Modal Parameters and norm of the eigenvectors

Summarized:

In this document, one describes:
- various possibilities in *Code_Aster* to normalize the eigen modes,
- the important modal parameters associated with the eigen modes.

Contents

1 Definition of the problem with the eigenvalues

1.1 General information

Is the problem with the eigenvalues according to:

To find

$$(\lambda, \boldsymbol{\Phi}) \in \mathbb{C} \, x \, \mathbb{C}^n / \qquad (\lambda^2 \mathbf{B} + \lambda \, \mathbf{C} + \mathbf{A}) \boldsymbol{\Phi} = 0 \qquad \text{éq 1.1-1}$$

where $\mathbf{A}, \mathbf{C}, \mathbf{B}$ are positive symmetric real matrixes of order n .

Two cases are distinguished:

- quadratic problem: $\mathbf{C} \neq 0$,
- generalized problem: $\mathbf{C} = 0$.

λ is called eigenvalue and $\boldsymbol{\Phi}$ eigenvector. In the continuation, one will speak about eigen mode for $\boldsymbol{\Phi}$ and one will introduce the notion of eigenfrequency.

To solve this problem, several methods are available in *Code_Aster* and one returns the reader to the documents [R5.01.01] and [R5.01.02].

1.2 Problem generalized

the generalized problem can be written in the form:

To find

$$(\lambda, \boldsymbol{\Phi}) \in \mathbb{R} \, x \, \mathbb{R}^n / \qquad (-\lambda^2 \mathbf{B} + \mathbf{A}) \boldsymbol{\Phi} = 0 \qquad \text{éq 1.2-1}$$

One introduces two other quantities which make it possible to characterize the eigen mode:

$$\lambda = \omega = (2 \pi f) \qquad \text{éq 1.2-2}$$

where

ω : own pulsation associated with the eigen mode $\boldsymbol{\Phi}$
f : eigenfrequency associated with the eigen mode $\boldsymbol{\Phi}$.

One also shows that the eigen modes are $\mathbf{A}$ and $\mathbf{B}$ orthogonal, i.e.:

$$\begin{cases} \boldsymbol{\Phi}^{iT} \mathbf{A} \boldsymbol{\Phi}^{\mathrm{j}} = \delta_{ij} \boldsymbol{\Phi}^{iT} \mathbf{A} \boldsymbol{\Phi}^{\mathrm{i}} \\ \boldsymbol{\Phi}^{iT} \mathbf{B} \boldsymbol{\Phi}^{\mathrm{j}} = \delta_{ij} \boldsymbol{\Phi}^{iT} \mathbf{B} \boldsymbol{\Phi}^{\mathrm{i}} \end{cases} \qquad \text{éq 1.2-3}$$

where $(\boldsymbol{\Phi}^{\mathbf{i}}, \boldsymbol{\Phi}^{\mathbf{j}})$ are two eigen modes.

1.3 Quadratic problem

the quadratic problem [éq 1.1-1] can be put in another form of double size (one speaks about linear reduction [R5.01.02]):

To find

$$(\lambda, F) \in \mathbb{C}\, x\, \mathbb{C}^n / \quad \left(\lambda \begin{bmatrix} \mathbf{0} & \mathbf{B} \\ \mathbf{B} & \mathbf{C} \end{bmatrix} + \begin{bmatrix} -\mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix} \right) \begin{pmatrix} \lambda \boldsymbol{\Phi} \\ \boldsymbol{\Phi} \end{pmatrix} = 0 \qquad \text{éq 1.3-1}$$

One poses in the continuation: $\hat{\mathbf{B}} = \begin{bmatrix} \mathbf{0} & \mathbf{B} \\ \mathbf{B} & \mathbf{C} \end{bmatrix} \quad \hat{\mathbf{A}} = \begin{bmatrix} -\mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix}$.

As the matrixes $\mathbf{A}, \mathbf{C}, \mathbf{B}$ are real, the values and eigen modes imaginary are combined two to two.

