Nonlinear thermal

Abstract

operator THER_NON_LINE [U4.54.02] makes it possible to solve the problems of transient thermal in solids in the presence of non-linearities of the properties of the materials (heat capacity and conductivity), or of the boundary conditions (heat exchange of standard radiation). One presents here the formulation and the algorithm employed, this last being close to that related to operator STAT_NON_LINE [R5.03.01]. The various computation options necessary were presented in the plane, axisymmetric and three-dimensional structural elements [U3.22.01], [U3.23.01] and [U3.24.01].

Contents

1 Statement of the equation of heat in nonlinear thermal

1.1 Equation of heat for a motionless solid

In this document, one treats only the thermal of the solid bodies, even if the liquid/solid phase change is taken into account. There is thus no heat transfer by convection but only by conduction.

The first principle of the thermodynamics connects the temporal variation of total energy dE_{totale} of a system included in a control volume Ω to the work of the external forces δW and heat δQ received by this same system:

$$dE_{totale}=d(E_{\mathrm{interne}}+E_{\mathrm{cintique}})=\delta W+\delta Q \quad \textbf{éq 1.1-1}$$

By injecting the theorem of kinetic energy in this equation, one reveals thus the power of the internal forces, function of the velocity field [bib1]:

$$\dot{E}_{\mathrm{interne}}=\dot{Q}-P_i(u) \quad \textbf{éq 1.1-2}$$

For the resolution of the problem of thermal, the system is supposed without motion. The power of the internal forces $P_i(u)$ is thus null. Indeed, in the majority of the applications concerned, the thermal and mechanical phenomena are decoupled; the density power density dissipated by plastic strains $P_i=\sigma_c.\dot{\epsilon}_{plastique}$, is neglected in front of the heat exchanged on the surface or the other voluminal heat sources.

The equation [éq 1.1-2] which expresses the variation of heat in volume Ω is written then:

$$\forall s\in\Omega \quad \rho\frac{d}{dt}\int_S e\,d\,\Omega=\dot{Q}=\int_S \left(r_{vol}-\mathrm{div}\,q\right)d\,\Omega \quad \textbf{éq 1.1-3}$$

where one noted:

e internal energy,
ρ density,
r_{vol} the voluminal rate of contribution external of heat,
$\boldsymbol{q}$ the vector heat flux.

Moreover, since the solid is motionless, for any control volume $\Omega(t)=\Omega$, one then obtains the local equation of conservation of heat:

$$\rho\frac{de}{dt}=r_{vol}-\mathrm{div}\,q \quad \textbf{éq 1.1-4}$$

If all the system is actuated by a rigid body motion, an additional term appears in the member of left, utilizing the velocity of solid and the gradient of energy. This situation is treated by the command `THER_NON_LINE_MO` [R5.02.04].

In the case of a reversible transformation, the equation [éq 1.1-4] becomes, with the assistance of the second principle of the thermodynamics which makes it possible to write in our case $dE_{interne}=TdS$:

$$\rho \ T\dot{s}=r_{vol}-\operatorname{div} q$$ **éq 1.1-5**

and finally the equation of heat in its classical form:

$$\rho \ C_P\dot{T}=r_{vol}-\operatorname{div} q$$ **éq 1.1-6**

with heat capacity with constant pressure defined by: $C_P=T\frac{\partial s}{\partial T}|_P$

As he is explained in chapter 1.4, he can be advantageous to write the term of left of the equation [éq 1.1-6] with the enthalpy β which does not depend whereas temperature:

$$\dot{\beta}=r_{vol}-\operatorname{div} q$$ **éq 1.1-7**

where $\beta(T)=\int_{T_0}^{T}\rho C_P dT$

1.2 Fourier analysis

In thermal conduction, the Fourier analysis provides an equation connecting heat flux to the gradient of the temperature (normal vector on the isothermal surface). This model reveals, in its most general form, a tensor of conductivity. In the case of an isotropic material, this tensor is reduced to a coefficient λ (being able to depend on the temperature), thermal conductivity:

$$q(x,t)=-\lambda(T)\nabla T(x,t)$$ **éq 1.2-1**

1.3 Equation of the nonlinear heat in the case of the model of transient thermal

By combining the equations [éq 1.1-5] and [éq 1.2-1], one obtains:

$$r_{vol}-\operatorname{div}(-\lambda(T)\nabla T)=\frac{d\beta}{dt}$$ **éq 1.3-1**

or, if heat capacity does not depend on the temperature:

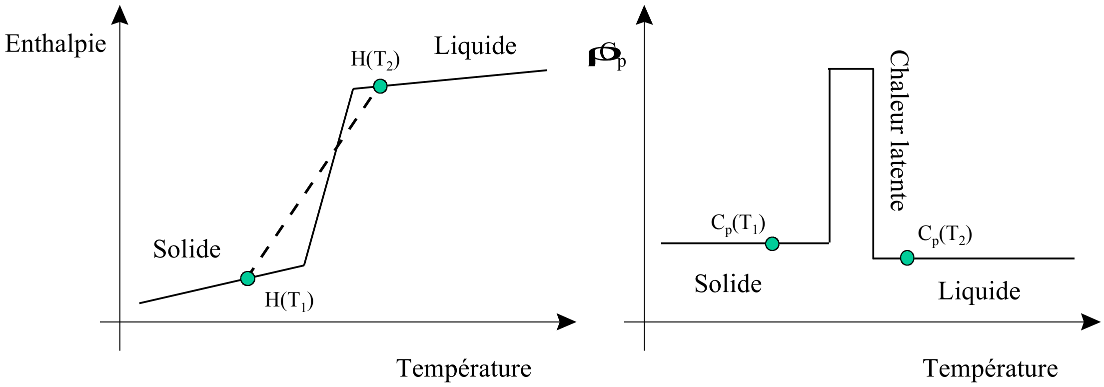
$$r_{vol}-\operatorname{div}(-\lambda(T)\nabla T)=\rho \ C_p\frac{dT}{dt}$$ **éq 1.3-2**

1.4 numerical Advantage of the formulation in enthalpy for the problems with phase change.

The relation between enthalpy and heat capacity is:

$$\beta(T)=\int_{T_0}^{T}\rho C_p(u)du$$

When this function enthalpy presents abrupt variations, it is more precise to handle $\beta(T)$ than its derivative. Indeed, the paces characteristic of these functions in the vicinity of the melting point are the following ones:



During an iteration, either because the thermal transient is violent, or because the beach of phase change is very small (pure substance), two the reiterated successive ones of the temperature can be located on both sides of discontinuity. The evaluating of the slope of the function enthalpy in the vicinity of the melting point will be very false if one considers $C_p(T_1)$, $C_p(T_2)$ or a weighted average of both. On the other hand, the slope of the right in dotted lines is always a correct approximation of $d\beta/dT$ at the melting point.

2 Boundary conditions, loading and initial condition

One will refer to [R5.02.01] for the thermal boundary conditions and the loadings leading to linear equations in temperature like for the initial condition.

2.1 Normal flux nonlinear

It is of the conditions of the Neumann type, defining flux entering the field.

$$-q(x,t).\boldsymbol{n}=g(x,T) \quad \text{on the border } \Gamma \qquad \textbf{éq 2.1-1}$$

where $g(x,T)$ is a function of the temperature and possibly of the variable of space x and/or time t and $\boldsymbol{n}$ indicates the norm external with the border Γ, q is the vector heat flux (directed according to the decreasing temperatures).

This statement makes it possible to introduce for example conditions of the type exchanges with a convective coefficient of heat exchange depend on the temperature:

$$-q(x,t).\boldsymbol{n}=g(x,T)=h(x,T)(T_{ext}(x,t)-T) \qquad \textbf{éq 2.1-2}$$

2.2 nonlinear normal Flux - condition of type radiation ad infinitum

a typical case of the preceding boundary conditions is the radiation ad infinitum of gray body which results in a typical case of function $g(x,T)$:

$$-q(x,t).\boldsymbol{n}=\sigma\epsilon\left[(T(x)+273.15)^4-(T_\infty+273.15)^4\right] \qquad \textbf{éq the 2.2-1}$$

characteristics to be defined at the time of the definition of this loading are emissivity ε, the constant of Stefan-Boltzmann $\sigma=5{,}73.10^{-8} usi$ and the temperature ad infinitum.

$T(r)$ and T_∞ are then expressed in degrees Celsius. $-273.15\,°C$ is the temperature of the absolute zero.

3 Variational formulation of the problem

We will restrict ourselves here to present the problem with only the boundary conditions of imposed temperature [R5.02.01 §2.1], of imposed normal flux [R5.02.01 §2.3], of exchange [R5.02.01 §2.4], nonlinear flux [§2.1] and radiation [§2.2].

