Behavior viscoplastic with damage of Summarized

HAYHURST:

The model viscoplastic coupled with the isotropic damage called of Hayhurst is particularly adapted to carry out structural analyzes in creep. The viscoplastic part of the model was proposed by Hayhurst et al. in [bib1] while the damage model was proposed by Charles Pétry in [bib2]. This model allows a satisfactory prediction of the fall of ductility in creep via the application of limiting criteria on the strain and the damage. This model has for time mainly used at EDF R & D /MMC for predictions of life duration in creep on the steel of rank 92. Via the identification of specific parameters and the realization of structural analyzes, this model also makes it possible to predict in a satisfactory way behavior in creep and the life duration of welded junctions [bib3].

This model is established in *Code_Aster* under the name of `HAYHURST` ; the equations of velocity are integrated numerically by an explicit diagram of Runge-Kutta of order 2 with automatic cutting under-PAS buildings according to an estimate of the error of integration (method of Runge-Kutta encased, confer [R5.03.14]) or by an implicit integration method of Newton.

Test SSNV225 validates the integration of this model and is presented in the document of validation [V6.04.225] which also provides experimental references in keeping with the case test.

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1 Introduction

the applications of the thermal sector to flame and certain nuclear technologies (GENIV, AGR) require to be able to predict the behavior of materials in creep at the same time in term of viscoplastic strain but also in term of damage of creep [bib1].

In addition, from a point of view “material”, the mechanisms of creep must be taken into account in the most physical possible way so that the constitutive law is valid at the same time in the field of creep-dislocation (forced high) and in the field of the flow-diffusion (forced weak). In the field of lower stresses, one generally observes a fall of ductility for steels martenistic temperings containing between 9 and 12% of chromium. This fall of ductility can be modelled via an isotropic variable of damage causing the fracture of the material before plastic strains significant did not have time to develop. In these situations, the criteria in maximum strain cannot apply, just like the laws of evolution of damage strongly related to the strain like the model of LEMAITRE.

More generally, of the cavities of creep appear in many families of metallic materials and can be observed on site by carrying out extractive counterparts on the surface of the investigated components. These cavities, associated with a reduction of effective surface resisting the forces in the material, can be directly correlated with a damage of the Kachanov type.

In response with these needs for modelization, while remaining in a simple phenomenologic mechanical frame, a constitutive law of the Hayhurst type (in reference to its viscoplastic flow) was proposed in [bib2] and was applied to computations of creep to a P92 steel usually used in the modern components of thermo plants with flame.

This model, implemented in *Code_Aster,* is a viscoplastic model of behavior to double isotropic hardening, viscosity out of model sine hyperbolic and coupled to a damage of Kachanov.

One will note that models HAYHURST and VENDOCHAB are both of the viscoplastic models to isotropic damage, however the model of HAYHURST has his own advantages detailed in the continuation of this document.

Nota bene:

One will find in the reference [bib3] the application of this model to computations on joints welded out of P92 steel, and in the reference [bib4] of preliminary works to the extension of this model to take into account the interaction of type fatigue-creep onto this same material.

2 Formulation of the model

2.1 Tallies theoretical

In the initial formulation suggested in [bib1], two distinct variables of damage having each one own kinetics are proposed. A variable ϕ is in particular associated with the evolutions of the microstructure depending only on time (i.e static aging of the material), whereas the variable describes ω the mechanisms of cavitation developing under the combined influence of the viscoplastic strain and the triaxiality of the stresses. In

the modelization retained in Code_Aster, the variable formula ϕ is preserved. In practice, its identification is delicate, and it is possible not to utilize the microstructural aging while putting at zero the coefficient (cf k_c equations in section 2.2). In addition, the variable formula ω renamed because D its law of evolution is different from that proposed by Hayhurst (cf [bib1]): the model is here in hyperbolic sine. The advantages of this formulation are detailed in [bib2]. Equations