One introduces three other quantities which make it possible to characterize the eigen mode:

$$\lambda = a + ib = -\frac{\xi \omega}{\sqrt{1-\xi^2}} + i\omega = -\frac{\xi (2\pi f)}{\sqrt{1-\xi^2}} + i(2\pi f) \qquad \text{éq 1.3-2}$$

where ω : own pulsation associated with the eigen mode Φ
f : eigenfrequency associated with the eigen mode Φ
ξ : reduced damping.

One also shows that the eigen modes are $\begin{bmatrix} \mathbf{0} & \mathbf{B} \\ \mathbf{B} & \mathbf{C} \end{bmatrix}$ and $\begin{bmatrix} -\mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix}$ orthogonal, i.e.:

$$\begin{cases} -\lambda_i \lambda_j \boldsymbol{\Phi}^{iT} \mathbf{B} \boldsymbol{\Phi}^{\mathbf{j}} + \boldsymbol{\Phi}^{iT} \mathbf{A} \boldsymbol{\Phi}^{\mathbf{j}} = \boldsymbol{\delta}_{ij} (-\lambda_i^2 \boldsymbol{\Phi}^{iT} B \boldsymbol{\Phi}^{\mathbf{i}} + \boldsymbol{\Phi}^{iT} A \boldsymbol{\Phi}^{\mathbf{i}}) \\ (\lambda_i + \lambda_j) \boldsymbol{\Phi}^{iT} \mathbf{B} \boldsymbol{\Phi}^{\mathbf{j}} + \boldsymbol{\Phi}^{iT} \mathbf{C} \boldsymbol{\Phi}^{\mathbf{j}} = \boldsymbol{\delta}_{ij} (2\lambda_i \boldsymbol{\Phi}^{iT} B \boldsymbol{\Phi}^{\mathbf{i}} + \boldsymbol{\Phi}^{iT} C \boldsymbol{\Phi}^{\mathbf{i}}) \end{cases} \qquad \text{éq 1.3-3}$$

where (λ_i, λ_j) are the eigenvalues respectively associated with the eigen modes $(\boldsymbol{\Phi}^{\mathbf{i}}, \boldsymbol{\Phi}^{\mathbf{j}})$.

Note:

the eigen modes are thus not $\mathbf{A}, \mathbf{B}\, \mathrm{ou}\, \mathbf{C}$ *orthogonal.*

2 Eigen modes of the generalized problem One

normalizes supposes to have calculated a couple $(\lambda, \boldsymbol{\Phi})$ solution of the problem [éq 1.2-1]: λ is the eigenvalue associated with the eigen mode $\boldsymbol{\Phi}$. One considers for time only the case of the generalized problem.

In *Code_Aster*, command `NORM_MODE` [U4.52.11] makes it possible to impose a kind of standardization for all the modes.

2.1 Components of an eigen mode

Is an eigen mode $\boldsymbol{\Phi}$ of components $(\boldsymbol{\Phi}_{\mathbf{j}})_{j=1,n}$.

Among these components, one distinguishes:

- components or degrees of freedom called "physics" (they are for example the degrees of freedom of displacement (DX, DY, DZ), the degrees of freedom of rotation (DRX, DRY, DRZ), the potential characterizing an irrotational fluid (PHI) ,…),
- components of Lagrange (the parameters of Lagrange are additional unknowns which are added with the "physical" problem initial so that the boundary conditions are checked [R3.03.01]).

In *Code_Aster*, one has three families of norms:

- euclidian norm,
- norm: "larger component with 1" among a group of degrees of freedom defined,
- norm masses or unit generalized stiffness.

They successively are described.