That is to say Ω open of R^3, border $\Gamma=\Gamma_1\cup\Gamma_2\cup\Gamma_3\cup\Gamma_4\cup\Gamma_5$.
One must solve the equation [éq 1.1-4] in T on $\Omega\times]0,t[$ with the boundary conditions:

$$\begin{cases} T=T^d(r,t) & & \text{sur } \Gamma_1 \\ \lambda(T)\dfrac{\partial T}{\partial n} = f(r,t) & & \text{sur } \Gamma_2 \\ \lambda(T)\dfrac{\partial T}{\partial n} = h(r,t)(T_{ext}(r,t)-T) & & \text{sur } \Gamma_3 \\ \lambda(T)\dfrac{\partial T}{\partial n} = g(r,T) & & \text{sur } \Gamma_4 \\ \lambda(T)\dfrac{\partial T}{\partial n} = \sigma\epsilon\left[(T+273.15)^4-(T_\infty+273.15)^4\right] & & \text{sur } \Gamma_5 \end{cases} \quad \textbf{éq 3-1}$$

and with, possibly, of the initial conditions $T(t=0)$. If these last are not specified, one solves as a preliminary the steady problem, i.e. the equation [éq 1.3-1] without the term of temporal evolution.

That is to say v a sufficiently regular function cancelling itself on Γ_1, while noticing:

$$\frac{d}{dt}\left(\int_\Omega \beta(T).v.d\Omega\right)=\int_\Omega \dot{\beta}(T).v.d\Omega$$
$$\int_\Omega \lambda(T)\nabla T.\nabla v.d\Omega= -\int_\Omega div(\lambda(T)\nabla T).v.d\Omega+\int_\Gamma \lambda(T)\frac{\partial T}{\partial n}.v.d\Gamma \quad \textbf{éq the 3-2}$$

weak formulation of the equation of heat can then be written:

$$\frac{d}{dt}\left(\int_\Omega \beta(T).v.d\Omega\right)+\int_\Omega \lambda(T)\nabla T.\nabla v d\Omega-\int_\Gamma \lambda(T)\frac{\partial T}{\partial n}.v.d\Gamma=\int_\Omega r_{vol}.v.d\Omega \quad \textbf{éq 3-3}$$

One from of deduced the variational formulation from problem:

$$\int_{\Omega} \frac{d\beta(T)}{dt}.v.d\Omega + \int_{\Omega} \lambda(T)\nabla T.\nabla v.d\Omega + \int_{\Gamma_3} hT.v.d\Gamma_3 =$$
$$\int_{\Omega} r_{vol}.v.d\Omega + \int_{\Gamma_2} f.v.d\Gamma_2 + \int_{\Gamma_3} hT_{ext}.vd\Gamma_3 +$$
$$\int_{\Gamma_4} g.v.d\Gamma_4 + \int_{\Gamma_5} \sigma\epsilon.\left[(T+273.15)^4 - (T_{\infty}+273.15)^4\right].v.d\Gamma_5$$

éq 3-4

4 Discretization in time of the differential equation

4.1 Introduction of Θ - method

a classical way to discretize a first order differential equation is it Θ - the method. Let us consider the following differential equation:

$$\begin{cases} \dot{y}(t)=\phi(t,y(t)) \\ y(0)=y_0 \end{cases}$$ **éq 4.1-1**

Θ the - method consists in discretizing the equation [éq 4.1-1] by a diagram with the finite differences

$$\frac{1}{\Delta t}(y_{n+1}-y_n)=\theta.\phi(t_{n+1},y_{n+1})+(1-\theta).\phi(t_n,y_n)$$ **éq 4.1-2**

where y_{n+1} is an approximation of $y(t_{n+1})$, y_n being supposed known and θ is the parameter of the method $\theta\in[0,1]$.

Note:

if $\theta=0$ the diagram is known as explicit,
if $\theta\neq 0$ the diagram is known as implicit.

