Behavior viscoplastic with effect of memory and restoration of Summarized

Chaboche:

This document describes the integration of the model of behavior élasto-visco-plastic of J.L. Chaboche with an isotropic hardening comprising a ratchet effect of hardening and two nonlinear kinematic hardenings, with taking into possible account of the restoration and viscosity. This model is usable by relation VISCOCHAB of key word COMP_INCR. The model established has an effect of hardening on the tensorial variables of recall and takes into account all the variations of the coefficients with the température1à[1]. This constitutive law is integrated by the resolution of a system of equations nonlinear. This model is available in 3D, plane strain, axisymetry. The modelization in plane stress is taken into account.

1 share for the computation of the jacobienne

Contents

1 Models élasto-visco-plastic of J.L. Chaboche with ratchet effect and of restoration

The model elastoviscoplastic of J.L.Chaboche [bib1] most complete superimposes on isotropic hardening two variables of kinematic hardening, and makes it possible to take into account the effects of cyclic hardening, creep, the restoration and the memory of largest hardening. It is thus well adapted to the cyclic loadings. Under certain

conditions of loading, the structures can be prone to a phenomenon of progressive strain, being able to harm the functional capacities of the device. In its original form, the model largely over-estimates the progressive strain; to improve its representativeness, it was modified by introducing terms of radial evanescence, in order to better model this phenomenon [bib2]. The model resulting, named VISCOCHAB, is described in this document. Several studies

consisted in testing this constitutive law with respect to its capacity with modelling the progressive strains [bib3, bib4], while comparing with tests [bib5, bib6, bib7, bib8] and with other models [bib12] in particular that of Taheri [R5.03.05]. In particular, identifications of steels 316L and 304L were made (resp. in [bib4] and [bib9]). Recently, the studies of thermal fatigue required the use of VISCOCHAB, in particular to take into account the effect of memory. The model

comprises 25 parameters (+ 2 elastic parameters) introduced into command DEFI_MATERIAU : VISCOCHAB (

```
VISCOCHAB_FO) = _F (# hardening
                        isotropic ♦K=, ◊B
                        =, ◊A                 k
                        _R=,    (    default       b
                           :             α_R 1  .) # kinematic
                     hardening

                        ♦C1=, ♦
                        C2=,    ♦             C_1
                        G1_0=  ,              C_2  ♦
                        G2_0=  ,            γ_1^0  ◊
                           A_I=, (   default      γ_2^0
                           :             a_∞ 1   .) # viscosity

                        ♦K_0=, ♦
                        N=,  #      exponential      K_0
                                             n

                        flow ◊A_K=, (default
                           :             α_K 0  .) ◊ALP=, (default
                           :             α  0      .) # effect of

                        memory ◊ETA=, (default
                           :             η  0.5    ) ♦MU=, (default
                           :             μ 0       .) ♦Q_M=, ♦
                        Q_0=   ,       #      Q_m
                        progressive                Q_0
```

```
strains (Burlet) ◊D1=, (default
    :            δ1 1      .) ◊D2=, (default
    :            δ 2 1  .) # restoration

◊M_R=, (default
    :            mr 1  .) ◊G_R=, (default
    :            γ r 0     .) ◊M_1=, (default
    :            m1 1  .) ◊M_2=, (default
    :            m2 1  .) ◊G_X1=, (default
    :          γ X1 0 .) ◊G_X2=, (default
    :          γ X2 0 .) ◊QR_0=, (default
    :          Q*r 0   .)) These parameters
```

are real constants. All these parameters can depend on the temperature (key words VISCOCHAB_FO) and the expected values are of standard function. There is no value by default in this case. It will be noted that if C1 or C2 depends on the temperature, the computation of the jacobian matrix is not exact, which can make convergence local difficult if C1 or C2 strongly varies with. The use T

of this constitutive law is accessible in commands STAT_NON_LINE or DYNA_NON_LINE by the key word VISCOCHAB from COMP_INCR. This complete

model can be "degraded" by cancelling the effect of certain mechanisms (for example the effect of restoration). For example, by cancelling the effect of the restoration and the terms of radial évanescence of kinematic hardening (materials parameters by default), one finds the model of Chaboche with effect of memory VISC_CIN2_MEMO [bib12]. For details

concerning the choice of materials parameters leading to the cancellation of certain effects, one will be able to refer to documentation of the command DEFI_MATERIAU . In the continuation

of this document, one describes VISCOCHAB precisely the model. One presents then the detail of his numerical integration in restrain with the construction of the coherent tangent matrix. The model

2 VISCOCHAB in `Code_Aster` *Description*

2.1 of the model At any moment

, the state of the material is described by the strain, the temperature $\boldsymbol{\varepsilon}$, the plastic strain T , the cumulated $\boldsymbol{\varepsilon}^{\mathbf{p}}$ plastic strain and the tensor p of recall. The equations of state $\mathbf{X}$ then define according to these variables of state the stress (broken up $\boldsymbol{\sigma}=\sigma^H \mathbf{Id}+\tilde{\boldsymbol{\sigma}}$ into hydrostatics parts and deviatoric), the isotropic share of hardening and the kinematical R share: with éq $\mathbf{X}$ 2.1

$$\sigma^H=\frac{1}{3}tr(\boldsymbol{\sigma})=K\,tr(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^{\mathbf{th}})$$ **- 1 éq** $$\boldsymbol{\varepsilon}^{\mathbf{th}}=\alpha(T-T^{ref})\,\mathbf{Id}$$ **2.1-2 éq**

$$\tilde{\boldsymbol{\sigma}}=\boldsymbol{\sigma}-\sigma^H\,\mathbf{Id}=2\,\mu(\tilde{\boldsymbol{\varepsilon}}-\boldsymbol{\varepsilon}^{\mathbf{p}})$$ **2.1-3 éq**

$$R=R(p)$$ **2.1-4 where**

$$\mathbf{X}=(\mathbf{p},\boldsymbol{\varepsilon}^{\mathbf{p}})$$ **and the**

coefficients K,μ,α of and are $\mathbf{X}(p)$ characteristics $R(p)$ of the material which can depend on the temperature. More precisely, they are respectively the moduli of compressibility and shears, the thermal coefficient of thermal expansion, the functions of isotropic and kinematical hardening. As for, it is T^{ref} the reference temperature, for which one regards the thermal strain as being null. Flow surface

2.1.1 - viscoplastic potential flow surface

associated with model VISCOCHAB `is represented` within the space of principal stresses by: its center

- , tensor D $\mathbf{X}$ “kinematic hardening, its size
- , being its $\alpha_R R+k$ initial k size and the variable R D” isotropic hardening giving the evolution of this size, modulated by the coefficient, its form α_R given
- by the criterion of Von Mises steady to. is the deviator $\tilde{\boldsymbol{\sigma}}$**-X** $\tilde{\boldsymbol{\sigma}}$ of the stresses. The evolution

of the plastic strain is controlled by a normal flow model to a plasticity criterion of Von Mises: éq 2.1-5

$$F(\boldsymbol{\sigma},R,\mathbf{X})=(\tilde{\boldsymbol{\sigma}}-\mathbf{X})_{eq}-\alpha_R R(p)-k \quad \text{avec } A_{eq}=\sqrt{\frac{3}{2}\tilde{A}:\tilde{A}}$$ **the viscoplastic**

potential of dissipation for the model of Chaboche is written: éq 2.1-6 where

$$\Omega^p=\frac{K_0+\alpha_k R}{\alpha(n+1)}\exp\left[\alpha\left\langle\frac{F}{K_0+\alpha_k R}\right\rangle^{n+1}\right]$$ **is**

the positive $\langle F\rangle$ part of. One from of deduced F

the law of evolution from the plastic strain: éq 2.1-7 while

$$\dot{\boldsymbol{\varepsilon}}^p=\frac{\partial\Omega^p}{\partial\boldsymbol{\sigma}}=\dot{p}\,\frac{\partial F}{\partial\boldsymbol{\sigma}}=\frac{3}{2}\dot{p}\,\frac{\tilde{\boldsymbol{\sigma}}-\mathbf{X}}{(\tilde{\boldsymbol{\sigma}}-\mathbf{X})_{eq}}$$ **having posed**

: éq 2.1-8 Classically

$$\dot{p}=\left\langle\frac{F}{K_0+\alpha_k R}\right\rangle^n\exp\left[\alpha\left\langle\frac{F}{K_0+\alpha_k R}\right\rangle^{n+1}\right]$$

