Behavior models élasto-visco-plastic of Summarized

Chaboche:

This document describes the integration of the model of behavior élasto-visco-plastic of Chaboche with nonlinear and isotropic kinematic hardening, with taking into possible account of viscosity. The model established has one or two kinematical variables, and takes into account all the variations of the coefficients with the temperature, and has an effect of hardening on the tensorial variables of recall. This version also makes it possible to model (in an optional way) the viscous character of the material (viscosity of Norton). It is integrated by the solution of only one scalar equation nonlinear. This model is available in 3D, plane strain, axisymetry. The modelization in plane stress uses a method of condensation static (of Borst). One gives also elements to identify the coefficients of the behavior model.

Contents

1 Models élasto-visco-plastics of Chaboche available in *Code_Aster*

For the structural analysis subjected to loadings cyclic, hardenings isotropic (linear or not) and linear kinematics classics [R5.03.02] and [R5.03.16] are not sufficient any more. In particular, one cannot correctly describe the stabilized cycles obtained in experiments on a tensile specimen subjected to an alternated imposed strain or a traction and compression.

If one seeks to precisely describe the effects of a cyclic loading, it is desirable to adopt modelizations more sophisticated (but easy to use) such as the model of Saïd Taheri, for example, cf [R5.03.05], or if the number of cycles is restricted the model of Jean-Louis Chaboche who is introduced here.

Actually, the model of Chaboche can be more or less sophisticated. The models developed in *Code_Aster* comprise either a kinematical variable (`VMIS_CIN1_CHAB` and `VISC_CIN1_CHAB`) or two (`VISC_CIN2_CHAB` and `VMIS_CIN2_CHAB`), and isotropic hardening.

The choice to use two kinematical variables complicates certainly the model, but makes it possible to correctly identify the uniaxial tests in a broader range of strains [bib2], [bib7]. A certain number of identifications of the parameters of this model were carried out mainly for the stainless steels A316 and A304 ([bib7], [bib8]).

The models comprise 8 parameters (only one kinematical variable) or 10 (two kinematical variables), introduced into command `DEFI_MATERIAU` :

```
CIN1_CHAB (CIN1_CHAB_FO)              = _F (
                              ♦ R_0 =R_0     ,
                              ◊ R_I =R_I     ,    (useless if B=0)
                              ◊ B =b           ,  (default: 0.)

                              ♦ C_I =C_I     ,
                              ◊ K =k           ,  (default: 1.)
                              ◊ W =w           ,  (default: 0.)

                              ♦ G_0 =G_0     ,
                              ◊ A_I =A_I     ,    (default: 0.)
                            )
CIN2_CHAB (CIN2_CHAB_FO)              = _F (
                              ♦ R_0 =R_0     ,
                              ◊ R_I =R_I     ,
                                 useless if B=0 or if effect of memory)
                              ◊ B =b           ,  (default: 0.)

                              ♦ C1_I =C1_I   ,
                              ♦ C2_I =C2_I   ,
                              ◊ K =k           ,    (default: 1.)
                              ◊ W =w           ,    (default: 0.)

                              ♦ G1_0 =G1_0   ,
                              ♦ G2_0 =G2_0   ,
                              ◊ A_I =A_I     ,    (default: the 0.)
                            )
```

8 or 10 parameters are real constants. All these parameters can depend on the temperature (key words `CIN1_CHAB_FO` or `CIN2_CHAB_FO`) and the expected values are of standard `function`.

If one wants to introduce besides viscosity (models `VISC_CIN1_CHAB` and `VISC_CIN2_CHAB`), it is also necessary to provide in command `DEFI_MATERIAU`, under key word `LEMAITRE` (or `LEMAITRE_FO`) the parameters `N` and `UN_SUR_K`, which can depend on the temperature.

```
LEMAITRE (LEMAITRE_FO)                = _F (
                        ♦ N =n         ,
                        ♦ UN_SUR_K =1/K
```

parameter `UN_SUR_M` of key word `LEMAITRE` (respectively `LEMAITRE_FO`) must obligatorily be put at zero (respectively with the function identically null).

It is possible also to take into account an effect of memory of the more plastic large deformation using models (`VISC_CIN2_MEMO` and `VMIS_CIN2_MEMO`) . The keywords with informing are:

```
MEMO_ECRO (MEMO_ECRO_FO)                  = _F (
                        ♦ Q_M =Qm    ,
                        ♦ Q_0 =Q0    ,
                        ♦ MU =mu     ,
                        ♦ ETA =eta   , (default: 0.5)
```

In the event of loading nonproportional, it is necessary to enrich the model, by the data of two additional parameters:

CIN2_NRAD = _F (
◊ DELTA1= δ_1 (défaut= 1.E+0),
◊ DELTA2= δ_2 (défaut= 1.E+0),
with $0\leq\delta_1\leq 1$, $0\leq\delta_2\leq 1$

the constitutive laws are accessible in all the commands using key word `COMP_INCR` with the following relations:
`VISC_CIN1_CHAB`, `VISC_CIN2_CHAB`, `VISC_CIN2_MEMO`, `VISC_CIN2_NRAD`, `VISC_MEMO_NRAD`, `VMIS_CIN1_CHAB`, `VMIS_CIN2_CHAB`, `VMIS_CIN2_MEMO`, `VMIS_CIN2_NRAD`, `VMIS_MEMO_NRAD`.

Note: the model `VISCOCHAB` [R5.03.14] also makes it possible to represent the effects described in this document. It comprises moreover of the terms of additional restoration and hardening. But its use in structural analyzes is more expensive in time computation (because one must solve either by the method of Runge-Kutta or by the method of Newton a system of 27 equations to 27 unknowns). Moreover, it poses problems of robustness when time step is large, because the method of Newton can fail. That involves many subdivisions of time step.

The models described in this document are optimized, insofar as they result in solving only one scalar resolution, and the method of very robust resolution used (method of Brent or secant, cf [R5.03.14]); it is thus a model able to integrate quickly the large ones time step.

In the continuation of this document, one describes the characteristics of the various models. One presents then the detail of their numerical integration in restrain with the construction of the coherent tangent matrix. Lastly, one also gives some elements for the identification of the characteristics of the material.

2 Description of the models

2.1 Description of the models

At any moment, the state of the material is described by the strain ε , the temperature T , the plastic strain ε^p , the cumulated plastic strain p and the tensor of recall X . The equations of state then define according to these variables of state the stress $\sigma=\sigma^H \mathbf{Id}+\tilde{\sigma}$ (broken up into hydrostatics parts and deviatoric), the isotropic share of hardening R and the kinematical share X :

$$\sigma^H = \frac{1}{3} tr(\sigma) = K\, tr(\varepsilon - \varepsilon^{\mathbf{th}}) \text{ with } \varepsilon^{\mathbf{th}} = \alpha (T\text{-}T^{ref}) \mathbf{Id}$$ **éq 2.1-1**

$$\tilde{\sigma} = \sigma - \sigma^H \mathbf{Id} = 2\mu (\tilde{\varepsilon} - \varepsilon^p)$$ **éq 2.1-2**

$$R = R(p)$$ **éq 2.1-3**

$$X = X(p, \varepsilon^p) = X_1(p, \varepsilon^p) + X_2(p, \varepsilon^p)$$ **éq 2.1-4**

where K, μ, α and the coefficients of $X(p)$ and $R(p)$ are characteristics of the material which can depend on the temperature. More precisely, they are respectively the moduli of compressibility and shears, the thermal coefficient of thermal expansion, the functions of isotropic and kinematical hardening. As for $T^{réf}$, it is the reference temperature, for which one regards the thermal strain as being null.

Note:

For the model `VISC_CIN1_CHAB` one thus considers only the only tensorial $X_1(p)$ variable $X_2(p) = 0$. This remains valid for all the continuation: one will describe the two models formally in the same way, the model `VISC_CIN1_CHAB` resulting from `VISC_CIN2_CHAB` while supposing $X_2(p) = 0$.

The evolution of the plastic strain is controlled by a normal flow model to a plasticity criterion of von Mises:

$$F(\sigma, R, X) = (\tilde{\sigma} - X_1 - X_2)_{eq} - R(p) \text{ with } A_{eq} = \sqrt{\frac{3}{2} \tilde{A} : \tilde{A}}$$ **éq 2.1-5**

$$\dot{\varepsilon}^p = \dot{\lambda} \frac{\partial F}{\partial \sigma} = \frac{3}{2} \dot{\lambda} \frac{\tilde{\sigma} - X_1 - X_2}{(\tilde{\sigma} - X_1 - X_2)_{eq}}$$ **éq 2.1-6**

$$\dot{p} = \dot{\lambda} = \sqrt{\frac{2}{3} \dot{\varepsilon}^p : \dot{\varepsilon}^p}$$ **éq 2.1-7**

As for the plastic multiplier $\dot{\lambda}$, it is obtained by the condition of coherence:

$$\begin{cases} \text{si } F < 0 \text{ ou } \dot{F} < 0 & \dot{\lambda} = 0 \\ \text{si } F = 0 \text{ et } \dot{F} = 0 & \dot{\lambda} \geq 0 \end{cases}$$ **éq 2.1-8**

Note::

The evolution of the variables $\mathbf{X_1}$ and $\mathbf{X_2}$ is given by:

$$\begin{aligned} X_1 &= \frac{2}{3} C_1(p) \alpha_1 \\ X_2 &= \frac{2}{3} C_2(p) \alpha_2 \\ \dot{\alpha}_1 &= \dot{\varepsilon}^p - \gamma_1(p) \alpha_1 \dot{p} \\ \dot{\alpha}_2 &= \dot{\varepsilon}^p - \gamma_2(p) \alpha_2 \dot{p} \end{aligned}$$ **éq the 2.1-9**

functions $C(p)$, $\gamma(p)$ and $R(p)$ are defined, in accordance with [bib2] by:

$$R(p)=R_\infty+(R_0-R_\infty)e^{-bp}$$
$$C_1(p)=C_1^\infty\left(1+(k-1)e^{-wp}\right)$$
$$C_2(p)=C_2^\infty\left(1+(k-1)e^{-wp}\right)$$
$$\gamma_1(p)=\gamma_1^0\left(a_\infty+(1-a_\infty)e^{-bp}\right)$$
$$\gamma_2(p)=\gamma_2^0\left(a_\infty+(1-a_\infty)e^{-bp}\right)$$

The evolution of these coefficients makes it possible to represent hardening in several ways: classical isotropic hardening (monotonous or cyclic) by $R(p)$, "hardening" of the coefficients relating to the kinematical terms by $C(p)$ and $\gamma(p)$. (cf [biberon1122]). The statements into exponential are similar to the definition of nonlinear kinematic hardening (eq.2, 1,9), and (in their principle) represent a variation of the coefficients from the subscripted value by 0 (for $p=0$) to the subscripted value by ∞ when p becomes large.
This implies that the coefficients b and w are supposed to be positive. In the contrary case, an alarm message is transmitted, because the solution calculated risk to be not physics.

