Model of Rousselier for the ductility fracture

Summarized

The model of Rousselier describes the damage due to the plastic growth of cavities in a metal. It makes it possible to model cracking and the ductility fracture. The behavior model is elastoplastic or viscoplastic with isotropic hardening. It allows the changes of plastic volume and is written in small strains. The writing in large deformations with a formulation of Simo and Miehe modified, in the elastoplastic case only, is described in [R5.03.06].
This model is available in command STAT_NON_LINE via the key word RELATION = "ROUSS_PR" or "ROUSS_VISC" under factor key word the COMP_INCR and with the key word DEFORMATION = "PETIT_REAC".
This model is established for the modelizations three-dimensional (3D), axisymmetric (AXIS), in plane stresses and plane strains (C_PLAN, D_PLAN).

One presents the writing and the digital processing of this model.

Contents

1 Introduction

the mechanisms at the origin of the ductility fracture of metals are associated with the development of cavities within the material. Three phases are generally distinguished:

germination: it is starting or nucleation the cavities, into cubes sites which correspond preferentially to the particles of second phase present in the material,
growth: it is the phase which corresponds to the development itself of the cavities, controlled primarily by the yielding of the metal matrix which surrounds these cavities,
coalescence: it is the phase which corresponds to the localization of the strain between the cavities to create macroscopic cracks.

The model of Rousselier [bib1], [bib2], [bib3] presented here is based on microstructural assumptions which introduces a microstructure made up of cavities and of a matrix whose elastic strain negligible are compared with plastic strains. In this case, and in the absence of nucleation of new cavities, porosity f, definite like the relationship between the volume of the cavity V^c and the total volume V of representative ground volume, is directly connected to the macroscopic plastic strain by:

$$\frac{\rho_0}{\rho}=\frac{1-f_0}{1-f} \text{ with } f=\frac{V^c}{V}\Leftrightarrow \dot{f}=(1-f)\,\mathrm{tr}\,\dot{\boldsymbol{\varepsilon}}^p \qquad \text{1-1}$$

where f_0 indicates initial porosity, ρ_o and ρ are respectively the density in the configurations initial and current (one takes in the continuation $\rho_o=1$) and $\dot{\varepsilon}^p$ the plastic strain rate of total volume V.
The construction of the model rests on a thermodynamic and phenomenologic analysis which brings to write the plastic *potential* F in the following form:

$$F(\boldsymbol{\tau},\ p,\ f)=\tau_{eq}+\sigma_1 D_1 f\exp\left(\frac{\tau_m}{\sigma_1}\right)-R(p) \qquad \text{éq 1-2}$$

where $\boldsymbol{\tau}=\boldsymbol{\sigma}/\rho$ is the stress of Kirchhoff, $\boldsymbol{\sigma}$ is the stress of Cauchy, R isotropic hardening function of the cumulated plastic strain p, σ_1 and D_1 the parameters of the material. The presence in the plastic potential of the hydrostatic stress τ_m authorizes the changes of plastic volume.
In the event of nucleation of new cavities, one considers that the voluminal fraction created is proportional to the cumulated plastic strain. It is thus enough to replace f by $f+A_n p$ in the equations of the model. A_n is a parameter of the material. The equation [éq 1-1] is not modified.
In the viscoplastic case, one writes the viscoplastic *potential* F^{vp} like a function of the plastic potential F :

$$F^{vp}=\Lambda(F,p,f) \qquad \text{éq 1-3}$$

One will consider only the typical case such as:

$$\dot{p}=\frac{\partial\Lambda}{\partial F}=\dot{\varepsilon}_0\left[sh\left(\frac{F}{\sigma_0}\right)\right]^m \qquad \text{éq 1-4}$$

which is reduced to a function power (model of the type Norton) when two parameters of the material $\dot{\varepsilon}_0$ and σ_0 is very large.
Thereafter, one presents the behavior models of the model of Rousselier and his numerical integration.