, is the cumulated $\dot{p}$ plastic strainrate. The parameters, and are K_0 relating α_k α to the viscosity of the material (viscosity of Norton). The equation [éq 2.1-7] can be also written in an equivalent way in the following form: éq 2.1-9

$$\dot{p}=\sqrt{\frac{2}{3}\dot{\boldsymbol{\varepsilon}}^p:\dot{\boldsymbol{\varepsilon}}^p}$$ **the unit**

norm at flow surface is noted: éq the 2.1-10

$$\mathbf{n}=\frac{\partial F}{\partial \boldsymbol{\sigma}}\left\|\frac{\partial F}{\partial \boldsymbol{\sigma}}\right\|^{-1}=\sqrt{\frac{2}{3}}\frac{\partial F}{\partial \boldsymbol{\sigma}}=\sqrt{\frac{3}{2}}\frac{\tilde{\boldsymbol{\sigma}}-\mathbf{X}}{(\tilde{\boldsymbol{\sigma}}-\mathbf{X})_{eq}}$$ **laws of**

evolution of isotropic hardening and kinematic hardening are obtained starting from the thermodynamic potential and from the potential of dissipation. Formulation

2.1.2 of isotropic hardening the evolution

of the isotropic variable of hardening is given R by: éq 2.1-11

$$\dot{R}=b(Q-R)\dot{p}+\gamma_r|Q_r-R|^{m_r}\operatorname{sgn}(Q_r-R)$$ **This model D**

"evolution of L" isotropic hardening utilizes a first linear term according to the cumulated viscoplastic strainrate. This term $\dot{p}$ is useful to describe the evolution of the loop (softening or hardening) in cyclic loading. This term provides an asymptotic value of (corresponding R at the stabilized state) equal to the variable. This variable Q represents Q cyclic hardening (the effect of memory of the maximum strains). It is not a constant but it depends on the maximum amplitude of the strain (effect of memory): éq 2.1-12

$$Q=Q_0+(Q_m-Q_0)(1-e^{-2\mu q})$$ **One defines**

within the space of strains a flow surface inside which is a constant Q (field of NON-hardening): éq 2.1-13

$$f(\boldsymbol{\varepsilon}^{\mathbf{p}},\boldsymbol{\xi},q)=\frac{2}{3}(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}-q\leq 0$$ **This criterion**

defines a field characterizing the plastic strains maximum ones, of which measures the radius q and the center, $\boldsymbol{\xi}$ calculated according to a model of normality: éq 2.1-14

$$\dot{\boldsymbol{\varepsilon}}^p:\frac{\partial f}{\partial \boldsymbol{\varepsilon}^{\mathbf{p}}}>0$$ **the unit**

norm at flow surface is noted: éq the 2.1-15

$$\mathbf{n}^*=\frac{\partial f}{\partial \boldsymbol{\sigma}}\left\|\frac{\partial f}{\partial \boldsymbol{\sigma}}\right\|^{-1}=\sqrt{\frac{3}{2}}\frac{\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi}}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{\mathbf{eq}}}$$ **laws of**

evolution of the variables and are given q $\boldsymbol{\xi}$ in the form: éq 2.1-16

$$\dot{q}=\eta H(f)\langle\mathbf{n}:\mathbf{n}^*\rangle\dot{p}$$ **éq 2.1-17**

$$\dot{\boldsymbol{\xi}}=\sqrt{\frac{3}{2}}(1-\eta)H(f)\langle\mathbf{n}:\mathbf{n}^*\rangle\dot{p}\ \mathbf{n}^*$$ **the parameter**

(which does not exist η in the initial formulation 5) makes it possible 5to partially take into account the effect of memory. If it is equal to 1/2, the initial formulation is found. If it is worth 1, to the norm of q the more plastic large deformation attack is equal. If it is much lower than 1/2, the effect of memory is taken into account partly only. Note:

The same

statements for, and Q f are used $\dot{q}$ $\dot{\xi}$ in models VMIS/VISC `_CIN1` / `2_MEMO` `[feeding-bottle 11]` `which` are simplified versions (and optimized) of VISCOCHAB cf `[R5.03.04`]. The law of evolution of isotropic hardening [éq 2.1-11] utilized a second term, allowing to take into account the effect of the restoration. The variable is given Q_r by the equation: éq 2.1-18

$$Q_r=Q-Q_r^*\left[1-\left(\frac{Q_m-Q}{Q_m}\right)^2\right]$$ **Let us note that**

in the model initial of Chaboche [bib1], the coefficient in the equation m_r [éq 2.1-11] is worth 1. Formulation

2.1.3 of kinematic hardening Before giving

the form of the model of hardening of model VISCOCHAB, `one points out` the various stages which allowed its development. The simplest

model is a linear hardening of the form: éq 2.1-19

$$\dot{\mathbf{X}}=\frac{2}{3}C\,\dot{\boldsymbol{\varepsilon}}^p$$ **With this model**

, one can add a nonlinear term of recall providing an effect of evanescent memory of the way of loading (models initial of Chaboche): éq 2.1-20

$$\dot{\mathbf{X}}=\frac{2}{3}C\,\dot{\boldsymbol{\varepsilon}}^{\mathbf{p}}-\gamma\,\mathbf{X}\,\dot{p}$$ **with éq 2.1**

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$$\gamma=\gamma^0\left[a_\infty+\left(1-a_\infty\right)e^{-bp}\right]$$ **It**

was shown that such a model largely over-estimates the phenomenon of progressive strain. This brings to introduce a term with radial evanescence (term due to Burlet and Cailletaud): éq 2.1-22

$$\dot{\mathbf{X}}=\frac{2}{3}C\,\dot{\varepsilon}^p-\gamma\left(\mathbf{X}:\mathbf{n}\right)\mathbf{n}\;\dot{p}$$ **In fact, this**

model underestimates the phenomenon of progressive strain now. One can then combine the two equations [éq 2.1.20] and [éq 2.1.22] with the parameter of weighting, in order to better $\delta\in\left[0,1\right]$ consider the strains progressive: éq 2.1-23

$$\dot{\mathbf{X}}=\frac{2}{3}C\,\dot{\boldsymbol{\varepsilon}}^p-\left[\delta\left(\gamma\,\mathbf{X}\,\dot{p}\right)+\left(1-\delta\right)\left(\gamma\left(\mathbf{X}:\mathbf{n}\right)\mathbf{n}\;\dot{p}\right)\right]$$ **In the model**

initial of Chaboche, one finds also a term additional which makes it possible to introduce the effects of the restoration into kinematic hardening, which gives to final the following model: éq 2.1-24

$$\dot{\mathbf{X}}=\frac{2}{3}C\,\dot{\boldsymbol{\varepsilon}}^p-\gamma\left[\delta\;\mathbf{X}+\left(1-\delta\right)\left(\mathbf{X}:\mathbf{n}\right)\mathbf{n}\right]\dot{p}-\gamma_X\left[\left(\mathbf{X}\right)_{eq}\right]^{m-1}\mathbf{X}$$ **with given**

by γ the equation [éq 2.1-21]. For an exact

taking into account of the dependence of materials parameters compared to the temperature, it is necessary to add an additional term in, which gives $\dot{T}$: éq 2.1-25

$$\dot{\mathbf{X}}=\frac{2}{3}C\,\dot{\boldsymbol{\varepsilon}}^p-\gamma\left[\delta\;\mathbf{X}+\left(1-\delta\right)\left(\mathbf{X}:\mathbf{n}\right)\mathbf{n}\right]\dot{p}-\gamma_X\left[\left(\mathbf{X}\right)_{eq}\right]^{m-1}\mathbf{X}+\frac{1}{C}\frac{\partial C}{\partial T}\mathbf{X}\,\dot{T}$$ **with given**

by γ the equation [éq 2.1-21]. The model

VISCOCHAB proposed comprises in fact 2 variables of kinematic hardening and whose $\mathbf{X_1}$ laws $\mathbf{X_2}$ of evolutions are given by the equation [éq 2.1-25]. Assessment:

The law of
evolution of hardening is form: éq 2.1-26

$$\begin{cases} \mathbf{X}=\mathbf{X_1}+\mathbf{X_2} \\ \dot{\mathbf{X}}_i=\frac{2}{3}C_i\dot{\boldsymbol{\varepsilon}}^p-\gamma_i\left[\delta_i\,\mathbf{X_i}+\left(1-\delta_i\right)\left(\mathbf{X_i}:\mathbf{n}\right)\mathbf{n}\right]\dot{p} \\ \quad -\gamma_{Xi}\left[\left(\mathbf{X_i}\right)_{eq}\right]^{m_i-1}\mathbf{X_i}+\frac{1}{C_i}\frac{\partial C_i}{\partial T}\mathbf{X_i}\dot{T} \quad i=1,2 \end{cases}$$ **with éq 2.1**

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$$\gamma_i=\gamma_i^0\left[a_\infty+\left(1-a_\infty\right)e^{-bp}\right] \quad i=1,2$$ **implicit**

