Viscoplastic behavior model of Summarized

Taheri

One presents in this document the behavior model installation of Taheri viscoplastic, available for all the isoparametric elements (continuum 2D and 3D) except for the plane stresses. After a presentation of the equations of evolution of this model, one describes the system obtained by implicit discretization; it is shown in particular that he always admits a solution.

This model is well adapted to describe the response of austenitic steels under cyclic requests, and in particular the phenomenon of progressive strain. On the other hand, because of its complexity (two surfaces of load, semi-discrete local variable), it does not appear desirable to employ it for different applications (way of monotonic loading, for example).

Contents

1 Description of the model

the behavior model suggested by Taheri [bib5] makes it possible to describe the response of austenitic steels under cyclic requests: it is indeed well adapted to represent the phenomenon of progressive strain. Before stating the equations themselves, one can specify that this model differs from plasticity classical (criterion of von Mises with kinematic hardening and isotropic) by two characteristics, sources of difficulties in the numerical formulation. On the one hand, the evolution of the dissipative variables rests on two criteria of load instead of one: the first, classic, condition the appearance of plastic strain, the second makes it possible to keep a trace of the "maximum" hardening reached by the material to give an account of the phenomenon of ratchet. In addition, to represent the progressive strain satisfactorily, a semi-discrete local variable was introduced. Constant when the behavior is dissipative, it evolves only in the elastic mode of the material. Of original appearance, this model does not rest any less on physical bases, always exposed in Taheri [bib5]. It is accessible, in a viscoplastic wide version (necessary to describe the behavior under high temperatures), by the command `STAT_NON_LINE` under the key word `RELATION : VISC_TAHERI`.

1.1 Behavior plastic

a detailed description of the constitutive law is given in Taheri and al. [bib6]. Briefly, the state of the material is described by its strain state, its temperature like four local variables:

$\boldsymbol{\varepsilon}$	strain tensor total
T	temperature
p	cumulated plastic strain
$\boldsymbol{\varepsilon}^p$	plastic strain tensor
σ^p	forced of peak, memory of tensor maximum
$\boldsymbol{\varepsilon}_n^p$	hardening plastic strain with the last discharge (variable semi-discrete).

The equations of state which express the thermodynamic forces associated according to the variables with state write:

$$\boldsymbol{\sigma}=K\,\mathrm{Tr}\left(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^{th}\right)\mathbf{Id}+2\mu\left(\tilde{\boldsymbol{\varepsilon}}-\boldsymbol{\varepsilon}^{p}\right) \qquad \boldsymbol{\varepsilon}^{th}=\alpha\left(T-T^{réf}\right)\mathbf{Id}$$ **éq 1.1-1.1-11.1-1**

$$R=\mathrm{D}\left[R^0+\left(\frac{2}{3}\right)^a A\left(\boldsymbol{\varepsilon}^p-\boldsymbol{\varepsilon}_n^p\right)_{eq}^a\right] \qquad \mathrm{D}=1-m\,e^{-b\,p\left(1-\frac{\sigma^p}{S}\right)}$$ **éq 1.1-1.1-21.1-2**

$$\mathbf{X}=\mathrm{C}\left[S\,\boldsymbol{\varepsilon}^p-\sigma^p\,\boldsymbol{\varepsilon}_n^p\right] \qquad \mathrm{C}=C_\infty+C_1 e^{-b\,p\left(1-\frac{\sigma^p}{S}\right)}$$ **éq 1.1-1.1-31.1-3**

$\tilde{\mathbf{a}}$	deviatoric part of a variable $\mathbf{a}$
R	tensor of variable isotropic
$\mathbf{X}$	hardening of kinematic hardening
K , μ	moduli of compressibility and thermal
α	shears coefficient of thermal expansion
$T^{ré f}$	forced
S	reference temperature of ratchet
$b , R^0 , A , \mathbf{a} , m , C_\infty , C_1$	other characteristics of hardening of the material

Let us note that the elasticity moduli and the thermal coefficient of thermal expansion are indicated by user by the command `DEFI_MATERIAU`, key word `ELAS`, while the characteristics of hardening are built-in by the key word `TAHERI`. These characteristics can depend on the temperature, by employing key words `ELAS_FO` and `TAHERI_FO`. Also let us specify that an example of identification of the characteristics of hardening on uniaxial situations is given in Geyer [bib2].

The evolution of the local variables is defined by two criteria. The first controls traditional with hardenings kinematics and isotropic plasticity compounds:

$$\mathrm{F}=\left(\tilde{\boldsymbol{\sigma}}-\mathbf{X}\right)_{eq}-R\leq 0 \text{ et } \boldsymbol{\sigma}^0=\frac{\tilde{\boldsymbol{\sigma}}-\mathbf{X}}{\left(\tilde{\boldsymbol{\sigma}}-\mathbf{X}\right)_{eq}} \qquad \textbf{éq 1.1-1.1-41.1-4}$$

$(\cdot)_{eq}$	equivalent norm: $\mathbf{a}_{eq}=\left(\frac{3}{2}\tilde{\mathbf{a}}:\tilde{\mathbf{a}}\right)^{\frac{1}{2}}$
F	norm
$\mathbf{s}^0$	plasticity criterion external with the criterion F

This criterion is matched classical condition of load/discharge:

$$\begin{cases}\text{si } \mathrm{F}<0 \text{ ou } \dot{\boldsymbol{\sigma}}:\mathbf{s}^0\leq 0 & \dot{p}=0 & (\text{élasticité})\\ \text{si } \mathrm{F}=0 \text{ et } \dot{\boldsymbol{\sigma}}:\mathbf{s}^0>0 & \dot{p}\geq 0 \text{ tel que } \dot{\mathrm{F}}=0 & (\text{plasticité})\end{cases} \qquad \textbf{éq 1.1-1.1-51.1-5}$$