4.2 Application to the equation of heat

Let us use it Θ - method in the variational formulation of the equation of heat, where one posed:

$$\begin{array}{llll} T^+=T(r,t+\Delta t) & T^-=T(r,t) & h^+=h(r,t+\Delta t) & h^-=h(r,t) \\ f^+=f(r,t+\Delta t) & f^-=f(r,t) & T^+_{ext}=T_{ext}(r,t+\Delta t) & T^-_{ext}=T_{ext}(r,t) \\ r^+_{vol}=r_{vol}(r,t+\Delta t) & r^-_{vol}=r_{vol}(r,t) & T^{d+}=T^d(r,t+\Delta t) & T^{d-}=T^d(r,t) \\ g^+=g(r,T^+) & g^-=g(r,T^-) & & \end{array}$$

where $T^d(r,t)$ the temperature imposed on the border of the field represents, according to time and of space.

Let us introduce following spaces of functions:

$$V_{t^+}=\left\{v\in H^1(\Omega)\, v_{/\Gamma_1}=T^{d+}\right\}$$
$$V_{t^-}=\left\{v\in H^1(\Omega)\, v_{/\Gamma_1}=T^{d-}\right\}$$
$$V_0=\left\{v\in H^1(\Omega)\, v_{/\Gamma_1}=0\right\}$$

The field $T^-\in V_{t^-}$ being supposed known, one seeks $T^+\in V_{t^+}$ such as $\forall\, v\in V_0$:

$$
\begin{aligned}
&\int_\Omega \frac{\beta(T^+)-\beta(T^-)}{\Delta t} v.d\Omega \qquad +\int_\Omega (\theta\lambda(T^+)\nabla T^+.\nabla v+(1-\theta)\lambda(T^-)\nabla T^-.\nabla v)d\Omega\\
&-\int_{\Gamma_2}(\theta f^+ +(1-\theta)f^-)v.d\Gamma_2 \quad -\int_{\Gamma_3}(\theta h^+T^+_{ext}+(1-\theta)h^-T^-_{ext}-\theta h^+T^+-(1-\theta)h^-T^-)v.d\Gamma_3\\
&-\int_{\Gamma_4}(\theta g^+ +(1-\theta)g^-)v.d\Gamma_4 \qquad =\\
&\int_\Omega(\theta r^+_{vol}+(1-\theta)r^-_{vol})v.d\Omega \qquad +\int_\Omega(\theta r_v(T^+)+(1-\theta)r_v(T^-))v.d\Omega
\end{aligned}
$$

éq 4.2-1

not to excessively weigh down the writing and insofar as the process is identical to the other terms, one did not make appear the term of radiation in these equations (integral on Γ_5).

While posing:

$$
\begin{aligned}
&(hT_{ext})^\theta=\theta h^+T^+_{ext}+(1-\theta)h^-T^-_{ext}\\
&f^\theta=\theta f^+ +(1-\theta)f^-\\
&r^\theta=\theta r^+_{vol}+(1-\theta)r^-_{vol}
\end{aligned}
$$

one obtains finally:

$$
\begin{aligned}
&\int_\Omega \frac{\beta(T^+)}{\Delta t}v.d\Omega+\theta\int_\Omega\lambda(T^+)\nabla T^+.\nabla v.\,d\Omega+\theta\int_{\Gamma_3}h^+T^+v.d\Gamma_3\\
&-\theta\int_{\Gamma_4}g(T^+).v.d\Gamma_4-\theta\int_\Omega r_v(T^+).v.d\Omega=L_1(v,T^-)\\
&\forall v\in V_0
\end{aligned}
$$

éq 4.2-2

where one posed:

$$
\begin{aligned}
L_1(v,T^-)=&\int_\Omega\frac{\beta(T^-)}{\Delta t}v.d\Omega-\int_\Omega(1-\theta)\lambda(T^-)\nabla T^-.\nabla v.d\Omega+\int_{\Gamma_2}f^\theta v.d\Gamma_2\\
&+\int_{\Gamma_3}((hT_{ext})^\theta-(1-\theta)h^-T^-)v.d\Gamma_3+\int_\Omega r^\theta v.d\Omega\\
&+(1-\theta)\int_{\Gamma_4}g(T^-)v.d\Gamma_4+(1-\theta)\int_\Omega r_v(T^-)v.d\Omega
\end{aligned}
$$

éq 4.2-3

A a time of given computation, this term is known. Indeed, only the temperature at previous time T^- , as well as the values at the known time running of *function* of time, intervene.

