

## Integration of the viscoelastic behavior models in operator STAT\_NON\_LINE

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### Summarized

This document describes the viscoelastic behaviors in the case of the ingredients necessary to the implementation of nonlinear algorithm `STAT_NON_LINE` describes in [R5.03.01]. The data input of all the viscoelastic behavior models integrated in *Aster* have in a general way the same form. Only the way of introducing the principal data (the function viscous strainrate) varies: it is presented according to different the key word which makes it possible to the user to choose the desired behavior model.

These quantities are calculated by an semi-implicit or implicit integration method. From the initial state, or from the time of preceding computation, one calculates the stress field resulting from an increment of strain.

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## 1 Introduction

One describes here the implementation of the nonlinear model of viscoelasticity of Lemaître, which can be brought back for certain particular values of the parameters to a viscoelastic behavior model of Norton.

An alternative (depend on the fluence) of this model of Lemaître was added, for the modelization of the fuel assemblies (key word `LEMAITRE_IRRA`). This model has as a characteristic to comprise an additional unelastic strain: strain of growth.

A viscoelastic model with threshold was added. It is about a material whose behavior is purely elastic until a threshold then once this exceeded threshold, the behavior model becomes a cas particulier of the relation of Lemaître ( `LEMA_SEUIL`).

A model developed specifically to represent the nonlinear viscoelastic behavior of the pastilles of dioxide-to Uranium was more recently introduced. This model, heading `GATT_MONERIE`, are of the interest to be readjusted on a broad experimental basis (compression tests on various products in a broad range of temperature, load and velocity of request). The effects of porosities of fabrication, of size of grain and temperature on the velocity of creep steady of the pastilles, in particular, could be identified on these tests.

Lastly, a viscoelastic model with creep in logarithm of the fluence was established, it is accessible by keyword `VISC_IRRA_LOG`.

For each one of these models, one supposes that the material is isotropic (except for the strain of growth, which, it, is uniaxial). They can be used in 3D, plane strains (`D_PLAN`) and axisymmetric (`AXIS`).

One presents in this note the constitutive equations of the models and their establishment in *Code\_Aster*.

## 2 Relation One

continues places oneself on the assumption of the small disturbances and one divides the tensor of the strains into an elastic part, a thermal part, an unelastic part (known) and a viscous part. The equations are then:

$$\begin{aligned}\boldsymbol{\varepsilon}_{tot} &= \boldsymbol{\varepsilon}_e + \boldsymbol{\varepsilon}_{th} + \boldsymbol{\varepsilon}_a + \boldsymbol{\varepsilon}_v \\ \boldsymbol{\sigma} &= A(T) \boldsymbol{\varepsilon}_e \\ \dot{\boldsymbol{\varepsilon}}_v &= g(\sigma_{eq}, \lambda, T) \frac{3}{2} \frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\end{aligned}$$

with:

$$\lambda : \text{cumulated viscous strain} \quad \dot{\lambda} = \sqrt{\frac{2}{3}} \dot{\boldsymbol{\varepsilon}}_v : \dot{\boldsymbol{\varepsilon}}_v$$

$$\tilde{\boldsymbol{\sigma}} : \text{deviator of the stresses} \quad \tilde{\boldsymbol{\sigma}} = \boldsymbol{\sigma} - \frac{1}{3} Tr(\boldsymbol{\sigma}) I$$

$$\sigma_{eq} : \text{equivalent stress} \quad \sigma_{eq} = \sqrt{\frac{3}{2}} \tilde{\boldsymbol{\sigma}} : \tilde{\boldsymbol{\sigma}}$$

$$A(T) : \text{natural}$$

## 3 elasticity tensor of the function G for each behavior model

### 3.1 Relation LEMAITRE

In this case,  $g$  is expressed explicitly ( $\sigma$  is a scalar here):

$$g(\sigma, \lambda, T) = \left( \frac{1}{K} \frac{\sigma}{\lambda^{1/m}} \right)^n \quad \text{with} \quad \frac{1}{K} \geq 0, \frac{1}{m} \geq 0, n > 0$$

the data of the characteristics of materials are those provided under the key keys factors LEMAITRE or LEMAITRE\_FO of operator DEFI\_MATERIAU.

$$\text{LEMAITRE} = \_F \left( N=n, \quad \text{UN\_SUR\_K} = \frac{1}{K}, \quad \text{UN\_SUR\_M} = \frac{1}{m} \right)$$

the Young modulus  $E$  and the Poisson's ratio  $\nu$  are those provided under the key keys factors ELAS or ELAS\_FO.

### 3.2 Relation of LEMAITRE depending on fluence (LEMAITRE\_IRRA)

This paragraph J describes the dependence with respect to the fluence (and its processing) of the viscoplastic behavior model of. Lemaître, introduced for the modelization of the fuel assemblies and applicable to the elements 2D and 3D massive and the elements PIPE, under key word LEMAITRE\_IRRA.

#### 3.2.1 Formulation of the model

the equations are the following ones:

$$\left\{ \begin{array}{l} \dot{\varepsilon}_v = \frac{3}{2} \dot{p} \frac{\sigma^D}{\sigma_{eq}} \\ \dot{p} = \left[ \frac{\sigma_{eq}}{p^{1/m}} \right]^n \left( \frac{1}{K} \frac{\phi}{\phi_0} + L \right)^\beta e^{-\frac{Q}{R(T+T_0)}} \\ \underbrace{(A^{-1}(T)\sigma)} = \dot{\varepsilon}_{tot} - \dot{\varepsilon}_v - \dot{\varepsilon}_g - \dot{\varepsilon}_{th} \end{array} \right.$$

with:

$$T_0 = 273,15^\circ C$$

$$n > 0 \quad 1/K \geq 0 \quad 1/m \geq 0$$

$$\phi_0 > 0 \quad Q/R \geq 0 \quad L \geq 0, \quad \beta \text{ unspecified reality}$$

the coefficients are provided under key words LEMAITRE\_IRRA and ELAS of DEFI\_MATERIAU and  $\phi$  is the neutron flux (quotient of the increment of fluence, definite by the key word AFFE\_VARC of AFFE\_MATERIAU, by the increment of time).