Previously, one defines L a family of indices which contains m terms:

$$L=\{l_k, k=1,m \,\text{avec}\, 1\leq l_k\leq n\} \text{ et } 1\leq m\leq n.$$

2.2 Euclidian norm

One defines the following norm: $\|\boldsymbol{\Phi}\|_2=\left(\sum_{k=1}^{m}\left(\boldsymbol{\Phi}_{\mathbf{l_k}}\right)^2\right)^{1/2}$

The normalized vector is then obtained $\hat{\boldsymbol{\Phi}}$: $\hat{\boldsymbol{\Phi}}=\frac{1}{\|\boldsymbol{\Phi}\|_2}\boldsymbol{\Phi} \qquad (\hat{\boldsymbol{\Phi}}_j=\frac{1}{\|\boldsymbol{\Phi}\|_2}\boldsymbol{\Phi}_j \qquad j=1,n)$.

In *the Code_Aster* , two norms of this family are available:

- `NORME=' EUCL'` : L corresponds to all the indices which characterize a physical degree of freedom,
- `NORME=' EUCL_TRAN'` : L corresponds to all the indices which characterize a physical degree of freedom of displacement in translation (DX, DY, DZ) .

2.3 "Larger component with 1" One

normalizes defines the following norm: $\|\boldsymbol{\Phi}\|_\infty=\max_{k=1,m}|\boldsymbol{\Phi}_{\mathbf{l}_k}|$

The normalized vector is then obtained $\hat{\boldsymbol{\Phi}}$: $\hat{\boldsymbol{\Phi}}=\frac{1}{\|\boldsymbol{\Phi}\|_\infty}\boldsymbol{\Phi}$ $\quad (\hat{\boldsymbol{\Phi}}_j=\frac{1}{\|\boldsymbol{\Phi}\|_\infty}\boldsymbol{\Phi}_j \quad j=1,n)$.

In *Code_Aster* , five norms of this family are available:

- `NORME=' SANS_CMP=LAGR'` : L corresponds to all the indices which characterize a physical degree of freedom,
- `NORME=' TRAN'` : L corresponds to all the indices which characterize a physical degree of freedom of displacement in translation (DX, DY, DZ),
- `NORME=' TRAN_ROTA'` : L corresponds to all the indices which characterize a physical degree of freedom of displacement in translation and rotation $(DX, DY, DZ, DRX, DRY, DRZ)$,
- `NORME=' AVEC_CMP'` or "`SANS_CMP`" : L is built either by taking all the indices which correspond to types of components stipulated by the user (for example standard displacement following the axis x : "`DX`") (`NORME=' AVEC_CMP'`), is by taking the complementary one to all the indices which correspond to types of components stipulated by user (`NORME=' SANS_CMP'`),
- `NORME=' NOEUD_CMP'`: L corresponds to only one index which characterizes a component of a node of the mesh. The name of the node and the component are specified by the user (keys - key `NOM_CMP` and `NOEUD` of the command `NORM_MODE` [U4.52.11]).

By defaults the modes are normalized with norm "`SANS_CMP=LAGR`".

2.4 Normalizes mass or unit generalized stiffness

Is a positive definite matrix of order n . The following norm is defined: $\|\boldsymbol{\Phi}\|_E=\left(\boldsymbol{\Phi}^{\mathrm{T}}\mathbf{E}\boldsymbol{\Phi}\right)^{1/2}$

The normalized vector is then obtained $\hat{\boldsymbol{\Phi}}$: $\hat{\boldsymbol{\Phi}}=\frac{1}{\|\boldsymbol{\Phi}\|_{\mathbf{E}}}\boldsymbol{\Phi}$ $\quad (\hat{\boldsymbol{\Phi}}_j=\frac{1}{\|\boldsymbol{\Phi}\|_{\mathbf{E}}}\boldsymbol{\Phi}_j \quad j=1,n)$. .

In *the Code_Aster* , two norms of this family are available:

- `NORME=' MASSE_GENE'`: $\mathbf{E}=\mathbf{B}$. In a classical problem of vibration, B is the mass matrix.
- `NORME=' RIGI_GENE'` : $\mathbf{E}=\mathbf{A}$. In a classical problem of vibration, A is the stiffness matrix.