2 Notations

One will note by:

$\mathbf{Id}$	tensor second-order identity
$\mathbf{II}$	tensor identity of the fourth order
$\mathrm{tr}\,\mathbf{A}$	traces second-order tensor $\mathbf{A}$
$\tilde{\mathbf{A}}$	left deviatoric the tensor $\mathbf{A}$ definite by $\tilde{\mathbf{A}}=\mathbf{A}-(\frac{1}{3}\mathrm{tr}\,\mathbf{A})\mathbf{Id}$
A_m	hydrostatic part of the tensor $\mathbf{A}$ definite by $A_m=\frac{\mathrm{tr}\,\mathbf{A}}{3}$
A_{eq}	equivalent value of von Mises defined by $A_{eq}=\sqrt{\frac{3}{2}\tilde{\mathbf{A}}:\tilde{\mathbf{A}}}$
:	doubly contracted product: $\mathbf{A}:\mathbf{B}=\sum_{i,j}A_{ij}B_{ij}=\mathrm{tr}(\mathbf{A}\mathbf{B}^{\mathbf{T}})$
$\otimes$	tensor product: $(\mathbf{A}\otimes\mathbf{B})_{ijkl}=A_{ij}B_{kl}$
λ,μ,E,ν,K	moduli of the isotropic elasticity
$\dot{p}$	equivalent plastic strainrate $\dot{p}=\sqrt{\frac{2}{3}\dot{\tilde{\varepsilon}}^{p}:\dot{\tilde{\varepsilon}}^{p}}$

In addition, in the frame of a discretization in time, all the quantities Q evaluated at previous time are subscripted by $^-$, the quantities evaluated at time $t=t^-+\Delta t$ are not subscripted and the increments are indicated par. Δ One has as follows:

$$Q=Q^-+\Delta Q$$

The numerical resolution is carried out by one θ - method, with $0\leq\theta\leq 1$. For all the quantities, one defines:

$$Q^{\theta}=Q^-+\theta\Delta Q$$

3 Model of Rousselier

We now describe derivative of the equations of the model of Rousselier presented in introduction.

3.1 Derivative of the equations of the model

One supposes that the specific free energy breaks up into three parts: a hyper elastic part which depends only on the elastic strain, a part related to the mechanism of hardening and a part related to the damage:

$$\Phi\left(\boldsymbol{\varepsilon}^{\mathbf{e}},\ p,f\right)=\Phi^{e}\left(\boldsymbol{\varepsilon}^{\mathbf{e}}\right)+\Phi^{p}\left(p\right)+\Phi^{f}\left(f\right) \qquad \text{éq 3.1-1}$$

the inequality of Clausius-Duhem is written (one does not consider the thermal part):

$$\boldsymbol{\tau}:\dot{\boldsymbol{\varepsilon}}-\dot{\Phi}\geq 0 \qquad \text{éq 3.1-2}$$

statement in which $\dot{\boldsymbol{\varepsilon}}=\dot{\boldsymbol{\varepsilon}}^{e}+\dot{\boldsymbol{\varepsilon}}^{p}$ strain rate represents.

Dissipation is still written:

$$\left(\boldsymbol{\tau}-\frac{\partial\Phi}{\partial\boldsymbol{\varepsilon}^{\mathbf{e}}}\right):\dot{\boldsymbol{\varepsilon}}^{e}+\boldsymbol{\tau}:\dot{\boldsymbol{\varepsilon}}^{p}-\frac{\partial\Phi}{\partial p}\dot{p}-\frac{\partial\Phi}{\partial f}\dot{f}\geq 0 \qquad \text{éq 3.1-3}$$

the second principle of the thermodynamics then requires the following statement for the elastic relation stress-strain:

$$\boldsymbol{\tau}=\frac{\partial\Phi}{\partial\boldsymbol{\varepsilon}^{e}} \qquad \text{éq 3.1-4}$$

One defines the thermodynamic forces associated with the elastic strain, the cumulated plastic strain and porosity in accordance with the frame of the generalized standard materials:

$$\boldsymbol{\tau}\left(\boldsymbol{\varepsilon}^{e}\right)=\frac{\partial\Phi}{\partial\boldsymbol{\varepsilon}^{e}} \qquad \text{éq 3.1-5}$$

$$A\left(p\right)=\frac{\partial\Phi}{\partial p} \qquad \text{éq 3.1-6}$$

$$B\left(f\right)=\frac{\partial\Phi}{\partial f} \qquad \text{éq 3.1-7}$$

It remains then for dissipation:

$$\boldsymbol{\tau}:\dot{\boldsymbol{\varepsilon}}^{p}-A\,\dot{p}\ -B\,\dot{f}\geq 0 \qquad \text{éq 3.1-8}$$

the principle of maximum dissipation applied starting from the viscoplastic *potential* $F^{\mathrm{vp}}(\boldsymbol{\tau},\mathrm{A},\mathrm{B})$ makes it possible to deduce the laws of evolution from them from the plastic strain, of the cumulated plastic strain and of porosity, that is to say:

$$\dot{\boldsymbol{\varepsilon}}^{p}=\frac{\partial F^{vp}}{\partial\boldsymbol{\tau}} \qquad \text{éq 3.1-9}$$

$$\dot{p}=-\frac{\partial F^{vp}}{\partial A}$$ éq 3.1-10

$$\dot{f}=-\frac{\partial F^{vp}}{\partial B}$$ éq 3.1-11

One supposes that $F^{\mathrm{vp}}(\boldsymbol{\tau}, A, B)$ is a function of the plastic *potential* $F(\boldsymbol{\tau}, A, B)$ and that this last breaks up into two terms depending respectively on the second invariant on $\boldsymbol{\tau}$ coupled to A and of the first invariant on $\boldsymbol{\tau}$ coupled to B :

$$F^{vp}=\Lambda(F)=\Lambda\left(F_{vM}(\tau_{eq}, A)+F_m(\tau_m, B)\right)$$ éq 3.1-12

Per assumption, the first term breaks up in an additive way like the potential of von Mises:

$$F_{vM}(\tau_{eq}, A)=\tau_{eq}-A(p)-R_0=\tau_{eq}-R(p)$$ éq 3.1-13

not to obtain result commonplace, the decomposition of the second term must be multiplicative:

$$F_m(\tau_m, B)=g(\tau_m)h(B)$$ éq 3.1-14

Taking into account the equation [éq 1-1], the laws of evolution for $\mathrm{tr}\,\dot{\boldsymbol{\varepsilon}}^p$ and $\dot{f}$ leads to the equality:

$$\frac{g'(\tau_m)}{g(\tau_m)}=\left(\frac{-1}{1-f}\right)\frac{h'(B(f))}{h(B(f))}$$ éq the 3.1-15

two members of this equation are functions of the two independent variables τ_m and f, therefore it is equal to a constant of dimension the reverse of a stress, it is the parameter of the material $1/\sigma_1$. The parameter without dimension D_1 appears in the integration of g'/g :

$$g(\tau_m)=D_1\sigma_1\exp\left(\frac{\tau_m}{\sigma_1}\right)$$ éq 3.1-16

the function $B(f)$ and the function reverses $f=h_1(B)$ are unknown. The most natural choice simplest and is to take $h_1\equiv h$, which gives:

$$h(B)\equiv h_1(B)=f$$ éq 3.1-17

$$h'(B)=\frac{df}{\mathrm{dB}}=-\frac{1}{\sigma_1}f(1-f)$$ éq 3.1-18

the plastic potential is written finally:

$$F=\tau_{eq}+\sigma_1 D_1 f\exp\left(\frac{\tau_m}{\sigma_1}\right)-R(p)$$ éq 3.1-19

the law of evolution for $\dot{p}$ gives:

$$\dot{p}=\frac{d\Lambda(F)}{\mathrm{dF}}=V(F)$$ éq 3.1-20

the function $V(F)$ defines the viscosity of the material. One will consider only the typical case such as:

$$V(F)=\dot{\varepsilon}_0\left[sh\left(\frac{F}{\sigma_0}\right)\right]^m$$ éq 3.1-21

which is reduced to a function power (model of the type Norton) when two parameters of the material $\dot{\varepsilon}_0$ and σ_0 is very large. Conversely one a:

$$F-S(\dot{p})=0$$ éq 3.1-22

$$S(\dot{p})=\sigma_0 sh^{-1}\left[\left(\frac{\dot{p}}{\dot{\varepsilon}_0}\right)^{\frac{1}{m}}\right]$$ éq 3.1-23

In the case of plasticity independent of time, the preceding equation becomes $F=0$ (criterion or threshold of plasticity) and $\dot{p}$ is given by the equation of consistency $\dot{F}=0$ if $F=0$ and $\dot{p}=0$ if $F<0$.

The equations of the model are now completely defined, in the case without nucleation of new cavities. In the event of nucleation of new cavities, one considers that the voluminal fraction created is proportional to the cumulated plastic strain. It is thus enough to replace f by $f+A_n p$ in the equations of the model. A_n is a parameter of the material. The equation [éq 1-1] is not modified.