The model of growth is:  $\varepsilon_g(t) = f(T, \Phi_t) \varepsilon_g^0$  with  $\varepsilon_g^0$  uniaxial strain unit in a reference  $R_1$  given by the user using key word MASSIF (see [U4.42.01] and [U4.43.01]) and  $f(T, \Phi_t)$  also a function defined by the user in DEFI\_MATERIAU ([U4.43.01]). Note:

The fluence, time and the flux must  $\Phi_0$  be expressed into cubes units such as the ratio is  $\Phi/\Phi_0$  without dimension. is  $Q/R$  in Kelvin. is  $T$  in. °C Processing

### 3.2.2 of the dependence with respect to the fluence The model

described above corresponds in fact to a normal model of Lemaître, defined by the three coefficients,  $n$  and  $1/K'$  with  $1/m$  : It

$$\frac{1}{K'} = \left( \frac{1}{K} \frac{\Phi}{\Phi_0} + L \right)^{\beta/n} e^{-\frac{Q}{nR(T+T_0)}}$$

is thus enough to calculate and  $1/K'$  to provide it like data to computation instead of.  $1/K$  In addition, in the computation of the elastic stress, one adds to the unelastic strains (null by default) the strain of growth expressed above, after having carried out the change of reference between the local coordinate system and the reference.  $R_1$  Relation

### 3.3 LEMA\_SEUIL For

this behavior,  $G$  is expressed also explicitly (since it is about a cas particulier of the relation of LEMAITRE presented Ci above): So

then  $D \leq 1$  (purely  $g(\sigma, \lambda, T) = 0$  elastic behavior) So

then  $D > 1$  with  $g(\sigma, \lambda, T) = A \left( \frac{2}{\sqrt{3}} \sigma \right) \Phi$ ,  $A \geq 0$  With  $\Phi \geq 0$

$$: D = \frac{1}{S} \int_0^t \sigma_{eq}(u) du$$

The data materials with informing by the user must be provided under key word LEMA\_SEUIL or LEMA\_SEUIL\_FO of operator DEFI\_MATERIAU: LEMA\_SEUIL

= \_F (A=A, S= S) As for

the parameter,  $\Phi$  it acts of flux of neutron which bombards the material (quotient of the increment of fluence, definite by the key word AFFE\_VARC of AFFE\_MATERIAU, by the increment of time).

The Young modulus and  $E$  the Poisson's ratio are  $\nu$  those provided under the key keys factors ELAS or ELAS\_FO. Relation

### 3.4 VISC\_IRRA\_LOG For

this relations,  $g$  is not expressed explicitly. The behavior is represented by a unidimensional creep test, with constant stress, which utilizes the time passed since time when the stress is applied. The behavior model is defined here by the data of four functions describing  $f_1, g_1, f_2, g_2$  the evolution of the viscous strain in the course of time:  $\varepsilon_q$

$$\varepsilon_v = \lambda = f_1(t) g_1(\sigma, T) + f_2(t) g_2(\sigma, T)$$

3.4-1

the function  $g$  is calculated then numerically by eliminating time in the following way  $t$  : for

- 1) a given triplet,  $(\sigma, \lambda, T)$  one solves in  $t$  the equation [éq 3.2-1] by the method of Newton (see [bib2]). One finds an approximation of the solution,  $t(\sigma, \lambda, T)$  one
- 2) obtains the value of the function by  $g$  deriving  $(\sigma, \lambda, T)$  from it compared to time the equation [éq 3.2-1] (see [bib1]): and

$$\dot{\varepsilon}_v = \dot{\lambda} = g(\sigma, \lambda, T) = f'_1(t)g_1(\sigma, T) + f'_2(t)g_2(\sigma, T)$$

in substituent in this new equation the value of previously  $t(\sigma, \lambda, T)$  found. One finds the formulation uniaxial following:

$$\dot{\varepsilon}_v = \dot{\lambda} = g(\sigma, \lambda, T) = f'_1(t(\sigma, \lambda, T))g_1(\sigma, T) + f'_2(t(\sigma, \lambda, T))g_2(\sigma, T)$$

The form of the four functions is  $f_1, g_1, f_2, g_2$  preset and the user introduces only some parameters into the command file. For

VISC\_IRRA\_LOG , one a:

$$f_1(t) = \ln(1 + \omega \cdot \Phi \cdot t)$$

$$g_1(\sigma, T) = A \cdot \sigma \cdot e^{-\frac{Q}{T}}$$

$$f_2(t) = \Phi \cdot t$$

$$g_2(\sigma, T) = B \cdot \sigma \cdot e^{-\frac{Q}{T}}$$

the parameter  $\Phi$  is the flux of neutrons. It is either indicated in DEF1\_MATERIAU, or taken equal to 1 and must then be indicated under factor key word the AFFE\_VARC .

The parameters  $A$  ,  $B$  and  $\omega$  are  $Q$  those provided under the key word factor VISC\_IRRA\_LOG of operator DEF1\_MATERIAU. It

will be noted that, for all the functions:

$t$  express yourself in hours

$T$  expresses itself in  $^{\circ}C$

$\sigma$  expresses itself in It  $MPa$

is thus necessary to return of the coherent data with these units in the command file and mesh file.

The Young modulus and  $E$  the Poisson's ratio are  $\nu$  those provided under the key words factors ELAS or ELAS\_FO . Relation

## 3.5 GATT\_MONERIE Because of

residual porosities of fabrication affecting the pastilles fuel worked out by sintering, the viscous strainrate included a component of thermal expansion, depend on the shears and average constraint according to: with

$$\dot{\varepsilon}_v = g(\sigma_{eq}, \sigma_m, \lambda, f, T) \frac{3}{2} \frac{\tilde{\sigma}}{\sigma_{eq}} + g_d(\sigma_{eq}, \sigma_m, \lambda, f, T) \frac{1}{3} I$$

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::

$f$  voluminal fraction of porosity:

$$\sigma_m \text{ average constraint } \sigma_m = \frac{1}{3} Tr(\sigma)$$

the functions derive  $(g, g_d)$  from a potential of dissipation according to  $\Psi(\sigma_{eq}, \sigma_m, \lambda, f, T)$  :

$$g = \frac{\partial \Psi}{\partial \sigma_{eq}} \cdot g_d = \frac{\partial \Psi}{\partial \sigma_m} \text{ This}$$

potential of dissipation does not depend on the cumulated plastic strain (see [3]) and is written: with

$$\Psi(\sigma_{eq}, \sigma_m, f, T) = (1 - \theta(\sigma_Y, T)) \Psi_1(\sigma_{eq}, \sigma_m, f, T) + \theta(\sigma_Y) \Psi_2(\sigma_{eq}, \sigma_m, f, T)$$