2.2 Integration to carry out

the integration of the constitutive law numerically, one carries out a discretization in time and one adopts a diagram of implicit, famous Eulerian adapted for elastoplastic behavior models. It is the method used by default. It is also possible to carry out an explicit integration (see §2.3) by choosing2.3 key word ALGO_INTE=' RUNGE_KUTTA'. Henceforth
, the following notations will be employed: etreprésentent A^-, A^+ ΔA respectively the values of a quantity at the beginning and at the end of time step considered thus that its increment during the step. There is thus the relation: . The problem $\Delta A=A^+-A^-$ is then the following: knowing the state at time as well as t^- the increments of strain (resulting from $\Delta\boldsymbol{\varepsilon}$ the phase of prediction (cf STAT_NON_LINE [R5.03.01]) and of temperature, to determine ΔT the state of the local variables at time as well as t the stresses. System of equations $\boldsymbol{\sigma}$

2.2.1 implicit

discretization of the problem led to a system of 27 equations: Relation stress-strain

6 éq éq 2.2	$\Delta\boldsymbol{\sigma}-H\left(\Delta\boldsymbol{\varepsilon}-\Delta\boldsymbol{\varepsilon}^{\mathbf{th}}-\Delta p\frac{\partial F}{\partial\boldsymbol{\sigma}^+}\right)=0$	- 1 Kinematic hardening	
12 éq éq	$\begin{cases} i=1,2 \\ \Delta\mathbf{X_i}-\frac{2}{3}C_i\Delta\boldsymbol{\varepsilon}^{\mathbf{p}}+\gamma_i^+\left[\delta_i\,\mathbf{X_i^+}+\left(1-\delta_i\right)\left(\mathbf{X_i^+}:\mathbf{n}^+\right)\mathbf{n}^+\right]\Delta p \\ \quad +\gamma_{Xi}\left[\left(\Delta\mathbf{X_i^+}\right)_{eq}\right]^{m_i-1}\Delta\mathbf{X_i^+}\Delta t-\frac{1}{C_i}\frac{\partial C_i}{\partial T}\mathbf{X_i^+}\Delta T=0 \end{cases}$	2.2-2 cumulated	
Plasticity 1 éq éq 2.2	$\Delta p-\Delta t\left\langle\frac{F^+}{K_0+\alpha_k R^+}\right\rangle^n\exp\left[\alpha\left\langle\frac{F^+}{K_0+\alpha_k R^+}\right\rangle^{n+1}\right]=0$	- 3 isotropic	
Hardening 1 éq éq 2.2	$\Delta R-b\left(Q^+-R^+\right)\Delta p-\gamma_r\left\|Q_r^+-R^+\right\|^{m_r}\operatorname{sgn}\left(Q_r^+-R^+\right)\Delta t=0$	- 4 Effect	**of memory**

1 éq éq 2.2	$\Delta\ q-\eta H(f)\langle\mathbf{n}:\mathbf{n}^*\rangle^+\Delta p=0$	- 5 6	**éq éq 2.2**
	$\Delta\xi-\sqrt{\frac{3}{2}}(1-\eta)H(f)\langle\mathbf{n}:\mathbf{n}^*\rangle^+\Delta p\ (\mathbf{n}^*)^+=0$	- 6 In	**this system**

, is given γ_i^+ according to by the equation p^+ [éq. 2.1-27]; and are obtained Q^+ Q_r^+ according to by the equations q^+ [éq. 2.1-12] and [éq. 2.1-18]. The 27 unknowns

are: $\Delta\boldsymbol{\sigma}$, $\Delta\mathbf{X_1}$ and $\Delta\mathbf{X_2}$. Δp Note: ΔR Δq $\Delta\boldsymbol{\xi}$

Contrary
to VISC_CIN2_MEMO [R5.03.04], the equation of checking of the threshold of the maximum strains: (see [éq. $f(\boldsymbol{\varepsilon}^{\mathbf{p}},\boldsymbol{\xi},q)=\frac{2}{3}(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}-q=0$ 2.1-13]) is not taken into account. General outline

2.2.2 of resolution One calculates

the stress by making the assumption of a purely elastic increment (). éq 2.3 - $\Delta p=0$ 1 One

$$\Delta\boldsymbol{\sigma}=H\Delta\boldsymbol{\varepsilon}\ \text{avec}\ \Delta\boldsymbol{\varepsilon}^{\mathbf{p}}=\Delta\mathbf{X_i}=\Delta\boldsymbol{\xi}=0$$
$$\Delta R=\Delta p=\Delta q=0$$

calculates

the function threshold. So then $F^+=(\tilde{\boldsymbol{\sigma}}-\mathbf{X})_{eq}^+-\alpha_R R^+-k$ the increment $F^+\leq 0$ is purely elastic and integration is finished. If not of the viscoplastic corrections are carried out by solving the (S1) system according to: Relation stress-strain

6 éq éq 2.3	$\Delta\boldsymbol{\sigma}-H\left(\Delta\boldsymbol{\varepsilon}-\Delta\boldsymbol{\varepsilon}^{\mathbf{th}}-\Delta p\frac{\partial F}{\partial\boldsymbol{\sigma}^+}\right)=0$	- 2 Kinematic hardening	
12 éq éq	$\begin{cases} i=1,2 \\ \Delta\mathbf{X_i}-\frac{2}{3}C_i\Delta\boldsymbol{\varepsilon}^{\mathbf{p}}+\gamma_i^+\left[\delta_i\ \mathbf{X_i^+}+(1-\delta_i)(\mathbf{X_i^+}:\mathbf{n}^+)\mathbf{n}^+\right]\Delta p \\ \quad +\gamma_{Xi}\left[(\mathbf{X_i^+})_{eq}\right]^{m_i-1}\mathbf{X_i^+}\Delta t-\frac{1}{C_i}\frac{\partial C_i}{\partial T}\mathbf{X_i}\Delta T=0\end{cases}$	2.3-3 cumulated	
Plasticity 1 éq éq 2.3	$\Delta p-\Delta t\left(\frac{F^+}{K_0+\alpha_k R^+}\right)^n\exp\left[\alpha\left(\frac{F^+}{K_0+\alpha_k R^+}\right)^{n+1}\right]=0$	- 4 isotropic	
Hardening 1 éq éq 2.3	$\Delta R-b(Q^+-R^+)\Delta p-\gamma_r\lvert Q_r^+-R^+\rvert^{m_r}\operatorname{sgn}(Q_r^+-R^+)\Delta t=0$	- 5 Effect	**of memory**
1 éq éq 2.3	$\Delta\ q=0$	- 6 6	**éq éq 2.3**

	$\Delta \boldsymbol{\xi}=0$	- the 7	**20 unknowns**

are: $\Delta\boldsymbol{\sigma}$. $\Delta\mathbf{X_1}$ It $\Delta\mathbf{X_2}$ is noticed Δp ΔR that and are not part Δq of $\Delta\boldsymbol{\xi}$ the unknowns of it because their solution is commonplace. It is also noticed

that, at this stage, one can check that, and like $F^+>0$, and, one $K_0>0$ $\alpha_k>0$ formally $R^+>0$ replaced the hooks (left positive) of the equation [éq 2.2.3] by simple brackets in the equation [éq 2.3-4]. One calculates

then the dual function "envelope surface f of the maximum strains": éq 2.3-8 So

$$f^+=\frac{2}{3}\left(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi}\right)^+_{eq}-q^+$$ **then L**

"integration $f^+\leq 0$ is finished. If not, one solves the (S2) system according to: Relation stress-strain