3.2 Equations of the model

One summarizes the equations of the model deduced from the thermodynamic and phenomenologic analysis which precedes:

$$\Phi_{vp}=\tau_{eq}+\sigma_1 D_1\left(f+A_n p\right)\exp\left(\frac{\tau_m}{\sigma_1}\right)-R(p)-\sigma_0 sh^{-1}\left[\left(\frac{\dot{p}}{\dot{\varepsilon}_0}\right)^{\frac{1}{m}}\right]=0$$ **éq 3.2-1**

$$\boldsymbol{\tau}=\frac{\boldsymbol{\sigma}}{\rho}=\left[\lambda\left(\mathbf{Id}\otimes\mathbf{Id}\right)+2\mu\,\mathbf{II}\right]:\boldsymbol{\varepsilon}^e$$ éq 3.2-2

$$\rho=\frac{1-f-A_n p}{1-f_0}$$ éq 3.2-3

$$\tilde{\boldsymbol{\varepsilon}}^p=\dot{p}\frac{3\tilde{\boldsymbol{\sigma}}}{2\sigma_{eq}}=\dot{p}\frac{3\tilde{\boldsymbol{\tau}}}{2\tau_{eq}}$$ éq 3.2-4

$$tr\,\dot{\boldsymbol{\varepsilon}}^p=\dot{p}\,D_1\left(f+A_n p\right)\exp\left(\frac{\tau_m}{\sigma_1}\right)$$ éq 3.2-5

$$\dot{f}=A_1\left(1-f\right)tr\,\dot{\boldsymbol{\varepsilon}}^p$$ éq 3.2-6

with $A_1=1$, this parameter being introduced only for numerical reasons.

4 Numerical formulation

4.1 Keywords, material characteristics and local variables

For the foreseeable applications, the model was established under two distinct keywords: "ROUSS_PR `for the model plastic with nucleation of cavities or "ROUSS_VISC `for the model viscoplastic without nucleation. That allows D" to avoid useless numerical computations. The corresponding simplified equations are obtained starting from the general equations while posing respectively $\sigma_0=0$ or $A_n=0$.

L" together of the parameters of the model is provided under the key keys factors "ROUSSELIER `or "ROUSSELIER_FO `and "TENSION `(to define curve of tension) of the command DEFI_MATERIAU ([U4.43.01]). The parameters of the viscoplastic model ( $\sigma_0$ , $\dot{\varepsilon}_0$ and $m$ ) are provided by the key word "VISC_SINH `.

The local variables produced in *Code_Aster* are:

V1, cumulated plastic strain p ,
V2, porosity f ,
V3 with V8, the elastic strain tensor ε^e ,
V9, L" indicating of plasticity (0 if the last calculated increment is elastic, 1 if regular plastic solution, 2 if singular plastic solution).

We now present L" numerical integration of the constitutive law and give L" form of the tangent matrix (options FULL_MECA and RIGI_MECA_TANG).

4.2 Form of the model discretized

the numerical resolution is carried out by one θ - method, with $0\leq\theta\leq 1$, and in an incremental way. For all the quantities Q , one defines:

$$Q=Q^-+\Delta Q$$
$$Q^\theta=Q^-+\theta\,\Delta Q$$

The incremental writing requires the taking into account of the possible variation of the properties material (of the fact, for example, of a change of temperature during time step).
Resourcefulness" equations discretized is:

$$\tilde{\boldsymbol{\tau}}^\theta=2\,\mu\,\theta\,\Delta\tilde{\boldsymbol{\varepsilon}}^e+\frac{2\mu\,\theta+(1-\theta)\,2\mu^-}{2\mu^-}\tilde{\boldsymbol{\tau}}^-=2\,\mu\,\theta\left(\Delta\tilde{\boldsymbol{\varepsilon}}-\Delta\tilde{\boldsymbol{\varepsilon}}^p\right)+\frac{2\mu\,\theta+(1-\theta)\,2\mu^-}{2\mu^-}\tilde{\boldsymbol{\tau}}^- \qquad \text{éq 4.2-1}$$

$$\tau_m^\theta=K\,\theta\,\mathrm{tr}\,\Delta\boldsymbol{\varepsilon}^e+\frac{3\mathrm{K}*\theta+(1-\theta)\,3\mathrm{K}^-}{3\mathrm{K}^-}\tau_m^-=K\,\theta\left(\mathrm{tr}\,\Delta\boldsymbol{\varepsilon}-\mathrm{tr}\,\Delta\boldsymbol{\varepsilon}^p\right)\frac{3\mathrm{K}*\theta+(1-\theta)\,3\mathrm{K}^-}{3\mathrm{K}^-}\tau_m^- \qquad \text{éq 4.2-2}$$