2.2 of the model

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the equations of the models are written: where

$$
\begin{cases}
\textit{élasticité}: \\
\sigma=(1-D)\,\boldsymbol{C}\,\varepsilon^{e} \text{ et } \varepsilon^{e}=\varepsilon-\varepsilon^{th}-\varepsilon^{p} \\
\\
\textit{viscoplasticité}: \\
\dot{\varepsilon}^{p}=\frac{3}{2}\,\dot{p}\,\frac{\tilde{\sigma}}{\sigma_{eq}} \text{ avec } \dot{p}=\varepsilon_0 \sinh\left(\frac{\sigma_{eq}(1-H)}{K(1-D)(1-\phi)}\right) \\
\dot{\phi}=\frac{k_c}{3}(1-\phi)^4 \\
\\
\textit{écrouissage}: \\
H=H_1+H_2 \\
\dot{H}_i=\frac{h_i}{\sigma_{eq}}\left(H_i^{*}-\delta_i H_i\right)\dot{p} \quad \text{pour } i=1,2 \\
\\
\textit{endommagement}: \\
\text{si } \alpha_{\sigma}=0 \quad \dot{D}=A_0 \sinh\left(\frac{\alpha_D<\sigma_1>_{+}+\sigma_{eq}(1-\alpha_D)}{\sigma_0}\right) \\
\text{si } \alpha_{\sigma}=1 \quad \dot{D}=A_0 \sinh\left(\frac{\alpha_D<tr(\sigma)>_{+}+\sigma_{eq}(1-\alpha_D)}{\sigma_0}\right)
\end{cases}
$$

: ,

ε formula ε^{e} formula ε^{th} formula ε^{p}	respectively the deflections total, elastic, thermal and plastic, is
$<x>_{+}$	the positive part of, x is
σ_1	the maximum principal stress, formula
$\tilde{\sigma}=\sigma-\frac{1}{3}Tr(\sigma)I$	the deviatoric part of the tensor of the stresses, is
$\sigma_{eq}=\sqrt{\frac{3}{2}\tilde{\sigma}_{ij}\tilde{\sigma}_{ij}}$	the deviatoric stress of Von-Put, is
$\boldsymbol{C}$	the elastic tensor of stiffness, is
p	the cumulated plastic strain
H , H_1 are H_2	the variables of viscoplastic isotropic hardening, is
D	the scalar variable of isotropic damage, is
ϕ	the scalar variable of microstructural damage, is
α_{σ}	the parameter allowing for choice of to calculate the damage compared to for (σ_{eq},σ_1) formula $\alpha_{\sigma}=0$ formula $(\sigma_{eq}, Tr(\sigma))$ formula $\alpha_{\sigma}=1$ formula
α_D	the parameter making it possible to adjust the sensitivity to the triaxiality formulates $(\alpha_D=1)$ the sensitivity to maximum principal stress formulates $(\alpha_D=0)$ the computation of the damage, formula
δ_i	0 or 1 according to whether one wishes a linear or nonlinear isotropic hardening, respectively. Nota bene

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The parameters of the model formula $K, \varepsilon_{0,} \sigma_{0,} h_{1,} h_{2,} A_{0,} \alpha_D, \text{et}\, k_c$ be functions of the temperature (in formula $°C$ In [bib2], the identified parameters vary according to the temperature according to a model of Arrhenius. Note:

The preceding system of equations can be reduced: indeed, those which are relating to the evolution of hardening are integrated in the following way: and

$$H_i = \frac{H_i^*}{\delta_i}\left[1-\exp\left(\frac{-h_i\delta_i}{\sigma_{eq}}p\right)\right]$$

the equation relating to the microstructural damage returns to: It

$$\phi = 1-\frac{1}{(1+k_c t)^{1/3}}$$

is this statement which will be used in the continuation of the document. Parameters

3 of the model

the material parameters necessary for the use of the model in Code_Aster via command DEFI _MATERIAU (cf Doc. U4.43.01) are the following: ASTER

Symbol	Definition	EPS
0 K	ε_0	Parameter acting on the viscoplastic kinetics of strain
formula	K	governing the behavior in hyperbolic sine of the viscoplastic model H1
formula	h_1	H2
formula	h_2	DELTA
1 formulate s	δ_1	of the type of hardening DELTA
2 formulate s	δ_2	of the type of hardening H1
ST formulate s	H_1^*	with saturation of hardening, in the nonlinear case H2
ST formulate s	H_2^*	with saturation of hardening, in nonlinear case BIGA
formulate s	A_0	acting on the kinetics of damage SIG
0 formulate s	σ_0	governing the behavior in hyperbolic sine of damage model ALPHAD
KC	α_D	Coefficient exploiting the effective stress for the computation of the damage
formula	k_c	governing the kinetics of microstructural damage S_

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EQUI_D formulates	α_{σ}	of stress hydrostatic or principal maximum

the parameters formulates α_{D} formula k_c and formula α_{σ} optional and have zero values by default.

