# Behavior viscoplastic with damage of Summarized

**CHABOCHE:**

The model viscoplastic coupled with the isotropic damage of Chaboche (developed at the origin to successfully predict the life duration and the cracking of the paddles of the modern turbojets) is used for computations of prediction of the time of failure of structures at high temperatures.

This model is established in *Code_Aster* under the name of VENDOCHAB ; the equations of velocity are integrated numerically by an explicit diagram of Runge-Kutta of order 2 with automatic cutting under-PAS buildings according to an estimate of the error of integration (method of Runge-Kutta encased, confer [R5.03.14]), or by an implicit integration method [R5.03.01].

Tests SSNV126 and SSNV183 validate the integration of this model. The document of validation [V6.04.126] provides the analytical solution for an isothermal uniaxial creep test.

## Contents

# 1 Introduction

computations by finite elements carried out into the frame as of studies on the serious accidents of the nuclear reactors highlighted the need to use models of damage in order to lay down the failure of a structure such as the tank subjected to the severe and complex thermal conditions (high temperatures going until fusion, high thermal gradients in space or time, etc) which would impose to him the corium [bib1].

The major interest of this choice lies in the fact that the value of the variable of damage to the fracture (or cracking) can be regarded as an intrinsic parameter of the material which is accessible, although that is difficult and delicate, by physical measurements (ultrasounds, diffraction $X$, etc). The rupture criterion with the theory of the damage is then more “physical” that the criteria in maximum strain used sometimes in viscoplastic computations without damage or the criteria of damage not coupled (rule of addition of time actually passed under certain conditions $(\sigma, T)$ divided by the time of failure for these same conditions).

The model implemented in *Code_Aster* is a viscoplastic model of behavior to viscosity-multiplicative hardening coupled with the isotropic damage (models due to Chaboche [bib2]).

**Nota bene:**

*One will find in the reference [bib3] a detailed description of the capacities of the model, a methodology for the identification of the parameters and the values of these parameters for steel 22 MoNiCr 3 7.*

# 2 Formulation of the model

## 2.1 Tallies theoretical

In this sub-chapter, one insists on the specificity of model VENDOCHAB (i.e. the damage) compared to the usual viscoplastic models. For more details, one will refer to [bib2].

The theory of the damage describes the evolution of the phenomena between the virgin state and the macroscopic crack initiation in a material by means of a continuous variable (scalar or tensorial) describing the progressive deterioration of this material. Up this idea, due to Kachanov which was the first to use it to model the creep rupture of metals in uniaxial request, was taken in France in the Seventies by Lemaitre and Chaboche. The evolution of the material of its virgin state in its damaged state is not always easy to distinguish from the phenomenon of strain generally accompanying it and is due to several different mechanisms of which creep is part. The viscoplastic damage of creep corresponds to intergranular decoherences accompanying the strains viscoplastic for metals with average temperatures and high.

To define what is this variable of damage, let us consider the area $S$ of one of the sides of a volume element $\Omega$ located by its norm directed towards outside $\mathbf{n}$ . On this section, the microscopic cracks and the cavities which constitute the damage leave traces of various forms. That is to say $\tilde{S}$ the effective resistant area and $S_D$ the total area of all the traces.

One a:

$$S_D=S-\tilde{S}$$

and one defines the variable of damage by:

$$D_n=\frac{S_D}{S}$$

$D_n$ is the measurement of the local damage compared to the direction $n$ . From a physical point of view, the variable of damage $D_n$ is thus the area relative of cracks and cavities cut by the normal plane to the direction $\vec{n}$ . From a mathematical point of view, while making tend $S$ towards 0, the variable $D_{\vec{n}}$ is the surface density of discontinuities of the matter in the normal plane with $n$ . $D_n=0$ corresponds at the virgin state not damaged. $D_n=1$ corresponds to the volume element broken in two parts according to a normal plane to $n$ .

The assumption of isotropy implies that the cracks and cavities are uniformly distributed in directional sense in a point of the material. In this case, the variable of damage becomes a scalar which does not depend any more directional sense and is noted $D$ . One a:

$$D=D_n \forall n$$

We will consider here only the isotropic variable of damage.