And the model of flow associated with the criterion F is:

$$\dot{\boldsymbol{\varepsilon}}^p=\frac{3}{2}\dot{p}\,\mathbf{s}^0 \text{ and thus } \dot{p}=\frac{2}{3}\dot{\varepsilon}^p_{\mathrm{eq}} \qquad \textbf{1.1-1.1-61.1-6}$$

the second criterion controls the evolution of the stress of peak. Geometrically within the space of deviators of the stresses, it translates the fact that the first surface of load $(\mathrm{F}=0)$, represented by a sphere of center $\mathbf{X}$ and radius R, remains inside a sphere of center the origin and σ^p. He is written simply:

$$\mathrm{G}=X_{eq}+R-\sigma^p\leq 0 \qquad \textbf{éq 1.1-1.1-71.1-7}$$

G criterion of maximum hardening

According to the preceding geometrical considerations, the evolution of the stress of peak is:

$$\begin{cases} \text{si } \mathrm{G}<0 \text{ ou } \dot{X}_{eq}+\dot{R}\le 0 & \dot{\sigma}^p=0 \\ \text{si } \mathrm{G}=0 \text{ et } \dot{X}_{eq}+\dot{R}>0 & \dot{\sigma}^p\ge 0 \text{ tel que } \dot{G}=0 \end{cases}$$ **éq 1.1-1.1-81.1-8**

It should be noticed that in the natural state of the material, the stress of peak is not null but is worth the initial elastic limit, namely:

$$\sigma^p(\text{initial})=(1-m)R^0$$

Until now, we did not evoke the evolution of the semi-discrete local variable $\boldsymbol{\varepsilon}_n^p$. In fact, it evolves only in elastic mode. More exactly, this variable takes account of the plastic strain state during the last discharge; in other words, at the beginning of each discharge, this variable should take the value of the current plastic strain instantaneously. However, to preserve a continuous behavior, one regularizes the evolution of $\boldsymbol{\varepsilon}_n^p$ in the following way:

In elastic mode:

$$\dot{\boldsymbol{\varepsilon}}_n^p=\dot{\xi}(\boldsymbol{\varepsilon}_n^p-\boldsymbol{\varepsilon}^p)\begin{cases} \text{si } \boldsymbol{\varepsilon}_n^p=\boldsymbol{\varepsilon}^p & \dot{\xi}=0 & (\text{élasticité classique}) \\ \text{si } \boldsymbol{\varepsilon}_n^p\ge\boldsymbol{\varepsilon}^p & \dot{\xi}\le 0 \text{ tq } \dot{F}=0 & (\text{pseudo-décharge}) \end{cases}$$ **éq 1.1-1.1-91.1-9**

In plastic mode:

$$\dot{\boldsymbol{\varepsilon}}_n^p=0$$

The behavior is thus completely given. Before passing to the introduction of viscosity, the observation of two surfaces of load calls an important remark. One could think that surface $\mathrm{G}=0$ is actually activated only in plastic mode. In practice, it of it is nothing. One can for example quote the case of a thermal loading: a cooling involves (generally) a thermal expansion of the surface of load $\mathrm{F}=0$, so that the stress of peak is brought to evolve to preserve $\mathrm{G}\le 0$, and this same in elastic mode.

1.2 Taken into account of viscosity

to model the behavior of the stainless steels under cyclic loading when the temperature is about $550\,°C$, it is not possible any more to neglect the terms of creep. To give an account of these effects of viscosity while preserving the properties of the preceding model, a simple method consists in making viscous the evolution of the plastic strain. In other words, viscosity intervenes only in plastic mode: no direct influence on the semi-discrete local variable nor on the surface of load $\mathrm{G}=0$. For that, while following Lemaitre and Chaboche [bib3], one replaces the condition of coherence [éq 1.1-5] by:

$$\dot{p}=\left(\frac{\langle \mathrm{F}\rangle}{K\,p^{1/M}}\right)^N$$ **éq 1.2-1.2-11.2-1**

$\langle \mathrm{F}\rangle$	positive part of F (hooks of Macauley)
K, N, M	characteristic of viscosity of the material

the characteristics of viscosity of the material are indicated in command `DEFI_MATERIAU`, is by the key word `LEMAITRE` if they do not depend on the temperature, that is to say by the key word `LEMAITRE_FO` in the contrary case. In the absence of one of these key words, the behavior is supposed plastic.

Unchanged all the other equations of the model are left. It will be seen that such an introduction of viscosity involves only minor modifications of the implicit algorithm of integration of the constitutive law.

1.3 Description of the local variables calculated by *Code_Aster*

the local variables calculated by *Code_Aster* are 9. They are arranged in the following order:

1	p	cumulated plastic strain
2	σ^p	stress of peak
3 to 8	$\boldsymbol{\varepsilon}_n^p$	plastic strain tensor with the last discharge (arranged in the order xx yy zz xy xz , yz)
9	χ	discharge/loadmeter (cf [§2.3]) 0 elastic discharge 1 classical plastic load 2 plastic load on two surfaces 3 pseudo-discharge

As for the tensor of the viscoplastic strains, it is not arranged among the local variables but can be calculated in postprocessing via command `CALC_CHAMP`, options "`EPSP_ELGA`" or "`EPSP_ELNO`", (cf [U4.61.02]).