$$\Delta\tilde{\boldsymbol{\varepsilon}}^p=\Delta\,p\,\frac{3\tilde{\boldsymbol{\tau}}^\theta}{2\,\tau_{eq}^\theta} \qquad \text{éq 4.2-3}$$

$$\mathrm{tr}\;\Delta\boldsymbol{\varepsilon}^p=\Delta\,pD_1\left(f^\theta+A_n\,p^\theta\right)\exp\left(\frac{\tau_m^\theta}{\sigma_1}\right) \qquad \text{éq 4.2-4}$$

$$\Delta f=A_1\left(1-f^\theta\right)\mathrm{tr}\,\Delta\boldsymbol{\varepsilon}^p \qquad \text{éq 4.2-5}$$

$$\Phi_{vp}^\theta=\tau_{eq}^\theta+\sigma_1\,D_1\left(f^\theta+A_n\,p^\theta\right)\exp\left(\frac{\tau_m^\theta}{\sigma_1}\right)-R(p^\theta)-\sigma_0\,sh^{-1}\left[\left(\frac{\Delta\,p}{\dot{\varepsilon}_0\,\Delta\,t}\right)^{\frac{1}{m}}\right]=0 \qquad \textbf{éq 4.2-6}$$

This system is reduced to the solution of only one equation scalar for the unknown Δf , knowing $\Delta \varepsilon$, Δt and the quantities Q^- . It is noted that ρ does not intervene in the algorithm, on the other hand it will intervene in the computation of the coherent tangent matrix. One calculates successively:

$$\tau_m^\theta = \frac{3\mathrm{K}*\theta+(1-\theta)3\mathrm{K}^-}{3\mathrm{K}^-}\tau_{m^-}+K\theta\left(\mathrm{tr}\,\Delta\varepsilon-\frac{\Delta f}{A_1(1-f^\theta)}\right) \qquad \text{éq 4.2-7}$$

Δp is the positive root of the quadratic equation:

$$A_n\theta(\Delta p)^2+\left(f^\theta+A_n p^-\right)\Delta p-\frac{\Delta f}{A_1(1-f^\theta)}\frac{1}{D_1\exp\left(\tau_m^\theta/\sigma_1\right)}=0 \qquad \text{éq 4.2-8}$$

$$\tilde{\boldsymbol{\tau}}^\theta=\left(1-\frac{3\mu\theta\Delta p}{\left[\frac{2\mu\theta+(1-\theta)2\mu^-}{2\mu^-}\tilde{\boldsymbol{\tau}}^-+2\mu\theta\Delta\tilde{\varepsilon}\right]_{eq}}\right)\left(\frac{2\mu\theta+(1-\theta)2\mu^-}{2\mu^-}\tilde{\boldsymbol{\tau}}^-+2\mu\theta\Delta\tilde{\varepsilon}\right) \qquad \text{éq 4.2-9}$$

$$\tau_{eq}^\theta=\left[\frac{2\mu\theta+(1-\theta)2\mu^-}{2\mu^-}\tilde{\boldsymbol{\tau}}^-+2\mu\theta\Delta\tilde{\varepsilon}\right]_{eq}-3\mu\theta\Delta p \qquad \text{éq 4.2-10}$$

the scalar equation for Δf is the equation [éq 4.2-6] $\Phi_{vp}^\theta=0$.

Notice 1 :

As Δf is very weak in most of structure, it would be preferable to use Δp as principal unknown. But in this case it is not possible to be brought back to a scalar equation, which makes more difficult the use of a method of the Newton type. It is also one of the reasons why the equations [éq 1-1], [éq 3.2-6] and [éq 4.2-5] *were not modified by the introduction of the nucleation of the cavities.*

Notice 2 :

The equation [éq 3.2-6] *can be integrated exactly:*

$$\mathrm{tr}\,\varepsilon^p=\frac{1}{A_1}\ln\left(\frac{1-f_0}{1-f}\right)$$

from where:

$$\mathrm{tr}\,\Delta\varepsilon^p=\frac{1}{A_1\theta}\ln\left(\frac{1-f^-}{1-f^\theta}\right)$$

As the numerical parameter A_1 can be modified in a discontinuous way, the derived form [éq 4.2-5] *was preserved, including in the computation of the coherent tangent matrix. If the use of the parameter A_1 were to be abandoned in a later version, it would be necessary to consider the use of the integrated form.*

Notice 3 :

The integrated form $\Phi_{vp}^\theta=0$ is used, including in plasticity instead of the relation of consistency $\dot{F}=0$ which gives $\dot{p}$. The coherent tangent matrix is calculated with this integrated form.