The presence of viscosity can model in a simple way [bib2] by replacing the condition of coherence [éq 2.1-8] by:

$$\dot{p}=\left(\frac{\langle F\rangle}{K}\right)^N \qquad \textbf{éq 2.1-10}$$

$\langle F\rangle$ positive part of F (hooks of Macauley), K, N characteristic of viscosity (Norton) of the material. Unchanged all the other equations of the model are left. It will be seen that such an introduction of viscosity involves only minor modifications of the implicit algorithm of integration of the constitutive law.

The effect of memory consists in replacing the evolution of isotropic hardening by:

$$F(\sigma,R,X)=(\tilde{\sigma}-X_1-X_2)_{eq}-R_0-R(p)$$
$$\dot{R}=b(Q-R)\dot{p}$$
$$Q=Q_0+(Q_m-Q_0)\left(1-e^{-2\mu q}\right)$$

$f(\varepsilon^p,\xi,q)=\frac{2}{3}J_2(\varepsilon^p-\xi)-q\le 0$ defining a field characterizing the plastic strains maximum ones, of which q measures the radius and ξ the center, calculated according to a model of normality i.e. with the law of evolution: $\dot{\xi}=\frac{1-\eta}{\eta}\dot{q}n^*$. The parameter η (which does not exist in the initial formulation [bib.2]), I makes it possible to partially take into account the effect of memory. If it is equal to 0.5, the initial formulation is found. If it is worth 1, q to the norm of the more plastic large deformation attack is equal. If it is much lower than 0.5, the effect of memory is taken into account partly only.

Note:

- *The definition of* $\mathbf{X}_1$ *and* $\mathbf{X}_2$ *in the form [éq 2.1-9]:*
  - *allows in the same way to keep a formulation which takes into account the variations of the parameters with the temperature without introducing term* $\dot{T}$ *in as into [bib.4], that the model of viscoplastic Chaboche. These terms are necessary because their not taken into account would lead to inaccurate results [bib4].*
  - *allows to have a coherent writing with the thermodynamic statement of the plastic potential [bib2] (p.221).*

- *It is noted that the functions $C_1(p), \gamma_1(p), C_2(p), \gamma_2(p), R(p)$ intervening in the preceding equations allow all the three to model various nonlinear effects of hardening . The introduction of hardening, either on the level of the kinematical part, by $C(p)$, or on the level of the term of recall, by the function $\gamma(p)$, does not have the same effect on the classification tests [bib2]. The use of a model with $\gamma(p)$ makes it possible in particular to identify more easily of strong cyclic hardenings. Several works of identification of the coefficients of the models of Chaboche besides were carried out on the basis of the model with a hardening represented by $\gamma(p)$ ([bib5], [bib6]), in particular for the stainless steels.*

2.2 Addition of the effect of memory

implicit discretization of the problem with effect of memory led to a system of 20 equations to 20 unknowns [7]:

6 éq: $\tilde{\sigma}=\frac{\mu}{\mu^-}\tilde{\sigma}^- + 2\mu\left(\Delta\tilde{\varepsilon}-\Delta\varepsilon^p\right)$

1 eq :

$$\left(\tilde{\sigma}-\frac{2}{3}C_1\alpha_1^- - \frac{2}{3}C_2\alpha_2^- - \frac{2}{3}C_1\Delta\alpha_1(\Delta\varepsilon^p) - \frac{2}{3}C_2\Delta\alpha_2(\Delta\varepsilon^p)\right)_{eq} = R_0 + R^- + \Delta R + K\left(\frac{\Delta p}{\Delta t}\right)^{1/N}$$

6 eq : $$\Delta\varepsilon^p=\frac{3}{2}\Delta p\,\frac{\tilde{\sigma}-\frac{2}{3}C_1\alpha_1^- - \frac{2}{3}C_2\alpha_2^- - \frac{2}{3}C_1\Delta\alpha_1 - \frac{2}{3}C_2\Delta\alpha_2}{R_0+R(p)+K\left(\frac{\Delta p}{\Delta t}\right)^{1/N}} = \Delta p\ n$$

1 eq: $f(\varepsilon^p, \boldsymbol{\xi}, q)=\frac{2}{3}J_2(\varepsilon^p-\xi)-q=\frac{2}{3}\sqrt{\frac{3}{2}(\varepsilon^p-\xi):(\varepsilon^p-\xi)}-q\le 0$

6 eq: $\Delta\xi=(1-\eta)\frac{\Delta q}{\eta}n^*$

with $\Delta R=b(Q-R)\Delta p$ $\qquad Q=Q_0+(Q_m-Q_0)\left(1-e^{-2\mu(q^-+\Delta q)}\right)$

$\Delta q=\eta H(F)\langle\mathbf{n}:\mathbf{n}^*\rangle\Delta p$

$$\Delta\alpha_i=\frac{\Delta\varepsilon^p-\gamma_i\alpha_i^-\Delta p}{1+\gamma_i\Delta p} \qquad n^*=\frac{3}{2}\frac{\varepsilon^p-\xi}{J_2(\varepsilon^p-\xi)} \qquad n=\frac{3}{2}\frac{\tilde{\sigma}-\frac{2}{3}C_1\alpha_1-\frac{2}{3}C_2\alpha_2}{\left(\tilde{\sigma}-\frac{2}{3}C_1\alpha_1-\frac{2}{3}C_2\alpha_2\right)_{eq}}$$

the 20 unknowns are: $\tilde{\sigma}, \Delta\varepsilon^p, \Delta\xi, \Delta p, \Delta q$

2.3 Insertion of the effect of nonproportionality of the loading

Of way similar to model VISCOCHAB, one can insert in VISC_CIN2_CHAB/MEMO the equations translating the effect nonproportional. The model obtained here VISC/VMIS_CIN2_NRAD, or VISC/VMIS_MEMO_NRAD is called (according to whether one takes into account or not the effect of memory).

$\dot{\alpha}_1=\dot{\varepsilon}^p-\gamma_1(p)\alpha_1\dot{p}$
$\dot{\alpha}_2=\dot{\varepsilon}^p-\gamma_2(p)\alpha_2\dot{p}$ becomes: $\dot{\boldsymbol{\alpha}}_1=\dot{\boldsymbol{\varepsilon}}^p-\gamma_1(p)\left(\delta_1\boldsymbol{\alpha}_1+(1-\delta_1)(\boldsymbol{\alpha}_1:\boldsymbol{n})\boldsymbol{n}\right)\dot{p}$
$\dot{\boldsymbol{\alpha}}_2=\dot{\boldsymbol{\varepsilon}}^p-\gamma_2(p)\left(\delta_2\boldsymbol{\alpha}_2+(1-\delta_2)(\boldsymbol{\alpha}_2:\boldsymbol{n})\boldsymbol{n}\right)\dot{p}$

formulate $\boldsymbol{n}=\sqrt{\frac{3}{2}}\frac{\tilde{\boldsymbol{\sigma}}-\boldsymbol{X}_1-\boldsymbol{X}_2}{(\tilde{\boldsymbol{\sigma}}-\boldsymbol{X}_1-\boldsymbol{X}_2)_{eq}}$ formula $\boldsymbol{n}:\boldsymbol{n}=1$ and in particular $\dot{\boldsymbol{\varepsilon}}^p=\sqrt{\frac{3}{2}}\Delta p\boldsymbol{n}$

It is easy to check that this new statement of the evolution of the local variables $\boldsymbol{\alpha}_i$ cost to the preceding statement if $\delta_i=1$, or in the event of **radial situation** , where one can pose $\boldsymbol{\alpha}_i=\xi\boldsymbol{n}$.

It comes then: $\dot{\boldsymbol{\alpha}}_i=\dot{\boldsymbol{\varepsilon}}^p-\gamma_i\dot{p}\left(\delta_i\xi\boldsymbol{n}+(1-\delta_i)\xi\boldsymbol{n}\right)=\dot{\boldsymbol{\varepsilon}}^p-\gamma_i\dot{p}\boldsymbol{\alpha}_i$.

3 Integration of the behavior models

to carry out the integration of the constitutive law numerically, one carries out a discretization in time and one adopts a diagram of implicit, famous Eulerian adapted for elastoplastic behavior models. Henceforth, the following notations will be employed: A^- , A and ΔA represent respectively the values of a quantity at the beginning and at the end of time step considered thus that its increment during the step. The problem is then the following: knowing the state at time t^- as well as the increments of strain $\Delta\boldsymbol{\varepsilon}$ (resulting from the phase of prediction (cf documentation of reference of STAT_NON_LINE [R5.03.01])) and of temperature ΔT , to determine the state of the local variables at time t as well as the stresses $\boldsymbol{\sigma}$.