2 Numerical formulation of the behavior model

In order to be able to treat in the same frame plasticity and viscoplasticity, one chooses to carry out an implicit discretization of the behavior models, (cf [R5-03-02]). Let us note moreover that an explicit procedure of integration is delicate for two reasons: on the one hand, the processing of the semi-discrete variable is necessarily implicit and can lead to numerical oscillations (one pseudo - discharge, therefore one solves $\mathrm{F}=0$, and as a result, F can be (very weak but) higher than zero, from where load with the step following instead of continuing the discharge), and on the other hand, the equation [éq 1.1-2] is not differentiable when $\boldsymbol{\varepsilon}^p=\boldsymbol{\varepsilon}_n^{p-}$

2.1 implicit Discretization of the equations of behavior

Henceforth, one adopts the convention of following notation. If u a quantity indicates, then:

u^-	quantity u at the beginning of time step
Δu	the increment of the quantity u during time step
u	the quantity u at the end of the step of time (not of exhibitor $+$)

Let us start by introducing the elastic stress, i.e. the stress in the absence of increment of plastic strain. One can notice besides that only the term deviatoric cheek a role in the nonlinear part of the behavior:

$$\boldsymbol{\sigma}^e=K\,\mathrm{tr}\left(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^{\mathrm{th}}\right)\mathbf{Id}+\underbrace{2\mu\left(\tilde{\boldsymbol{\varepsilon}}-\boldsymbol{\varepsilon}^{p^-}\right)}_{\tilde{\boldsymbol{\sigma}}^e}\ \text{ et }\ \tilde{\boldsymbol{\sigma}}=\tilde{\boldsymbol{\sigma}}^e-2\mu\,\Delta\boldsymbol{\varepsilon}^p \qquad \textbf{éq 2.1-2.1-12.1-1}$$

By taking account as of equations of state [éq 1.1-1] and [éq 1.1-3] and of the flow model [éq 1.1-6], one a:

$$\mathbf{s}\overset{\mathrm{def}}{=}\tilde{\boldsymbol{\sigma}}-\mathbf{X}=\tilde{\boldsymbol{\sigma}}^e-\mathrm{C}\left(S\,\boldsymbol{\varepsilon}^{p^-}-\sigma^p\,\boldsymbol{\varepsilon}_n^p\right)-\frac{3}{2}\left(2\mu+\mathrm{C}S\right)\Delta\,p\,\mathbf{s}^0 \qquad \textbf{éq 2.1-2.1-22.1-2}$$

By noting that $\mathbf{s}^0$ other than $\mathbf{s}$ is not normalized, one from of deduced immediately:

$$\left[s_{eq}+\frac{3}{2}(2\mu+\mathrm{C}\,S)\Delta p\right]\mathbf{s}^0=\underbrace{\tilde{\boldsymbol{\sigma}}^e-\mathrm{C}\left(S\,\boldsymbol{\varepsilon}^{p-}-\sigma^p\,\boldsymbol{\varepsilon}_n^p\right)}_{\mathbf{s}^e}$$ **éq 2.1-2.1-32.1-3**

Consequently, $\mathbf{s}$ is entirely determined by:

$$\mathbf{s}=s_{eq}\mathbf{s}^0 \quad \text{avec} \quad \mathbf{s}^0=\frac{\mathbf{s}^e}{s_{eq}^e} \quad \text{et} \quad s_{eq}=s_{eq}^e-\frac{3}{2}(2\mu+\mathrm{C}\,S)\Delta p$$ **éq 2.1-2.1-42.1-4**

Finally, the loading functions are:

$$\mathrm{F}=s_{eq}-\mathrm{D}\left[R^0+\left(\frac{2}{3}\right)^a A\left(\boldsymbol{\varepsilon}^{p-}-\boldsymbol{\varepsilon}_n^p+\frac{3}{2}\Delta p\,\mathbf{s}^0\right)_{eq}^a\right]$$ **éq 2.1-2.1-52.1-5**

$$G=\mathrm{C}\left[S\,\boldsymbol{\varepsilon}^{p-}-\sigma^p\,\boldsymbol{\varepsilon}_n^p+\frac{3}{2}S\,\Delta p\,\mathbf{s}^0\right]_{eq}+\mathrm{D}\left[R^0+\left(\frac{2}{3}\right)^a A\left(\boldsymbol{\varepsilon}^{p-}-\boldsymbol{\varepsilon}_n^p+\frac{3}{2}\Delta p\,\mathbf{s}^0\right)_{eq}^a\right]-\sigma^p$$ **éq 2.1-2.1-62.1-6**

2.2 Taking into account of the viscous terms

In the absence of viscous terms, the relation of discretized coherence is:

$$\begin{array}{ll}\text{Régime élastique :} & \mathrm{F}\le 0 \ \text{ et } \ \Delta p=0\\ \text{Régime plastique :} & \mathrm{F}=0 \ \text{ et } \ \Delta p\ge 0\end{array}$$ **éq 2.2-2.2-12.2-1**

On the other hand, in the presence of viscosity, the condition of coherence is replaced by the equation [éq 1.2 - 1] which, discretized, is written:

$$\frac{\Delta p}{\Delta t}=\left(\frac{\langle \mathrm{F}\rangle}{K\,p^{1/M}}\right)^N \iff \langle \mathrm{F}\rangle=K\,p^{1/M}\left(\frac{\Delta p}{\Delta t}\right)^{\frac{1}{N}}$$ **éq 2.2-2.2-22.2-2**

In other words, while posing:

$$\tilde{\mathrm{F}}=\mathrm{F}-K\,p^{1/M}\left(\frac{\Delta p}{\Delta t}\right)^{\frac{1}{N}}$$ **éq 2.2-2.2-32.2-3**

the viscoplastic increment of cumulated strain is determined by:

$$\begin{array}{ll}\text{Régime élastique :} & \tilde{\mathrm{F}}\le 0 \ \text{ et } \ \Delta p=0\\ \text{Régime viscoplastique :} & \tilde{\mathrm{F}}=0 \ \text{ et } \ \Delta p\ge 0\end{array}$$ **éq 2.2-2.2-42.2-4**

Finally, by adopting an implicit, the only difference between the constitutive laws figure and viscoplastic discretization resides in the form of the loading function F : one observes a complementary term in the event of viscosity there. In fact, incremental plasticity appears as the borderline case (without associated numerical difficulty) of incremental viscoplasticity when viscosity

K tends towards zero. Let us note that this remark was already mentioned by Chaboche and al. [bib1].