Note:

> *For a mode* $\boldsymbol{\Phi}$ *of rigid body, one a:* $\|\boldsymbol{\Phi}\|_E=\|\boldsymbol{\Phi}\|_A=0$

3 Normalizes eigen modes of the quadratic problem

3.1 Euclidian norms and "larger component with 1"

For the quadratic problem, one has the same norms as for the generalized problem. The eigen modes being complex, one works with the hermitian product. The various "classical" norms become:

- hermitian norm: $\|\boldsymbol{\Phi}\|_2=\left(\sum_{k=1}^{m}|\boldsymbol{\Phi}_{\mathbf{l_k}}|^2\right)^{1/2}=\left(\sum_{k=1}^{m}\left(\bar{\boldsymbol{\Phi}}_{\mathbf{l_k}}\boldsymbol{\Phi}_{\mathbf{l_k}}\right)\right)^{1/2}$ where $\bar{\boldsymbol{\Phi}}_{\mathbf{l_k}}$ is combined of $\boldsymbol{\Phi}_{\mathbf{l_k}}$ (the absolute value in the real field becomes the modulus in the field complexes),
- norm "larger component with 1": $\|\boldsymbol{\Phi}\|_\infty=\max_{k=1,m}|\boldsymbol{\Phi}_{\mathbf{l_k}}|=\max_{k=1,m}\left(\left(\bar{\boldsymbol{\Phi}}_{l_k}\boldsymbol{\Phi}_{\mathbf{l_k}}\right)^{1/2}\right)$.

3.2 Normalizes mass or unit stiffness generalized

With regard to the norm "masses or generalized stiffness", denomination by analogy with the generalized problem, one uses like matrix associated with the norm, that which intervenes in the writing of the quadratic problem put in the reduced form [éq 1.3-1].

One has then:

- generalized mass normalizes:

$$\|\boldsymbol{\Phi}\|_{\hat{B}}=\left(\lambda\boldsymbol{\Phi}^{\mathbf{T}},\boldsymbol{\Phi}^{\mathbf{T}}\right)\hat{\mathbf{B}}\begin{pmatrix}\lambda\boldsymbol{\Phi}\\ \boldsymbol{\Phi}\end{pmatrix}=\left(\lambda\boldsymbol{\Phi}^{\mathbf{T}},\boldsymbol{\Phi}^{\mathbf{T}}\right)\begin{bmatrix}\mathbf{0} & \mathbf{B}\\ \mathbf{B} & \mathbf{C}\end{bmatrix}\begin{pmatrix}\lambda\boldsymbol{\Phi}\\ \boldsymbol{\Phi}\end{pmatrix}=2\lambda\boldsymbol{\Phi}^{\mathbf{T}}\mathbf{B}\boldsymbol{\Phi}+\boldsymbol{\Phi}^{\mathbf{T}}\mathbf{C}\boldsymbol{\Phi},$$

$$\hat{\boldsymbol{\Phi}}=\frac{1}{\|\boldsymbol{\Phi}\|_{\hat{\mathbf{B}}}}\boldsymbol{\Phi}\ ,$$

- norm generalized stiffness:

$$\|\boldsymbol{\Phi}\|_{\hat{B}}=\left(\lambda\boldsymbol{\Phi}^{\mathbf{T}},\boldsymbol{\Phi}^{\mathbf{T}}\right)\hat{\mathbf{A}}\begin{pmatrix}\lambda\boldsymbol{\Phi}\\ \boldsymbol{\Phi}\end{pmatrix}=\left(\lambda\boldsymbol{\Phi}^{\mathbf{T}},\boldsymbol{\Phi}^{\mathbf{T}}\right)\begin{bmatrix}-\mathbf{B} & \mathbf{0}\\ \mathbf{0} & \mathbf{A}\end{bmatrix}\begin{pmatrix}\lambda\boldsymbol{\Phi}\\ \boldsymbol{\Phi}\end{pmatrix}=-\lambda^2\boldsymbol{\Phi}^{\mathbf{T}}\mathbf{B}\boldsymbol{\Phi}+\boldsymbol{\Phi}^{\mathbf{T}}\mathbf{A}\boldsymbol{\Phi},$$

$$\hat{\boldsymbol{\Phi}}=\frac{1}{\|\boldsymbol{\Phi}\|_{\hat{\mathbf{A}}}}\boldsymbol{\Phi}\ .$$

4 Modal parameters associated for the generalized problem

One in the case of places a classical generalized problem of vibration. One a:

- $\mathbf{A}=\mathbf{K}$ is the stiffness matrix,
- $\mathbf{B}=\mathbf{M}$ is the mass matrix.