One compared to the takes into account the variations of the characteristics temperature by noticing that:

$$\sigma^H=\frac{K}{K^-}\sigma^{H^-}+K\,\mathrm{tr}\left(\Delta\varepsilon-\Delta\varepsilon^{\mathbf{th}}\right) \quad \textbf{éq 2.2-1}$$

$$\tilde{\sigma}=\frac{\mu}{\mu^-}\tilde{\sigma}^-+2\mu\left(\Delta\tilde{\varepsilon}-\Delta\varepsilon^p\right)=\tilde{\sigma}^\varepsilon-2\mu\Delta\varepsilon^p \quad \textbf{éq 2.2-2}$$

with

$$\tilde{\sigma}^\varepsilon=\frac{\mu}{\mu^-}\tilde{\sigma}^-+2\mu\Delta\tilde{\varepsilon}$$

Within sight of the equation [éq 2.2-1], one notes that the hydrostatic behavior is purely elastic if K is constant. Only the processing of the deviatoric component is delicate.

In the absence of viscous term, the relation of discretized coherence is:

Elastic mode: $F\le 0$ and $\Delta p=0$
plastic Mode: $F=0$ and $\Delta p\ge 0$

On the other hand, in the presence of viscosity, the condition of coherence is replaced by the equation [éq 2.1 - 10] which, discretized, is written:

$$\frac{\Delta p}{\Delta t}=\left(\frac{\langle F\rangle}{K}\right)^N\Leftrightarrow\langle F\rangle=K\left(\frac{\Delta p}{\Delta t}\right)^{1/N}$$

In other words, while posing:

$$\tilde{F}=F-K\left(\frac{\Delta p}{\Delta t}\right)^{1/N}$$

the viscoplastic increment of cumulated strain is determined by:

$$\begin{array}{rl}\text{Régime élastique :} & \tilde{F}\le 0 \text{ et } \Delta p=0\\ \text{Régime viscoplastique :} & \tilde{F}=0 \text{ et } \Delta p\ge 0\end{array}$$ **éq 2.2-3**

Finally, by adopting an implicit, the only difference between the constitutive laws figure and viscoplastic discretization resides in the form of the loading function F : one observes a complementary term in the event of viscosity there. In fact, incremental plasticity appears as the borderline case of incremental viscoplasticity when K tends towards zero. This convergence was already described by J.L. Chaboche and G. Cailletaud in [bib3].

In the continuation of this paragraph, one will thus detail the integration of the viscoplastic model. To find the case of the plastic behavior, it is enough to take $K=0$ in the equations below (it is pointed out that the user to place itself in this case must obligatorily remove key word LEMAITRE or LEMAITRE_FO of the command DEFI_MATERIAU).

$$\tilde{\sigma}-X_1-X_2=\tilde{\sigma}^e-\frac{2}{3}C_1\alpha_1^- -\frac{2}{3}C_2\alpha_2^- -2\mu\Delta\varepsilon^p-\frac{2}{3}\left(C_1\Delta\alpha_1+C_2\Delta\alpha_2\right)$$

Equations of flow [éq 2.1-6] and [éq 2.1-7], once discretized, and the condition of coherence [éq 2.2-3] are written (by noticing that $p=\lambda$):

$$\Delta\varepsilon^p=\frac{3}{2}\Delta p\frac{\tilde{\sigma}^e-\frac{2}{3}C_1\alpha_1^- -\frac{2}{3}C_2\alpha_2^- -2\mu\Delta\varepsilon^p-\frac{2}{3}C_1\Delta\alpha_1-\frac{2}{3}C_2\Delta\alpha_2}{\left(\tilde{\sigma}^e-\frac{2}{3}C_1\alpha_1^- -\frac{2}{3}C_2\alpha_2^- -2\mu\Delta\varepsilon^p-\frac{2}{3}C_1\Delta\alpha_1-\frac{2}{3}C_2\Delta\alpha_2\right)_{eq}}$$ **2.2-4**

$$\tilde{F}\le 0 \quad \Delta p\ge 0 \quad \tilde{F} \quad \Delta p=0$$ **éq 2.2-5**

the processing of the condition of coherence (preceding equation) is classical. One starts with test elastic ($\Delta p=0$) which is well the solution if the plasticity criterion is not exceeded, i.e. if:

$$\left(\tilde{\sigma}^e-\frac{2}{3}C_1\left(p^-\right)\alpha_1^- -\frac{2}{3}C_2\left(p^-\right)\alpha_2^-\right)_{eq}-R\left(p^-\right)<0$$ **2.2-6**

In the contrary case, the solution is plastic ($\Delta p>0$) and the condition of coherence is reduced to $\tilde{F}=0$. To solve it, it is shown that one can bring back oneself to a scalar problem while expressing $\Delta\varepsilon^p$ and $\Delta\alpha_1,\Delta\alpha_2$ according to Δp . By gathering the equations of the problem resulting from the implicit discretization, one obtains the system of equations:

$$\left(\tilde{\sigma}^e-\frac{2}{3}C_1\alpha_1^- -\frac{2}{3}C_2\alpha_2^- -2\mu\Delta\varepsilon^p-\frac{2}{3}C_1\Delta\alpha_1-\frac{2}{3}C_2\Delta\alpha_2\right)_{eq}=R(p)+K\left(\frac{\Delta p}{\Delta t}\right)^{1/N} \quad \textbf{éq 2.2-7}$$

$$\Delta\varepsilon^p=\frac{3}{2}\Delta p\frac{\tilde{\sigma}^e-\frac{2}{3}C_1\alpha_1^- -\frac{2}{3}C_2\alpha_2^- -2\mu\Delta\varepsilon^p-\frac{2}{3}C_1\Delta\alpha_1-\frac{2}{3}C_2\Delta\alpha_2}{R(p)+K\left(\frac{\Delta p}{\Delta t}\right)^{1/N}} \quad \textbf{éq 2.2-8}$$

$$\begin{aligned}\Delta\alpha_1&=\Delta\varepsilon^p-\gamma_1\alpha_1\Delta p\\ \Delta\alpha_2&=\Delta\varepsilon^p-\gamma_2\alpha_2\Delta p\end{aligned} \quad \textbf{éq 2.2-9}$$

In this writing, it should well be noted that $p=p^-+\Delta p$ and $\alpha_i=\alpha_{i^-}+\Delta\alpha_i$ that C_i,γ_i are functions of p. By considering the three last equations, this linear system in $\Delta\varepsilon^p$ and $\Delta\alpha_i$ can be solved to express these quantities according to Δp. Indeed, it is equivalent to:

$$\Delta\varepsilon^p\left(R(p)+3\mu\Delta p+K\left(\frac{\Delta p}{\Delta t}\right)^{1/N}\right)=\Delta p\left(\frac{3}{2}\tilde{\sigma}^e-C_1\alpha_1^- -C_2\alpha_2^- -C_1\Delta\alpha_1-C_2\Delta\alpha_2\right) \quad \textbf{éq 2.2-10}$$

$$\begin{aligned}\Delta\alpha_1(1+\gamma_1\Delta p)&=\Delta\varepsilon^p-\gamma_1\alpha_1^-\Delta p\\ \Delta\alpha_2(1+\gamma_2\Delta p)&=\Delta\varepsilon^p-\gamma_2\alpha_2^- Dp\end{aligned} \quad \textbf{éq 2.2-11}$$

While calculating $C_1\Delta\alpha_1$ and $C_2\Delta\alpha_2$ by replacing them in the statement of $\Delta\varepsilon^p$ one obtains a statement of $\Delta\varepsilon^p$ according to Δp only:

$$\begin{aligned}C_1\Delta\alpha_1&=\left(\frac{C_1}{1+\gamma_1\Delta p}\right)\Delta\varepsilon^p-\left(\frac{C_1\gamma_1\alpha_1^-\Delta p}{1+\gamma_1\Delta p}\right)=M_1(p)\Delta\varepsilon^p-M_1(p)\gamma_1\Delta p\,\alpha_1^-\\ C_2\Delta\alpha_2&=\left(\frac{C_2}{1+\gamma_2\Delta p}\right)\Delta\varepsilon^p-\left(\frac{C_2\gamma_2\alpha_2^-\Delta p}{1+\gamma_2\Delta p}\right)=M_2(p)\Delta\varepsilon^p-M_2(p)\gamma_2\Delta p\,\alpha_2^-\\ &\text{avec } M_i(p)=\frac{C_i(p)}{1+\gamma_i(p)\Delta p}\end{aligned} \quad \textbf{2.2-12}$$

By deferring this statement in the statement of $\Delta\varepsilon^p$ one finds:

$$\Delta\varepsilon^p=\frac{1}{\left(R(p)+(3\mu+M_1+M_2)\Delta p+K\left(\frac{\Delta p}{\Delta t}\right)^{1/N}\right)}\left(\frac{3}{2}\Delta p\,\tilde{\sigma}^e-\Delta p\left((C_1-M_1\gamma_1\Delta p)\,\alpha_1^-+(C_2-M_2\gamma_2\Delta p)\,\alpha_2^-\right)\right)$$

what is simplified in:

$$\Delta\varepsilon^p=\frac{1}{D(p)}\left(\frac{3}{2}\Delta p\,\tilde{\sigma}^e-\Delta p\left(M_1\alpha_1^-+M_2\alpha_2^-\right)\right)$$ **éq 2.2-13**

with:

$$D(p)=R(p)+\left(3\mu+M_1(p)+M_2(p)\right)\Delta p+K\left(\frac{\Delta p}{\Delta t}\right)^{1/N}$$