Total mechanical measurements (modification of the characteristics of elasticity, plasticity or viscoplasticity) are easier to interpret in term of variable of damage thanks to the notion of effective stress introduced by Rabotnov. The effective stress represents the stress reported to the section which resists the forces indeed. In the case of the isotropic damage, she is written:

$$\tilde{\sigma}=\frac{\sigma}{(1-D)}$$

And it a:

1) $\tilde{\sigma}=\sigma$ for a virgin material
2) $\tilde{\sigma}\to+\infty$ at the instant of the failure

the principle of equivalence in strain is implied that any behavior with the strain, unidimensional or three-dimensional of a damaged material is translated by the constitutive laws of the virgin material in which one replaces the usual stress by the effective stress.

One distinguishes 2 types of variables to characterize the medium:

Observable variables (measurable):

- the temperature $T$
- the total deflection $\underline{\underline{\varepsilon}}$ which breaks up as indicated below:

$$\underline{\underline{\varepsilon}}=\underline{\underline{\varepsilon}}^{e}+\underline{\underline{\varepsilon}}^{vp}+\underline{\underline{\varepsilon}}^{th}$$

Local variables:

- the viscoplastic strain $\underline{\underline{\varepsilon}}^{vp}$
- the isotropic variable of hardening $r$
- the isotropic variable of damage $D$

Is $\Psi=\Psi\left(\underline{\underline{\varepsilon}},\underline{\underline{\varepsilon}}^{vp},T,r,D\right)$ , the potential of state, the state models describing this potential are:

$$\begin{cases}\underline{\underline{\sigma}}=\rho\dfrac{\partial\Psi}{\partial\underline{\underline{\varepsilon}}}\\ R=-\rho\dfrac{\partial\Psi}{\partial r}\\ s=-\rho\dfrac{\partial\Psi}{\partial T}\\ Y=-\rho\dfrac{\partial\Psi}{\partial D}\end{cases}$$

According to the model of normality, one has, with $\Phi$ , the potential of dissipation:

$$\begin{cases}\dot{\underline{\underline{\varepsilon}}}^{vp}=\dfrac{\partial\Phi}{\partial\underline{\underline{\sigma}}}\\ \dot{r}=\dfrac{\partial\Phi}{\partial R}\\ \dot{D}=\dfrac{\partial\Phi}{\partial Y}\end{cases}$$

The modelization of the hardening and the damage of the material is done via local variables (or hidden). In the case of the model VENDOCHAB, the local variables introduced into *Code_Aster* are:

- $\underline{\underline{\varepsilon}}^{vp}$ : tensor of the inelastic strains
- $p$ : cumulated plastic strain
- $r$ : variable of hardening viscosity
- $D$ : scalar variable of isotropic damage

## 2.2 Equations of the model

the equations of the models are written then:

$$
\begin{cases}
\underline{\underline{\varepsilon}}=\underline{\underline{\varepsilon}}^{e}+\underline{\underline{\varepsilon}}^{th}+\underline{\underline{\varepsilon}}^{vp} \\
\underline{\underline{\sigma}}=(1-D)\underline{\underline{\underline{\underline{\Lambda}}}}\,\underline{\underline{\varepsilon}}^{e} \\
\dot{\underline{\underline{\varepsilon}}}^{vp}=\frac{3}{2}\dot{p}\frac{\underline{\underline{\sigma}}'}{\sigma_{eq}} \\
\dot{p}=\frac{\dot{r}}{(1-D)} \\
\dot{r}=\left\langle\frac{\sigma_{eq}-\sigma_{y}(1-D)}{(1-D)K r^{1/M}}\right\rangle^{N} \\
\dot{D}=\left\langle\frac{\chi(\underline{\underline{\sigma}})}{A}\right\rangle^{R}(1-D)^{-k(\chi(\underline{\underline{\sigma}}))}
\end{cases}
$$

with:

$$\chi(\underline{\underline{\sigma}})=\alpha J_0(\underline{\underline{\sigma}})+\beta J_1(\underline{\underline{\sigma}})+(1-\alpha-\beta)J_2(\underline{\underline{\sigma}})$$

où : $J_0(\underline{\underline{\sigma}})$ est la contrainte principale maximale

$$J_1(\underline{\underline{\sigma}})=Tr(\underline{\underline{\sigma}})$$

$$J_2(\underline{\underline{\sigma}})=\sigma_{eq}=\sqrt{\frac{3}{2}\tilde{\sigma}'_{ij}\tilde{\sigma}'_{ij}}$$