The identification of the coefficients of the viscoplastic part of the model is carried out from creep tests at various levels of stress. Once

this identification carried out, the coefficients controlling the damage can be identified from long creep tests to low stress for which a fall of ductility is observed (fall of the strains with fracture).

A typical example of identification is detailed in [bib2]. Establishment

4 in Code_Aster

the use of model HAYHURST `is` possible in modelizations 3D, axisymmetric, and plane strain (3D , AXIS , D_PLAN , respectively)

the possible models of strain are PETIT , PETIT_REAC , GDEF_HYPO_ELAS , GDEF_LOG .

The algorithm used is of the total-room type. The total iterations use the elastic stiffness matrix calculated from the matrix of Hooke damaged: Integration $\underline{\underline{\Delta}}=(1-D)\underline{\underline{\Delta}}^0$

4.1 clarifies On the level of

the local iterations (i.e. in each Gauss point), the numerical integration of the equations of velocity is carried out by an explicit diagram of Runge-Kutta of order 2 with automatic cutting under-PAS buildings according to an estimate of the error of integration (method of Runge-Kutta encased) (cf [R5.03.14]). Test SSNV225A illustrates this method. Implicit

4.2 integration For

implicit integration, one will employ the following notations: ,
A^- and A represent ΔA respectively the values of a quantity at the beginning and at the end of time step considered thus that its increment during the step. The system is discretized according to a theta-method: $\Delta A=\Delta t\, g\left(A^-+\theta\Delta A\right)=\Delta t\, g\left(A^{\theta}\right)\quad 0<\theta\leq 1$

The problem discretized to solve is then the following: knowing the state at time formulates t^- the increments of strain formulates $\Delta\boldsymbol{\varepsilon}$ from the phase of prediction, cf [R5.03.01]) and of temperature formulates ΔT the state of the local variables at time formulates t the stresses formulates $\boldsymbol{\sigma}$

taking well into account the variation of the parameters of elasticity with the temperature, it is necessary to discretize (in an implicit way) the elastic relation stress-strain in the following way (see for example [R5.03.02]): with

$$\boldsymbol{C}^{-1}\boldsymbol{\sigma}=\frac{(1-D)}{(1-D^-)}(\boldsymbol{C}^-)^{-1}\boldsymbol{\sigma}^-+(1-D)(\Delta\varepsilon-\Delta\varepsilon^{\mathrm{th}})-(1-D)\Delta\varepsilon^p\qquad \Delta\varepsilon^p=\Delta p\frac{3}{2}\frac{\tilde{\boldsymbol{\sigma}}}{\boldsymbol{\sigma}_{eq}}=\Delta p\,\boldsymbol{n}$$

simplifying the statements and decreasing the number of operations during the resolution, it is also possible to write the first equation according to the elastic strain; that supposes to store the elastic strain or plastic like local variables. One obtains then: with

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$\Delta \boldsymbol{\varepsilon}^e - \left(\Delta \varepsilon - \Delta \varepsilon^{\text{th}}\right) + \Delta p\, \boldsymbol{n}^\theta = 0$ formula $\boldsymbol{n}^\theta = \frac{3}{2} \frac{\tilde{\boldsymbol{\sigma}}^\theta}{\sigma_{eq}^\theta}$ F_e

forced are calculated then using plastic strains at time formulates t^- :

$$\boldsymbol{\sigma} = (1-D)\,\boldsymbol{\sigma}^{nd} = (1-D)\,\boldsymbol{C}\,\varepsilon^e = (1-D)\,\boldsymbol{C}\left((\varepsilon)^- - (\varepsilon^{th})^- - (\varepsilon^p)^- + \theta\,\Delta\varepsilon^e\right)$$

The following equations result from the forms of derivatives of the local variables: (

$$\Delta p - \Delta t\, \varepsilon_0 \sinh\left(\frac{\sigma_{eq}^\theta (1 - H_1^\theta - H_2^\theta)}{K\left(1 - D^\theta\right)(1-\phi)}\right) = 0$$) F_p ,