2.3 Discretization of the conditions of coherence

Before discretizing the conditions of coherence and describing the various possible modes of behavior, a remark is essential as for the processing of the semi-discrete variable. As ξ only intervenes "to control" $\boldsymbol{\varepsilon}_n^p$, one can always be reduced during time step to:

$$\boldsymbol{\varepsilon}_n^p = \xi\,\boldsymbol{\varepsilon}_n^{p-} + (1-\xi)\,\boldsymbol{\varepsilon}^{p-} \qquad 0 \le \xi \le 1 \qquad \textbf{éq 2.3-2.3-12.3-1}$$

the value of ξ is then fixed by the conditions of coherence, which translates the equation of evolution [éq 1.1-9] on the continuous level. Such a parameter setting with each time step makes it possible to be freed from storage from ξ , with condition well-sure preserving the values of $\boldsymbol{\varepsilon}_n^p$.

After this opening remark, one can be interested in the conditions of coherence. For the criterion G which controls the evolution of the stress of peak, the discretized form of the condition of coherence is:

$$\mathrm{G}(\Delta p, \Delta\sigma^p, \xi) \le 0 \quad \Delta\sigma^p \ge 0 \quad \Delta\sigma^p\,\mathrm{G}(\Delta p, \Delta\sigma^p, \xi) = 0 \qquad \textbf{éq 2.3-2.3-22.3-2}$$

the condition of coherence relating to F is more delicate insofar as it controls the evolution of the plastic strain in plastic mode of load and the evolution of ξ in mode of discharge. Once discretized, she is written:

In plastic mode of load $(\xi=1)$:

$$\mathrm{F}(\Delta p, \Delta\sigma^p, \xi=1) = 0 \qquad \Delta p \ge 0 \qquad \Delta p\,\mathrm{F}(\Delta p, \Delta\sigma^p, \xi=1) = 0 \qquad \textbf{éq 2.3-2.3-32.3-3}$$

In mode of discharge $(\Delta p=0)$:

$$\mathrm{F}(\Delta p=0, \Delta\sigma^p, \xi) = 0 \qquad 0 \le \xi \le 1\mathrm{x} \qquad \xi\,\mathrm{F}(\Delta p=0, \Delta\sigma^p, \xi) = 0 \qquad \textbf{éq 2.3-2.3-42.3-4}$$

to be able to select the mode of behavior of the material, and thus the equations to be solved, the first question is:

Do we Somme in plastic or elastic situation?

In fact, there exists a solution in elastic mode (pseudo-discharge $\xi > 0$ or classical elasticity $\xi = 0$) if one can find an increment of stress of peak such as:

Incremental condition of discharge (scalar equation in $\Delta\sigma^p$):

$$\begin{aligned} &\mathrm{F}(\Delta p=0, \Delta\sigma^p, \xi=1) \le 0 \\ &\mathrm{G}(\Delta p=0, \Delta\sigma^p, \xi=1) \le 0 \qquad \Delta\sigma^p \ge 0 \qquad \Delta\sigma^p\,\mathrm{G}(\Delta p=0, \Delta\sigma^p, \xi=1) = 0 \end{aligned} \qquad \textbf{éq 2.3-2.3-52.3-5}$$

In the event of plastic load, i.e. when it does not exist $\Delta\sigma^p$ satisfactory [éq 2.3-5], one has then to solve the nonlinear system in Δp and $\Delta\sigma^p$ following:

Plastic mode (nonlinear system in Δp and $\Delta\sigma^p$):

$$\mathrm{F}(\Delta p, \Delta \sigma^p, \xi=1)=0 \qquad \Delta p \geq 0$$
$$\mathrm{G}(\Delta p, \Delta \sigma^p, \xi=1) \leq 0 \qquad \Delta \sigma^p \geq 0 \qquad \Delta \sigma^p \mathrm{G}(\Delta p, \Delta \sigma^p, \xi=1)=0$$

éq 2.3-2.3-62.3-6

On the other hand, in elastic situation, two choices are still possible: pseudo-discharge $(\xi>0)$ or classical elasticity $(\xi=0)$. The second case being more favorable, one starts by examining whether it is not realizable, i.e. if there exists an increment of stress of peak such as:

Incremental condition of classical elastic mode (scalar equation in $\Delta \sigma^p$):

$$\mathrm{F}(\Delta p=0, \Delta \sigma^p, \xi=0) \leq 0$$
$$\mathrm{G}(\Delta p=0, \Delta \sigma^p, \xi=0) \leq 0 \qquad \Delta \sigma^p \geq 0 \qquad \Delta \sigma^p \mathrm{G}(\Delta p=0, \Delta \sigma^p, \xi=0)=0$$

éq 2.3-2.3-72.3-7

Lastly, if it were to be a question of a discharge pseudo-elastic, it remains to solve the nonlinear system in ξ and $\Delta \sigma^p$ following:

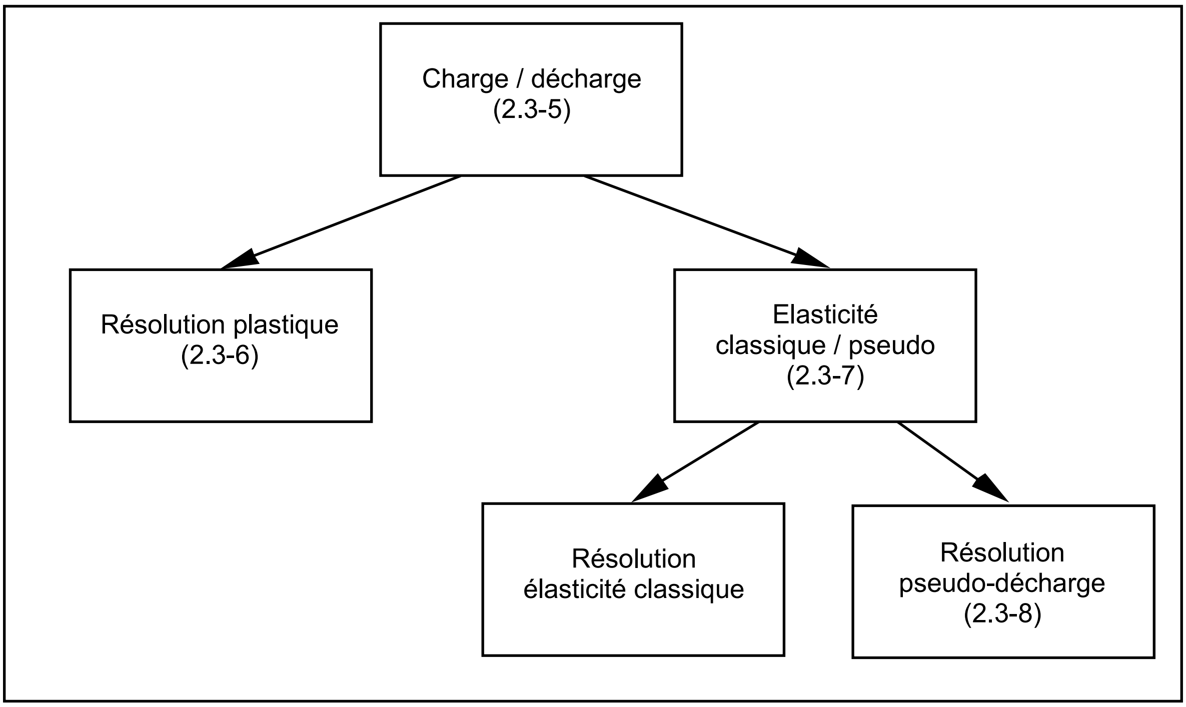
Discharge pseudo-elastic (nonlinear system in ξ and $\Delta \sigma^p$):

$$\mathrm{F}(\Delta p=0, \Delta \sigma^p, \xi)=0 \qquad 0 \leq \xi \leq 1$$
$$\mathrm{G}(\Delta p=0, \Delta \sigma^p, \xi) \leq 0 \qquad \Delta \sigma^p \geq 0 \qquad \Delta \sigma^p \mathrm{G}(\Delta p=0, \Delta \sigma^p, \xi)=0$$

éq 2.3-2.3-82.3-8

Let us note as of now that the nonlinear systems [éq 2.3-6] and [éq 2.3-8] can be reduced to the solution of a simple scalar equation if $\Delta \sigma^p=0$ allows to obtain a solution.

One can summarize the algorithm of choice of the equations to be solved by the decision tree below.



Appear 2.3-2.3-a2.3-a : Decision tree to choose the mode of behavior

2.4 Framing of the solutions

To the reading of the preceding paragraph, one could note the need for solving (numerically) a certain number of scalar equations or of nonlinear systems. For that, it is always interesting to have an interval on which to seek the solution. On the one hand, a framing of the solution shows its existence (what strongly reinforces the chances of success of an algorithm of resolution!), and on the other hand, it allows a suitable digital processing, therefore surer.

Concerning $\Delta\sigma^p$, one undervaluing is of course 0. In addition, the stress of ratchet S represents a limit beyond which the model any more meaning does not have. In fact, with the examination of constant the materials obtained by identification, cf Taheri and al. [bib6], G becomes indeed negative when $\sigma^p=S$ if the difference between the plastic strain and the plastic strain with the last discharge is not too important (a few %):

$$\frac{\mathrm{G}(\Delta p,\sigma^p=S,\xi)}{S}=\underbrace{(C_\infty+C_1)}_{\approx 13}\underbrace{(\boldsymbol{\varepsilon}^p-\boldsymbol{\varepsilon}_n^p)_{eq}}_{\leq 3\,\%}+\underbrace{(1-m)}_{\approx 0,75}\left(\underbrace{\frac{R^0}{S}}_{\approx 20\,\%}+\underbrace{\left(\frac{2}{3}\right)^a\frac{A}{S}}_{\approx 0,5}\underbrace{(\boldsymbol{\varepsilon}^p-\boldsymbol{\varepsilon}_n^p)_{eq}^a}_{\approx 0,7}\right)-1\leq 0$$

éq **2.4-2.4-12.4-1**

One can also seek one raising for Δp. By examining the statement of F :

$$\begin{aligned}\mathrm{F}(\Delta p,\Delta\sigma^p,\xi)&\leq s_{eq}-\mathrm{D}\,R^0\\&\leq s_{eq}^e-\frac{3}{2}(2\mu+\mathrm{C}\,S)\,\Delta p-\mathrm{D}\,R^0\\&\leq s_{\max}^e-\frac{3}{2}(2\mu+C_\infty S)\,\Delta p-\mathrm{D}(p^-)\,R^0\end{aligned}$$