: corresponding

$$\sigma_Y = \sqrt{\frac{B_1}{B_1 + \frac{A_1}{4}} \sigma_{eq}^2 + \frac{9A_1}{4B_1 + A_1} \sigma_m^2}$$

$(\Psi_1, \Psi_2)$  with two viscous flow modes distinct (low stress and strong stress) defined by: functions

$$\Psi_i(\sigma_{eq}, \sigma_m, f) = \frac{\dot{\varepsilon}_{0i} \sigma_{0i}}{n_i + 1} \left[ A_i(f) \left( \frac{3\sigma_m}{2\sigma_{0i}} \right)^2 + B_i(f) \left( \frac{\sigma_{eq}}{\sigma_{0i}} \right)^2 \right]^{\frac{n_i+1}{2}}$$

$$\dot{\varepsilon}_{0i} = \dot{\varepsilon}_{0i} \chi_i(d) e^{-\frac{Q_i}{RT}}$$

$\chi_i$  of the size of grain such as: ,  $\chi_1(d) = d^{-2}$   $\chi_2(d) = 2d_0^2 \left( 1 - \cos\left(\frac{d}{d_0}\right) \right)$

the coefficients  $(A_i, B_i)$  are deduced from a micromechanical analysis: ,

$$A_i(f) = \left( n_i \left( f^{-\frac{1}{n_i} - 1} \right) \right)^{\frac{-2n_i}{n_i+1}}, \quad B_i(f) = \left( 1 + \frac{2}{3} f \right) (1 - f)^{\frac{-2n_i}{n_i+1}} \text{ function}$$

$\theta$  of coupling depend on the first invariant of the stresses and the temperature

the law of evolution of porosity is given by:  $\dot{f} = (1 - f) Tr(\dot{\varepsilon}_v)$

The final statement of the function is  $g$  : whereas

$$g(\sigma_{eq}, \sigma_m) = (1 - \theta) \frac{d\Psi_1}{d\sigma_{eq}} + \theta \frac{d\Psi_2}{d\sigma_{eq}} + \frac{d\theta}{d\sigma_{eq}} (\Psi_2 - \Psi_1)$$

$$\frac{d\Psi_i}{d\sigma_{eq}} = \dot{\varepsilon}_{0i} B_i(f) \left( \frac{\sigma_{eq}}{\sigma_{0i}} \right) \left[ A_i(f) \left( \frac{3\sigma_m}{2\sigma_{0i}} \right)^2 + B_i(f) \left( \frac{\sigma_{eq}}{\sigma_{0i}} \right)^2 \right]^{\frac{n_i-1}{2}}$$

$$\frac{d\theta}{d\sigma_{eq}} = \frac{1}{2} \left[ 1 - \tanh^2(\phi(\sigma_I)) \right] \frac{d\phi}{d\sigma_{eq}}$$



$$\begin{cases} \Phi(\sigma_Y) = \frac{T - \tilde{T}(\sigma_Y)}{h} \\ \tilde{T}(\sigma_Y) = w \sigma_Y^q \end{cases} \Rightarrow \frac{d\Phi}{d\sigma_{eq}} = \frac{d\Phi}{d\sigma_Y} \frac{d\sigma_Y}{d\sigma_{eq}} = \frac{-B_1 q w \sigma_{eq} \sigma_Y^{q-2}}{\left(B_1 + \frac{A_1}{4}\right) h}$$

that of the function is  $g_d$  :

$$g_d(\sigma_{eq}, \sigma_m) = (1 - \theta) \frac{d\phi_1}{d\sigma_m} + \theta \frac{d\phi_2}{d\sigma_m} + \frac{d\theta}{d\sigma_m} (\phi_2 - \phi_1)$$

$$\frac{d\Psi_i}{d\sigma_m} = \dot{\varepsilon}_{0i} A_i(f) \left( \frac{9\sigma_m}{4\sigma_{0i}} \right) \left[ A_i(f) \left( \frac{3\sigma_m}{2\sigma_{0i}} \right)^2 + B_i(f) \left( \frac{\sigma_{eq}}{\sigma_{0i}} \right)^2 \right]^{\frac{n_i-1}{2}}$$

$$\frac{d\theta}{d\sigma_m} = \frac{1}{2} \left[ 1 - \tanh^2(\phi(\sigma_Y)) \right] \frac{d\phi}{d\sigma_m}$$

$$\begin{cases} \varphi(\sigma_Y) = \frac{T - \tilde{T}(\sigma_Y)}{h} \\ \tilde{T}(\sigma_Y) = w \sigma_Y^q \end{cases} \Rightarrow \frac{d\Phi}{d\sigma_m} = \frac{d\Phi}{d\sigma_Y} \frac{d\sigma_Y}{d\sigma_m} = \frac{-9 A_1 q w \sigma_m \sigma_Y^{q-2}}{(4 B_1 + A_1) h}$$

The values of the various constants of the model are:

$$\begin{aligned} n_1 &= 1.0, Q_1 = 377 \cdot 10^3 \text{ J/mol}, \tilde{\varepsilon}_{01} = 2,725 \cdot 10^{-10} \text{ Pa/h} \\ n_2 &= 8.0, Q_2 = 462 \cdot 10^3 \text{ J/mol}, \tilde{\varepsilon}_{02} = 9,14 \cdot 10^{-41} \text{ Pa/h}, d_0 = 15 \text{ microns} \\ \sigma_{01} &= \sigma_{02} = 1 \text{ Pa} \\ h &= 600 \text{ K}, q = -0.189, w = 47350.4 \end{aligned}$$

The positive parameters as well as  $\tilde{\varepsilon}_{01}, \tilde{\varepsilon}_{02}, d$  the initial value of the voluminal fraction of pores are those provided under the key word factor GATT\_MONE of operator DEF1\_MATERIAU: GATT\_MONERIE

```
= _F (EPSI_01= ,  $\tilde{\varepsilon}_{01}$  EPSI
      _02= ,  $\tilde{\varepsilon}_{02}$  PORO
      _INIT=, f(0) GRAIN
      _COMB=) d Integration
```