If the distribution of temperature in the system at initial time is not provided, the steady problem is solved. The terms of evolution disappear $\Theta=1$; the field of temperature at initial time is given by:

$$
\begin{aligned}
&\int_\Omega\lambda(T^{t=0})\nabla T^{t=0}.\nabla v.d\Omega+\int_{\Gamma_3}h^{t=0}T^{t=0}v.d\Gamma_3-\int_{\Gamma_4}g(T^{t=0})v.d\Gamma_4\\
&=\int_{\Gamma_2}f^{t=0}v.d\Gamma_2+\int_{\Gamma_3}h^{t=0}T^{t=0}_{ext}v.d\Gamma_3+\int_\Omega r^{t=0}v.d\Omega\\
&\forall v\in V_0
\end{aligned}
$$

éq 4.2-4

the problem is written finally in the condensed form:

$$\begin{cases} Soit\ T^- \in V_{t^-}\ connu, trouver\ T^+ \in V_{t^+}\ tel\ que \\ \forall v \in V_0 : a(v, T^+) = L_1(v, T^-) \end{cases}$$ **éq 4.2-5**

5 spatial Discretization and adaptation of the algorithm of Newton with the problem

the principle of the method of Newton is very detailed in [R5.03.01], one will expose here only the adaptations specific to the nonlinear algorithm of thermal.

5.1 Spatial discretization

Is P_h a space division Ω , indicate by N the number of nodes of the mesh, p_i the shape function associated with the node i . One indicates by J all the nodes belonging to the border G_1.

Are:

$$\begin{aligned} V_{t^+}^h &= \left\{ v = \sum_{i=1,N} v_i\, p_i(x)\ ;\ v_j = T^d(x_j, t^+)\quad j \in J \right\} \\ V_{t^-}^h &= \left\{ v = \sum_{i=1,N} v_i\, p_i(x)\ ;\ v_j = T^d(x_j, t^-)\quad j \in J \right\} \\ V_0^h &= \left\{ v = \sum_{i=1,N} v_i\, p_i(x)\ ;\ v_j = 0\quad j \in J \right\} \end{aligned}$$ **éq 5.1-1**

the problem [éq 4.2-5] can be replaced by the problem discretized with many unknowns finished according to:

That is to say $T^- \in V_t^h$ known, to find $T^+ \in V_{t^+}^h$ such as

$$v_h \in V_0^h a(v_h, T^+) = L_1(v_h, T^-)$$

éq 5.1-2

that one can also write, with the same formalism as `STAT_NON_LINE` [R5.03.01], in vectorial form:

$$v^T R(T^+, t^+) = v^T L(T^-, t^+) \quad \forall v \text{ such as } Bv = 0$$
$$BT^+ = T^d(t^+)$$

éq 5.1-3

where the operator $\boldsymbol{B}$ expresses the boundary condition of imposed temperature $T^+ \in V_{t^+}^h$. It is defined by:

$$(\mathbf{Bv})_j = \begin{cases} 0\ si\ j \notin J \\ v_j\ si\ j \in J \end{cases}$$ **éq the 5.1-4**

cases where the application R is linear is treated by the command `THER_LINEAIRE` [R5.02.01].

The dualisation of the boundary conditions, detailed in [R3.03.01], led to the nonlinear problem in T^+ :

$$\begin{cases} R(T^+, t^+) + B^T \lambda^+ = L(T^-, t^+) \\ BT^+ = T^d(t^+) \end{cases}$$ **éq the 5.1-5**

unknowns are the couple (T^+, λ^+), where λ^+ represents the "Lagrange multipliers" of the boundary conditions of Dirichlet.

To solve the system [éq 5.1-5] amounts cancelling in (T_i^+, λ_i^+) the vector $F(T^+, \lambda^+)$, called residue, defined by:

$$F(T^+, \lambda^+) = \begin{pmatrix} L(T^-, t^+) - R(T^+, t^+) - B^T \lambda^+ \\ T^d(t^+) - BT^+ \end{pmatrix}$$ **éq 5.1-6**

the method of Newton consists in building a vector series $\{x^n\}_n$ converging towards the solution of $F(x)=0$ using the tangent linear application of F.

5.2 Steady computation

the variational problem is that of the equation [éq 4.2-4]. A to note: in steady computation, the enthalpy does not intervene in the application R.