It now only remains to replace $\Delta\varepsilon^p$ in the statements of $C_1\Delta\alpha_1$ and $C_2\Delta\alpha_2$ to express this term according to Δp by:

$$C_1\Delta\alpha_1=\frac{M_1}{D}\left(\frac{3}{2}\Delta p\,\tilde{\sigma}^e-\Delta p\left(M_1\alpha_1^-+M_2\alpha_2^-\right)\right)-M_1\gamma_1\Delta p\,\alpha_1^-$$
$$C_2\Delta\alpha_2=\frac{M_2}{D}\left(\frac{3}{2}\Delta p\,\tilde{\sigma}^e-\Delta p\left(M_1\alpha_1^-+M_2\alpha_2^-\right)\right)-M_2\gamma_2\Delta p\,\alpha_2^-$$

then to substitute the statement obtained thus that $\Delta\varepsilon^p$ according to Δp in the equation $\tilde{F}=0$, and one obtains a scalar equation in Δp to solve, namely:

$$\tilde{F}(p)=\left(\tilde{\sigma}^e-\frac{2}{3}C_1\alpha_1^--\frac{2}{3}C_2\alpha_2^--2\mu\Delta\varepsilon^p-\frac{2}{3}C_1\Delta\alpha_1-\frac{2}{3}C_2\Delta\alpha_2\right)_{eq}-R(p)-K\left(\frac{\Delta p}{\Delta t}\right)^{1/N}=0$$

what is simplified in:

$$\tilde{F}(p)=\frac{R(p)+K\left(\frac{\Delta p}{\Delta t}\right)^{1/N}}{D(p)}\left(\tilde{\sigma}^e-\frac{2}{3}M_1\alpha_1^--\frac{2}{3}M_2\alpha_2^-\right)_{eq}-R(p)-K\left(\frac{\Delta p}{\Delta t}\right)^{1/N}=0$$ **éq 2.2-14**

This scalar equation in Δp is solved numerically, by a research method of zero of function (method of secants which one briefly describes in the appendix 2).

It is normalized in the following way:

$$\tilde{\tilde{F}}(p)=1-\frac{D(p)}{\left(\tilde{\sigma}^e-\frac{2}{3}M_1\alpha_1^--\frac{2}{3}M_2\alpha_2^-\right)_{eq}}=0$$ 2.2-15

Once determined Δp, one can calculate $\Delta\varepsilon^p$ using the equation [éq 2.2-13] then $\Delta\alpha_1$ and $\Delta\alpha_2$ using the equations [éq 2.2-11]. It any more but does not remain to calculate the tensor of the stresses, by the equations [éq 2.2-1] and [éq 2.2-2], and to bring up to date the local variables α_1 and α_2.

Note:

- *an interesting borderline case (for the validation of this model) arises while posing $\gamma_i=0$. One finds oneself then exactly in the situation of linear kinematic hardening (if $R(p)=\sigma_y$, [R5.03.02]) or of mixed hardening for $R(p)$ unspecified (cf [R5.03.16]),*
- *these models are also available in plane stresses, by a global method (static condensation due to R. of Borst) [R5.03.03].*

3.1 Integration of the terms taking into account it not radiality

the discretization leads to: $\Delta\boldsymbol{\alpha}_i=\Delta\boldsymbol{\varepsilon}^p-\gamma_i\Delta p\left[\delta_i(\boldsymbol{\alpha}_i^-+\Delta\boldsymbol{\alpha}_i)+(1-\delta_i)((\boldsymbol{\alpha}_i^-+\Delta\boldsymbol{\alpha}_i):\boldsymbol{n})\boldsymbol{n}\right]$

Let us calculate

$$\Delta\boldsymbol{\alpha}_i:\boldsymbol{n}=\sqrt{\frac{3}{2}}\Delta p-\gamma_i\Delta p\left(\delta_i\sqrt{\frac{3}{2}}\beta_i+\delta_i\Delta\boldsymbol{\alpha}_i:\boldsymbol{n}+(1-\delta_i)\sqrt{\frac{3}{2}}\beta_i+(1-\delta_i)(\Delta\boldsymbol{\alpha}_i:\boldsymbol{n})\right)$$

while having posed $\boldsymbol{\alpha}_i^-:\boldsymbol{n}=\sqrt{\frac{3}{2}}\beta_i$. O N can thus express $\Delta\boldsymbol{\alpha}_i:\boldsymbol{n}$ according to Δp and β_i

$\Delta\boldsymbol{\alpha}_i:\boldsymbol{n}(1+\gamma_i\Delta p)=\sqrt{\frac{3}{2}}\Delta p(1-\gamma_i\beta_i)$ is $\Delta\boldsymbol{\alpha}_i:\boldsymbol{n}=\dfrac{\sqrt{\frac{3}{2}}\Delta p(1-\gamma_i\beta_i)}{(1+\gamma_i\Delta p)}$

One can thus express $\Delta\boldsymbol{\alpha}_i$ only according to Δp and $\beta_i=\sqrt{\frac{2}{3}}\boldsymbol{\alpha}_i^-:\boldsymbol{n}$ and propagate these modifications in the method of resolution used previously:

$$\Delta\boldsymbol{\alpha}_i(1+\gamma_i\delta_i\Delta p)=\Delta\boldsymbol{\varepsilon}^p-\gamma_i\Delta p\,\delta_i\boldsymbol{\alpha}_i^--\gamma_i\Delta p(1-\delta_i)(\boldsymbol{\alpha}_i^-:\boldsymbol{n})\boldsymbol{n}-\gamma_i\Delta p(1-\delta_i)(\Delta\boldsymbol{\alpha}_i:\boldsymbol{n})\boldsymbol{n}$$

By means of the statement of $\Delta\boldsymbol{\alpha}_i:\boldsymbol{n}$ according to Δp and β_i ,

$$\Delta\boldsymbol{\alpha}_i(1+\gamma_i\delta_i\Delta p)=\Delta\boldsymbol{\varepsilon}^p-\gamma_i\Delta p\,\delta_i\boldsymbol{\alpha}_i^--\gamma_i\Delta p(1-\delta_i)\sqrt{\frac{3}{2}}\beta_i\boldsymbol{n}-\gamma_i\Delta p(1-\delta_i)\frac{\sqrt{\frac{3}{2}}\Delta p(1-\gamma_i\beta_i)}{(1+\gamma_i\Delta p)}\boldsymbol{n}$$

$$\Delta\boldsymbol{\alpha}_i(1+\gamma_i\delta_i\Delta p)=\Delta\boldsymbol{\varepsilon}^p-\gamma_i\Delta p\,\delta_i\boldsymbol{\alpha}_i^--\gamma_i(1-\delta_i)\frac{\beta_i+\Delta p}{1+\gamma_i\Delta p}\Delta\boldsymbol{\varepsilon}^p$$

$\Delta\boldsymbol{\alpha}_i(1+\gamma_i\delta_i\Delta p)=\Delta\boldsymbol{\varepsilon}^p N_i(\Delta p,\beta_i)-\gamma_i\Delta p\,\delta_i\boldsymbol{\alpha}_i^-$ with

$$N_i(\Delta p,\beta_i)=\frac{1+\gamma_i\Delta p\,\delta_i-\gamma_i(1-\delta_i)\beta_i}{1+\gamma_i\Delta p}$$

There still, one can check that if $\delta_i=1$, one finds the equations without effect of nonradiality.

To continue to solve, it is necessary to calculate:

$C_i\Delta\boldsymbol{\alpha}_i=M_i N_i\Delta\boldsymbol{\varepsilon}^p-\gamma_i\Delta p\,\delta_i M_i\boldsymbol{\alpha}_i^-$ avecformule $M_i=\dfrac{C_i}{(1+\gamma_i\delta_i\Delta p)}$

S I although the computation of the increase in plastic strain is similar to the classical case:

$$\Delta\boldsymbol{\varepsilon}^p=\sqrt{\frac{3}{2}}\Delta p\,\boldsymbol{n} \text{ with } \boldsymbol{n}=\sqrt{\frac{3}{2}}\frac{\tilde{\boldsymbol{\sigma}}-\frac{2}{3}C_1\boldsymbol{\alpha}_1-\frac{2}{3}C_2\boldsymbol{\alpha}_2}{\left(\tilde{\boldsymbol{\sigma}}-\frac{2}{3}C_1\boldsymbol{\alpha}_1-\frac{2}{3}C_2\boldsymbol{\alpha}_2\right)_{eq}}.$$

By means of statements calculated previously as well as the statement of the criterion:

$$\left(\tilde{\sigma}^e-\frac{2}{3}C_1\alpha_1^- -\frac{2}{3}C_2\alpha_2^- -2\mu\Delta\varepsilon^p-\frac{2}{3}C_1\Delta\alpha_1-\frac{2}{3}C_2\Delta\alpha_2\right)_{eq}=R(p)+K\left(\frac{\Delta p}{\Delta t}\right)^{1/N}$$ it comes:

$$\Delta\boldsymbol{\varepsilon}^p\left(R(p)+3K\left(\frac{\Delta p}{\Delta t}\right)^{1/N}+\Delta p(3\mu+M_1N_1+M_2N_2)\right)=\frac{3}{2}\Delta p\left(\tilde{\boldsymbol{\sigma}}^e-\frac{2}{3}M_1\boldsymbol{\alpha}_1^- -\frac{2}{3}M_2\boldsymbol{\alpha}_2^-\right)$$

thus $\boldsymbol{n}=\sqrt{\frac{3}{2}}\dfrac{\tilde{\boldsymbol{\sigma}}^e-\frac{2}{3}M_1\boldsymbol{\alpha}_1^- -\frac{2}{3}M_2\boldsymbol{\alpha}_2^-}{D}$ with

$$D(\Delta p;\beta_1;\beta_2)=R(p)+K\left(\frac{\Delta p}{\Delta t}\right)^{1/N}+\Delta p(3\mu+M_1N_1+M_2N_2)$$

Note: : L with still, O N can check that if one does not take account of the nonradial effect $\delta_i=1$, which involves $N=1$. One finds well the classical statement of the norm $\boldsymbol{n}$.