## 4 of the behavior model Establishment

### 4.1 of the scalar equation for the implicit scheme and with constant elastic coefficients One

indicates by  $\varepsilon_{tot}$  the total deflection at time and  $t + \Delta t$   $\Delta \varepsilon_{tot}$  the variation of total deflection during time step running. One

calls  $\varepsilon_o$  the strain at time resulting  $t + \Delta t$  from thermal thermal expansion and the unelastic strains (among which possibly the strains of growth appear, cf [§3.2]). One thus has: where

$$\Delta \varepsilon_o = \left[ \alpha(t + \Delta t) (T(t + \Delta t) - T_{ref}) - \alpha(t) (T(t) - T_{ref}) \right] I_3 + \varepsilon_a(t + \Delta t) - \varepsilon_a(t)$$

is  $I_3$  the tensor identity of order 2 in dimension 3. One

poses As  $\Delta \varepsilon = \Delta \varepsilon_{tot} - \Delta \varepsilon_o$

one supposes here that is  $\mu$  constant, one has the following relation between the deviators of and  $\Delta \sigma$  :  $\Delta \varepsilon$  éq

$$\Delta \tilde{\sigma} = 2\mu (\Delta \tilde{\varepsilon} - \Delta \varepsilon_v) \quad \text{4.1-1 Gold}$$

, the model of flow is written, an implicit way: éq

$$\frac{\Delta \varepsilon_v}{\Delta t} = \frac{3}{2} g(\sigma_{eq}, \lambda^- + (\Delta \varepsilon_v)_{eq}, T) \frac{\tilde{\sigma}}{\sigma_{eq}} \quad \text{4.1-2 One}$$

thus has, while eliminating between  $\Delta \varepsilon_v$  [éq 4.1-1] and [éq 4.1-2]: éq

$$2\mu \Delta \tilde{\varepsilon} = \Delta \tilde{\sigma} + 3\mu \Delta t \cdot g(\sigma_{eq}, \lambda^- + (\Delta \varepsilon_v)_{eq}, T) \frac{\tilde{\sigma}}{\sigma_{eq}} \quad \text{4.1-3 While}$$

$$(\tilde{\sigma}^- + 2\mu \Delta \tilde{\varepsilon}) = \left( 1 + 3\mu \Delta t \frac{g(\sigma_{eq}, \lambda^- + (\Delta \varepsilon_v)_{eq}, T)}{\sigma_{eq}} \right) \tilde{\sigma}$$

posing,  $\tilde{\sigma}^e = \tilde{\sigma}^- + 2\mu \Delta \tilde{\varepsilon}$  one thus has: éq

$$\sigma_{eq}^e = 3\mu \Delta t . g\left(\sigma_{eq}, \lambda^- + (\Delta \varepsilon_v)_{eq}, T\right) + \sigma_{eq} \quad \text{4.1-4 Gold}$$

, one has according to [éq 4.1-2]: From where

$$(\Delta \varepsilon_v)_{eq} = \Delta t . g\left(\sigma_{eq}, \lambda^- + (\Delta \varepsilon_v)_{eq}, T\right)$$

: In

$$\begin{aligned} \sigma_{eq}^e &= 3\mu (\Delta \varepsilon_v)_{eq} + \sigma_{eq} \\ (\Delta \varepsilon_v)_{eq} &= \frac{1}{3\mu} (\sigma_{eq}^e - \sigma_{eq}) \end{aligned}$$

substituent this last statement in [éq 4.1-4], one a: If

$$\sigma_{eq}^e = 3\mu \Delta t . g\left(\sigma_{eq}, \lambda^- + \frac{1}{3\mu} (\sigma_{eq}^e - \sigma_{eq}), T\right) + \sigma_{eq}$$

one poses, and  $\sigma_{eq}^e, \lambda^-, T$  being  $\Delta t$  known: one

$$f(x) = 3\mu \Delta t . g\left(x, \lambda^- + \frac{1}{3\mu} (\sigma_{eq}^e - x), T\right) + x - \sigma_{eq}^e$$

can then calculate the quantity as  $\sigma_{eq} = (\sigma^- + \Delta \sigma)_{eq}$  being the solution of the scalar equation: where  $f(x) = 0$ ,  $x = \sigma_{eq}$  convention adopted for the following paragraphs. In the case of a model of Lemaître with threshold, LEMA\_IRRA\_SEUIL, the equations preceding are useful only once the crossed threshold. Indeed in-on this side threshold the behavior is elastic. One

discretizes the threshold implicitly: In the same way

$$D(\sigma^- + \Delta \sigma) = \frac{1}{S} \int_0^t (\sigma^- + \Delta \sigma)_{eq}(u) du$$

that for the elastoplastic integration of the constitutive laws of Von-Put, one distinguishes two cases then. then

$$D(\sigma^- + \Delta \sigma) \leq 1 \quad g(\sigma^- + \Delta \sigma, \lambda, T) = 0$$

$$D(\sigma^- + \Delta \sigma) > 1 \quad \text{It} \quad g(\sigma^- + \Delta \sigma, \lambda, T) = A \cdot \left(\frac{2}{\sqrt{3}} \sigma_{eq}\right) \varphi$$

results from it from the equation above: imply

$$g(\sigma^- + \Delta \sigma, \lambda, T) \neq A \cdot \left(\frac{2}{\sqrt{3}} \sigma_{eq}\right) \varphi \quad \text{Gold} \quad D(\sigma^- + \Delta \sigma) \leq 1$$

$g$  can take only the value or 0 thus  $A \cdot \left( \frac{2}{\sqrt{3}} \sigma_{eq} \right) \varphi$  implies

$g(\sigma^- + \Delta \sigma, \lambda, T) \neq A \cdot \left( \frac{2}{\sqrt{3}} \sigma_{eq} \right) \varphi$  and  $D(\sigma^- + \Delta \sigma) \leq 1$  is  $\Delta \sigma = A \cdot \Delta \varepsilon$

then  $g(\sigma^- + \Delta \sigma, \lambda, T) = A \cdot \left( \frac{2}{\sqrt{3}} \sigma_{eq} \right) \varphi$   $D(\sigma^- + A \cdot \Delta \varepsilon) > 1$

the criterion of crossing of the threshold can thus be written Gold  $D(\sigma^- + A \cdot \Delta \varepsilon) > 1$