$\langle x\rangle$ partie positive de $x$

where:

| | |
|---|---|
| $\underline{\underline{\varepsilon}}, \underline{\underline{\varepsilon}}^{e}, \underline{\underline{\varepsilon}}^{th}$ and $\underline{\underline{\varepsilon}}^{vp}$ | are respectively the deflections total, elastic, thermal and plastic, |
| $\underline{\underline{\underline{\underline{\Lambda}}}}=(\Lambda_{ijkl})$ | is the elastic tensor of stiffness, |
| $\underline{\underline{\sigma}}'=\underline{\underline{\sigma}}-\frac{1}{3}Tr(\underline{\underline{\sigma}})\underline{\underline{Id}}$ | is the deviatoric part of the tensor of the stresses, |
| $p$ | is the cumulated plastic strain, |
| $r$ | is the variable of isotropic hardening viscoplastic |
| $D$ | is the scalar variable of isotropic damage |

**Nota bene:**

*All the parameters of the model* $\alpha, \beta, N, M, K, A, R$ et $k$ can be functions of the temperature (in $°C$). $k$ can be constant, depend on the temperature or $\chi(\underline{\underline{\sigma}})$ (in $MPa$) and the temperature.

In addition, it is seen that this model considers that it can exist a viscoplastic threshold $\sigma_y$ which depends on the temperature.

It is seen that this model is reduced to the viscoplastic model of Lemaitre if it is considered that $D=0$ and if one neglects the equation of evolution of $D$. $M, N,$ et $K$ are coefficients characteristic of the purely viscoplastic behavior of the material.

The evolution of the damage is governed by a model with three parameters: $A, R$, et $k$ . The equivalent stress $\chi(\underline{\underline{\sigma}})$ makes it possible to take account of a possible effect of the spherical part of the tensor of the stresses on the damage (a little as in the models growth cavities at the base the models Gurson and Rousselier). The fact that the maximum principal stress can play a part in $\chi(\underline{\underline{\sigma}})$ is difficult to imagine for materials as steel but makes more general the model.

# 3 Computation of the material parameters

the parameters of the constitutive law can be calculated from creep tests carried out for various levels of stresses and temperature. For that one uses a unidimensional constitutive law because the request of a cylindrical test-tube in tension can be modelled in dimension 1. The tensor of the stresses is reduced to its axial component.

$$\underline{\underline{\sigma}}=\begin{pmatrix}\sigma_0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix} \text{ and } \underline{\underline{\sigma}}'=\sigma_0\begin{pmatrix}\frac{2}{3} & 0 & 0\\ 0 & -\frac{1}{3} & 0\\ 0 & 0 & -\frac{1}{3}\end{pmatrix}$$

formulates thus has: $J_0(\underline{\underline{\sigma}})=J_1(\underline{\underline{\sigma}})=J_2(\underline{\underline{\sigma}})=\sigma_0$

$$\chi(\underline{\underline{\sigma}})=\sigma_0 \quad \forall(\alpha,\beta)$$

The system of equations to be solved is then:

$$\underline{\underline{\dot{\varepsilon}}}^{vp}=\dot{p}\begin{pmatrix}1 & 0 & 0\\ 0 & -\frac{1}{2} & 0\\ 0 & 0 & -\frac{1}{2}\end{pmatrix}$$

$$\dot{p}=\frac{\dot{r}}{(1-D)}$$

$$\dot{r}=\left\langle\frac{\sigma_0-\sigma_y(1-D)}{(1-D)K\,r^{\frac{1}{M}}}\right\rangle^N$$

$$\dot{D}=\left\langle\frac{\sigma_0}{A}\right\rangle^R (1-D)^{-k(\sigma_0)}$$

This system of equations is integrable, which makes it possible to have only one cumulated equation for viscoplastic strain rate (that one can compare to the total deflection by neglecting the elastic strain).