In this case; there are 3 scalar unknowns: Δp β_1 β_2 . In fact, it is possible to express β_1 and β_2 according to Δp by noticing that :

$\boldsymbol{n}=\sqrt{(\frac{3}{2})}\dfrac{\tilde{\boldsymbol{\sigma}}^e-\frac{2}{3}M_1\boldsymbol{\alpha}_1^- -\frac{2}{3}M_2\boldsymbol{\alpha}_2^-}{\left(\tilde{\boldsymbol{\sigma}}^e-\frac{2}{3}M_1\boldsymbol{\alpha}_1^- -\frac{2}{3}M_2\boldsymbol{\alpha}_2^-\right)_{eq}}$. One can thus determine $\boldsymbol{n}$ according to Δp only,

then to calculate directly $\beta_i=\sqrt{\frac{2}{3}}\boldsymbol{\alpha}_i^-:\boldsymbol{n}$, which becomes explicit functions then of Δp . To solve, it is enough to replace the statements above in the criterion (what amounts writing $\boldsymbol{n}:\boldsymbol{n}=1$):

$$\tilde{\hat{F}}(p)=\left(\tilde{\boldsymbol{\sigma}}^e-\frac{2}{3}M_1\boldsymbol{\alpha}_1^- -\frac{2}{3}M_2\boldsymbol{\alpha}_2^-\right)_{eq}-D(\Delta p;\beta_1(\Delta p);\beta_2(\Delta p))=0$$

3.2 Integration of the effect of memory

In the case of the effect of memory, the function $R(p)$ is not known any more explicitly, but via the system of equations:

1 eq : $f(\varepsilon^p,\xi,q)=\frac{2}{3}J_2(\varepsilon^p-\xi)-q=\frac{2}{3}\sqrt{\frac{3}{2}(\varepsilon^p-\xi):(\varepsilon^p-\xi)}-q\le 0$

6 eq: $\Delta\xi=(1-\eta)H(F)\langle\mathbf{n}:\mathbf{n}^*\rangle\Delta p\,n^*=(1-\eta)\frac{\Delta q}{\eta}n^*$

With

$\Delta R=b(Q-R)\Delta p$ $\quad Q=Q_0+(Q_m-Q_0)\left(1-e^{-2\mu(q^-+\Delta q)}\right)$ $\quad n^*=\frac{3}{2}\frac{\varepsilon^p-\xi}{J_2(\varepsilon^p-\xi)}$

Knowing Δp , one starts by calculating $f(\varepsilon^p,\xi^-,q^-)$.

If this quantity is negative, then the solution of the system managing the effect of memory is: $\Delta q=0, \Delta\xi=0$.

In the contrary case, knowing Δp , it is necessary to find Δq and $\Delta\xi$ such as:

$$f(\varepsilon^p,\xi,q)=\frac{2}{3}J_2(\varepsilon^p-\xi^- -\Delta\xi)-q^- -\Delta q=0$$

$$\Delta\xi=\frac{(1-\eta)}{\eta}\Delta q\, n^*=\frac{(1-\eta)}{\eta}\Delta q\frac{3}{2}\frac{\varepsilon^{p-}+\Delta\varepsilon^p-\xi^- -\Delta\xi}{\frac{3}{2}(q^- +\Delta q)}$$

Because $\Delta\boldsymbol{\varepsilon}^{\mathbf{p}}=\frac{1}{D(p)}\left(\frac{3}{2}\Delta p\,\tilde{\boldsymbol{\sigma}}^e-\Delta p\left(M_1\boldsymbol{\alpha}_1^- +M_2\,\boldsymbol{\alpha}_2^-\right)\right)$ can be calculated explicitly from Δp .

It remains:

$$\Delta\xi\left(1+\frac{(1-\eta)\Delta q}{\eta(q^- +\Delta q)}\right)=\frac{(1-\eta)}{\eta}\Delta q\frac{\varepsilon^{p-}+\Delta\varepsilon^p-\xi^-}{(q^- +\Delta q)}\Rightarrow$$

$$\Delta\xi(\eta q^- +\Delta q)=(1-\eta)\Delta q(\varepsilon^p-\xi^-)\Rightarrow\Delta\xi=\frac{(1-\eta)\Delta q(\varepsilon^p-\xi^-)}{\eta q^- +\Delta q}$$

while deferring in the equation of surface threshold: $f(\varepsilon^p,\xi,q)=0$

$$\frac{2}{3}J_2(\varepsilon^p-\xi^- -\Delta\xi)-q^- -\Delta q=0=\frac{2}{3}J_2(\varepsilon^p-\xi^-)\left|\left(1-\frac{(1-\eta)\Delta q}{\eta q^- +\Delta q}\right)\right|-q^- -\Delta q=0$$

$$\Leftrightarrow\frac{2}{3}J_2(\varepsilon^p-\xi^-)|\eta(q^- +\Delta q)|-(q^- +\Delta q)(\eta q^- +\Delta q)=0\ \ si\ \ \eta q^- +\Delta q>0$$

what makes it possible to calculate explicitly Δq from Δp :

$$\boxed{\Delta q=\eta\frac{2}{3}J_2(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi}^-)-\eta q^-}$$

It then remains to modify the function of isotropic hardening while calculating:

$$Q=Q_0+(Q_m-Q_0)\left(1-e^{-2\mu(q^- +\Delta q)}\right)\qquad\text{then }\ \Delta R=b(Q-R)\Delta p$$

One can thus use the resolution of the scalar equation in Δp (éq 2.2-14) by means of the statements above.

Note:

•In [bib2] one also finds the statement *:* $dq=\eta H(f)\langle\mathbf{n}:\mathbf{n}^*\rangle dp$.
This last equation results from the statement of velocity of the multiplier. In the implicit discretization carried out here, it is not used for the resolution (since then the system would comprise more equations than unknowns). Moreover, the 3 equations given in [bib2] are redundant: indeed, knowing $\Delta\varepsilon^p$ it is necessary to determine a tensorial variable $\Delta\xi$ and a scalar variable. Δq However we have a tensorial equation and two scalar equations.
This is due to the fact that the equation $dq=\eta H(f)\langle\mathbf{n}:\mathbf{n}^*\rangle dp$ is resulting from the condition of coherence $df=0$ (what is specified in [bib2]) but is not used for the implicit resolution of the

problème.formule

$$df\left(\varepsilon^p,\xi,q\right)=\frac{\varepsilon^p-\xi}{J_2\left(\varepsilon^p-\xi\right)}d\,\varepsilon^p-\frac{\varepsilon^p-\xi}{J_2\left(\varepsilon^p-\xi\right)}d\,\xi-dq=n:n^*dp-n^*:n^*dq-dq=n:n^*dp-2\,dq=0$$

would be useful for an explicit resolution, by expressing derivatives compared to the time of all the sought variables.

•an interesting criterion, given in [bib2] makes it possible to adjust the parameters of the effect of memory. Indeed, by considering a simple loading of traction and compression, one must find (while $q=\frac{1}{2}\Delta\varepsilon^{p_{\max}}$ choosing) $\eta=\frac{1}{2}$. For a material point in uniaxial load, the fields (uniform) have as components: formuleDans

$$\boldsymbol{\sigma}=\sigma\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix}\quad\boldsymbol{\varepsilon}^p=p\begin{pmatrix}1&0&0\\0&-\frac{1}{2}&0\\0&0&-\frac{1}{2}\end{pmatrix}$$

this case, at the time of the first uniaxial load in the direction: x formuleDans

$$\xi^-=0$$
$$q^-=0$$
$$\Delta q=\eta\,\varepsilon_x^p$$

this case, $q=\frac{1}{2}\Delta\varepsilon^{p_{\max}}$ implies that and

$\eta=\frac{1}{2}$ formuleDe $\Delta\xi=\frac{1}{2}\left(\varepsilon^p\right)$

more, in the case of a cycle of symmetric tension compression (in plastic strain), one obtains, during the first symmetric discharge (with) $\eta=\frac{1}{2}$: formuleformulece

$$\xi^-=\frac{1}{2}\varepsilon^{p_{\max}}$$
$$q^-=\frac{1}{2}\varepsilon_{xx}^{p_{\max}}$$
$$\Delta q=\eta\left(\frac{2}{3}J_2\left(\varepsilon^p\right)-q^-\right)=\eta\left(\left|\varepsilon_{xx}^{p_{\min}}-\xi^-\right|-\frac{1}{2}\varepsilon_{xx}^{p_{\max}}\right)=\frac{1}{2}\left|\varepsilon_{xx}^{p_{\min}}\right|$$
$$q=q^-+\Delta q=\varepsilon_{xx\max}^p=\frac{1}{2}\Delta\varepsilon_{xx}^p$$
$$\Delta\xi=\frac{\left(1-\eta\right)\Delta q\left(\varepsilon^p-\xi^-\right)}{\eta\,q^-+\Delta q}=-\frac{1}{2}\Delta\varepsilon_{xx\max}^p$$
$$\xi=\xi^0+\Delta\xi=0$$

which corresponds well to result waited (cf [bib2]): field centered $F=0$ on the origin, and of radius the half-amplitude of plastic strain. Computation

3.3 of the tangent stiffness In order to

allow a resolution of the total problem (balance equations) by a method of Newton [R5.03.01], it is necessary to determine the coherent tangent matrix of the incremental problem. This

matrix is composed classically of an elastic contribution and a plastic contribution: éq

$$\frac{\delta\sigma}{\delta\varepsilon}=\frac{\delta\sigma^e}{\delta\varepsilon}-2\mu\frac{\delta\Delta\varepsilon^p}{\delta\varepsilon}$$ **2.3-1 with**