$\Delta H_i - \frac{h_i}{\sigma_{eq}}\left(H_i^* - \delta_i (H_i^- + \theta\,\Delta H_i)\right)\Delta p = 0$ for i=1,2 formula

(F_{H_i})

$$\Delta D - \Delta t\, A_0 \sinh\left(\frac{\alpha_D < \sigma_p^\theta >_+ + \sigma_{eq}^\theta (1-\alpha_D)}{\sigma_0}\right) = 0$$ ($\sigma_p^\theta = \max_I \sigma_I^\theta \text{ ou } tr(\sigma^\theta)$) F_D One

can formally write this system: formulate $F(\Delta Y) = 0$ and $\Delta Y = \left(\Delta \boldsymbol{\varepsilon}^{\mathrm{e}}, \Delta p, \Delta H_{1,} \Delta H_2, \Delta D\right)^t$

formulates $F(\Delta Y) = \left(F_e, F_p, F_{H_1},, F_{H_2},, F_D\right)^{\mathrm{t}}$

nonlinear system is solved by the iterative method of Newton [R5.03.14]): formulate

$F(\Delta Y_k) + \left(\frac{\partial F}{\partial \Delta Y}\right)_k \left(\Delta Y_{k+1} - \Delta Y_k\right)$ reiterating in jusqu k "with convergence.

The jacobian matrix of the system, necessary to the resolution by the method of Newton, can be calculated either numerically (`ALGORITHME_INTE=' NEWTON_PERT", cf` test SSNV `225B)`, or analytically. In
this last case the form of derivatives is: with

$\left(\frac{\partial F_e}{\partial \Delta \boldsymbol{\varepsilon}^e}\right) = \boldsymbol{I}_d + \Delta p \frac{\partial \boldsymbol{n}^\theta}{\partial \Delta \boldsymbol{\varepsilon}^e}$ formula $\frac{\partial \boldsymbol{n}^\theta}{\partial \Delta \boldsymbol{\varepsilon}^e} = 2\mu\theta \frac{(1-D^\theta)}{\sigma_{eq}}\left[\boldsymbol{I}_{dev} - \boldsymbol{n} \otimes \boldsymbol{n}\right]$ formula

$$\boldsymbol{I}_{dev} = \frac{3}{2}\left(\boldsymbol{I}_4 - \frac{1}{3} I_2 \otimes I_2\right)$$

$\left(\frac{\partial F_e}{\partial \Delta p}\right) = \boldsymbol{n}^\theta$ $\left(\frac{\partial F_e}{\partial \Delta D}\right) = 0$

$$\left(\frac{\partial F_p}{\partial \Delta \boldsymbol{\varepsilon}^{\mathrm{e}}}\right) = -\Delta t\, \varepsilon_0 \cosh\left(\frac{\sigma_{eq}^\theta (1 - H_1^\theta - H_2^\theta)}{K\left(1 - D^\theta\right)(1-\phi)}\right) \frac{(1 - H_1^\theta - H_2^\theta)}{K\left(1-D^\theta\right)(1-\phi)} \frac{\partial \sigma_{eq}^\theta}{\partial \Delta \boldsymbol{\varepsilon}^{\mathrm{e}}}$$

$$\left(\frac{\partial F_p}{\partial\, \Delta p}\right) = 1$$

$$\left(\frac{\partial F_p}{\partial \Delta H_i}\right) = \Delta t\, \varepsilon_0 \cosh\left(\frac{\sigma_{eq}^\theta (1 - H_1^\theta - H_2^\theta)}{K\left(1 - D^\theta\right)(1-\phi)}\right) \theta \frac{\sigma_{eq}^{nd}}{K(1-\phi)}$$