One can then correlate this statement with the experimental data to adjust the coefficients, but the number of parameters and the non-linearities make that difficult (moreover there is not unicity).
It is thus necessary to use a method of correlation calling on “physical” assumptions on the phenomenon of creep whose curve is represented hereafter.

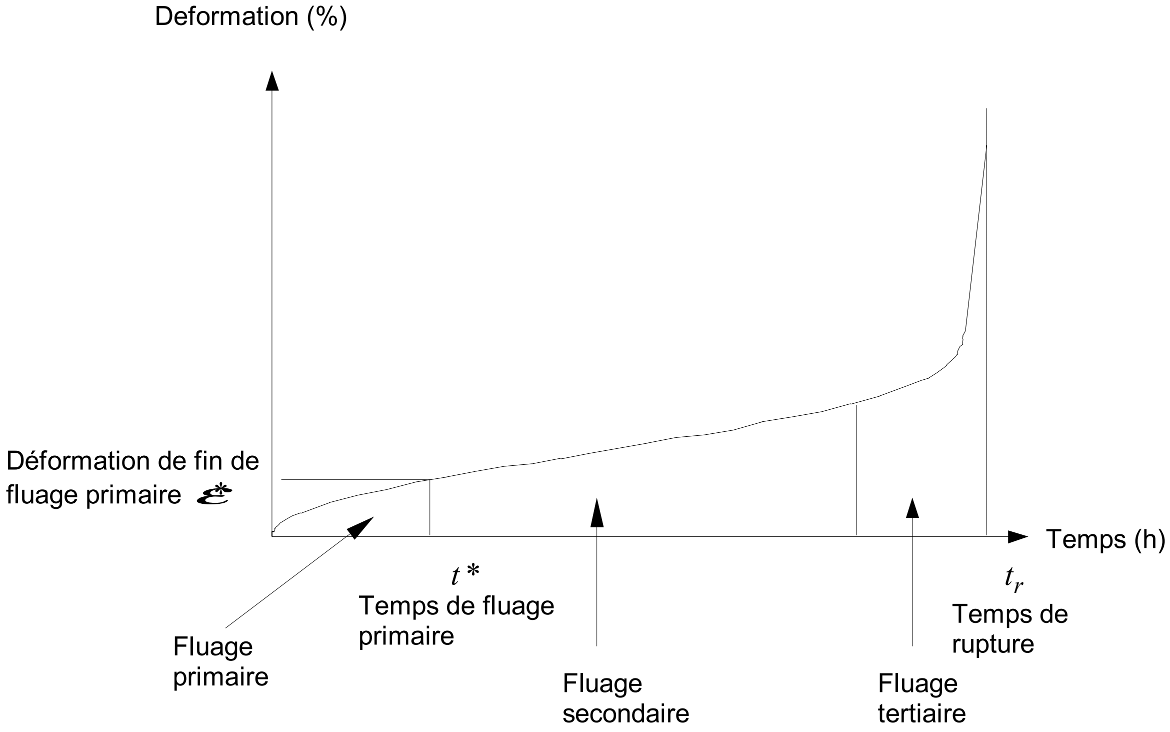



**Figure 3-a : The various phases of creep on a curve of creep**

the curve of strain according to time obtained after a creep test breaks up into three parts:

- a part known as of primary education creep where the damage is negligible.
- a part known as of secondary creep where the strainrate is appreciably constant.
- a part known as of tertiary creep where hardening is saturated and where phenomena of damage are dominating.

A method of calculating of the parameters using the experimental data $(\varepsilon, t)$ (one also uses $(\dot{\varepsilon}, t)$ who from of deduced by a numerical procedure) for various levels of stresses and various temperatures was elaborate to the French atomic energy agency. It uses the statements found in the case of higher a homogeneous and unidimensional stress by making assumptions according to the part of the curve where the data are taken. For example, in the primary education phase of creep, one makes the assumption $D=0$ and in the secondary phase of creep, one uses the fact that $\dot{\varepsilon}$ is constant.

One will find in the references [bib3] and [bib4] the complete description and of the examples of computations carried out on the German steel of tank 22 MoNiCr 3 7.