By  $D(\sigma^- + A \cdot \Delta \varepsilon) = \frac{1}{S} \int_0^t (\sigma^- + A \cdot \Delta \varepsilon)_{eq}(u) du$

discretizing time, one has then: Resolution

$$D(\sigma^- + A \cdot \Delta \varepsilon) = \frac{1}{S} \int_0^{t^-} (\sigma^- + A \cdot \Delta \varepsilon)_{eq}(u) du + \frac{1}{S} \int_{t^-}^t (\sigma^- + A \cdot \Delta \varepsilon)_{eq}(u) du$$

$$D(\sigma^- + A \cdot \Delta \varepsilon) = \frac{1}{S} \left( D^- S + \frac{\sigma_{eq}^- + (\sigma^- + A \cdot \Delta \varepsilon)_{eq}(t - t^-)}{2} (t - t^-) \right)$$

## 4.2 of the scalar equation: principle of routine ZEROF 2 One

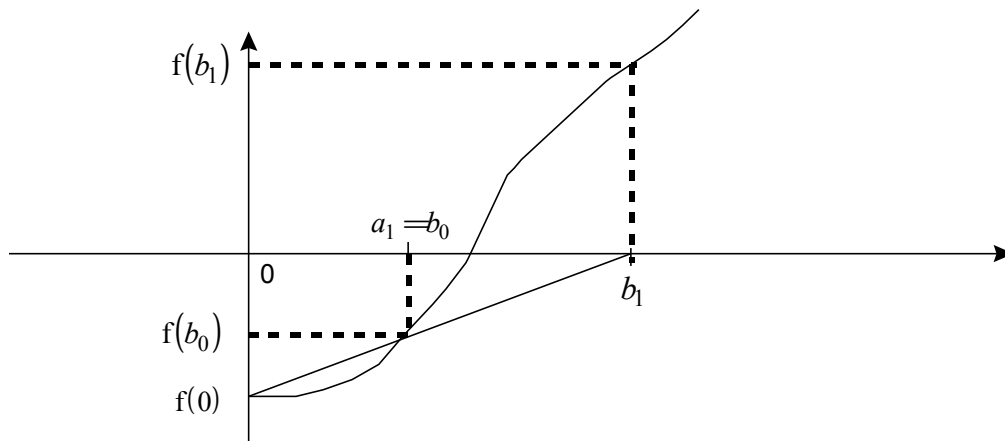
shows easily that, if the requirements in the paragraph [§3] on the characteristics of the materials are checked, the function is  $f$  strictly increasing and the equation admits  $f(x)=0$  a single solution. If

,  $\sigma_{eq}^e = 0$  then the solution is.  $x=0$  If not, one a:  $f(0) = -\sigma_{eq}^e < 0$

the problem thus consists in finding for an unspecified function  $f$  the solution of the equation knowing  $f(x)=0$  that this solution exists, that and  $f(0) < 0$  that is  $f$  strictly increasing.

The algorithm adopted in ZEROF 2 is the following: one

- leaves and  $a_0=0$  where  $b_0=x_{ap}$  is  $x_{ap}$  an approximation of the solution. If it is necessary (i.e. if),  $f(b_0) < 0$  one brings back oneself by the method of the secants (then  $z_n = \frac{a_n f(b_n) - b_n f(a_n)}{f(b_n) - f(a_n)}$  and  $a_{n+1} = b_n$ )  $b_{n+1} = z_n$  in one or more iterations if and  $f(a) < 0 : f(b) > 0$  (In



the case of the figure above, this first sentence was done in an iteration: and  $a_1 = b_0$  ).  
 $f(b_1) > 0$  one

- calculates  $= N_d$  left whole where  $(\sqrt{N_{\max}})$  is  $N_{\max}$  the maximum number of iterations whom one gave oneself. One then solves the equation by the method of the secants by means of however the method of dichotomy with each time is  $n$  multiple of:  $N_d - 1$ )

```

So
divided  $N_d$  if not  $n$ 

$$z_n = \frac{a_n + b_n}{2}$$

finsi

$$z_n = \frac{a_n f(b_n) - b_n f(a_n)}{f(b_n) - f(a_n)}$$

if
 $n = n + 1$ 
 $|f(z)| > \epsilon$ 
if not  $f(z) < 0$ 
 $a_{n+1} = z_n$      $b_{n+1} = b_n$ 
finsi
 $a_{n+1} = a_n$      $b_{n+1} = z_n$ 
to go
in 1) if not
    
```

the solution is: finis  $x = z_n \rightarrow FIN$

This

second part of the algorithm makes it possible to treat in a reasonable nombre of iterations the cases where is  $f$  very strongly nonlinear, whereas the method of the secants would have converged too slowly. These cases of strong non-linearity meet in particular with the model of LEMAITRE, for values the large ones  $\frac{n}{m}$  . Computation

## 4.3 of the stress at the end of time step running According to

[éq 4.1-3], if is  $x$  the solution of the scalar equation, while posing: one

$$b(x, \sigma_{eq}^e) = \frac{1}{1 + 3\mu \Delta t \frac{g\left(x, \lambda^- + \frac{1}{3\mu}(\sigma_{eq}^e - x), T\right)}{x}} = \frac{x}{\sigma_{eq}^e}$$

a: éq

$$\tilde{\sigma} = b(x, \sigma_{eq}^e) \tilde{\sigma}^e$$

**4.3-1 In the case where**

,  $\sigma_{eq}^e = 0$  which is equivalent according to the scalar equation to,  $x=0$  one prolongs by  $b$  continuity. For that, one poses and  $y(x) = \lambda^- + \frac{1}{3\mu}(\sigma_{eq}^e - x)$ .  $G(x) = g(x, y(x), T)$  The derivative of  $G$  is expressed according to derivatives partial of with  $g$  point:  $(x, y(x), T)$

$$G'(x) = \frac{\partial g}{\partial x}(x, y(x), T) - \frac{1}{3\mu} \frac{\partial g}{\partial y}(x, y(x), T)$$

The prolongation from  $b$  continuity gives then: and

$$b(0,0) = \frac{1}{1 + 3\mu \Delta t G'(0)}$$

one has, always in the case where  $\sigma_{eq}^e = 0$ .  $\tilde{\sigma} = 0$  Once

one calculated,  $\tilde{\sigma}$  one obtains by  $\sigma$  the relation ( $K$  is supposed to be constant here): éq

$$\sigma = \sigma^- + \Delta \sigma = \tilde{\sigma} + \left( \frac{1}{3} Tr(\sigma^-) + K Tr(\Delta \varepsilon) \right) I_3$$

**4.3-2 semi-implicit**

## 4.4 Diagram With

an implicit numerical diagram [éq 4.1-2], in the case, for example, where  $g$  does not depend on  $\lambda$  only intervenes by the computation of  $\Delta \varepsilon_v$  the value of the stress at the end of time step. It can result from it from the important numerical errors if the stress strongly varies in the course of time (see [bib2]).