6 éq éq 2.3	$\Delta\boldsymbol{\sigma}-H\left(\Delta\boldsymbol{\varepsilon}-\Delta\boldsymbol{\varepsilon}^{\mathbf{th}}-\Delta p\dfrac{\partial F}{\partial\boldsymbol{\sigma}^+}\right)=0$	- 9 Kinematic hardening	
12 éq éq	$\begin{cases} i=1,2 \\ \Delta\mathbf{X_i}-\frac{2}{3}C_i\Delta\boldsymbol{\varepsilon}^{\mathbf{p}}+\gamma_i^+\left[\delta_i\,\mathbf{X_i^+}+(1-\delta_i)(\mathbf{X_i^+}:\mathbf{n}^+)\mathbf{n}^+\right]\Delta p \\ \quad +\gamma_{Xi}\left[(\mathbf{X_i^+})_{eq}\right]^{m_i-1}\mathbf{X_i^+}\Delta t-\dfrac{1}{C_i}\dfrac{\partial C_i}{\partial T}\mathbf{X_i}\Delta T=0 \end{cases}$	2.3-10	**cumulated**
Plasticity 1 éq éq 2.3	$\Delta p-\Delta t\left(\dfrac{F^+}{K_0+\alpha_k R^+}\right)^n\exp\left[\alpha\left(\dfrac{F^+}{K_0+\alpha_k R^+}\right)^{n+1}\right]=0$	- 11	**isotropic**
Hardening 1 éq éq 2.3	$\Delta R-b(Q^+-R^+)\Delta p-\gamma_r\lvert Q_r^+-R^+\rvert^{m_r}\operatorname{sgn}(Q_r^+-R^+)\Delta t=0$	- 12	**Effect of memory**
	$\Delta q-\eta\langle\mathbf{n}:\mathbf{n}^*\rangle^+\Delta p=0$	- 13	**6 éq éq 2.3**
1 éq éq 2.3	$\Delta\boldsymbol{\xi}-\sqrt{\dfrac{3}{2}}(1-\eta)\langle\mathbf{n}:\mathbf{n}^*\rangle^+\Delta p\,(\mathbf{n}^*)^+=0$	- the 14	**27 unknowns**

are: $\Delta\boldsymbol{\sigma}$, $\Delta\mathbf{X_1}$ and $\Delta\mathbf{X_2}$. Δp One ΔR notices Δq $\Delta\boldsymbol{\xi}$
that since, then one $f^+>0$ could replace equations $H(f)$ [éq 2.2-5] and [éq 2.2-6] by in the equations 1 [éq 2.3-13] and [éq 2.3-14]. The system

(S1) (resp. (S2)) with 20 (resp is a nonlinear implicit system. 27) equations and 20 (resp. 27) unknown. One can formally
write his systems: , with: for $\phi(\Delta Y)=0$ (S1),

- $\Delta Y=\left(\Delta\boldsymbol{\sigma},\Delta\mathbf{X_1},\Delta\mathbf{X_2},\Delta p,\Delta R\right)^t$ and (S
- 2). $\Delta Y=\left(\Delta\boldsymbol{\sigma},\Delta\mathbf{X_1},\Delta\mathbf{X_2},\Delta p,\Delta R,\Delta q,\Delta\boldsymbol{\xi}\right)^t$ These nonlinear

systems are solved by the iterative method of Newton (in L" environment PLASTI describes for example in [R5.03.10]): while reiterating

$$\phi\left(\Delta Y_k\right)+\left(\frac{\partial\phi}{\partial\Delta Y}\right)_{\Delta Y_k}\left(\Delta Y_{k+1}-\Delta Y_k\right)=0$$

in until k convergence and starting with an initial solution ensuring this convergence. Computation of

2.2.3 Jacobienne According to whether

one has to solve the (S1) system or (S2), one breaks up the system into subsystems $\phi\left(\Delta Y\right)=0$: for (S1) and

$$\phi\left(\Delta Y\right)=\begin{bmatrix}g\left(\Delta Y\right)\\l\left(\Delta Y\right)\\j\left(\Delta Y\right)\\f\left(\Delta Y\right)\\r\left(\Delta Y\right)\end{bmatrix}$$ for (S2). $$\phi\left(\Delta Y\right)=\begin{bmatrix}g\left(\Delta Y\right)\\l\left(\Delta Y\right)\\j\left(\Delta Y\right)\\f\left(\Delta Y\right)\\r\left(\Delta Y\right)\\h\left(\Delta Y\right)\\c\left(\Delta Y\right)\end{bmatrix}$$ Where: represent

g the relation stress-strain represents
l the equations of kinematic hardening represents $\mathbf{X_1}$
j the equations of kinematic hardening represents $\mathbf{X_2}$
f the equation defining cumulated plasticity represents p
r the equation defining isotropic hardening represents $R\left(p\right)$
h the equation defining the effect of memory represents q
c the equations defining the effect of memory the jacobian matrix $\boldsymbol{\xi}$

of the system is the matrix written per J blocks: Each term

$$J=\begin{bmatrix}
\frac{\partial g}{\partial(\Delta\sigma)} & \frac{\partial g}{\partial(\Delta X_1)} & \frac{\partial g}{\partial(\Delta X_2)} & \frac{\partial g}{\partial(\Delta p)} & \frac{\partial g}{\partial(\Delta R)} & \frac{\partial g}{\partial(\Delta q)} & \frac{\partial g}{\partial(\Delta\xi)}\\
\frac{\partial l}{\partial(\Delta\sigma)} & \frac{\partial l}{\partial(\Delta X_1)} & \frac{\partial l}{\partial(\Delta X_2)} & \frac{\partial l}{\partial(\Delta p)} & \frac{\partial l}{\partial(\Delta R)} & \frac{\partial l}{\partial(\Delta q)} & \frac{\partial l}{\partial(\Delta\xi)}\\
\frac{\partial j}{\partial(\Delta\sigma)} & \frac{\partial j}{\partial(\Delta X_1)} & \frac{\partial j}{\partial(\Delta X_2)} & \frac{\partial j}{\partial(\Delta p)} & \frac{\partial j}{\partial(\Delta R)} & \frac{\partial j}{\partial(\Delta q)} & \frac{\partial j}{\partial(\Delta\xi)}\\
\frac{\partial f}{\partial(\Delta\sigma)} & \frac{\partial f}{\partial(\Delta X_1)} & \frac{\partial f}{\partial(\Delta X_2)} & \frac{\partial f}{\partial(\Delta p)} & \frac{\partial f}{\partial(\Delta R)} & \frac{\partial f}{\partial(\Delta q)} & \frac{\partial f}{\partial(\Delta\xi)}\\
\frac{\partial r}{\partial(\Delta\sigma)} & \frac{\partial r}{\partial(\Delta X_1)} & \frac{\partial r}{\partial(\Delta X_2)} & \frac{\partial r}{\partial(\Delta p)} & \frac{\partial r}{\partial(\Delta R)} & \frac{\partial r}{\partial(\Delta q)} & \frac{\partial r}{\partial(\Delta\xi)}\\
\frac{\partial h}{\partial(\Delta\sigma)} & \frac{\partial h}{\partial(\Delta X_1)} & \frac{\partial h}{\partial(\Delta X_2)} & \frac{\partial h}{\partial(\Delta p)} & \frac{\partial h}{\partial(\Delta R)} & \frac{\partial h}{\partial(\Delta q)} & \frac{\partial h}{\partial(\Delta\xi)}\\
\frac{\partial c}{\partial(\Delta\sigma)} & \frac{\partial c}{\partial(\Delta X_1)} & \frac{\partial c}{\partial(\Delta X_2)} & \frac{\partial c}{\partial(\Delta p)} & \frac{\partial c}{\partial(\Delta R)} & \frac{\partial c}{\partial(\Delta q)} & \frac{\partial c}{\partial(\Delta\xi)}
\end{bmatrix}$$

of this nonsymmetrical matrix is clarified in Appendix [§7.1]. It will be noted7.1 that the terms, and $\frac{\partial l}{\partial(\Delta X_1)}$ do not take $\frac{\partial l}{\partial(\Delta X_2)}$ $\frac{\partial j}{\partial(\Delta X_1)}$ $\frac{\partial j}{\partial(\Delta X_2)}$ account of the dependence in (see [éq ΔT 2.3-10]). Explicit

2.3 integration to carry out

the explicit integration of the constitutive law, one uses the method of Runge-Kutta [R5.03.14]. One thus integrates directly by this method the system of 27 differential equations according to: Flow