4.3 Resolution of the nonlinear scalar equation

the resolution of the equation $\Phi_{vp}^{\theta}(\Delta f)=0$ is carried out by an algorithm of Newton on terminals controlled in routine LCROUS. $\Phi_{vp}^{\theta}(\Delta f)$ and its derivative compared to Δf are calculated in routine RSLPHI called by LCROUS. The initial values of the limits are:

- lower limit: $\Delta f_1=0$ since $\Phi_{vp}^{\theta}(0)<0$ (it was checked as a preliminary that the elastic branch (negative threshold) is not solution),
- higher limit: Δf_2 as $\Phi_{vp}^{\theta}(0)>0$ sought by dichotomy between 0 and $1-f^-$ (first value for this search: $\frac{1-f^-}{2}$).

The algorithm of Newton begins with the value $\Delta f=0$. Whatever the value found for Δf one thus notes for the continuation that the function $\Phi_{vp}^{\theta}(\Delta f)$ and its derivative compared to Δf are at least calculated for $\Delta f=0$ and $\frac{1-f^-}{2}$.

The developments carried out to improve convergence and the robustness of the algorithm are described in [bib5].

4.4 Form of the tangent matrix of the behavior

One gives the form of the tangent matrix here (option FULL_MECA during iterations of Newton, option RIGI_MECA_TANG for the first iteration).

For the option RIGI_MECA_TANG, the tangent operator are the same one as that which connects $\boldsymbol{\varepsilon}^e$ to $\boldsymbol{\sigma}$ in [éq 3.2-2].

For the option FULL_MECA, the tangent matrix are obtained by linearizing the system of equations which governs the constitutive law: [éq 4.2-1] with [éq 4.2-6]. It is thus about a coherent tangent *matrix*.

To simplify the statements, one notes in this paragraph [§4.5]: Q for Q^{θ} , quantities all being expressed at time $t^{\theta}=t^-+\theta\Delta t$. The coherent tangent matrix is:

$$\frac{\delta\boldsymbol{\sigma}}{\delta\boldsymbol{\varepsilon}}=\rho\left[a_3\mathbf{II}+\mathbf{Id}\otimes\left(\frac{a_1-a_3}{3}\mathbf{Id}+a_2\tilde{\boldsymbol{\tau}}\right)+\tilde{\boldsymbol{\tau}}\otimes\left(a_4\tilde{\boldsymbol{\tau}}+\frac{a_5}{3}\mathbf{Id}\right)+\boldsymbol{\tau}\otimes\left(y_4\left(\frac{a_1}{3\mathrm{K}}-1\right)\mathbf{Id}+\frac{y_5}{\mathrm{K}}\tilde{\boldsymbol{\tau}}\right)\right] \quad \text{éq 4.4-1}$$

This operator is calculated in routine RSLJPL. The coefficients are calculated as follows:

$$a_1=3\mathrm{K}+y_1K\tau_{eq}(z_7+z_2\theta\Delta p) \quad \text{éq 4.4-2}$$

$$a_2=\mu(y_1+y_3)\sigma_1 \quad \text{éq 4.4-3}$$

$$a_3=\frac{2\mu\tau_{eq}}{z_5} \quad \text{éq 4.4-4}$$

$$a_4=3\mu y_2x_2 \quad \text{éq 4.4-5}$$

$$a_5=3\mu y_1\sigma_1 \quad \text{éq 4.4-6}$$

$$a_6=3\mu K\theta\Delta p-a_2\tau_{eq}\sigma_1 \quad \text{éq 4.4-7}$$

$$y_1=-\frac{3Kz_6z_1(f+A_np)}{x_1\tau_{eq}} \quad \text{éq 4.4-8}$$

$$y_2=-\frac{3\mu}{x_1z_5\tau_{eq}^2} \quad \text{éq 4.4-9}$$

$$y_3=-\frac{3Kz_6z_1A_n\theta\Delta p}{x_1\tau_{eq}} \quad \text{éq 4.4-10}$$

$$y_4=\frac{A_1z_8}{z_1}+\frac{z_9\sigma_1}{z_7+z_2\theta\Delta p} \quad \text{éq 4.4-11}$$