One introduces the matrix of the tangent linear application of the function $R(T^n)$:

$$K^n = \frac{\partial R}{\partial T}\Big|_{T_n}$$

That of the function $F(T^n, \lambda^n)$ is then:

$$\begin{bmatrix} K^n & B^T \\ B & 0 \end{bmatrix}$$

In the case of steady computation, one must reiterate from a uniform value of initialization of the field of temperature; in fact $T_0=0$ in any node. The first iteration of computation, known as iteration of prediction, consists in solving the following system:

$$\begin{bmatrix} K(T_0) & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} T_1 - T_0 \\ \lambda_1 - \lambda_0 \end{bmatrix} = \begin{bmatrix} L - R(T_0) - B^T \lambda_0 \\ T^d - BT_0 \end{bmatrix}$$ **éq 5.2-1**

As one can see it in the equation of the steady problem [éq 4.2-4], the temperature does not appear to the second member: one writes L and not $L(T_0)$.

If the problem is linear $R(T_0) = K(T_0) T_0 = K.T_0$. All the terms disappear T_0 from it by simplification. The solution is obtained in an iteration by inversion of a system identical to that described in [R5.02.01 §6].

The following iterations are iterations of Newton, with reactualization or not of the tangent matrix K.

$$\begin{bmatrix} K(T_{(i)}) & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} T_{i+1}-T_i \\ \lambda_{i+1}-\lambda_i \end{bmatrix} = \begin{bmatrix} L-R(T_i)-B^T\lambda_i \\ 0 \end{bmatrix}$$ **éq 5.2-2**

For the iteration of prediction, the writing of the lower subsystem of the equation [éq 5.2-1], after simplification, ensures us that $BT_1 = T^d$. The reiterated first and all the following thus check the conditions of Dirichlet.

The brackets around the index of iteration in the statement $K(T_{(i)})$ mean that one can reactualize or not the tangent matrix with the wire of the iterations.

Note:

The temperature of initialization T_0 has of influence only for one nonlinear steady computation. While being of about size of the expected temperatures, it would make it possible "to leave" less far from the solution that a null field everywhere; and thus the nombre of iterations would decrease. Today, it is not possible to enter a value of T_0 . The vector temperature is initialized, into tough, to zero.

5.3 Transient computation

For the first iteration of time step, known as iteration of prediction, one "makes as if" the problem describes in [éq 5.1-5] were linear. This formulation must make it possible to directly obtain the solution to a linear problem of thermal. But here, the situation is a little different from steady computation because of the formulation in enthalpy. The linearization of [éq 5.1-5] gives:

$$\begin{cases} R(T^-,t^+)+K(T^-,t^+)(T^+-T^-)+B^T\lambda^+=L(T^-,t^+) \\ BT^+=T^d(t^+) \end{cases}$$ **éq 5.3-1**

What amounts solving, for the problem presented in matric form:

$$\begin{bmatrix} K(T^-) & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} T_1^+ \\ \lambda_1^+ \end{bmatrix} = \begin{bmatrix} L(T^-,t^+)+K(T^-)T^- - R(T^-) \\ T^d(t^+) \end{bmatrix}$$ **éq 5.3-2**

the function enthalpy is known with a constant of integration close which appears in the relation binding $R(T^-)$ to $K(T^-)T^-$. This same constant is found in the statement of $L(T^-,t^+)$. One can then eliminate it while leading to the following system of equations:

$$\begin{bmatrix} K(T^-) & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} T_1^+ \\ \lambda_1^+ \end{bmatrix} = \begin{bmatrix} \tilde{L}(T^-,t^+) \\ T^d(t^+) \end{bmatrix}$$ **éq 5.3-3**

where $\tilde{L}(T^-,t^+)$ is the second member calculated with heat capacity and not the enthalpy (option `CHAR_THER_EVOLNI` [§6.2]).

Lastly, as for the steady case seen in the preceding chapter, the following iterations are iterations of Newton:

$$\begin{bmatrix} K(T_{(i)}, t^+) & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} T_{i+1}^+ - T_i^+ \\ \lambda_{i+1}^+ - \lambda_i^+ \end{bmatrix} = \begin{bmatrix} L(T^-, t^+) - R(T_i, t^+) - B^T \lambda_i \\ 0 \end{bmatrix} \quad \textbf{éq 5.3-4}$$

This times, on the other hand, $L(T^-, t^+)$ is calculated with the enthalpy and not heat capacity to be coherent with $R(T_i^+)$.

5.4 Convergence

Since time intervenes in the form of the tangent matrix, and also time step, one prefers systematically to bring up to date this one at the beginning of each step not to degrade the velocity of convergence too much. On the other hand, freedom is left to the user time step control his frequency of computation during one.