6 éq éq 2.3	$\dot{\boldsymbol{\varepsilon}}^p=\frac{\partial\Omega^p}{\partial\boldsymbol{\sigma}}=\dot{p}\,\frac{\partial F}{\partial\boldsymbol{\sigma}}=\frac{3}{2}\dot{p}\,\frac{\tilde{\boldsymbol{\sigma}}-\mathbf{X}}{(\tilde{\boldsymbol{\sigma}}-\mathbf{X})_{eq}}$	- 10	**Kinematic hardening**
12 éq éq	$\dot{\boldsymbol{\alpha}}_i=\dot{\boldsymbol{\varepsilon}}^p-\gamma_i\left[\delta_i\,\boldsymbol{\alpha}_\mathbf{i}+(1-\delta_i)(\boldsymbol{\alpha}_\mathbf{i}:\mathbf{n})\mathbf{n}\right]\dot{p}-\gamma_{Xi}\left[(\mathbf{X}_\mathbf{i})_{eq}\right]^{m_i-1}\boldsymbol{\alpha}_\mathbf{i}\quad i=1,2$	2.3-11	**cumulated**
Plasticity 1 éq éq 2.3	$\dot{p}=\langle\frac{F}{K_0+\alpha_k R}\rangle^n\exp\left[\alpha\langle\frac{F}{K_0+\alpha_k R}\rangle^{n+1}\right]$	- 12	**isotropic**
Hardening 1 éq éq 2.3	$\dot{R}=b(Q-R)\dot{p}+\gamma_r\lvert Q_r-R\rvert^{m_r}\operatorname{sgn}(Q_r-R)$	- 13	**Effect of memory**
1 éq éq 2.3	$\dot{q}=\eta\,H(f)\langle\mathbf{n}:\mathbf{n}^*\rangle\,\dot{p}$	- 14	**6 éq éq 2.3**

$$\dot{\boldsymbol{\xi}}=\sqrt{\frac{3}{2}}(1-\eta)H(f)\langle\mathbf{n}:\mathbf{n}^*\rangle\,\dot{p}\;\mathbf{n}^*$$ - 15

with: Meaning

$$\tilde{\boldsymbol{\sigma}}=\boldsymbol{\sigma}-\sigma^H\,\mathbf{Id}=2\mu(\tilde{\boldsymbol{\varepsilon}}-\boldsymbol{\varepsilon}^{\mathbf{p}})$$

$$A_{eq}=\sqrt{\frac{3}{2}\tilde{\mathbf{A}}:\tilde{\mathbf{A}}}$$

$$F(\boldsymbol{\sigma},R,\mathbf{X})=(\tilde{\boldsymbol{\sigma}}-\mathbf{X})_{eq}-\alpha_R R(p)-k$$

$$\mathbf{X}=\mathbf{X_1}+\mathbf{X_2}$$

$$\mathbf{X_1}=\frac{2}{3}C_1\boldsymbol{\alpha_1}\qquad\mathbf{X_2}=\frac{2}{3}C_2\boldsymbol{\alpha_2}$$

$$Q=Q_0+(Q_m-Q_0)(1-e^{-2\mu q})$$

$$Q_r=Q-Q_r^*\left[1-\left(\frac{Q_m-Q}{Q_m}\right)^2\right]$$

$$f(\boldsymbol{\varepsilon}^{\mathbf{p}},\boldsymbol{\xi},q)=\frac{2}{3}(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}-q$$

$$\mathbf{n}^*=\sqrt{\frac{3}{2}}\frac{\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi}}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{\mathbf{eq}}}$$

2.4 of the local variables the local variables

of model VISCOCHAB to `Gauss points` (VARI_ELGA) are: `in` 3D implicit

in 2D implicit	Runge-Kutta	V1 = V1 =
V1 = X_{1xx}	V2 = X_{1xx}	V2 = ε^{p}_{xx}
V2 = X_{1yy}	V3 = X_{1yy}	V3 = ε^{p}_{yy}
V3 = X_{1zz}	V4 = X_{1zz}	V4 = ε^{p}_{zz}
V4 = X_{1xy}	V5 = X_{1xy}	V5 = ε^{p}_{xy}
V5 = X_{1xz}	V6 = X_{2xx}	V6 = ε^{p}_{xz}
V6 = X_{1yz}	V7 = X_{2yy}	V7 = ε^{p}_{yz}
V7 = X_{2xx}	V8 = X_{2zz}	V8 = α_{1xx}
V8 = X_{2yy}	V9 = X_{2xy}	V9 = α_{1yy}
V9 = X_{2zz}	V 10 p =	V 10 α_{1zz}
= V 10 X_{2xy}	= V11 R	= V11 α_{1xy}
= V11 X_{2xz}	= V12 q	= V12 α_{1xz}
= V12 X_{2yz}	= V13 ξ_{xx}	= V13 α_{1yz}
= V13 p	= V 14 ξ_{yy}	= V 14 α_{2xx}
= V 14 R	= V 15 ξ_{zz}	= V 15 α_{2yy}
= V 15 q	= V 16 ξ_{xy}	= V 16 α_{2zz}
= V 16 ξ_{xx}	= V 17 ζ	= V 17 α_{2xy}
= V 18 ξ_{yy}		= V 18 α_{2xz}
= V 19 ξ_{zz}		= V 19 α_{2yz}
= V 20 ξ_{xy}		= V 20 ξ_{xx}
= V21 ξ_{xz}		= V21 ξ_{yy}
= V22 ξ_{yz}		= V22 ξ_{zz}
= V23 ζ		= V 24 ξ_{xy}
		= V 25 ξ_{xz}
		= V26 ξ_{yz}
		= V27 R
		= V 27 q

		= the indicator p
		0

- is worth 1 if ζ the Gauss point plasticized during L" increment or 0 if not represent
- p the cumulated equivalent plastic strain (positive or null) Functionalities

3 and checking the constitutive law

is defined by the key word VISCOCHAB (key word factor COMP_INCR of commands STAT_NON_LINE , DYNA_NON_LINE, SIMU_POINT_MAT,…). It is associated with materials VISCOCHAB and VISCOCHAB_FO (command DEFI_MATERIAU). Model VISCOCHAB

is checked in particular by the cases following tests: COMP002I [V

6.07.102]	elementary Test	of robustness and reliability of the viscoplastic behaviors. COMP010I [V
6.07.110]	elementary	Validation of the taking into account of the temperature in the viscoplastic behaviors. HSNV125D [V
7.22.125]	Volume element	in tension/shears and temperature variables (comparison with other codes) SSND105B [V
6.08.105]	Constitutive law	visco-élasto-plastic with effect of memory. Comparison with VISC_CIN2_MEMO SSND111A [V
6.08.111]	Effect of memory	in a cyclic test. Comparison with VISC_CIN2_MEMO SSNV118 [V6.04
.118]	Traction test	shears with the model viscoplastic of Chaboche. Comparison with software SIDOLO Identification

4 of the parameters of the model The model

suggested being very close to that of Chaboche, one will be able to refer to [bib4] for the identification of the parameters of the initial model of Chaboche. For the identification

of the additional parameters, the reference [bib4] presents the tests used to supplement the identification of steel 316 SPH. More recently

, the identification of steel 304L was carried out without taking account of the phenomena of restoration and progressive strain [bib10]. Bibliography

5 J.LEMAITRE,

1. J.L.CHABOCHE, Mechanics of the solid materials. Dunod 1996 L. BONNAFOUX, "Modification
2. of the viscoplastic model of Chaboche for the modelization of the phenomenon of progressive strains and establishment in the code finite elements Aster", Internal report EDF R & D, 1993 P. GEYER, "Modelization
3. of the phenomena of progressive strain by the model elastoplastic of Chaboche modified by Burlet and Cailletaud", HT-26/93/052/A (H-T20-1994-00254-FR in Eureka) Notes, 1994 P. GEYER, "Study and

4. elastoviscoplastic modification of the model of Chaboche to improve the description of the ratchet 2d". Note HT-26/95/032/A (H-T20-1995-05189-FR in Eureka), 1995 P. GEYER, "Confrontation
5. of the model of Chaboche modified by Burlet and Cailletaud with the tests of pressure-tension of the French atomic energy agency", HT-26/94/006/A (H-T20-1994-02201-FR in Eureka) Notes, 1994 P. GEYER, "Computational simulation
6. of a test bi--tube with the elastoplastic models of Chaboche and Taheri", HT-26/94/023/A (H-T20-1995-00954-FR in Eureka) Notes, 1995 P. GEYER, "Evaluating
7. of the models of Chaboche and Burlet by the computation of test COTHAA nA8", HT-C2/96/010/A (H-T00-1997-00320-FR in Eureka) Notes, 1997 P. GEYER, "Evaluating
8. of the models of Chaboche and Burlet by the computation of the cast solid sleeve tested on the loop CUMULUS in progressive strain", HT-C2/96/047/A (H-T00-1997-02043-FR in Eureka) Notes, 1997 P. GEYER, "Characterization
9. of steel 304L used during the tests "progressive strain" on CUMULUS and identification of the parameters of the model of Chaboche", HT-26/93/040/A (H-T20-1993-03153-FR in Eureka) Notes, 1993 F. CURTIT, "Identification
10. of a constitutive law of the Chaboche type with effect of memory of hardening for steel 304L with 20°C and 300°C", Notes H-T26-2007-3264, 2008 J.M. PROIX, "Behavior models
11. élasto-visco-plastic of Chaboche, Document de Référence de Code_Aster", [R5.03.04], 2007 P.GEYER, J.M.PROIX, S.JAYET-GENDROT
12. , S.TAHERI, "A 3D elastoplastic cyclic consitutive law for the description of ratchetting of 316 stainless steel", SMIRT 1995 Description of the versions of the document