, $\sigma^e=\sigma+2\mu\Delta\varepsilon^p$ which gives again in particular One $\tilde{\sigma}^e=\frac{\mu}{\mu^-}\tilde{\sigma}^-+2\mu\Delta\tilde{\varepsilon}$

from of deduced immediately that in elastic mode (classical or pseudo-discharge), the tangent matrix is reduced to the elastic matrix: éq

$$\frac{\delta\sigma}{\delta\varepsilon}=\frac{\delta\sigma^e}{\delta\varepsilon}$$ **2.3-2 For**

that, one once more adopts the convention of writing of the symmetric tensors of order 2 in the form of vectors with 6 components. Thus, for a tensor: a éq

$$a={}^t\left[a_{xx}\quad a_{yy}\quad a_{zz}\;\sqrt{2}a_{xy}\quad\sqrt{2}a_{xz}\quad\sqrt{2}a_{yz}\right]$$ **2.3-3 If**

one introduces moreover the hydrostatic vector and 1 the matrix of deviatoric projection: P éq

$$1={}^t\left[1\quad 1\quad 1\,0\quad 0\quad 0\right]$$ **2.3-4 éq**

$$P=\mathbf{Id}-\frac{1}{3}1\otimes 1$$ **2.3-5 where**

is $\otimes$ the tensor product Then

the coherent tangent stiffness matrix is written for an elastic behavior: éq

$$\frac{\partial\sigma^e}{\partial\Delta\varepsilon}=K\,1\otimes 1+2\mu\,P$$ **2.3-6 On the other hand**

, in plastic mode, the variation of the plastic strain is not null any more. One

derives compared to, $\tilde{\sigma}^e$ knowing that one a: formuleéq

$$\frac{\delta\Delta\varepsilon^p}{\delta\varepsilon}=\frac{\delta\Delta\varepsilon^p}{\delta\tilde{\sigma}^e}.\frac{\delta\tilde{\sigma}^e}{\delta\varepsilon}=2\mu\frac{\delta\Delta\varepsilon^p}{\delta\tilde{\sigma}^e}.P$$ **2.3-7 space**

s of the symmetric tensors projector
P on the deviators To compute:

, $\frac{\delta\Delta\varepsilon^p}{\delta\tilde{\sigma}^e}$ one uses the statement of according to $\Delta\varepsilon^p$ and $\tilde{\sigma}_e$: p formulate

$$\Delta \varepsilon^p = \frac{1}{D(p)}\left(\frac{3}{2}\Delta p\,\tilde{\sigma}_e - \Delta p\left(M_1\alpha_1^- + M_2\alpha_2^-\right)\right)$$

is written in the form: thus

$$\Delta \varepsilon^p = A(p)\,\tilde{\sigma}_e + B_1(p)\alpha_1^- + B_2(p)\alpha_2^-$$

:

$$\frac{\delta \Delta \varepsilon^p}{\delta \tilde{\sigma}^e} = A(p)\,\mathbf{Id} + \tilde{\sigma}^e \otimes \frac{\delta A(p)}{\delta \tilde{\sigma}^e} + \frac{\delta B_1(p)}{\delta \tilde{\sigma}^e}\otimes \alpha_1^- + \frac{\delta B_2(p)}{\delta \tilde{\sigma}^e}\otimes \alpha_2^-$$

The quantities of the type $\frac{\delta A(p)}{\delta \tilde{\sigma}^e}$ are calculated using: do not formulate $\frac{\delta A(p)}{\delta \tilde{\sigma}^e} = \frac{\delta A(p)}{\delta p}\frac{\delta p}{\delta \tilde{\sigma}^e}$

, it but does not remain any more to calculate the variation of: p One $\frac{\delta p}{\delta \tilde{\sigma}^e}$

uses for that: formuleéq $\tilde{F}(p, \tilde{\sigma}^e) = 0$

$$\tilde{F}(p, \tilde{\sigma}_e) = \frac{R(p) + K\left(\frac{\Delta p}{\Delta t}\right)^{1/N}}{D(p)}\left(\tilde{\sigma}^e - \frac{2}{3}M_1\alpha_1^- - \frac{2}{3}M_2\alpha_2^-\right)_{eq} - R(p) - K\left(\frac{\Delta p}{\Delta t}\right)^{1/N} = 0$$

$$\tilde{F}_{,p}(p, \tilde{\sigma}^e)\,\delta p = -\tilde{F}_{,\tilde{\sigma}^e}(p, \tilde{\sigma}^e)\,\delta \tilde{\sigma}^e \Rightarrow \frac{\delta p}{\delta \tilde{\sigma}^e} = -\frac{\tilde{F}_{,\tilde{\sigma}^e}(p, \tilde{\sigma}^e)}{\tilde{F}_{,p}(p, \tilde{\sigma}^e)} \qquad \textbf{2.3-8}$$

the detail of computations is given in appendix 1.

The initial tangent matrix, used by option RIGI `_MECA_TANG` is obtained by adopting the behavior of the preceding step (elastic or plastic, meant by local variable being worth ξ 0 or 1) and while making tend towards Δp zero in the preceding equations. Meaning

3.4 of the local variables

the local variables of the two models to Gauss points (VELGA) `are`: V1

- = : p cumulated plastic strain (positive or null) V2
- = : ξ being worth (nombre of iterations n interns) if the Gauss point plasticized during the increment or 0 if not.

The following local variables are, for the modelization 3D : `For`

- the model VMIS `/VISC_CIN1_CHAB V3`
  - `= V4` α_{1xx}
  - `= V5` α_{1yy}
  - `= V6` α_{1zz}
  - `= V7` α_{1xy}
  - `= V8` α_{1xz}

   - = For $\alpha_{1\,yz}$

- the model VMIS /VISC_CIN2_CHAB V3
   - = V4 $\alpha_{1\,xx}$
   - = V5 $\alpha_{1\,yy}$
   - = V6 $\alpha_{1\,zz}$
   - = V7 $\alpha_{1\,xy}$
   - = V8 $\alpha_{1\,xz}$
   - = V9 $\alpha_{1\,yz}$
   - = V10 $\alpha_{2\,xx}$
   - = V11 $\alpha_{2\,yy}$
   - = V12 $\alpha_{2\,zz}$
   - = V13 $\alpha_{2\,xy}$
   - = V14 $\alpha_{2\,xz}$
   - = For $\alpha_{2\,yz}$
   -

  modelizations C_PLAN , D_PLAN , and AXIS : V7

   - = 0 V8
   - = 0 V13
   - = 0 V14
   - = 0 For

- the model VMIS /VISC_CIN2_MEMO V3
   - = V4 $\alpha_{1\,xx}$
   - = V5 $\alpha_{1\,yy}$
   - = V6 $\alpha_{1\,zz}$
   - = V7 $\alpha_{1\,xy}$
   - = V8 $\alpha_{1\,xz}$
   - = V9 $\alpha_{1\,yz}$
   - = V10 $\alpha_{2\,xx}$
   - = V11 $\alpha_{2\,yy}$
   - = V12 $\alpha_{2\,zz}$
   - = V13 $\alpha_{2\,xy}$
   - = V14 $\alpha_{2\,xz}$
   - = V15 $\alpha_{2\,yz}$
   - = V16 $R(p)$
   - = V17 q
   - = V18 ξ_{xx}
   - = V19 ξ_{yy}
   - = V20 ξ_{zz}
   - = V21 ξ_{xy}

- = V22 ξ_{xz}
- = V23 ξ_{yz}
-
- = V24 $\varepsilon^{p}{}_{xx}$
- = V25 $\varepsilon^{p}{}_{yy}$
- = V26 $\varepsilon^{p}{}_{zz}$
- = V27 $\varepsilon^{p}{}_{xy}$
- = V28 $\varepsilon^{p}{}_{xz}$
- = Principle $\varepsilon^{p}{}_{yz}$

4 of the identification of the parameters of the model. In

the simplest case (only one kinematical variable,) $\gamma_1 = cste, C_1 = cste \quad , R(p) = \sigma_y$ the coefficients of the model can γ_1, C_1 be identified on a uniaxial simple traction test, or of course a cyclic curve of hardening. Indeed

in the uniaxial case, the model is reduced in 1D to [biberon2]: formulate

$$dX_1 = C_1 d\varepsilon^p - \gamma_1 X_1 \xi d\varepsilon^p, \xi = \pm 1$$
$$|\sigma - X_1| = \sigma_y$$

one can integrate (in monotonic loading) in the following way: whose

$$X_1 = \xi \frac{C_1}{\gamma_1} + \left(X_1^0 - \xi \frac{C_1}{\gamma_1} \right) \exp\left(-\xi \gamma_1 \left(\varepsilon^p - \varepsilon_0^p \right) \right), \xi = \pm 1$$
$$\sigma = \xi \sigma_y + X_1$$

asymptote of curve of tension makes it possible to obtain by $\frac{C_1}{\gamma_1}$: formuleformule

$\varepsilon^p \to \infty \quad X_1 \to \xi \frac{C_1}{\gamma_1}$ formula $\sigma \to \xi \left(\sigma_y + \frac{C_1}{\gamma_1} \right)$

whose slope in the beginning provides (if C_1) $X_1^0 = 0$: formulate

$\varepsilon^p \to 0 \quad \dot{X}_1 \to C_1 - y_1 X_1^0 \xi \quad X_1^0 = C_1 - y_1 X_1 \xi$

a model has two kinematical variables, without isotropic hardening, a curve of tension still allows to find these relations: and

$\varepsilon^p \to \infty \ \sigma \to \xi \left(\sigma_y + \left(\frac{C_1}{\gamma_1} + \frac{C_2}{\gamma_2} \right) \right)$ the slope in the beginning is worth But $C_1 + C_2$

apart from these simple cases a numerical identification is necessary to obtain the parameters. One will be able to make this identification for example on traction tests compression with imposed strain. (cf 10 22Elements

5 of validation.