To cure that and to improve the resolution, one discretizes the flow model in way semi - implicit: éq

$$\frac{\Delta \varepsilon_v}{\Delta t} = \frac{3}{2} g \left( \left( \sigma^- + \frac{\Delta \sigma}{2} \right)_{eq}, \lambda^- + \frac{(\Delta \varepsilon_v)_{eq}}{2}, T^- + \frac{\Delta T}{2} \right) \frac{\left( \tilde{\sigma}^- + \frac{\Delta \tilde{\sigma}}{2} \right)}{\left( \sigma^- + \frac{\Delta \sigma}{2} \right)_{eq}} \quad 4.4-1$$

to transform in the most economic way what was programmed previously (while following the implicit formulation [éq 4.1-2]), it is enough to divide each member of the equation [éq 4.4-1] by 2: and

$$\frac{(\Delta \varepsilon_v / 2)}{\Delta t} = \frac{3}{2} \frac{g \left( \left( \sigma^- + \frac{\Delta \sigma}{2} \right)_{eq}, \lambda^- + \frac{(\Delta \varepsilon_v)_{eq}}{2}, T^- + \frac{\Delta T}{2} \right) \left( \tilde{\sigma}^- + \frac{\Delta \tilde{\sigma}}{2} \right)}{2 \left( \sigma^- + \frac{\Delta \sigma}{2} \right)_{eq}}$$

to make the same thing with the relation [éq 4.1-1]: It

$$\frac{\Delta \tilde{\sigma}}{2} = 2 \mu \left( \frac{\Delta \tilde{\varepsilon}}{2} - \frac{\Delta \varepsilon_v}{2} \right)$$

is noted that this system is same form as that consisted the equations [éq 4.1-1] and [éq 4.1-2], the data being instead of  $\frac{\Delta \varepsilon}{2}$ ,  $\Delta \varepsilon$  the unknowns being respectively and  $\frac{\Delta \sigma}{2}$  instead of  $\frac{\Delta \varepsilon_v}{2}$  and

$\Delta \sigma$   $\Delta \varepsilon_v$  the function replacing  $\frac{g}{2}$  the function.  $g$  One

can thus use the resolution of the paragraphs [§4.1] with [§4.3] as well as the algorithm corresponding while introducing and  $\frac{\Delta \varepsilon}{2}$  by dividing the function by  $g$  2. It then remains to multiply the results and  $\frac{\Delta \sigma}{2}$  by  $\frac{\Delta \varepsilon_v}{2}$  2 to obtain the increments of stress and viscous strain calculated by the semi-implicit diagram (it and  $\Delta \sigma$  of  $\Delta \varepsilon_v$  the equation [éq 4.4-1]). It

will be noticed that the computation of the tangent operator is not affected by this modification of the numerical diagram. Indeed, one has obviously: Casparticulier

$$\frac{\partial \Delta \sigma}{\partial \Delta \varepsilon} = \frac{\partial \left( \frac{\Delta \sigma}{2} \right)}{\partial \left( \frac{\Delta \varepsilon}{2} \right)}$$

## 4.5 of model GATT\_MONERIE 1f

the reasoning of the §4.2 is taken again, one obtains L "equation: éq

$$\sigma_{eq}^e = 3\mu \cdot \Delta t \cdot g(\sigma_{eq}, \sigma_m, f, T) + \sigma_{eq} \quad 4.5-1$$

the fact that this model does not depend on the cumulated plastic strain thus simplifies this equation. On the other hand, an additional unknown is introduced: voluminal fraction of porosity. Another equation is thus necessary. To find it, it is enough D" to write the Hooke's law binding the spherical parts of the increments of stress and elastic strain according to: Knowing

$$Tr(\Delta \sigma) = 3K \left( Tr(\Delta \varepsilon) - Tr(\Delta \varepsilon_v) \right)$$

in addition that :

$$Tr(\Delta \varepsilon_v) = \frac{\Delta f}{1-f} \text{ one}$$

can express the average constraint according to the voluminal fraction of porosity, i.e.: , which

$$\Delta \sigma_m = K \left( Tr(\Delta \varepsilon) - \frac{\Delta f}{1-f} \right)$$

leads us to the second scalar equation: éq 4.5

$$\Delta f - (1-f) g_d \left( \sigma_{eq}, \sigma_m^- + 3K \left( Tr(\Delta \varepsilon) - \frac{\Delta f}{1-f} \right), f, T \right) = 0 \quad - 2 We$$

thus obtain two coupled scalar equations whose unknowns are the equivalent stress and the voluminal fraction of porosity: Let us note

$$\begin{cases} 3\mu \Delta t \cdot g \left( \sigma_{eq}, \sigma_m^- + 3K \left( Tr(\Delta \varepsilon) - \frac{\Delta f}{1-f} \right), f, T \right) + \sigma_{eq} - \sigma_{eq}^e = 0 \\ \Delta f - (1-f) g_d \left( \sigma_{eq}, \sigma_m^- + 3K \left( Tr(\Delta \varepsilon) - \frac{\Delta f}{1-f} \right), f, T \right) = 0 \end{cases}$$

the scalar  $f_d$  function definite by: , statement

$$f_d(x) = x - f^- - (1-x) g_d \left( \sigma_{eq}, \sigma_m^- + 3K \left( Tr(\Delta \varepsilon) - \frac{x-f^-}{1-x} \right), x, T \right)$$

in which the equivalent stress is regarded as a given parameter. It is noticed

that: . In addition

$$f_d(0) = -f^- \leq 0$$



, the sign of the trace viscoplastic strainrate is determined by the sign of the average constraint and this function of porosity: is monotonous

$$\sigma_m(x) = \sigma_m^- + 3K \left( Tr(\Delta \varepsilon) - \frac{x - f^-}{1 - x} \right)$$

decreasing on the interval and present  $[0; 1 - f^-[$  a vertical asymptote on the higher limit of this interval. Taking into account