6 Version Aster Author (S) or contributor

(S), organization	Description of modifications 9.4	S. Geniaut EDF/R & D /AMA initial
Text	, model VISCOCHAB 10.5	J.M.Proix EDF/R & D /AMA Correction of
the § meaning	of the local variables	and addition of the § Additional functionalities and checking Statement of the terms of the jacobian matrix

7

7.1 In computations presented here, one will omit

the exhibitor “+” to indicate the quantities at current time (fine of time step). The jacobian matrix of the system is

the matrix written per blocks: terms related to the elastic $\mathbf{J}$ relation

$$\mathbf{J}=\begin{bmatrix}
\frac{\partial g}{\partial(\Delta\boldsymbol{\sigma})} & \frac{\partial g}{\partial(\Delta\mathbf{X_1})} & \frac{\partial g}{\partial(\Delta\mathbf{X_2})} & \frac{\partial g}{\partial(\Delta p)} & \frac{\partial g}{\partial(\Delta R)} & \frac{\partial g}{\partial(\Delta q)} & \frac{\partial g}{\partial(\Delta\boldsymbol{\xi})}\\
\frac{\partial l}{\partial(\Delta\boldsymbol{\sigma})} & \frac{\partial l}{\partial(\Delta\mathbf{X_1})} & \frac{\partial l}{\partial(\Delta\mathbf{X_2})} & \frac{\partial l}{\partial(\Delta p)} & \frac{\partial l}{\partial(\Delta R)} & \frac{\partial l}{\partial(\Delta q)} & \frac{\partial l}{\partial(\Delta\boldsymbol{\xi})}\\
\frac{\partial j}{\partial(\Delta\boldsymbol{\sigma})} & \frac{\partial j}{\partial(\Delta\mathbf{X_1})} & \frac{\partial j}{\partial(\Delta\mathbf{X_2})} & \frac{\partial j}{\partial(\Delta p)} & \frac{\partial j}{\partial(\Delta R)} & \frac{\partial j}{\partial(\Delta q)} & \frac{\partial j}{\partial(\Delta\boldsymbol{\xi})}\\
\frac{\partial f}{\partial(\Delta\boldsymbol{\sigma})} & \frac{\partial f}{\partial(\Delta\mathbf{X_1})} & \frac{\partial f}{\partial(\Delta\mathbf{X_2})} & \frac{\partial f}{\partial(\Delta p)} & \frac{\partial f}{\partial(\Delta R)} & \frac{\partial f}{\partial(\Delta q)} & \frac{\partial f}{\partial(\Delta\boldsymbol{\xi})}\\
\frac{\partial r}{\partial(\Delta\boldsymbol{\sigma})} & \frac{\partial r}{\partial(\Delta\mathbf{X_1})} & \frac{\partial r}{\partial(\Delta\mathbf{X_2})} & \frac{\partial r}{\partial(\Delta p)} & \frac{\partial r}{\partial(\Delta R)} & \frac{\partial r}{\partial(\Delta q)} & \frac{\partial r}{\partial(\Delta\boldsymbol{\xi})}\\
\frac{\partial h}{\partial(\Delta\boldsymbol{\sigma})} & \frac{\partial h}{\partial(\Delta\mathbf{X_1})} & \frac{\partial h}{\partial(\Delta\mathbf{X_2})} & \frac{\partial h}{\partial(\Delta p)} & \frac{\partial h}{\partial(\Delta R)} & \frac{\partial h}{\partial(\Delta q)} & \frac{\partial h}{\partial(\Delta\boldsymbol{\xi})}\\
\frac{\partial c}{\partial(\Delta\boldsymbol{\sigma})} & \frac{\partial c}{\partial(\Delta\mathbf{X_1})} & \frac{\partial c}{\partial(\Delta\mathbf{X_2})} & \frac{\partial c}{\partial(\Delta p)} & \frac{\partial c}{\partial(\Delta R)} & \frac{\partial c}{\partial(\Delta q)} & \frac{\partial c}{\partial(\Delta\boldsymbol{\xi})}
\end{bmatrix}$$

stress-strain: terms related to kinematic hardening

$$g(\Delta Y)=\Delta\boldsymbol{\sigma}-H\left(\Delta\boldsymbol{\varepsilon}-\Delta\boldsymbol{\varepsilon}^{\mathbf{th}}-\Delta p\frac{\partial F}{\partial\boldsymbol{\sigma}^{+}}\right)$$

$$\frac{\partial g}{\partial(\Delta\boldsymbol{\sigma})}=I_d+H\left(\frac{\partial^2 F}{\partial^2\boldsymbol{\sigma}}\right)\Delta p$$

$$\frac{\partial g}{\partial(\Delta\mathbf{X_i})}=H\left(\frac{\partial^2 F}{\partial\mathbf{X_i}\partial\boldsymbol{\sigma}}\right)\Delta p$$

$$\frac{\partial g}{\partial(\Delta p)}=H\left(\frac{\partial F}{\partial\boldsymbol{\sigma}}\right)$$

$$\frac{\partial g}{\partial(\Delta R)}=0,\quad \frac{\partial g}{\partial(\Delta q)}=0,\quad \frac{\partial g}{\partial(\Delta\boldsymbol{\xi})}=0$$

-

: with: and terms related to cumulated

$$k_i(\Delta Y)=\Delta \mathbf{X_i}-\frac{2}{3}C_i\Delta \boldsymbol{\varepsilon}^{\mathbf{p}}+\gamma_i^+\left[\delta_i\,\mathbf{X_i^+}+(1-\delta_i)(\mathbf{X_i^+}:\mathbf{n^+})\mathbf{n^+}\right]\Delta p+\gamma_{Xi}\left[(\mathbf{X_i^+})_{eq}\right]^{m_i-1}\mathbf{X_i^+}\Delta t \quad i=1,2$$

plasticity $n=\sqrt{\frac{2}{3}}\frac{\partial F}{\partial \boldsymbol{\sigma}}=\sqrt{\frac{3}{2}}\frac{\tilde{\boldsymbol{\sigma}}-\mathbf{X}}{(\tilde{\boldsymbol{\sigma}}-\mathbf{X})_{eq}} \quad \Delta \boldsymbol{\varepsilon}^{\mathbf{p}}=\Delta p\frac{\partial F}{\partial \boldsymbol{\sigma}}$

$$\frac{\partial k_i}{\partial(\Delta \boldsymbol{\sigma})}=-\frac{2}{3}C_i\Delta p\frac{\partial^2 F}{\partial \boldsymbol{\sigma}^2}+\gamma_i\frac{2}{3}(1-\delta_i)\Delta p\left[\left(X_i:\frac{\partial F}{\partial \boldsymbol{\sigma}}\right)\frac{\partial^2 F}{\partial \boldsymbol{\sigma}^2}+\left(\frac{\partial^2 F}{\partial \boldsymbol{\sigma}^2}:\mathbf{X_i}\right)\otimes\frac{\partial F}{\partial \boldsymbol{\sigma}}\right]$$

$$\frac{\partial k_i}{\partial(\Delta \mathbf{X_i})}=\mathbf{I_d}-\frac{2}{3}C_i\Delta p\frac{\partial^2 F}{\partial \mathbf{X_i}\partial \boldsymbol{\sigma}}$$

$$+\gamma_i\Delta p\left\{\delta_i\mathbf{I_d}+\frac{2}{3}(1-\delta_i)\left[\left(\mathbf{X_i}:\frac{\partial F}{\partial \boldsymbol{\sigma}}\right)\frac{\partial^2 F}{\partial \mathbf{X_i}\partial \boldsymbol{\sigma}}+\frac{\partial F}{\partial \boldsymbol{\sigma}}\otimes\frac{\partial F}{\partial \boldsymbol{\sigma}}+\left(\frac{\partial^2 F}{\partial \mathbf{X_i}\partial \boldsymbol{\sigma}}:\mathbf{X_i}\right)\otimes\frac{\partial F}{\partial \boldsymbol{\sigma}}\right]\right\}$$

$$+\gamma_{Xi}\Delta t\left[(\mathbf{X_i})_{eq}\right]^{m_i-1}\mathbf{I_d}+\gamma_{Xi}\Delta t(m_i-1)\frac{\partial(\mathbf{X_i})_{eq}}{\partial \mathbf{X_i}}\left[(\mathbf{X_i})_{eq}\right]^{m_i-2}\mathbf{X_i}$$

$$\frac{\partial k_i}{\partial(\Delta \mathbf{X_j})_{(j\neq i)}}=-\frac{2}{3}C_i\Delta p\frac{\partial^2 F}{\partial \mathbf{X_j}\partial \boldsymbol{\sigma}}+\gamma_i\frac{2}{3}(1-\delta_i)\Delta p\left[\left(\mathbf{X_i}:\frac{\partial F}{\partial \boldsymbol{\sigma}}\right)\frac{\partial^2 F}{\partial \mathbf{X_j}\partial \boldsymbol{\sigma}}+\left(\frac{\partial^2 F}{\partial \mathbf{X_j}\partial \boldsymbol{\sigma}}:\mathbf{X_i}\right)\otimes\frac{\partial F}{\partial \boldsymbol{\sigma}}\right]$$

$$\frac{\partial k_i}{\partial(\Delta p)}=-\frac{2}{3}C_i\frac{\partial F}{\partial \boldsymbol{\sigma}}+\left(\gamma_i'(p)\Delta p+\gamma_i(p)\right)\left[\delta_i\mathbf{X_i}+\frac{2}{3}(1-\delta_i)\left(\mathbf{X_i}:\frac{\partial F}{\partial \boldsymbol{\sigma}}\right)\frac{\partial F}{\partial \boldsymbol{\sigma}}\right]$$

$$\frac{\partial k_i}{\partial(\Delta R)}=0,\quad \frac{\partial k_i}{\partial(\Delta q)}=0,\quad \frac{\partial k_i}{\partial(\Delta \boldsymbol{\xi})}=0$$