éq 2.4-2.4-22.4-2

One from of deduced one raising for Δp, such as $\mathrm{F}(\Delta p,\Delta\sigma^p,\xi)\leq 0$:

$$\Delta p\leq\frac{s_{\max}^e(\sigma^p)-\mathrm{D}(p^-,\sigma^p)\,R^0}{\frac{3}{2}(2\mu+C_\infty S)}$$

éq 2.4-2.4-32.4-3

$$s_{\max}^e(\sigma^p)=\max\begin{cases}\left[\tilde{\boldsymbol{\sigma}}^e-\mathrm{C}(p^-,\sigma^p)\left(S\,\boldsymbol{\varepsilon}^{p-}-\sigma^p\,\boldsymbol{\varepsilon}_n^p\right)\right]_{eq}\\\left[\tilde{\boldsymbol{\sigma}}^e-C_\infty\left(S\,\boldsymbol{\varepsilon}^{p-}-\sigma^p\,\boldsymbol{\varepsilon}_n^p\right)\right]_{eq}\end{cases}$$

éq 2.4-2.4-42.4-4

In particular, one can give one raising (coarse) Δp the independent one of $\Delta\sigma^p$:

$$\Delta p_{max}=\frac{s_{max}^{e\,max}-(1-m)\,R^0}{\frac{3}{2}(2\mu+C_\infty S)}$$

éq 2.4-2.4-52.4-5

$$s_{max}^{emax}=\sigma_{eq}^e+(C_1+C_\infty)\max\begin{cases}\left[S\,\boldsymbol{\varepsilon}^{p-}-\sigma^{p-}\,\boldsymbol{\varepsilon}_n^p\right]_{eq}\\\left[S\,\boldsymbol{\varepsilon}^{p-}-S\,\boldsymbol{\varepsilon}_n^p\right]_{eq}\end{cases}$$

éq 2.4-2.4-62.4-6

One can then notice that the systems [éq 2.3-6] and [éq 2.3-8] always admit a solution. Indeed, so for each system, one writes respectively $\Delta\sigma^p(\Delta p)$ et $\Delta\sigma^p(\xi)$ the solutions of $\mathrm{G}=0$, then one nonlinear

- a: System of plastic load:

$$\mathrm{F}\left(\Delta p=0,\Delta\sigma^{p}\left(\Delta p=0\right),\xi=1\right)\geq 0 \text{ et } \mathrm{F}\left(\Delta p_{\max},\Delta\sigma^{p}\left(\Delta p_{\max}\right),\xi=1\right)\leq 0$$ **éq 2.4-2.4-72.4-7**

- nonlinear System of pseudo-discharge:

$$\mathrm{F}\left(\Delta p=0,\Delta\sigma^{p}\left(\xi=1\right),\xi=1\right)\leq 0 \text{ et } \mathrm{F}\left(\Delta p=0,\Delta\sigma^{p}\left(\xi=0\right),\xi=0\right)\geq 0$$ **éq 2.4-2.4-82.4-8**

3 Methods of numerical resolution

the resolution of the incremental equations confronts us either with a nonlinear scalar equation, or with a nonlinear system with two unknowns. Below the numerical methods employed are exposed. One also examines the computation of the tangent matrix, possibly used by the total algorithm of `STAT_NON_LINE`, (cf [R5.03.01]).

3.1 Scalar equation: method of secants

It acts to solve a nonlinear scalar equation by seeking the solution in a confidence interval. For that, one proposes to couple a method of secant with a control of the interval of search. That is to say the following equation to solve:

$$\mathrm{f}(x)=0 \quad x\in[a,b] \quad \mathrm{f}(a)<0 \quad \mathrm{f}(b)>0$$ **éq 3.1-3.1-13.1-1**

the secant method consists in building a succession of points x^n which converges towards the solution. It is defined by recurrence (linear approximation of the function by its rope):

$$x^{n+1}=x^{n-1}-\mathrm{f}\left(x^{n-1}\right)\frac{x^n-x^{n-1}}{\mathrm{f}\left(x^n\right)-\mathrm{f}\left(x^{n-1}\right)}$$ **éq 3.1-3.1-23.1-2**

In addition, if x^{n+1} were to leave the interval, then one replaces it by the limit of the interval in question:

$$\begin{cases}\text{si } x^{n+1}<a \text{ alors } x^{n+1}:=a\\ \text{si } x^{n+1}>b \text{ alors } x^{n+1}:=b\end{cases}$$ **éq 3.1-3.1-33.1-3**

On the other hand, if x^{n+1} is in the interval running, then one reactualizes the interval:

$$\begin{cases}\text{si } x^{n+1}\in[a,b] \text{ et } \mathrm{f}\left(x^{n+1}\right)<0 \text{ alors } a:=x^{n+1}\\ \text{si } x^{n+1}\in[a,b] \text{ et } \mathrm{f}\left(x^{n+1}\right)>0 \text{ alors } b:=x^{n+1}\end{cases}$$ **éq 3.1-3.1-43.1-4**

One considers to have converged when f is sufficiently close to 0 (tolerance to informing). As for the first two leader characters, one can choose the limits of the interval, or, if one has an estimate of the solution, one can adopt this estimate and one of the limits of the interval.

3.2 Nonlinear systems: method of Newton and linear search

One presents here a method of Newton with which one associated a linear technique of search and a control of the direction of descent not to leave the field of search (limits on the unknowns).

