a direction distant from that of the preceding step)
- if $\Delta p(t)=0$:
  - if IND_DCHA>-1, one calculates the tensor $\boldsymbol{X}$ (center of the field of elasticity if one were in kinematics pure, therefore if the surface of initial load, represented by a circle in the deviative plane, had been relocated up to the point running) by:

$$\boldsymbol{X}=\boldsymbol{\sigma}(t)\frac{R(p)-R(0)}{R(p)}$$

  - if IND_DCHA=-1, one uses the tensor $\boldsymbol{X}$ already calculated.

  - one calculates the "kinematical" criterion $(\boldsymbol{\sigma}(t)-\boldsymbol{X})_{eq}\leq R(0)$
  - then If this criterion is checked, the discharge would be elastic also with a kinematic hardening, it is thus "licit": IND_DCHA=-1 (any point enters B and E on the figure 2). To apply the criterion to next time, one stores $\boldsymbol{X}$ (6 components)
    - if not, IND_DCHA=-2 and keeps this value (because the continuation of computation would be modified if hardening were kinematical). (points enters E and F, or enters C and This on the following figure). VAL_DCHA $=\frac{\|\boldsymbol{\sigma}-\boldsymbol{X}\|}{R_0}$

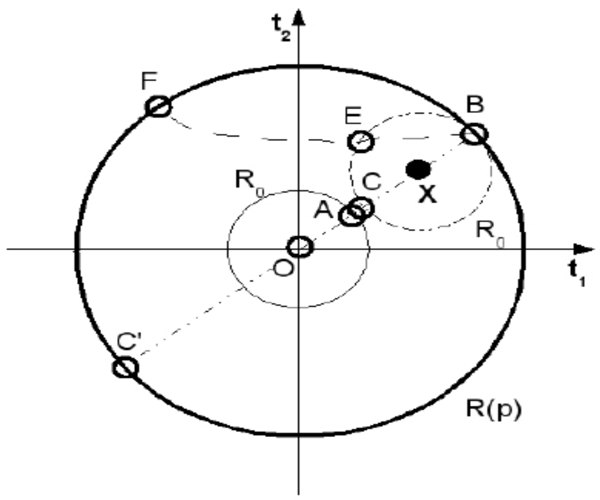


Figure 2 : Représentation de chemins de chargement dans le plan octaédral des contraintes

Appear 2.1.2-a : elastoplastic indicator of discharge

- **Comments** :

- The disadvantage of this method comes from what it is not systematic: it is necessary that the user "thinks" of calling option DERA_ELGA.
- The criterion obtained is relatively severe: it supposes that one replaces pure isotropic hardening by a pure kinematic hardening. An upgrading capability would consist in defining a radius of surface threshold larger than R_0.

2.2 Indicators of loss of radiality

2.2.1 local Indicator of radiality of the loading

This indicator measures at the point M and between time t and $t+\Delta t$, the variation of the direction of the stresses. He is written:

$I_2=1-\frac{|\boldsymbol{\sigma}(M,t).\Delta\boldsymbol{\sigma}|}{\|\boldsymbol{\sigma}(M,t)\|\|\Delta\boldsymbol{\sigma}\|}$, where the scalar product "." is associated with the one of the four preceding norms. This quantity is null when the radiality is preserved during the increment of time. This criterion can also be interpreted like the quantity $1-\cos(\theta)$, where θ is the angle enters $\boldsymbol{\sigma}$ and $\Delta\boldsymbol{\sigma}$. This indicator with actual value evolves between 0 for the radial loadings and 2. It is useful in particular to determine the validity of an elastoplastic solution in fracture mechanics,

2.2.2 Error indicator due to the temporal discretization

It provides a m esure of the error η due to the discretization in time, directly connected to the rotation of the norm on the surface of load. One calculates the angle enters $\boldsymbol{n}^-$, the norm with the plasticity criterion at the beginning of time step (urgent T), and $\boldsymbol{n}^+$, the norm with the plasticity criterion calculated at the end of time step in the following way: $I_\eta=\frac{1}{2}\|\Delta\boldsymbol{n}\|=\frac{1}{2}\|\boldsymbol{n}^+-\boldsymbol{n}^-\|=\left|\sin(\frac{\alpha}{2})\right|$. This indicator is directly related to the norm of the variation of the norm on convex of plasticity (this can spread easily with any elastoplastic model with normal flow), and it can be also interpreted like the sine of half of the angle the two norms. That provides a measurement of the error (see [4]). This criterion is operational for the elastoplastic behaviors of Von Mises with hardening isotropic, kinematical linear and mixed. It can be used to control the automatic subdivision of the time step.

3 Total indicators

These indicators are intended to detect if, during the history of structure and until the current time t, and for a zone of structure chosen by the modelisator, there were loss of proportionality of the loading (these indicators thus leave a trace of the history contrary to the local indicators which are instantaneous). They are only usable in the frame of an elastoplastic behavior with isotropic hardening (in 2D or 3D).