: terms related to isotropic hardening

$$f(\Delta Y)=\Delta p-\Delta t\left(\frac{F^+}{K_0+\alpha_k R^+}\right)^n \exp\left[\alpha\left(\frac{F^+}{K_0+\alpha_k R^+}\right)^{n+1}\right]$$

$$\frac{\partial f}{\partial(\Delta\boldsymbol{\sigma})}=-\Delta t\left(\frac{F}{K_0+\alpha_k R}\right)^{n-1}\exp\left[\alpha\left(\frac{F}{K_0+\alpha_k R}\right)^{n+1}\right]\frac{1}{K_0+\alpha_k R}\left[n+\alpha(n+1)\left(\frac{F}{K_0+\alpha_k R}\right)^{n+1}\right]\frac{\partial F}{\partial\boldsymbol{\sigma}}$$

$$\frac{\partial f}{\partial(\Delta\mathbf{X_i})}=-\Delta t\left(\frac{F}{K_0+\alpha_k R}\right)^{n-1}\exp\left[\alpha\left(\frac{F}{K_0+\alpha_k R}\right)^{n+1}\right]\frac{1}{K_0+\alpha_k R}\left[n+\alpha(n+1)\left(\frac{F}{K_0+\alpha_k R}\right)^{n+1}\right]\frac{\partial F}{\partial\mathbf{X_i}}$$

$$\frac{\partial f}{\partial(\Delta R)}=\Delta t\left(\frac{F}{K_0+\alpha_k R}\right)^{n-1}\exp\left[\alpha\left(\frac{F}{K_0+\alpha_k R}\right)^{n+1}\right]\frac{1}{K_0+\alpha_k R}$$
$$\times\left[n+\alpha(n+1)\left(\frac{F}{K_0+\alpha_k R}\right)^{n+1}\right]\left(\alpha_R+\alpha_k\frac{F}{K_0+\alpha_k R}\right)$$

$$\frac{\partial f}{\partial(\Delta p)}=1,\quad \frac{\partial g}{\partial(\Delta q)}=0,\quad \frac{\partial g}{\partial(\Delta\boldsymbol{\xi})}=0$$

: dependant terms in keeping with field relating

$$r(\Delta Y)=\Delta R-b\left(Q^+-R^+\right)\Delta p-\gamma_r\left|Q_r^+-R^+\right|^{m_r}\operatorname{sgn}\left(Q_r^+-R^+\right)\Delta t$$

$$\frac{\partial r}{\partial(\Delta\boldsymbol{\sigma})}=\mathbf{0}$$

$$\frac{\partial r}{\partial(\Delta\mathbf{X_i})}=\mathbf{0}$$

$$\frac{\partial r}{\partial(\Delta p)}=-b(Q-R)$$

$$\frac{\partial r}{\partial(\Delta R)}=1+b\Delta p+\gamma_r m_r\left|Q_r-R\right|^{m_r-1}\Delta t$$

$$\frac{\partial r}{\partial(\Delta q)}=-b\Delta p\,Q'(q)-\gamma_r m_r\left|Q_r-R\right|^{m_r-1}Q_r'(q)\Delta t$$

$$\text{avec}\quad\begin{cases}Q'(q)=2\mu\left(Q_m-Q_0\right)e^{-2muq}\\ Q_r'(q)=Q'(q)\left[1-2\mathrm{Q}_r^*\left(\dfrac{Q_m-Q}{Q_m^2}\right)\right]\end{cases}$$

$$\frac{\partial r}{\partial(\Delta\boldsymbol{\xi})}=\mathbf{0}$$

to the effect of memory: with Here, one makes an approximation, by

$$h(\Delta Y)=\Delta q-\eta\langle \mathbf{n}:\mathbf{n}^{*}\rangle^{+}\Delta p$$ considering $\mathbf{n}^{*}=\sqrt{\frac{3}{2}}\frac{\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi}}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}$

that. terms related to the center of the field relating $\langle \mathbf{n}:\mathbf{n}^{*}\rangle^{+}=(\mathbf{n}:\mathbf{n}^{*})^{+}$

$$\frac{\partial h}{\partial(\Delta\boldsymbol{\sigma})}=-\eta\Delta p\frac{1}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}[\frac{\partial^2 F}{\partial\boldsymbol{\sigma}^2}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})+\Delta p\frac{\partial^2 F}{\partial\boldsymbol{\sigma}^2}:\frac{\partial F}{\partial\boldsymbol{\sigma}}$$
$$-\frac{3}{2}\frac{\Delta p}{[(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}]^2}\left(\frac{\partial F}{\partial\boldsymbol{\sigma}}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})\right)\left(\frac{\partial^2 F}{\partial\boldsymbol{\sigma}^2}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})\right)]$$

$$\frac{\partial h}{\partial(\Delta\mathbf{X_i})}=-\eta\Delta p\frac{1}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}[\frac{\partial^2 F}{\partial\mathbf{X_i}\partial\boldsymbol{\sigma}}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})+\Delta p\frac{\partial^2 F}{\partial\mathbf{X_i}\partial\boldsymbol{\sigma}}:\frac{\partial F}{\partial\boldsymbol{\sigma}}$$
$$-\frac{3}{2}\frac{\Delta p}{[(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}]^2}\left(\frac{\partial F}{\partial\boldsymbol{\sigma}}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})\right)\left(\frac{\partial^2 F}{\partial X_i\partial\boldsymbol{\sigma}}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})\right)]$$

$$\frac{\partial h}{\partial(\Delta p)}=-\eta\frac{1}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}[\frac{\partial F}{\partial\boldsymbol{\sigma}}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})+\Delta p\left(\frac{\partial F}{\partial\boldsymbol{\sigma}}:\frac{\partial F}{\partial\boldsymbol{\sigma}}\right)-\frac{3}{2}\frac{\Delta p}{[(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}]^2}\left(\frac{\partial F}{\partial\boldsymbol{\sigma}}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})\right)^2]$$

$$\frac{\partial h}{\partial(\Delta R)}=0$$

$$\frac{\partial h}{\partial(\Delta q)}=1$$

$$\frac{\partial h}{\partial(\Delta\boldsymbol{\xi})}=\frac{\eta}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}\Delta p\left[\frac{\partial F}{\partial\boldsymbol{\sigma}}+\frac{3}{2}(\mathbf{n}:\mathbf{n}^{*})\frac{\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi}}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}\right]$$

to the effect of memory: Here, one makes an approximation, by considering

$$c(\Delta Y) = \Delta \boldsymbol{\xi} - \sqrt{\frac{3}{2}}(1-\eta)\langle \mathbf{n}:\mathbf{n}^*\rangle^+ \Delta p \left(\mathbf{n}^*\right)^+$$

that. Computation of the tangent stiffness the iterative $\langle \mathbf{n}:\mathbf{n}^*\rangle^+ = \left(\mathbf{n}:\mathbf{n}^*\right)^+$

$$\frac{\partial c}{\partial(\Delta\boldsymbol{\sigma})} = -\frac{3}{2}(1-\eta)\frac{\Delta p}{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})_{eq}}\left[\left(\frac{\partial^2 F}{\partial\boldsymbol{\sigma}^2}:(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})\right)\otimes\frac{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})}{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})_{eq}} + \Delta p(\mathbf{n}:\mathbf{n}^*)\frac{\partial^2 F}{\partial\boldsymbol{\sigma}^2}\right.$$
$$\left. -3\frac{\Delta p}{\left[(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})_{eq}\right]^2}(\mathbf{n}:\mathbf{n}^*)\left(\frac{\partial^2 F}{\partial\boldsymbol{\sigma}^2}:(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})\right)\otimes(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi}) + \Delta p\frac{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})}{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})_{eq}}\otimes\left(\frac{\partial F}{\partial\boldsymbol{\sigma}}:\frac{\partial^2 F}{\partial\boldsymbol{\sigma}^2}\right)\right]$$