A each iteration, one can carry out the search for an optimum step of progression towards the solution by some iterations (2 or 3) of linear search. This method is described in detail in [R5.03.01].

The computation famous is converged when the vector residue is null [éq 5.1-6]:

$$F(T_i^+, \lambda_i^+, t^+) = \begin{pmatrix} L(T^-, t^+) - R(T_i^+, t^+) - B^T \lambda_i^+ \\ T^d(t^+) - BT_i^+ \end{pmatrix} \quad \textbf{éq 5.4-1}$$

the lower part of the vector is always null (conditions of Dirichlet). One thus checks:

$$\frac{\| L(T^-, t^+) - R(T_i^+, t^+) - B^T \lambda_i^+ \|_2}{\| L(T^-, t^+) - B^T \lambda_i^+ \|_2} \leq \epsilon \quad \textbf{éq 5.4-2}$$

the user also has the possibility of stopping the iterations on an absolute criterion:

$$\| L(T^-, t^+) - R(T_i^+, t^+) - B^T \lambda_i^+ \|_\infty \leq \epsilon \quad \textbf{éq 5.4-3}$$

6 Principal options of nonlinear thermal calculated in *Code_Aster*

6.1 Boundary conditions and loadings

One will refer to [R5.02.01] for the boundary conditions and the linear loadings.

Nonlinear flux	CHAR_THER_FLUNL	$\int_{\Gamma_4} (1-\theta) g(T^-) v.d\Gamma_4$
nonlinear	Radiation CHAR_THER_RAYO_R	$\int_{\Gamma_4} \sigma\epsilon\left[(T+273.15)^4-(1-\theta)(T^-+273.15)^4\right].v.d\Gamma_4$
CHAR_THER _RAYO_F Source	CHAR_THER_SOURNL	$\int_{\Omega} (1-\theta) r_v(T^-) v.d\Omega$

6.2 Computation of the elementary matrixes and transitory term

thermal Inertia, conductivity	MTAN_RIGI_MASS	$\int_{\Omega} \frac{\rho\ \ Cp}{\Delta t} v.v.d\Omega + \int_{\Omega} \theta\lambda(T^+)\nabla v.\nabla v.d\Omega$
Radiation	MTAN_THER_RAYO_R MTAN_THER_RAYO_F	$\int_{\Gamma_4} \theta.4.\sigma.\epsilon(T^+ +273.15)^3 v.v.d\Gamma_4$
nonlinear	Coefficient of heat exchange MTAN_THER_COEF_R	$\int_{\Gamma_4} \theta h.v.vd\Gamma_4$
MTAN_THER_ COEF_F Flux	MTAN_THER_FLUXNL	$-\int_{\Gamma_4} \theta \frac{dg}{dT}(T^+) v.v.d\Gamma_4$
nonlinear Source	MTAN_THER_SOURNL	$-\int_{\Omega} \theta \frac{d r_v}{dT}(T^+) v.v.d\Omega$
transitory Term	CHAR_THER_EVOLNI	$\int_{\Omega} \frac{\beta(T^-)}{\Delta t}.v.d\Omega - \int_{\Omega}(1-\theta)\lambda(T^-)\nabla T^-.\nabla v.d\Omega$

$$\int_{\Omega} \frac{\rho\ \ CpT^-}{\Delta t}.v.d\Omega - \int_{\Omega}(1-\theta)\lambda(T^-)\nabla T^-.\nabla v.d\Omega$$

6.3 Computation of residue

	RESI_RIGI_MASS	$\int_{\Omega} \frac{1}{\Delta t}\beta(T^i) v.d\Omega + \int_{\Omega} \theta\lambda(T^i)\nabla T^i.\nabla v.d\Omega$
Radiation	RESI_THER_RAYO_R RESI_THER_RAYO_F	$\int_{\Gamma_4} \theta\sigma\epsilon(T^i+273.15)^4 v.d\Gamma_4$

nonlinear	Coefficient of heat exchange RESI_THER_COEF_R	$\int_{\Gamma_3} (\theta h^+ T^i) v.d\Gamma_3$
RESI_THER_COEF_F Flux	RESI_THER_FLUXNL	$-\int_{\Gamma_3} \theta g(T^i) v.d\Gamma_3$
nonlinear Source	RESI_THER_SOURNL	$-\int_{\Omega} \theta r_v(T^i) v.d\Omega$

7 Bibliography

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