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$\left(\frac{\partial F_p}{\partial \Delta D}\right)=0$: formula $\frac{\boldsymbol{\sigma}^\theta}{1-D^\theta}=\boldsymbol{\sigma}^{nd}$ independent of formula ΔD

$$\left(\frac{\partial F_{H_i}}{\partial \Delta \boldsymbol{\varepsilon}^e}\right)=\frac{h_i}{\sigma_{eq}^2}\Delta p\left(H_i^*-\delta_i H_i^\theta\right)\frac{\partial \sigma_{eq}^\theta}{\partial \Delta \boldsymbol{\varepsilon}^e}$$

$$\left(\frac{\partial F_{H_i}}{\partial \Delta p}\right)=-\frac{h_i}{\sigma_{eq}}\left(H_i^*-\delta_i H_i^\theta\right)$$

$$\left(\frac{\partial F_{H_i}}{\partial \Delta H_i}\right)=1+\frac{h_i}{\sigma_{eq}}\delta_i\theta\Delta p$$

$$\left(\frac{\partial F_{H_i}}{\partial \Delta D}\right)=\frac{-h_i}{\sigma_{eq}^2}\Delta p\left(H_i^*-\delta_i H_i^\theta\right)\theta\,\sigma_{eq}^{nd}$$

$$\left(\frac{\partial F_D}{\partial \Delta \boldsymbol{\varepsilon}^e}\right)=-\Delta t\frac{A_0}{\sigma_0}\cosh\left(\frac{\alpha_D<\sigma_p^\theta>_+ +\sigma_{eq}^\theta(1-\alpha_D)}{\sigma_0}\right)\left(\alpha_D\frac{\partial<\sigma_p^\theta>_+}{\partial \Delta \boldsymbol{\varepsilon}^e}+(1-\alpha_D)\frac{\partial \sigma_{eq}^\theta}{\partial \Delta \boldsymbol{\varepsilon}^e}\right)$$

$$\left(\frac{\partial F_D}{\partial \Delta D}\right)=1+\frac{\Delta t\,A_0\,\theta}{\sigma_0}\cosh\left(\frac{\alpha_D<\sigma_p^\theta>_+ +\sigma_{eq}^\theta(1-\alpha_D)}{\sigma_{nd}}\right)\left[\alpha_D<\sigma_p^0>+(1-\alpha_D)\sigma_{eq}^{nd}\right]$$

: formulate $\frac{\partial \sigma_{eq}^\theta}{\partial \Delta \boldsymbol{\varepsilon}^e}=2\mu\theta(1-D^\theta)\boldsymbol{n}^\theta$

formulates

$\frac{\partial<tr\,\sigma^\theta>}{\partial \Delta \boldsymbol{\varepsilon}^e}=\frac{<tr\,\sigma^\theta>}{tr\,\sigma^\theta}(3\lambda+2\mu)\theta(1-D^\theta)\boldsymbol{I_d}$ formula $\sigma_p^\theta=tr(\sigma^\theta)$

$\frac{\partial<\sigma_1^\theta>}{\partial \Delta \boldsymbol{\varepsilon}^e}=\frac{<\sigma_1^\theta>}{\sigma_1^\theta}\boldsymbol{I_H}$ the principal reference, if formula $\sigma_p^\theta=\sigma_1=\max_I \sigma_I^\theta$

Meaning $\boldsymbol{I_H}=\begin{bmatrix}\lambda+2\mu & \lambda & \lambda & 0 & 0 & 0\\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0\\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$

5 of the local variables

the local variables of the model to Gauss points (key word VARI_ELGA) `are` accessible by: formulate

1) $V1=\varepsilon_{vp}^{11}$
2) $V2=\varepsilon_{vp}^{22}$

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3) $V3=\varepsilon_{vp}^{33}$
4) $V4=\varepsilon_{vp}^{12}$
5) $V5=\varepsilon_{vp}^{13}$
6) $V6=\varepsilon_{vp}^{23}$
7) $V7=p$ the cumulated plastic strain formula
8) $V8=H_1$ the first variable of isotropic hardening viscoplastic formulates
9) $V9=H_2$ the second variable of isotropic hardening viscoplastic formulates
10) $V10=\phi$ the variable of microstructural damage formulates
11) $V11=D$ the variable of damage formulates
12) $V12=0$ not used into explicit, indicator of plasticity into implicit. Bibliography

6 MUSTATA

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2. , C. "Behavior in creep of P92 steel: experimental characterization and modelization", H-T24-2009-00594-FR Notes. PETRY

3. , C. "Behavior in creep of the welded junctions of P92 steel: experimental characterization and modelization", H-T24-2010-00225-FR LATOURTE Notes

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7 of the versions of the document Version

Aster Author	(S) Organization (S) Description	of the modifications 11.2
F.	LATOURTE EDF R & D /MMC initial	Text 11.4
J.M.PROIX EDF	R & D /AMA Addition of	implicit integration

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