$$\frac{\partial c}{\partial(\Delta\mathbf{X_i})} = -\frac{3}{2}(1-\eta)\frac{\Delta p}{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})_{eq}}\left[\left(\frac{\partial^2 F}{\partial\mathbf{X_i}\partial\boldsymbol{\sigma}}:(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})\right)\otimes\frac{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})}{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})_{eq}} + \Delta p(\mathbf{n}:\mathbf{n}^*)\frac{\partial^2 F}{\partial\mathbf{X_i}\partial\boldsymbol{\sigma}}\right.$$
$$\left. -3\frac{\Delta p}{\left[(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})_{eq}\right]^2}(\mathbf{n}:\mathbf{n}^*)\left(\frac{\partial^2 F}{\partial\mathbf{X_i}\partial\boldsymbol{\sigma}}:(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})\right)\otimes(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi}) + \Delta p\frac{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})}{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})_{eq}}\otimes\left(\frac{\partial F}{\partial\boldsymbol{\sigma}}:\frac{\partial^2 F}{\partial\mathbf{X_i}\partial\sigma}\right)\right]$$

$$\frac{\partial c}{\partial(\Delta p)} = -\frac{3}{2}(1-\eta)\frac{1}{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})_{eq}}\left[(\mathbf{n}:\mathbf{n}^*)(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi}) + \frac{\Delta p}{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})_{eq}}\left(\frac{\partial F}{\partial\boldsymbol{\sigma}}:\frac{\partial F}{\partial\boldsymbol{\sigma}}\right)(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})\right.$$
$$\left. + \Delta p(\mathbf{n}:\mathbf{n}^*)\frac{\partial F}{\partial\boldsymbol{\sigma}} - 3\frac{\Delta p}{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})_{eq}}(\mathbf{n}:\mathbf{n}^*)^2(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})\right]$$

$$\frac{\partial c}{\partial(\Delta R)} = 0$$

$$\frac{\partial c}{\partial(\Delta q)} = 0$$

$$\frac{\partial c}{\partial(\Delta\boldsymbol{\xi})} = \mathbf{I_d} + \frac{3}{2}(1-\eta)\frac{\Delta p}{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})_{eq}}\left[\frac{\partial F}{\partial\boldsymbol{\sigma}}\otimes\frac{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})}{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})_{eq}} + (\mathbf{n}:\mathbf{n}^*)\mathbf{I_d} - 3(\mathbf{n}:\mathbf{n}^*)\frac{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})}{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})_{eq}}\otimes\frac{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})}{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})_{eq}}\right]$$
$$= \mathbf{I_d} + \frac{3}{2}(1-\eta)\frac{\Delta p}{(\boldsymbol{\varepsilon}^\mathbf{p}-\boldsymbol{\xi})_{eq}}\left[(\mathbf{n}\otimes\mathbf{n}^*) + (\mathbf{n}:\mathbf{n}^*)I_d - 3(\mathbf{n}:\mathbf{n}^*)(\mathbf{n}^*\otimes\mathbf{n}^*)\right]$$

7.2 diagram of total Newton (

to ensure the equilibrium) requires to know the tangent operator of the system assembled at the end of each increment. The tangent operator is noted. It is possible to determine this operator $\mathbf{M_c} = \frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\varepsilon}}$ starting from the terms of the jacobienne of the local system, already calculated previously J (see §7.1). Indeed, the system is checked7.1 the end

of the increment and for $\phi(\Delta Y)=0$ a small variation of, by regarding this time as variable ϕ, the system remains with ε the equilibrium and thus one checks. By differentiation, one obtains: For $d\,\phi=0$
the (S1) system: For the (S2)

system: In the continuation,

$$\frac{\partial\phi}{\partial(\Delta\varepsilon)}d(\Delta\varepsilon)+\frac{\partial\phi}{\partial(\Delta\sigma)}d(\Delta\sigma)+\frac{\partial\phi}{\partial(\Delta \mathrm{X_i})}d(\Delta \mathrm{X_i}) +\frac{\partial\phi}{\partial(\Delta p)}d(\Delta p)+\frac{\partial\phi}{\partial(\Delta R)}d(\Delta R)=0$$

the presentation is limited

$$\frac{\partial\phi}{\partial(\Delta\varepsilon)}d(\Delta\varepsilon)+\frac{\partial\phi}{\partial(\Delta\sigma)}d(\Delta\sigma)+\frac{\partial\phi}{\partial(\Delta \mathrm{X_i})}d(\Delta \mathrm{X_i}) +\frac{\partial\phi}{\partial(\Delta p)}d(\Delta p)+\frac{\partial\phi}{\partial(\Delta R)}d(\Delta R)+\frac{\partial\phi}{\partial(\Delta q)}d(\Delta q)+\frac{\partial\phi}{\partial(\Delta \xi)}d(\Delta \xi)=0$$

to the (S1) system, but the approach is identical for the (S2) system. One rewrites the system by putting

the ends in the member of right: With the notations ε defined previously

$$\frac{\partial\phi}{\partial(\Delta\sigma)}d(\Delta\sigma)+\frac{\partial\phi}{\partial(\Delta \mathrm{X_i})}d(\Delta \mathrm{X_i})+\frac{\partial\phi}{\partial(\Delta p)}d(\Delta p)+\frac{\partial\phi}{\partial(\Delta R)}d(\Delta R)=-\frac{\partial\phi}{\partial(\Delta\varepsilon)}d(\Delta\varepsilon)$$

, this system is written in matric form: . However where the elastic modulus represents

$$J\,.\,d(\Delta Y)=-\frac{\partial\phi}{\partial(\Delta\varepsilon)}d(\Delta\varepsilon)$$

. From where $\frac{\partial\phi}{\partial(\Delta\varepsilon)}=\begin{pmatrix}\frac{\partial g}{\partial(\Delta\varepsilon)}\\ \frac{\partial k_i}{\partial(\Delta\varepsilon)}\\ \frac{\partial f}{\partial(\Delta\varepsilon)}\\ \frac{\partial r}{\partial(\Delta\varepsilon)}\end{pmatrix}=\begin{pmatrix}-\mathbf{H}\\0\\0\\0\end{pmatrix}$ $\mathbf{H}$. The idea then consists in writing this $\mathbf{J}.d(\Delta \mathrm{Y}) = \begin{pmatrix} \mathbf{H}\, d(\Delta \varepsilon) \\ 0 \\ 0 \\ 0 \end{pmatrix}$

system per blocks, while separating from the other variables, which gives: By $d(\Delta \boldsymbol{\sigma})$ calculating the complement $\mathbf{Z} = \left(d(\Delta \mathbf{X_i}), d(\Delta p), d(\Delta R)\right)^t$ of Schur of

$$\begin{bmatrix} \mathbf{J}_{\sigma\sigma} & \mathbf{J}_{\sigma\mathbf{Z}} \\ \mathbf{J}_{\sigma\mathbf{Z}} & \mathbf{J}_{\mathbf{ZZ}} \end{bmatrix} \begin{pmatrix} d(\Delta \boldsymbol{\sigma}) \\ \mathbf{Z} \end{pmatrix} = \begin{pmatrix} \mathbf{H}\,\mathbf{d}(\Delta \boldsymbol{\varepsilon}) \\ \mathbf{0} \end{pmatrix}$$

, one finds that: , from where the statement $\mathbf{J}_{\mathbf{ZZ}}$ of the tangent

$$\left[\mathbf{J}_{\sigma\sigma} - \mathbf{J}_{\sigma\mathbf{Z}} (\mathbf{J}_{\mathbf{ZZ}})^{-1} \mathbf{J}_{\mathbf{Z}\sigma}\right] d(\Delta \boldsymbol{\sigma}) = \mathbf{H}\, d(\Delta \boldsymbol{\varepsilon})$$

operator: Note: Since is not symmetric

$$\mathbf{M_c} = \frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\varepsilon}} = \frac{d(\Delta \boldsymbol{\sigma})}{d(\Delta \varepsilon)} = \left[\mathbf{J}_{\sigma\sigma} - \mathbf{J}_{\sigma\mathbf{Z}} (\mathbf{J}_{\mathbf{ZZ}})^{-1} \mathbf{J}_{\mathbf{Z}\sigma}\right]^{-1} \mathbf{H}$$

, the tangent

operator $\mathbf{J}$ is not it either. He however $\mathbf{M_c}$ is symmetrized in Code_Aster .