3.1 Indicator on the parameters of plasticity

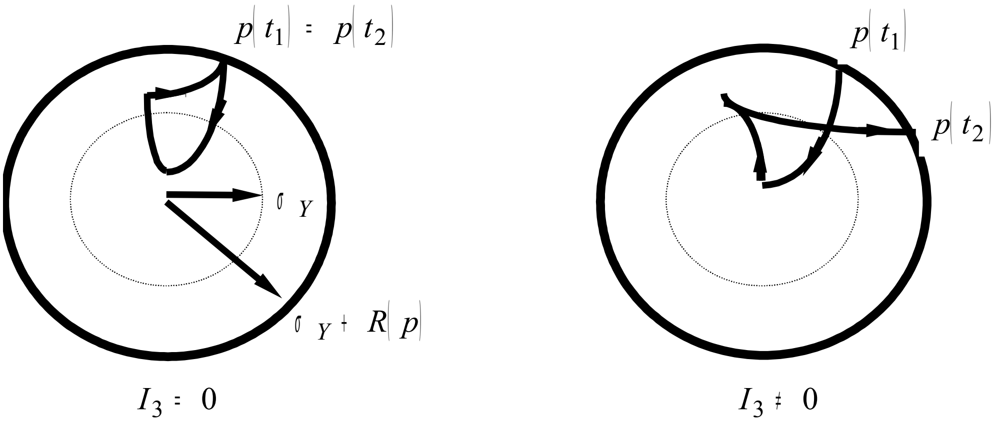
This quantity allows, in the case of the plasticity of Von Mises with isotropic hardening, on the one hand of knowing (on average on a zone $\boldsymbol{\Omega_S}$ of the field $\boldsymbol{\Omega}$) if the stresses and the plastic strains have the same directions and if the plastic threshold is reached at current time, and on the other hand so during the history plastic strain changed direction. This quantity is written:

$$I_3=\frac{1}{\boldsymbol{\Omega_S}}\int_{\Omega_s}(1-\frac{|\boldsymbol{\sigma}.\boldsymbol{\varepsilon}^p|}{(\sigma_Y+R(p))p})d\boldsymbol{\Omega},$$

where σ_Y is the initial plastic threshold, R extension of the surface of load related to hardening and p cumulated plastic strain. The scalar product is associated with the norm within the meaning of Von Mises. This indicator is standardized and has a value ranging between zero and one. It is null if the loading preserved its character proportional in each point of $\boldsymbol{\Omega}_S$ throughout the past history.

Notice 1:

The indicator is not affected so during the history, there were discharges then elastic refills without change of management of the stresses when one reconsiders the threshold (cf [Figure 3.1-a]).



Appear 3.1-a : Loading path enters $\mathbf{t_1}$ and $\mathbf{t_2}$
in the plane deviatoric of the stresses

Notices 2:

In the formulation of this indicator three ingredients related to plasticity intervene:

- *the variation enters the direction of the stresses and plastic strains current (* $\boldsymbol{\sigma}.\boldsymbol{\varepsilon}$¿ $p \neq \|\boldsymbol{\sigma}\|\|\boldsymbol{\varepsilon}^p\|$ *),*
- *the position of the stresses compared to the current threshold (* $\|\boldsymbol{\sigma}\| \leq (\sigma_Y + R(p))$ *),*
- *the difference between the norm of the current plastic strain and the cumulated plastic strain (* $\|\boldsymbol{\varepsilon}^p\| \neq p$ *).*

A loss of proportionality could occur during the history without the indicator not detecting it via the first two ingredients (i.e. one can have at the end of the loading coincidence of the directions of the stresses and of plastic strains and be on the plastic threshold). On the other hand, one will have, $\|\boldsymbol{\varepsilon}^p\| \neq p$ and the indicator will be obligatorily higher than zero, consequently the user will be informed loss of proportionality.

Notice 3:

If the indicator detects obligatorily a loss of proportionality in a zone, in practice it is necessary that the latter contains sufficient material points with loading nonproportional. Indeed, if one chooses a very vast zone with few points concerned, the standardization of the indicator carried out with division by the volume of the zone implies a certain "crushing" towards zero of the value of the quantity. Typically, for a structure containing a default source of nonproportionality, one may find it beneficial to choose a zone of integration $\boldsymbol{\Omega}_S$ *surrounding the default with a weak vicinity in order to obtain a significant value of the indicator.*