these elements, two cases arise: : in

$$\begin{aligned} \sigma_m^- + 3K Tr(\Delta \varepsilon) < 0 &\Rightarrow \sigma_m(x) < 0 \quad \forall x \in [0; 1 - f^-[ \text{ this case. with } f_d(f^-) > 0 \\ \sigma_m^- + 3K Tr(\Delta \varepsilon) > 0 &\Rightarrow \sigma_m(f_{rig}) = 0 : \text{ in this case, } f_{rig} = (1 - f^-) \frac{\sigma_m^- + 3K Tr(\Delta \varepsilon)}{1 + \sigma_m^- + 3K Tr(\Delta \varepsilon)} \text{ and.} \\ f_d(f^-) < 0 \text{ in } f_d(f_{rig}) > 0 \end{aligned}$$

all the cases, we thus have a framing of the solution. On the other hand, the strict monotony of the function to be cancelled  $f_d$  is not guaranteed. In order to

use routine ZEROF 2, we proceed to a chained solution of these two scalar equations. One indeed carries out two imbricated calls to ZEROF 2: the first call solves equation 4.5.2. A each

iteration of this resolution, the current increment of porosity makes it possible  $\Delta f^i$  to calculate according to  $Tr(\Delta \sigma^i)$  : , then

$$Tr(\Delta \sigma^i) = 3K \left( Tr(\Delta \varepsilon) - \frac{\Delta f^i}{1 - f^- + \Delta f^i} \right)$$

by resolution  $\sigma_{eq}^i$  of equation 4.5.1 (second call to ZEROF 2) : .

$$3 \mu \Delta t . g \left( \sigma_{eq(i)}, \sigma_m^- + 3K \left( Tr(\Delta \varepsilon) - \frac{\Delta f_i}{1 - f^- - \Delta f_i} \right), \Delta f_i, T \right) + \sigma_{eq(i)} - \sigma_{eq}^e = 0 \text{ A good}$$

approximation for the porosity adopted  $(x_{ap})$  at the beginning of these iterations is obtained according to: . Once

$$\begin{aligned} \sigma_m^- + 3K Tr(\Delta \varepsilon) < 0 &\Rightarrow x_{ap} = f^- \\ \sigma_m^- + 3K Tr(\Delta \varepsilon) > 0 &\Rightarrow x_{ap} = f_{rig} = (1 - f^-) \frac{\sigma_m^- + 3K Tr(\Delta \varepsilon)}{1 + \sigma_m^- + 3K Tr(\Delta \varepsilon)} \end{aligned}$$

convergence reached, the computation of the stresses must take account of the variation of volume induced by the variations of voluminal fraction of porosity. Equation 4.3-2 must thus be modified according to: Lastly,

$$\sigma = \sigma^- + \Delta \sigma = \tilde{\sigma} + \left( \frac{1}{3} Tr(\sigma^-) + K \left( Tr(\Delta \varepsilon) - \frac{\Delta f}{1 - f} \right) \right) I_3$$

the variations of porosity are neglected during the computation of the tangent operator so that only the derivative of the function compared to the  $g$  equivalent stress is necessary: As

$$\left\{ \begin{aligned} \frac{dG(\sigma_{eq})}{d\sigma_{eq}} &= (1-\theta) \frac{d^2\psi_1}{d\sigma_{eq}^2} + \theta \frac{d^2\psi_2}{d\sigma_{eq}^2} + \frac{d^2\theta}{d\sigma_{eq}^2} (\psi_2 - \psi_1) + 2 \frac{d\theta}{d\sigma_{eq}} \frac{d(\psi_2 - \psi_1)}{d\sigma_{eq}} \\ \frac{d^2\psi_i}{d\sigma_{eq}^2} &= \frac{1}{\sigma_{eq}} \frac{d\psi_i}{d\sigma_{eq}} + \frac{n_i-1}{\sigma_{0i}} \dot{\varepsilon}_{0i} \left( B_i(f) \left( \frac{\sigma_{eq}}{\sigma_{0i}} \right) \right)^2 \left[ A_i(f) \left( \frac{3\sigma_m}{2\sigma_{0i}} \right)^2 + B_i(f) \left( \frac{\sigma_{eq}}{\sigma_{0i}} \right)^2 \right]^{\frac{n_i-3}{2}} \\ \frac{d^2\theta}{d\sigma_{eq}^2} &= \frac{1}{2} \frac{d^2\varphi}{d\sigma_{eq}^2} [1 - th^2(\varphi(\sigma_I))] - \left( \frac{d\varphi}{d\sigma_{eq}} \right)^2 th(\varphi(\sigma_I)) [1 - th^2(\varphi(\sigma_I))] \\ \Rightarrow \frac{d^2\theta}{d\sigma_{eq}^2} &= \frac{d\theta}{d\sigma_{eq}} \left[ \frac{1}{\sigma_{eq}} + \frac{9B_1(q-2)\sigma_{eq}}{(4B_1+A_1)\sigma_I^2} - 2 \frac{d\varphi}{d\sigma_{eq}} th(\varphi(\sigma_I)) \right] \end{aligned} \right.$$

explained to the §4.4, the adaptation with the semi-implicit case is brought back to a simple division by two of the two flow functions. Note:  $(g, g^d)$

for computation of the coefficients and according to  $A1$   $A2$  porosity, the following statement was used: .  
This

$$A_i(f) = f^{\frac{2}{n_i+1}} \left( n_i \left( 1 - f^{\frac{1}{n_i}} \right) \right)^{\frac{-2n_i}{n_i+1}}$$

second statement indeed has the advantage of being defined for a porosity null. Taken

## 4.6 into account of the variation of the elastic coefficients according to the temperature One has,

if is  $A$  the elasticity tensor: with

$$\Delta \varepsilon = \Delta \varepsilon_v + \Delta (A^{-1} \sigma)$$

: This

$$\Delta (A^{-1} \sigma) = A^{-1} (T^- + \Delta T) (\sigma^- + \Delta \sigma) - A^{-1} (T^-) \sigma^-$$

is translated in the equations of [§4.4] by: While posing

$$2\mu \left( \frac{\Delta \tilde{\varepsilon}}{2} \right) - \left( \tilde{\sigma}^- + \frac{\Delta \tilde{\sigma}}{2} \right) = 3\mu \Delta t \frac{g \left( \left( \sigma^- + \frac{\Delta \sigma}{2} \right)_{eq}, \lambda^- + \frac{(\Delta \varepsilon_v)_{eq}}{2}, T^- + \frac{\Delta T}{2} \right) \left( \tilde{\sigma}^- + \frac{\Delta \sigma}{2} \right)}{2 \left( \sigma^- + \frac{\Delta \sigma}{2} \right)_{eq}} - \tilde{\sigma}^- \left( \frac{2\mu^- + 2\mu}{4\mu^-} \right)$$