The tests allowing the elementary validation of these behaviors are: test

titrates	behavior		(S) elementary
	tests of robustness comp		
001f test	of robustness constitutive law 3D VMIS_CIN1_CHAB VMIS_CIN1_CHAB	comp	
001g test	of robustness constitutive law 3D VMIS_CIN2_CHAB VMIS_CIN2_CHAB	comp	
002b test	of robustness constitutive law 3D VISC_CIN1_CHAB VISC_CIN1_CHAB	comp	
002c test	of robustness constitutive law 3D VISC_CIN2_CHAB VISC_CIN2_CHAB	comp	
002h test	of robustness constitutive law 3D VISC_CIN2_MEMO VISC_CIN2_MEMO	comp	
008g variation	temperature in behavior VMIS_CIN1_CHAB VMIS_CIN1_CHAB	comp	
008h variation	temperature in behavior VMIS_CIN2_CHAB VMIS_CIN2_CHAB	comp	
008j variation	temperature in behavior VISC_CIN1_CHAB VISC_CIN1_CHAB	comp	
008k variation	temperature in behavior VISC_CIN2_CHAB VISC_CIN2_CHAB	comp	
008i variation	temperature in behavior VMIS_CIN2_MEMO VMIS_CIN2_MEMO	comp	
008l variation	temperature in thermoplastic behavior VISC_CIN2_MEMO	VISC_CIN2_MEMO	
	tests of the IPSI hsnv		
124c test	phi2as number 1 VMIS_CIN1_CHAB	VMIS_CIN2_CHAB	hsnv
124d test	phi2as number 1 VMIS_CIN1_CHAB	VMIS_CIN2_CHAB	hsnv
125c test	phi2as number 2: tension, shears, temperature variables VMIS_CIN1_CHAB	VMIS_CIN2_CHAB	hsnv
125e test	phi2as number 2: tension, shears, temperature variables VMIS_CIN2_MEMO	retiming	
	SNA		
109a model	VISC_CIN2_CHAB with 550 degrees, viscosity prevalent VISC_CIN2_CHAB	SNA	
110a model	retiming VISC_CIN2_CHAB on 4 curves of tension VISC_CIN2_CHAB	effect	
	of memory ssnd		
105a traction test	with maximum memory of hardening VMIS_CIN2_MEMO	ssnd	
105b traction test	with maximum memory of hardening VISC_CIN2_MEMO	ssnd	
105c tension	with maximum memory of hardening axis VISC_CIN2_MEMO	ssnd	
111a validation	effect of memory VISC_CIN2_MEMO VISC_CIN2_MEMO	Tension	
	- shears ssnv		
101b test	of tension-shears in plane stresses (chaboche) VMIS_CIN1_CHAB	VMIS_CIN2_CHAB	ssnv
101c test	of tension-shears 3D (chaboche) VMIS_CIN1_CHAB	VMIS_CIN2_CHAB	ssnv
101d test	of tension-shears in plane strains (chaboche) VMIS_CIN1_CHAB	VMIS_CIN2_CHAB	ssnv
118d traction test	shears in 3D (viscochab/VISC_CIN2_MEMO) VISC_CIN1_CHAB	VISC_CIN2_CHAB	large deformations
	ssnd		
107b tensions	- multiple rotations gdef _log in kinematical 3D VMIS_CIN2_CHAB	VMIS_CIN2_MEMO	Effect
	of nonproportionality ssnd		
105d traction test	with maximum memory of hardening and not radiality VMIS	_CIN2_NRAD VISC	_CIN2_NRAD ssnd
115a test	of tension-torsion with loading nonproportional VMIS	_CIN2_NRAD	

a validation compared to experimental results was carried out in (cf 10 22 on traction tests compression and of tension-torsion. It makes it possible to highlight the effect of memory and nonproportionality. Bibliography

6 P.

1 MIALON, Elements of analysis and numerical resolution of the relations of elastoplasticity. EDF - Bulletin of the Management of the Studies and Searches - Series C - N° 3 1986, p. 57 - 89. J.

2 LEMAITRE, J.L.CHABOCHE, Mechanics of the solid materials. Dunod 1996 J.L.CHABOCHE, G .CAILLETAUD

3 , constitutive Integration methods for complex equations, Methods Computer in Applied Machanics Engineering, N°133 (1996), pp 125-155 J.L.CHABOCHE, Cyclic viscoplastic constitutive

4 equations, Newspaper of Applied Mechanics, Vol.60, December 1993, pp. 813-828 R.FORTUNIER, Constitutive law of Chaboche

5 : identification of the plastic parameters élasto - and élasto-visco-plastics of steel EDF-SPH between 20°C and 600°C. Note FRAMATOME/NOVATOME, NOVTUDD90011, October 1990 C.MIGNE, Retiming of the parameters of the model

6 of kinematical plasticity nonlinear of SYSTUS. Modelization of the phenomenon of progressive strain with cyclic consolidation of the material. Note FRAMATOME EE/R.DC.0286. September 1992. J.J.ENGEL, G.ROUSSELIER, Behavior in uniaxial

7 stress under cyclic loading of the austenitic stainless steel 17-12 Mo with very low carbon and nitrogenizes control. Identification of 20)C with 600°C of a model of elastoplastic behavior to nonlinear kinematic hardening. Note EDF/DER/EMA N°D599 MAT/T43 (1985) P.GEYER , C.COUTEROT, characterization of steel

8 304L used during the tests “strain, progressive” on CUMULUS and identification of the parameters of the model of Chaboche, Note EDF/DER/HT-26/93/040/A R. OF BORST “the zero normal stress condition

9 in plane stress and Shell elastoplasticity” Communications in applied numerical methods, Flight 7,29-33 (1991) J.M.PROIX “Behavior viscoplastic taking into account

10 it not proportionality of loading” EDF R & D - CR-AMA12-284, 12/12/12 J.L.CHABOCHE, A review of sum plasticity and viscoplasticity constitutive

11 theories, Inter. Newspaper of Plasticity, Vol.24, 2008, pp. 1642-1693 Description of the versions of the document Version Aster Author (S) or contributor

7 (S), organization Description of

the modifications	**5 P.Schoenberger EDF/R & D /MMN initial Text**	**, model of Chaboche 7e.Lorentz**
	, J.M.Proix EDF/ R & D /AMA Addition of models	VMIS_CIN1_CHAB, VMIS_CIN2_CHAB
	8 P. of Bonnières , J.M.Proix EDF/ R & D /AMA	Addition of viscosity : `models VISC_CIN1_CHAB`
	and VISC_CIN2_CHAB, and removal	model CHABOCHE. 9.3 J.M.Proix EDF/R & D /AMA `Addition of model VMIS/VISC_CIN2_MEMO`, taking into account `the effect of`

	of	
memory	of maximum	hardening. 11 3 J.M.Proix EDF/R & D /AMA Addition of model VMIS/VISC_CIN2_NRAD, taking into account the effect of nonproportionality
	of the loading	. 12.1 J.M.Proix EDF/R & D /AMA Addition of the remark on the positivity of the coefficients K and W, file 21019 Stamps
behavior	tangent	to obtain the tangent behavior in the elastoplastic case, it is necessary to calculate

Annexe 1 formula [éq 2.3-7]. One uses

for that the statement of formula according to formula and, which is written $\frac{d\,\Delta\boldsymbol{\varepsilon}^p}{d\,\tilde{\boldsymbol{\sigma}}^e}$ the form: formulate with formula One points out the following $\Delta\boldsymbol{\varepsilon}^p$: thus $\tilde{\boldsymbol{\sigma}}^e$: p formulate the quantities of the formula type

$$\Delta\varepsilon^p=\frac{3\,\Delta p}{2\mathrm{D}(p)}\tilde{\sigma}_e+B_1^*(p)\alpha_1^-+B_2^*(p)\alpha_2^-$$

are calculated

$$B_i^*(p)=-\Delta p\frac{M_i(p)}{D(p)}$$

$$M_i(p)=\frac{C_i(p)}{1+\delta_i\gamma_i(p)\Delta p}$$

$$D(p)=R(p)+\left(3\mu+M_1(p)N_1(p,\beta_1)+M_2(p)N_2(p,\beta_2)\right)\Delta p+K\left(\frac{\Delta p}{\Delta t}\right)^{1/N}$$

: formulate These various terms

$$R(p)=R_\infty+(R_0-R_\infty)e^{-bp}$$

$$C_i(p)=C_i^\infty\left(1+(k-1)e^{-wp}\right)$$

$$\gamma_i(p)=\gamma_i^0\left(a_\infty+(1-a_\infty)e^{-bp}\right)$$

express themselves

$$\frac{\delta\Delta\varepsilon^p}{\delta\tilde{\sigma}^e}=\frac{3\,\Delta p}{2\mathrm{D}(p)}\mathbf{Id}+\frac{\delta\left(\frac{3\,\Delta p}{2\mathrm{D}(p)}\right)}{\delta\tilde{\sigma}^e}\otimes\tilde{\sigma}^e+\frac{\delta B_1^*(p)}{\delta\tilde{\sigma}^e}\otimes\alpha_1^-+\frac{\delta B_2^*(p)}{\delta\tilde{\sigma}^e}\otimes\alpha_2^-$$

: formulate with formula $\frac{\delta A(p)}{\delta\tilde{\sigma}^e}$ the computation of: In the case $\frac{\delta A(p)}{\delta\tilde{\sigma}^e}=\frac{\delta A(p)}{\delta p}\frac{\delta p}{\delta\tilde{\sigma}^e}$ the effect of memory, it is enough to modify

- $\frac{\delta\left(\frac{3\Delta p}{2\mathrm{D}(p)}\right)}{\delta p}=\frac{3}{2}I(p)$ formulates $I(p)=\frac{1}{D(p)}-\frac{D'(p)}{D^2(p)}\Delta p$
- $\frac{\delta B_i^*(p)}{\delta p}=-\frac{M_i'(p)}{D(p)}\Delta p-M_i(p).I(p)=H_i(p)$

formuleformuleor formula D'