That is to say the following nonlinear system:

$$\begin{cases}\mathrm{F}(x,y)=0\\ \mathrm{G}(x,y)=0\end{cases} \text{ avec } \begin{cases}x_{\min}\le x\le x_{\max}\\ y_{\min}\le y\le y_{\max}\end{cases}$$ **éq 3.2-3.2-13.2-1**

If (x,y) is a point of the field of search, then one builds a succession of points $\left(x^n,y^n\right)$ which converges towards a solution (or at least, one hopes for it) by the following process.

- Determination of the direction of descent

  a direction of descent $(\delta x, \delta y)$ is given by the resolution of the linear system 2 X 2:

$$\begin{bmatrix} F^n_{,x} & F^n_{,y} \\ G^n_{,x} & G^n_{,y} \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \end{bmatrix} = - \begin{bmatrix} F^n \\ G^n \end{bmatrix}$$ **éq 3.2-3.2-23.2-2**

- Correction of the direction of descent

  One corrects the direction of descent $(\delta x, \delta y)$ so that the points considered are in the field of search (with ρ_{max} the maximum length which one is authorized to describe along the direction of descent):

$$\begin{cases} \text{si} \quad x+\rho_{\max}\delta x < x_{\min} \quad \delta x := \dfrac{x_{\min}-x}{\rho_{\max}} \quad \text{si} \quad x+\rho_{\max}\delta x > x_{\max} \quad \delta x := \dfrac{x_{\max}-x}{\rho_{\max}} \\ \text{si} \quad y+\rho_{\max}\delta y < y_{\min} \quad \delta y := \dfrac{y_{\min}-y}{\rho_{\max}} \quad \text{si} \quad y+\rho_{\max}\delta y > y_{\max} \quad \delta y := \dfrac{y_{\max}-y}{\rho_{\max}} \end{cases}$$ **éq 3.2-3.2-33.2-3**

- linear Search

  It any more but does not remain to minimize the quantity $\mathrm{E} = (\mathrm{F}^2 + \mathrm{G}^2)/2$ in the direction of descent. Let us note that the norm E that one minimizes thus is a measurement of the mistake made in the resolution of the system: it is null when (x, y) is solution of the system [éq 3.2-1]. To minimize E, one simply will seek to cancel his derivative, i.e. to solve the scalar equation:

$$\underbrace{\frac{\partial}{\partial \rho}\left[\mathrm{E}(x+\rho\,\delta x, y+\rho\,\delta y)\right]}_{(\mathrm{FF}_{,x}+\mathrm{GG}_{,x})\delta x + (\mathrm{FF}_{,y}+\mathrm{GG}_{,y})\delta y} = 0 \quad \text{et} \quad 0 \le \rho \le \rho_{\max}$$ **éq 3.2-3.2-43.2-4**

- Convergence criterion

  One considers to have converged when the error E is lower than a prescribed quantity. In addition, if the norm of the direction of descent becomes too weak (another quantity with informing), one can think that the algorithm does not manage to converge.

3.3 Stopping criteria

Until now, the values of stop and the maximum nombres of iterations of the preceding methods of resolution were not specified. Two cases should be distinguished.

- When one seeks to check the conditions of coherence (scalar equation or nonlinear system following the situation), one expects precise results, whose relative tolerance η is fixed by the user in command `STAT_NON_LINE` under key word `RESI_INTE_RELA`, (cf [U4.32.01]). According to whether one seeks to solve $\mathrm{F}=0, \mathrm{G}=0$ or simultaneously $\mathrm{F}=\mathrm{G}=0$, the stopping criteria are expressed respectively:

$$\left|\frac{\mathrm{F}}{R^0}\right| \le \eta \quad \text{ou} \quad \left|\frac{\mathrm{G}}{R^0}\right| \le \eta \quad \text{ou} \quad \frac{1}{R^0}\sqrt{\frac{\mathrm{F}^2+\mathrm{G}^2}{2}} \le \eta$$

R^0 initial elastic limit, provided by the user, cf [§ 1.1].

In addition, the user always specifies a maximum nombre of iterations in command `STAT_NON_LINE` under key word `ITER_INTE_MAXI`, (cf [U4.32.01]).

- When one carries out iterations of linear search, one seeks to obtain a faster convergence (or at least sourer). One should not therefore devoting an excessive time to it. This is why one fixed once for a a whole maximum nombre of iterations equal to 3, a maximum limit ρ_{max} equal to 2 and one relative stopping criteria of 1 %:

$$\left.\frac{\partial}{\partial\rho}\left[\mathrm{E}\left(x+\rho\,\delta x,\,y+\rho\,\delta y\right)\right]\right|_{\rho}\leq 10^{-2}\left.\frac{\partial}{\partial\rho}\left[\mathrm{E}\left(x+\rho\,\delta x,\,y+\rho\,\delta y\right)\right]\right|_{\rho=0}$$

3.4 Stamp tangent

In the optics of a resolution of the balance equations (total) by a method of Newton, it is essential to determine the consistent matrix of the tangent behavior, (cf Simo and al. [bib4]). This matrix is composed classically of an elastic contribution and a plastic contribution:

$$\frac{\delta\boldsymbol{\sigma}}{\delta\boldsymbol{\varepsilon}}=\frac{\delta\boldsymbol{\sigma}^e}{\delta\boldsymbol{\varepsilon}}-2\mu\frac{\delta\Delta\boldsymbol{\varepsilon}^p}{\delta\boldsymbol{\varepsilon}}$$ **éq 3.4-3.4-13.4-1**

One from of immediately deduced that in elastic mode (classical or pseudo-discharge), the tangent matrix is reduced to the elastic matrix:

Elastic mode:

$$\frac{\delta\boldsymbol{\sigma}}{\delta\boldsymbol{\varepsilon}}=\frac{\delta\boldsymbol{\sigma}^e}{\delta\boldsymbol{\varepsilon}}$$ **éq 3.4-3.4-23.4-2**

On the other hand, in plastic mode, the variation of the plastic strain is not null any more. The made up derivative rules make it possible to obtain:

$$\frac{\delta\Delta\boldsymbol{\varepsilon}^p}{\delta\tilde{\boldsymbol{\sigma}}^e}=\frac{3}{2}\left[\mathbf{s}^0\otimes\frac{\delta p}{\delta\tilde{\boldsymbol{\sigma}}^e}+\Delta p\frac{\delta\mathbf{s}^0}{\delta\tilde{\boldsymbol{\sigma}}^e}\right]=\frac{3}{2}\left[\mathbf{s}^0\otimes\frac{\delta p}{\delta\boldsymbol{\sigma}^e}+\frac{\Delta p}{s^e_{eq}}\left(\mathbf{Id}-\frac{3}{2}\mathbf{s}^0\otimes\mathbf{s}^0\right)\right]$$ **éq 3.4-3.4-33.4-3**

$\otimes$ tensor product

One can note that one preferred to derive compared to $\tilde{\boldsymbol{\sigma}}^{\mathbf{e}}$, knowing that one a:

$$\frac{\delta\Delta\boldsymbol{\varepsilon}^p}{\delta\boldsymbol{\varepsilon}}=\frac{\delta\Delta\boldsymbol{\varepsilon}^p}{\delta\boldsymbol{\sigma}^e}.\frac{\delta\boldsymbol{\sigma}^e}{\delta\boldsymbol{\varepsilon}}=2\mu\frac{\delta\Delta\boldsymbol{\varepsilon}^p}{\delta\tilde{\boldsymbol{\sigma}}^e}.\mathbf{P}\quad\text{avec}\quad\mathbf{P}:\begin{cases}S\rightarrow S\\ \boldsymbol{\varepsilon}\rightarrow\tilde{\boldsymbol{\varepsilon}}\end{cases}$$ **éq 3.4-3.4-43.4-4**

S space of the symmetric tensors
$\mathbf{P}$ projector on the deviators

Finally, it any more but does not remain to calculate the variation of p. For that, it is necessary to distinguish if it is about a classical mode of plasticity $\left(\Delta\sigma^p=0\right)$ or plasticity on two surfaces. As follows:

Classical plasticity: $\mathrm{F}\left(p,\tilde{\boldsymbol{\sigma}}^e\right)=0$

$$F_{,p}\left(p,\tilde{\boldsymbol{\sigma}}^e\right)\delta p=-F_{,\tilde{\boldsymbol{\sigma}}^e}\left(p,\tilde{\boldsymbol{\sigma}}^e\right)\delta\tilde{\boldsymbol{\sigma}}^e \quad\Rightarrow\quad \frac{\delta p}{\delta\tilde{\boldsymbol{\sigma}}^e}=-\frac{F_{,\tilde{\boldsymbol{\sigma}}^e}\left(p,\tilde{\boldsymbol{\sigma}}^e\right)}{F_{,p}\left(p,\tilde{\boldsymbol{\sigma}}^e\right)}$$ **éq 3.4-3.4-53.4-5**

Plasticity on two surfaces: $F\left(p,\sigma^p,\tilde{\boldsymbol{\sigma}}^e\right)=0$ et $G\left(p,\sigma^p,\tilde{\boldsymbol{\sigma}}^e\right)=0$

$$\begin{bmatrix}F_{,p} & F_{,\sigma^p}\\ G_{,p} & G_{,\sigma^p}\end{bmatrix}\begin{bmatrix}\delta p\\ \delta\sigma^p\end{bmatrix}=-\begin{bmatrix}F_{,\tilde{\boldsymbol{\sigma}}^e}\\ G_{,\tilde{\boldsymbol{\sigma}}^e}\end{bmatrix}.\delta\tilde{\boldsymbol{\sigma}}^e \quad\Rightarrow\quad \frac{\delta p}{\delta\tilde{\boldsymbol{\sigma}}^e}=\frac{\begin{vmatrix}F_{,\sigma^p} & F_{,\tilde{\boldsymbol{\sigma}}^e}\\ G_{,\sigma^p} & G_{,\tilde{\boldsymbol{\sigma}}^e}\end{vmatrix}}{\begin{vmatrix}F_{,p} & F_{,\sigma^p}\\ G_{,p} & G_{,\sigma^p}\end{vmatrix}}$$ **éq 3.4-3.4-63.4-6**

an attentive examination of the statements [éq 2.1-5] and [éq 2.1-6] makes it possible to note that the variations of F and G compared to $\tilde{\boldsymbol{\sigma}}^e$ are not necessarily colinéaires with $\mathbf{s}^0$. By keeping account of [éq 3.4 - 3], one from of deduced whereas the tangent matrix is in general not symmetric in plastic mode. Rather than to impose the use of an asymmetric solver, much more expensive in time computation, one prefers to symmetrize this matrix.

3.5 Plane stresses

the processing of the plane stresses adds a nonlinear equation to solve, coupled with the systems [éq 2.3 - 6] and [éq 2.3-8], (cf [R5-03-02]). In front of this considerable difficulty and the absence of expressed need, one preferred not to give the plane opportunity of forcing a stress state on the level of the constitutive law. In other words, modelization `C_PLAN` is not available for constitutive law `VISC_TAHERI`.

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5 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
4	4E.LorentzEDF -R&D/AMA	initial Text