: and one

$$\tilde{\sigma}^e = \left( \frac{2\mu^- + 2\mu}{4\mu^-} \right) \tilde{\sigma}^- + 2\mu \left( \frac{\Delta \tilde{\varepsilon}}{2} \right)$$

$$\text{Tr}(\sigma^e) = \left( \frac{3K^- + 3K}{6K^-} \right) \text{Tr}(\sigma^-) + 3K \text{Tr} \left( \frac{\Delta \varepsilon}{2} \right)$$

is reduced exactly to the preceding case [§4.4]. Computation

## 5 of the tangent operator If

and,  $\sigma_{eq}^e = 0$  one  $x = 0$  takes the elasticity tensor as tangent operator. If not  
, one obtains this operator by deriving the equation [éq 4.3-1] compared to: then  $\Delta \varepsilon$

$$\frac{\partial \tilde{\sigma}}{\partial \Delta \varepsilon} = \frac{\partial \Delta \tilde{\sigma}}{\partial \Delta \varepsilon} = \frac{\partial b(x, \sigma_{eq}^e)}{\partial \Delta \varepsilon} \tilde{\sigma}^e + b(x, \sigma_{eq}^e) \frac{\partial \tilde{\sigma}^e}{\partial \Delta \varepsilon}$$

while also deriving [éq 4.3-2] compared to: It  $\Delta \varepsilon$  will be noted

$$\frac{\partial \Delta \sigma}{\partial \Delta \varepsilon} = \frac{\partial \Delta \tilde{\sigma}}{\partial \Delta \varepsilon} + KI_3 \frac{\partial \text{Tr}(\Delta \varepsilon)}{\partial \Delta \varepsilon} = \frac{\partial \Delta \tilde{\sigma}}{\partial \Delta \varepsilon} + KI_3^t I_3$$

that, in these equations, the tensors of order 2 and order 4 are respectively compared to vectors and matrixes. is here  $I_3$  a tensor of a nature 2, compared to a vector: One has

$${}^t I_3 = (1, 1, 1, 0, 0, 0)$$

moreover: It is thus necessary

$$\frac{\partial b(x, \sigma_{eq}^e)}{\partial \Delta \varepsilon} = \frac{\partial b}{\partial x}(x, \sigma_{eq}^e) \frac{\partial x}{\partial \Delta \varepsilon} + \frac{\partial b}{\partial \sigma_{eq}^e}(x, \sigma_{eq}^e) \frac{\partial \sigma_{eq}^e}{\partial \Delta \varepsilon}$$

to calculate. For  $\frac{\partial x}{\partial \Delta \varepsilon}$  that, one derives the scalar equation implicitly compared to.  $\Delta \varepsilon$

To simplify, one will omit thereafter in the writing of and  $g$  his derivatives the parameter. One  $T$  has  
then: From where

$$\left[ 3\mu \Delta t G'(x) + 1 \right] \frac{\partial x}{\partial \Delta \varepsilon} + \Delta t \frac{\partial g}{\partial y}(x, y) \frac{\partial \sigma_{eq}^e}{\partial \Delta \varepsilon} = \frac{\partial \sigma_{eq}^e}{\partial \Delta \varepsilon}$$

: with

$$\frac{\partial x}{\partial \Delta \varepsilon} = \frac{1 - \Delta t \frac{\partial g}{\partial y}(x, y)}{1 + 3\mu \Delta t G'(x)} \frac{\partial \sigma_{eq}^e}{\partial \Delta \varepsilon}$$

$$\frac{\partial x}{\partial \Delta \varepsilon} = \frac{1 - \Delta t \frac{\partial g}{\partial y}(x, y)}{1 + 3\mu \Delta t G'(x)} \frac{3\mu}{\sigma_{eq}^e} \tilde{\sigma}^e$$

the statement of obtained  $G'(x)$  with [§4.3]. One obtains

finally the following statement of the tangent operator: with

$$\frac{\partial \Delta \sigma}{\partial \Delta \varepsilon} = K \mathbf{I}_3^t \mathbf{I}_3 + 2 \mu \left[ \gamma \tilde{\sigma}^{e'} \tilde{\sigma}^e + b(x, \sigma_{eq}^e) \mathbf{A} \right]$$

where

$$A = \frac{\partial \Delta \tilde{\varepsilon}}{\partial \Delta \varepsilon} = \mathbf{J}_6 - \frac{1}{3} \mathbf{I}_3^t \mathbf{I}_3 \text{ is } \mathbf{J}_6 \text{ the matrix identity of row 6. Note:}$$

$$\gamma = \frac{3}{2(\sigma_{eq}^e)^3} \left[ \sigma_{eq}^e \frac{1 - \Delta t \frac{\partial g}{\partial y}(x, y)}{1 + 3 \mu \Delta t G'(x)} - x \right]$$

In

the case of model VISC\_IRRA\_LOG , one checks easily that: where

$$G'(x) = \frac{1}{f_1' g_1 + f_2' g_2} \left[ g_1 g_1' (f_1'^2 - f_1 f_1'') + g_2 g_2' (f_2'^2 - f_2 f_2'') + g_1 g_2' (f_1' f_2' - f_1'' f_2) \right. \\ \left. + g_2 g_1' (f_1' f_2' - f_1 f_2'') - \frac{1}{3m} (f_1'' g_1 + f_2'' g_2) \right]$$

$$\frac{\partial g}{\partial \lambda}(x, y, T) = \frac{f_1'' g_1 + f_2'' g_2}{f_1' g_1 + f_2' g_2}$$

indicate  $f_1, f_1', f_1'', f_2, f_2', f_2''$  the values of and  $f_1, f_2$  their derivatives at the point and where  $t(x, y, T)$  indicate  $g_1, g_1', g_2, g_2'$  the values of and  $g_1, g_2$  their derivative compared to at the point  $\sigma$  (see  $(x, T)$  [bib1]). Bibliography

## 6 of BONNIERES

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## 7 of the versions of the document Version

Aster Author	(S) Organization (S) Description	of modifications 7.4 P.
of	BONNIERES initial	Text 8.4 S.
LECLERCQ	, R.MASSON Models	GATT-MONERIE and LEMA_SEUIL 9.3 P.
of	BONNIERES Suppression	ZIRC_CYRA2, ZIRC_EPRI.