That is to say a couple $(\lambda, \boldsymbol{\Phi})$ solution of problem:

$$\left(-\lambda^2 \mathbf{M}+\mathbf{K}\right)\boldsymbol{\Phi}=0 \qquad \text{éq 4-1}$$

In the continuation, one defines successively the following quantities:

- generalized quantities,
- effective modal mass and unit effective modal mass,
- participation factor.

4.1 Generalized quantities

4.1.1 Definition

One defines two generalized quantities:

- Generalized mass of the mode $\boldsymbol{\Phi}$: $m_{\Phi}=\boldsymbol{\Phi}^{\mathbf{T}}\mathbf{M}\boldsymbol{\Phi}$,
- Generalized Stiffness of the mode $\boldsymbol{\Phi}$: $k_{\Phi}=\boldsymbol{\Phi}^{\mathbf{T}}\mathbf{K}\boldsymbol{\Phi}$.

These quantities depend on standardization on F . These quantities are accessible in the result concept of the mode_meca `type` under names `MASS_GENE, RIGI_GENE`.

Notice 1:

One has the following relation between the pulsation (or the frequency) of the mode and the mass and stiffness generalized of the mode:

$$\lambda^2=\omega^2=(2\pi f)^2=\frac{\boldsymbol{\Phi}^{\mathbf{T}}\mathbf{K}\boldsymbol{\Phi}}{\boldsymbol{\Phi}^{\mathbf{T}}\mathbf{M}\boldsymbol{\Phi}}=\frac{k_{\Phi}}{m_{\Phi}} .$$

Notice 2:

From the physical point of view, the generalized mass (which is a positive value) can be interpreted as mass moving:

$m_{\Phi}=\boldsymbol{\Phi}^{\mathbf{T}}\mathbf{M}\boldsymbol{\Phi}=\int \rho\,\boldsymbol{\Phi}^{\mathbf{2}}$ *where* ρ *is the density of structure.*

The kinetic energy of structure vibrating according to the mode $\boldsymbol{\Phi}$ *is equal then to:*

$$E_c=\frac{1}{2}\omega^2 m_{\Phi}=\frac{1}{2}\omega^2\boldsymbol{\Phi}^{\mathbf{T}}\mathbf{M}\boldsymbol{\Phi} .$$

The potential energy of strain associated with the mode $\boldsymbol{\Phi}$ *is equal to:*

$$E_p=\frac{1}{2}k_{\Phi}=\frac{1}{2}\boldsymbol{\Phi}^{\mathbf{T}}\mathbf{K}\boldsymbol{\Phi} .$$

4.1.2 Use

During a computation by modal recombination [R5.06.01], one seeks a solution of the equation of the dynamics:

$$\mathbf{M}\ddot{\mathbf{x}}+\mathbf{C}\dot{\mathbf{x}}+\mathbf{K}\mathbf{x}=f(t) \ ,$$

in the form $x=\sum_{i=1,m}\boldsymbol{\alpha}_{\mathbf{i}}(t)\boldsymbol{\Phi}^{\mathbf{i}}$ where $\boldsymbol{\Phi}^{\mathbf{i}}$ is the real eigen mode associated with the eigenvalue λ_i , solution of the generalized problem (in general one has $m\leq n$ (n is the number of degree of freedom) because one takes into account only part of modal base):

$$\left(-\mathbf{M}\lambda_i^2+\mathbf{K}\right)\boldsymbol{\Phi}^{\mathbf{i}}=0$$