3.2 Energy indicator

This indicator has the same function that the precedent, but is founded on the density of energy. He is written:

$$I_4=\frac{1}{\Omega_S}\int_{\Omega_s}(1-\frac{\psi}{\omega})d\,\Omega\ ,$$

where ω is the density of strain energy defined by: $\omega(t)=\int_0^t \boldsymbol{\sigma}.\dot{\boldsymbol{\varepsilon}}\,d\tau$, and ψ is the density of elastic strain energy associated with curve of tension if the nonlinear elastic material were considered. More precisely, this quantity is written:

$$\psi(\boldsymbol{\varepsilon}(t))=\frac{1}{2}Ktr^2(\boldsymbol{\varepsilon})+\frac{2\mu}{3}\|\boldsymbol{\varepsilon}\|^2,\ \text{si}\ \|\boldsymbol{\sigma}\|<(\sigma_Y+R(p)),$$
$$\psi(\boldsymbol{\varepsilon}(t))=\frac{1}{2}Ktr^2(\boldsymbol{\varepsilon})+\frac{\mathrm{R}^2(P)}{6\mu}+\int_0^P\mathrm{R}(q)dq,\ \text{si}\ \|\boldsymbol{\sigma}\|=(\sigma_Y+R(p)),$$

with K the modulus of compressibility, μ the shear coefficient of Lamé, R the threshold of curve of tension associated with the norm with the plastic strain $P=\|\boldsymbol{\varepsilon}^p\|$ (this one can be different from the true plastic threshold, because $P\neq p$ if the loading is nonproportional). This indicator is also standardized between 0 and 1. It is null for a loading having always kept its character proportional $(\psi\neq\omega)$.

4 Functionalities and checking

the indicators presented here are usable in postprocessing of a mechanical computation and are available for the finite elements of the continuums in 2D (mode of plane strains, plane stresses or axisymmetric, meshes triangular or quadrangular) or 3D (meshes hexahedral, tetrahedral, pentaedric or pyramids). The telegraphic elements, beams, plates and shells are excluded from this application.

4.1 Local indicators

These indicators are accessible after a static or dynamic computation whatever the constitutive law from the material. Operator `CALC_CHAMP` presents options "`DERA_ELGA`" and "`DERA_ELNO`" for the indicator of discharge I_1 and the indicator of loss of radiality I_2 evaluated to the nodes or Gauss points of the element 9. In the general case, the first two norms described in paragraph 4 are calculated, the two last being used only if one carried out as a preliminary an elastoplastic computation with kinematic hardening.

The components of the field thus calculated are:

- `DCHA_T` : the indicator of discharge I_1 calculated starting from the tensor of the stresses;
- `DCHA_V` : the indicator of discharge I_1 calculated starting from the deviator of the stresses;
- `IND_DCHA` : the indicator of discharge I_{decha}
    - `IND_DCHA=0` initial value unconstrained
    - `IND_DCHA=1` if elastic load
    - `IND_DCHA=2` if plastic load
    - `IND_DCHA=-1` if licit elastic discharge (some is the type of hardening)
    - `IND_DCHA=-2` if abusive discharge (one would have plasticized with a kinematic hardening).
- `VAL_DCHA` : the indicator of the proportion of output of criterion
- `RADI_V` : the indicator of loss of radiality I_2 calculated starting from the deviator of the stresses.
- `ERR_RADI`: the error indicator I_η due to the temporal discretization

These indicators are checked by the cases following tests:

SSNP111	Transition of Gauss points with the nodes on quadratic elements	[V6.03.111]
SSNP14	Plates in tension-shears - Von Mises (Kinematic hardening)	[V6.03.014]
SSNP15	Plates in tension-shears - Von Mises (isotropic hardening)	[V6.03.015]

4.2 Indicating total

These indicators are accessible only after one elastoplastic computation with isotropic hardening. Operator `POST_ELEM` presents options "`INDIC_SEUIL`" and "`INDIC_ENER`" corresponding respectively to the total indicators I_3 and I_4. Those are evaluated on a group of mesh previously defined by user (for example by the command `DEFI_GROUP`).

These indicators are checked by the cases following tests:

HSNV100i	Thermoplasticity in tension simple	[V7.22.100]
SSNP15	Plates in tension-shears - Von Mises (isotropic hardening)	[V6.03.015]

5 Bibliography

[1] J. LEMAITRE, J. - L.CHABOCHE, Mechanics of the solid materials, Dunod 1985.

[2] Instruction manual of *Code_Aster*. Document [U4.61.02].

[3] CR-AMA-11.035: "Project LOCO - batch B2 - Indicating buildings of discharge and radiality" A.Foucault, 1/24/2011

[4] CR-AMA-11.30 "Study of a criterion of radiality" J.M.Proix Description

6 of the versions of the document Version

Aster Author (S	**) Organization (S) Description**	**of the modifications 4 G. DEBRUYNE**
4	*(EDF/IMA/MMN) initial Text*	J.M.PROIX EDF/R & D
11.1	/AMA Addition of the new	indicators of discharge. J.M.PROIX EDF/R & D /AMA Addition
11.2	of a new criterion	of radiality.