$$y_5=\frac{A_1a_2z_8}{z_1}-\frac{z_9a_6}{\tau_{eq}(z_7+z_2\theta\Delta p)} \quad \text{éq 4.4-12}$$

$$z_1=1+A_1\theta\Delta pD_1(f+A_np)\exp\left(\frac{\tau_m}{\sigma_1}\right) \quad \text{éq 4.4-13}$$

$$z_2=3\mu+R_{vp} \quad \text{éq 4.4-14}$$

$$z_3=K(f+A_np)z_1-A_1\sigma_1(1-f) \quad \text{éq 4.4-15}$$

$$z_4=R_{vp}\theta\Delta p-\tau_{eq} \quad \text{éq 4.4-16}$$

$$z_5=\tau_{eq}+3\mu\theta\Delta p \quad \text{éq 4.4-17}$$

$$z_6=D_1\exp\left(\frac{\tau_m}{\sigma_1}\right) \quad \text{éq 4.4-18}$$

$$z_7=z_6\sigma_1(f+A_np) \quad \text{éq 4.4-19}$$

$$z_8=\frac{1-f}{1-f-A_np} \quad \text{éq 4.4-20}$$

$$z_9=\frac{A_n}{1-f-A_np} \quad \text{éq 4.4-21}$$

$$x_1=z_3z_6(z_7+z_2\theta\Delta p)+z_1z_2\sigma_1-x_3 \quad \text{éq 4.4-22}$$

$$x_2=-z_3z_6\theta\Delta p(z_4+z_7)-z_1z_4\sigma_1+x_3\theta\Delta p \quad \text{éq 4.4-23}$$

$$x_3=A_nz_1z_6\sigma_1^2 \quad \text{éq 4.4-24}$$

$$R_{vp}=\frac{dR(p)}{dp}+\frac{1}{\theta\Delta t}\frac{dS(\Delta p/\Delta t)}{d\dot{p}} \quad \text{éq 4.4-25}$$

For the model plastic with nucleation of cavities "ROUSSELIER_PR `and for the model viscoplastic without nucleation "ROUSSELIER_VISC `, the corresponding simplified equations are obtained starting from the equations above while posing respectively $R_{vp}=dR(p)/dp$ and $A_n=0$.

5 Bibliography

1) ROUSSELIER G.: “Finite constitutive strain relations including ductile fracture ramming”, in Three-Dimensional Constitutive Relations and Ductile Fracture, ED. Nemat-Nasser, North Holland publishing company, pp. 331-355, 1981.

2) ROUSSELIER G.: “The Rousselier model for porous metal plasticity and ductile fracture”, in Handbook of Materials Behavior Models, ED. J. Lemaitre, Academic Close, pp. 436-445, 2001.

3) ROUSSELIER G.: “Dissipation in porous metal plasticity and ductile fracture”, J. Mech. Phys. Solids, vol. 49, pp. 1727-1746, 2001.

4) BARBER G.: “Models of Rousselier in *the Code_Aster* : new implementation”, EDF notes R & D HT-2C/98/007/A, 1998.

5) MASSON R., ROUSSELIER G., BONNAMY MR.: “Improvement of the convergence and evolution of L” implementation of the model of Rousselier in *Code_Aster*”, EDF notes R & D HT - 1/26/037 /A, 2002.

6) MIALON P.: “Elements D” analyzes and of numerical resolution of the relations of élasto - plasticity”, EDF, bulletin of DER, series C mathematical data processing, 3, pp. 57-89, 1986.

6 Features and checking

the constitutive laws described here are checked by the following tests:

For ROUSS_VISC :

CENTE01	CENTENARY. Validation of POST_ELEM option WEIBULL	[V1.02.001]
SSNP117	Models of Rousselier in 2D - DP	[V6.03.117]

For ROUSS_PR :

CENTE01	CENTENARY. Validation of POST_ELEM option WEIBULL	[V1.02.001]
SSNP117	Model of Rousselier in 2D - DP	[V6.03.117]
SSNV103	Traction test shears models of Rousselier	[V6.04.103]

7 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
6	*G. ROUSSELIER, R. MASSON, G. BARBER (EDF-R&D/MMC)*	initial Text
10	R.BARGELLINI (EDF-R&D/AMA)	Modification on the level of the algorithm discretized for taking into account of the changes of temperature