The generalized vector $\boldsymbol{\alpha}=\left(\boldsymbol{\alpha}_{\mathbf{i}}\right)_{i=1,m}$ is solution of:

$\tilde{\mathbf{M}}\ddot{\boldsymbol{\alpha}}+\tilde{\mathbf{C}}\dot{\boldsymbol{\alpha}}+\tilde{\mathbf{K}}\boldsymbol{\alpha}=\tilde{f}$ (problem of order m) with:

$$\begin{array}{ll}\tilde{\mathbf{M}}=\left(\tilde{\mathbf{M}}_{ij}\right)=\left(\boldsymbol{\Phi}^{iT}\mathbf{M}\boldsymbol{\Phi}^{\mathbf{j}}\right) & \tilde{\mathbf{C}}=\left(\tilde{\mathbf{C}}_{ij}\right)=\left(\boldsymbol{\Phi}^{iT}\mathbf{C}\boldsymbol{\Phi}^{\mathbf{j}}\right)\\ \tilde{\mathbf{K}}=\left(\tilde{\mathbf{K}}_{ij}\right)=\left(\boldsymbol{\Phi}^{iT}\mathbf{K}\boldsymbol{\Phi}^{\mathbf{j}}\right) & \tilde{f}=\left(\tilde{f}_i\right)=\left(\boldsymbol{\Phi}^{iT}f\right)\end{array} .$$

The modes of vibration of the generalized problem are $\mathbf{K}$ and $\mathbf{M}$ orthogonal [R5.01.01]. The matrixes $\tilde{\mathbf{M}}$ and $\tilde{\mathbf{K}}$ are then diagonal and are made up by the stiffness and generalized masses of each mode. The matrix $\tilde{\mathbf{C}}$ is usually full if one does not make additional assumptions on $\mathbf{C}$ [R5.05.04].

4.2 Effective modal masses and effective modal masses unit

4.2.1 effective modal Masses

Is $\mathbf{U_d}$ an unit vector in the direction d . In each node of the vector $\mathbf{U_d}$ having the components of displacement (DX, DY, DZ) one a:

$\left(\mathrm{DX}=x_d, \mathrm{DY}=y_d, \mathrm{DZ}=z_d\right)$ where $\left(x_d, y_d, z_d\right)$ are the cosine Directors of the direction d (one thus has: $x_d^2+y_d^2+z_d^2=1$).

For example, if d is the direction x , the vector $\mathbf{U_d}$ has all its components DX equal to 1 and its other components equal to 0.

One defines the effective modal masses in the direction d by:

$$m_{\Phi,d}=\frac{\left(\boldsymbol{\Phi}^{\mathrm{T}}\mathbf{M}\,\mathbf{U_d}\right)^{2}}{\left(\boldsymbol{\Phi}^{\mathrm{T}}\mathbf{M}\boldsymbol{\Phi}\right)} .$$

4.2.2 Property

Stated:

The sum of the effective modal masses in a direction d is equal to the total mass m_{totale} of structure. That is written:

$m_{totale}=\sum_{i=1,n}\frac{\left(\boldsymbol{\Phi}^{\mathbf{iT}}\mathbf{M}\,\mathbf{U}_{\mathbf{d}}\right)^2}{\left(\boldsymbol{\Phi}^{\mathbf{iT}}\mathbf{M}\boldsymbol{\Phi}^{\mathbf{i}}\right)}=\sum_{i=1,n} m_{\Phi^i,d}$ where n is the nombre total of modes associated with the problem [éq 4-1]

4.2.3 unit effective modal Masses

By means of the preceding property, one defines the unit effective modal masses:

$$\tilde{m}_{\Phi^i,d}=\frac{m_{\Phi^i,d}}{m_{totale}} ,$$

and one a: $\sum_{i=1,n}\tilde{m}_{\Phi^i,d}=1$.

The modal masses $\tilde{m}_{\Phi^i,d}$ and $m_{\Phi^i,d}$ are independent of the standardization of the mode Φ^i of vibration.