- thus formuleet formula In the case of not proportionality (or), $R'(p)$

derivatives $R=R^-+\Delta R=R^-+b\frac{\left(Q(\Delta p)-R^-\right)}{1+b\,\Delta p}\Delta p=R^-+\frac{b\,\Delta p}{1+b\,\Delta p}\left(Q_M+(Q_0-Q_M)e^{-2\mu q}-R^-\right)$

$$R'(p)=\frac{b}{1+b\,\Delta p}\left(\frac{Q-R^-}{1+b\,\Delta p}-2\mu\,\Delta p(Q-Q_M)\frac{\partial\Delta q}{\partial\Delta p}\right)=\frac{b}{1+b\,\Delta p}\left(\frac{Q-R^-}{1+b\,\Delta p}-2\mu\Delta\mathrm{p}(Q_0-Q_M)\frac{\partial\Delta q}{\partial\Delta p}\right)$$

$\Delta q=\eta\left(\frac{2}{3}J_2(\varepsilon^p-\xi^-)-q^-\right)$ It $\frac{\partial\Delta q}{\partial\Delta p}=\eta\frac{\varepsilon^p-\xi^-}{J_2(\varepsilon^p-\xi^-)}\frac{\partial\varepsilon^p}{\partial\Delta p}$

to calculate

$$\frac{\delta \Delta \varepsilon^p}{\delta \Delta p} = \frac{\delta \left(\frac{3 \Delta p}{2 D(p)} \right)}{\delta \Delta p} \tilde{\sigma}^e + \frac{\delta B_1^*(p)}{\delta \Delta p} \alpha_1^- + \frac{\delta B_2^*(p)}{\delta \Delta p} \alpha_2^- = \frac{3}{2} I(\Delta p) \tilde{\sigma}^e + H_1^*(\Delta p) \alpha_1^- + H_2^*(\Delta p) \alpha_2^-$$

•: formulate One thus uses, following [éq $\delta_1 \neq 1$ 2.3 $\delta_2 \neq 1$ - 8]: formulate formula with formula Then

$$M_i'(p) = \frac{C_i'(p)}{1+\delta_i \gamma_i(p) \Delta p} - \frac{C_i(p)}{(1+\delta_i \gamma_i(p) \Delta p)^2} (\gamma_i' \delta_i \Delta p + \gamma_i \delta_i)$$

$$D' = R' + \frac{K}{N \Delta t} \left(\frac{\Delta p}{\Delta t} \right)^{\frac{1}{N}-1} + 3\mu + M_1 N_1 + M_2 N_2 + \Delta p (M_1' N_1 + M_2' N_2 + M_1 N_1' + M_2 N_2')$$

, while $N_i' = \frac{1+\gamma_i'(\delta_i \Delta p + (\delta_i - 1)\beta_i) + \gamma_i(\delta_i + (\delta_i - 1)\beta_i') - N_i(\gamma_i + \gamma_i' \Delta p)}{1+\gamma_i \Delta p}$

posing: formulate with formula $\frac{\delta p}{\delta \tilde{\sigma}^e}$

, formula puts itself in the form $\frac{\delta p}{\delta \tilde{\sigma}^e} = - \frac{\tilde{F}_{,\tilde{\sigma}^e}(p, \tilde{\sigma}^e)}{\tilde{F}_{,p}(p, \tilde{\sigma}^e)}$

$$\tilde{F}(p, \tilde{\sigma}^e) = S_{eq}(p, \tilde{\sigma}^e) - R(p) - K \left(\frac{\Delta p}{\Delta t} \right)^{1/N} = G(p, \tilde{\sigma}^e) - R(p) - K \left(\frac{\Delta p}{\Delta t} \right)^{1/N}$$

$$S = A \tilde{\sigma}^e + B_1 \alpha_1^- + B_2 \alpha_2^- \qquad A = \frac{R(p) + K \left(\frac{\Delta p}{\Delta t} \right)^{1/N}}{D(p)} \quad B_i = -\frac{2}{3} \frac{M_i(p) \left(R(p) + K \left(\frac{\Delta p}{\Delta t} \right)^{1/N} \right)}{D(p)}$$

the equation F (Δ p) $R_v(p) = R(p) + K \left(\frac{\Delta p}{\Delta t} \right)^{1/N}$ =

$$\frac{\delta p}{\delta \tilde{\sigma}^e} = - \frac{G_{,\tilde{\sigma}^e}(p, \tilde{\sigma}^e)}{G_{,p}(p, \tilde{\sigma}^e) - R_v'(p)} = - \frac{\frac{3}{2} \frac{R_v(p)}{D(p)} \frac{S}{S_{eq}}}{\frac{3}{2} \frac{S}{S_{eq}} : S_{,p} - R_v'(p)} = -\frac{3}{2} \frac{\frac{R_v}{D S_{eq}} (A \tilde{\sigma}^e + B_1 \alpha_1^- + B_2 \alpha_2^-)}{\frac{3}{2} \frac{S}{S_{eq}} : S_{,p} - R_v'(p)}$$

$$= -\frac{3}{2} \frac{L_1(p) . \tilde{\sigma}^e + L_{21}(p) \alpha_1^- + L_{22}(p) \alpha_2^-}{L_3(p)}$$

¿
acts

$$L_1(p)=\frac{R_v^2(p)}{D^2(p)\times S_{eq}}=\frac{A^2(p)}{S_{eq}}$$

$$L_{21}(p)=\frac{R_v(p)}{D(p)}B_1(p)\frac{1}{S_{eq}} \qquad L_{22}(p)=\frac{R_v(p)}{D(p)}B_2(p)\frac{1}{S_{eq}}$$ ¿

$$L_3(p)=\frac{3}{2}\frac{S}{S_{eq}}:\left(A'(p)\tilde{\sigma}^e+B'_1(p)\alpha_1^-+B'_2(p)\alpha_2^-\right)-R'(p)-\frac{K}{N\,\Delta t}\left(\frac{\Delta p}{\Delta t}\right)^{\frac{1}{N}-1}$$

a nonlinear $\frac{\delta\,\Delta\varepsilon^p}{\delta\,\tilde{\sigma}^e}$ equation by

$$\frac{\delta\,\Delta\varepsilon^p}{\delta\,\tilde{\sigma}^e}=\frac{3}{2}\frac{\Delta p}{D(p)}\mathbf{Id}+\frac{3}{2}\left(I_S(p)\tilde{\sigma}^e+I_{a1}(p)\alpha_1^-+I_{a2}(p)\alpha_2^-\right)\otimes\tilde{\sigma}^e$$
$$+\left(H_s^1\tilde{\sigma}^e+H_{a1}^1\alpha_1^-+H_{a2}^1\alpha_2^-\right)\otimes\alpha_1^-$$
$$+\left(H_s^2\tilde{\sigma}^e+H_{a1}^2\alpha_1^-+H_{a2}^2\alpha_2^-\right)\otimes\alpha_2^-$$

seeking

$$I_S(p)=-\frac{3}{2}I(p).\frac{L_1(p)}{L_3(p)} \qquad I_{a1}(p)=-\frac{3}{2}\frac{I(p)L_{21}(p)}{L_3(p)}$$
$$I_{a2}(p)=-\frac{3}{2}\frac{I(p)L_{22}(p)}{L_3(p)}$$

$$H_s^1(p)=-\frac{3}{2}\frac{H_1(p).L_1(p)}{L_3(p)} \quad H_{a1}^1(p)=-\frac{3}{2}\frac{H_1(p)L_{21}(p)}{L_3(p)} \quad H_{a2}^1(p)=-\frac{3}{2}\frac{H_1(p)L_{22}(p)}{L_3(p)}$$
$$H_s^2(p)=-\frac{3}{2}\frac{H_2(p).L_1(p)}{L_3(p)} \quad H_{a1}^2(p)=-\frac{3}{2}\frac{H_2(p)L_{21}(p)}{L_3(p)} \quad H_{a2}^2(p)=-\frac{3}{2}\frac{H_2(p)L_{22}(p)}{L_3(p)}$$

Annexe 2 the solution in a confidence interval

. For that, one proposes to couple a method of secant with a control of the interval of search. That is to say the following equation to solve: , éq A2-1 the secant method consists in building a succession of points which converges towards the solution

$$f(x)=0 \quad x\in[a,b] \cdot \; f(a)<0 \text{ It } f(b)>0$$ **is defined**

by recurrence (linear approximation of the function by its rope x^n): éq A2-2 In addition, if were to leave the interval, then one replaces it by the limit of the interval in

$$x^{n+1}=x^{n-1}-f(x^{n-1})\frac{x^n-x^{n-1}}{f(x^n)-f(x^{n-1})}$$ **question**

: éq A2-3 On the other hand x^{n+1} , if is in the interval running, then one reactualizes the interval: éq A2-4 One considers

$$\begin{cases}\text{si } x^{n+1}<a \text{ alors } x^{n+1}:=a\\ \text{si } x^{n+1}>b \text{ alors } x^{n+1}:=b\end{cases}$$

to have converged when x^{n+1} is sufficiently close to 0 (tolerance to informing). As for

$$\begin{cases}\text{si } x^{n+1}\in[a,b] \text{ et } f(x^{n+1})<0 \text{ alors } a=x^{n+1}\\ \text{si } x^{n+1}\in[a,b] \text{ et } f(x^{n+1})>0 \text{ alors } b=x^{n+1}\end{cases}$$ **the first two**

leader characters, one can choose f the limits of the interval, or, if one has an estimate of the solution, one can adopt this estimate and one of the limits of the interval. Note: This method functions well if there is only one solution in the interval. Without that

being formally shown, one can note that. One seeks then such as. One leaves for $[a,b]$ *that If, one multiplies by 10 and one tests if, and so on* $f(0)>0$
, until finding b *a value* $f(b)<0$
such as. One is $b=\frac{\left(\tilde{s}^e\frac{2}{3}C_1\alpha_1^- - \frac{2}{3}C_2\alpha_2^-\right)_{eq} - R(p^-)}{3\mathrm{m}}$
sure $f(b)>0$ *that there is then* b *at least a solution* $f(b)>0$ *on.* b $f(b)<0$
$[a,b]$