4.2.4 “

Empirical” use Relation :

During a study “request seismic of a structure in a direction d” by a method of modal recombination, one must preserve the modes of vibration which have an important unit effective mass and it is of use in France to consider that one so has a good modal representation for all of the preserved modes one a:

$$\sum_{i=1,n}\tilde{m}_{\Phi^i,d}\geq 0{,}9$$

This relation empirical for example is stated in the RCC-G (Rules of design and construction applicable to the Civil engineer).

Note:

the sum of the effective modal masses is worth in fact the total mass which works on selected modal base. In other words, this working total mass is worth the total mass minus the contributions out of mass which are carried by clamped degrees of freedom (which thus do not work on modal base). Thus, for example, on a system with 1 mass-spring degree of freedom with a mass $M1$ at the top and another mass $M2$ at the level to erase it, then the working mass will be worth $M1$ and the total mass $M1+M2$. Consequently, the unit effective modal mass for the only mode of the system will be worth $M1/(M1+M2)$. The total office plurality will thus have the same value and, according to the ratio in $M1$ and $M2$, one will not be able thus inevitably to reach 90% of the total mass $(M1+M2)$, even by considering all the modes (there is only one only mode on this example). In practice, the model with the finite elements will be fine and realistic, more the difference between the working mass and the total mass will be weak.

4.2.5 Directions privileged in *Code_Aster*

In *Code_Aster*, one has three directions which are those of the reference of definition of mesh:

- d = direction X,
- d = direction Y,
- d = direction Z.

The effective modal masses and the unit effective modal masses are accessible in the result concept of the mode_meca `type` under names `MASS_EFFE_DX, MASS_EFFE_DY, MASS_EFFE_DZ, MASS_EFFE_UN_DX, MASS_EFFE_UN_DY, MASS_EFFE_UN_DZ`.

4.3 Participation factors

4.3.1 Definition

One defines other parameters called participation factor:

$$p_{F,d}^{i}=\frac{(\boldsymbol{\Phi}^{\mathbf{iT}}\mathbf{M}\mathbf{U_d})}{(\boldsymbol{\Phi}^{\mathbf{iT}}\mathbf{M}\boldsymbol{\Phi}^{\mathbf{i}})}.$$

This parameter depends on the standardization of the mode of vibration $\boldsymbol{\Phi}^{\mathbf{i}}$.

As for the effective masses, one has three directions d which are those of the reference of definition of the mesh.

The participation factors are accessible in the result concept of the mode_meca `type` under names `FACT_PARTICI_DX, FACT_PARTICI_DY, FACT_PARTICI_DZ`.

4.3.2 Property

Stated:

The participation factors associated with a direction d check the following relation:

$$m_{totale}=\sum_{i=1,n}\frac{\left(\boldsymbol{\Phi}^{iT}\mathbf{M}\mathbf{U_d}\right)^2}{\left(\boldsymbol{\Phi}^{iT}\mathbf{M}\boldsymbol{\Phi}^{\mathbf{i}}\right)}=\sum_{i=1,n}\left(\frac{\boldsymbol{\Phi}^{iT}\mathbf{M}\mathbf{U_d}}{\boldsymbol{\Phi}^{iT}\mathbf{M}\boldsymbol{\Phi}^{\mathbf{i}}}\right)^2\left(\boldsymbol{\Phi}^{iT}\mathbf{M}\boldsymbol{\Phi}^{\mathbf{i}}\right)=\sum_{i=1,n}\left(p_{\Phi^i,d}\right)^2 m_{\Phi^i}$$

where n is the nombre total of modes associated with the problem [éq 4-1].

This result is obtained easily by expressing the participation factor according to the effective modal mass and by means of result stated with [§ 4.2.3].

4.3.3 Use

These parameters are used in particular to compute: the response of a structure subjected to a seisme by spectral method. Please refer to document [R4.05.03].

4.4 Unit vector displacement

In what precedes, one considered a unit displacement vector $\mathbf{U_d}$ which relates to only the degrees of freedom of translation (DX, DY, DZ). This notion can be wide with rotations by considering the following definition. A matrix of $\mathbf{U}$ dimension is defined $(n\times 6)$. If all the nodes of the mesh support 3 degrees of freedom of translation and 3 others of rotation, the matrix $\mathbf{U}$ is formed by the stacking of the following $\mathbf{u}_{tr\,d}^{\mathbf{k}}(6\times 6)$ matrixes (the index k corresponds to the node of number k):

$$\mathbf{u}_{tr}^{\mathbf{k}}=\begin{bmatrix} 1 & 0 & 0 & 0 & (z_k-z_c) & -(y_k-y_c) \\ 0 & 1 & 0 & -(z_k-z_c) & 0 & (x_k-x_c) \\ 0 & 0 & 1 & (y_k-y_c) & -(x_k-x_c) & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where (x_k, y_k, z_k) are the coordinates of the node and (x_c, y_c, z_c) are the coordinates of the instantaneous center of rotation.

One can thus define effective modal masses, participation factors associated with degrees of freedom with rotation.

For time, the computation of these parameters is not available in Code_Aster.

5 Modal parameters associated for the quadratic problem

One writes the quadratic problem in the form: $(\lambda^2 \mathbf{M}+\lambda\,\mathbf{C}+\mathbf{K})\boldsymbol{\Phi}=0$.

For the quadratic problem, one calculates only three parameters which correspond to the following generalized quantities:

generalized mass (real quantity): $m_{\Phi}=\bar{\boldsymbol{\Phi}}^{\mathbf{T}}\mathbf{M}\boldsymbol{\Phi}$,

generalized stiffness (real quantity): $k_{\Phi}=\bar{\boldsymbol{\Phi}}^{\mathbf{T}}\mathbf{K}\boldsymbol{\Phi}$,

generalized damping (real quantity): $c_{\Phi}=\bar{\boldsymbol{\Phi}}^{\mathbf{T}}\mathbf{C}\boldsymbol{\Phi}$.

Attention, if one normalizes the eigen mode with the norm “generalized mass”, one does not have in the quadratic case: $m_{\Phi}=1$. One can pass the same remark concerning the generalized stiffness.

By means of the relations of orthogonality and the fact that the clean elements appear per combined pairs, one can write the following relations:

$$\frac{\bar{\boldsymbol{\Phi}}^{\mathbf{T}}\mathbf{C}\boldsymbol{\Phi}}{\bar{\boldsymbol{\Phi}}^{\mathbf{T}}\mathbf{M}\boldsymbol{\Phi}}=\frac{c_{\Phi}}{m_{\Phi}}=2\,\mathrm{Re}(\lambda)=-\frac{2\,\xi\,\omega}{\sqrt{1-\xi^2}}=-\frac{2\mathrm{x}(2\pi f)}{\sqrt{1-\xi^2}},$$

$$\frac{\bar{\boldsymbol{\Phi}}^{\mathbf{T}}\mathbf{K}\boldsymbol{\Phi}}{\bar{\boldsymbol{\Phi}}^{\mathbf{T}}\mathbf{M}\boldsymbol{\Phi}}=\frac{k_{\Phi}}{m_{\Phi}}=|\lambda|^2=\frac{\omega^2}{1-\xi^2}=\frac{(2\pi f)^2}{1-\xi^2}.$$

6 Bibliography

1) J.R. LEVESQUE, L. VIVAN, Fe WAECKEL: Seismic response by spectral method [R4.05.03].

2) D. SELIGMANN, B. QUINNEZ: Algorithms of resolution for the generalized problem [R5.01.01].

3) D. SELIGMANN, R. MICHEL: Algorithms of resolution for the quadratic problem [R5.01.02].

4) Operator `NORM_MODE` [U4.52.11].

7 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
04/01/00	*B. QUINNEZ J.R. LEVESQUE (EDF/IMA/MMN)*	initial Text