# 4 Establishment in *Code_Aster*

## 4.1 Algorithm of resolution

the algorithm used is of the total-room type.

The total iterations use the elastic stiffness matrix calculated from the matrix of Hooke damaged:

$$\underline{\underline{\Lambda}}=(1-D)\,\underline{\underline{\Lambda}}^0$$

On the level as of local iterations (i.e. in each Gauss point), the numerical integration of the equations of velocity can be carried out either by an explicit diagram of Runge-Kutta of order 2 with automatic cutting under-PAS buildings according to an estimate of the error of integration (method of Runge-Kutta encased), or by an implicit scheme of Eulerian solved by a method of Newton. One will refer to the references [bib3] for all the details concerning the numerical methods, and with [R5.03.14] for the explicit algorithms employed and their data-processing programming.

## 4.2 Implicit integration of the behavior model

A each total iteration of resolution of the variational problem of the equilibrium and for each point of elementary integration, it is necessary to integrate the equations of the model described into [§3] to obtain the tensor of the stresses and to possibly calculate the operator of tangent behavior.
The problem written in a generic form at the moment T is made up by the four following nonlinear systems of equations:

$$\begin{cases}\underline{Rb}\left(\underline{\beta},\underline{p},\underline{\varepsilon},\underline{\nu}_{etat}^{t-1}\right)=\underline{0}\\ \underline{Rp}\left(\underline{\beta},\underline{p},\underline{\varepsilon},\underline{\nu}_{etat}^{t-1}\right)=\underline{0}\end{cases} \quad \text{éq 4.2-1}$$

with

$$\begin{cases}\underline{\Omega}\left(\underline{\underline{\sigma}},\underline{\beta},\underline{p},\underline{\varepsilon},\underline{\nu}_{etat}^{t-1}\right)=\underline{0}\\ \underline{\Theta}\left(\underline{\nu}_{etat},\underline{\beta},\underline{p},\underline{\varepsilon},\underline{\nu}_{etat}^{t-1}\right)=\underline{0}\end{cases} \quad \text{éq 4.2-2}$$

**Rb** is a system of six equations (six unknowns) describing the unknowns associated with the stresses. One notes $\underline{\beta}$ the vector 6 components of these unknowns. Connection enters $\underline{\beta}$ and $\underline{\underline{\sigma}}$ is carried out by means of the system of equations $\underline{\Omega}$ and the vector $\underline{p}$ contains the variables $r$ and $D$.
**Rp** is a system of equations describing the internal unknowns. One chooses a system of 2 equations with $\dot{D}$ and $\dot{r}$ like internal unknowns. The evolution of the variables of state is described by the system of equations $\underline{\Theta}$.

The implicit scheme of Eulerian is used and the algorithm is presented in the following form:

| |
|---|
| Initialization of the unknowns of the discretized problem and recovery of the values of the variables of state obtained with the preceding step |
| iterations of the method of Newton (maximum number of pre iterations defined by the user):<br>1) Recovery of the values of the parameters intervening in the material model (the operator of elasticity)<br>2) Computation of the criteria of stress and their derivatives compared to the stresses<br>3) Recovery of the values of the parameter K intervening in the evolution of the damage and its derivative<br>4) Computation of the current price of the variables of state, the equations describing the internal unknowns and the equations describing the stresses<br>5) Computation of derivatives of the equations compared to the unknowns<br>6) Resolution of the linear system<br>$\begin{bmatrix} \frac{\partial \underline{\mathbf{Rb}}^n}{\partial \underline{\beta}} & \frac{\partial \underline{\mathbf{Rb}}^n}{\partial \underline{p}} \\ \frac{\partial \underline{\mathbf{Rp}}^n}{\partial \underline{\beta}} & \frac{\partial \underline{\mathbf{Rp}}^n}{\partial \underline{p}} \end{bmatrix} \begin{bmatrix} d\underline{\beta} \\ d\underline{p} \end{bmatrix} = - \begin{bmatrix} \underline{\mathbf{Rb}}^n \\ \underline{\mathbf{Rp}}^n \end{bmatrix}$ **éq 4.2-3**<br>• Test of convergence |
| Evaluating of the tangent operator |

### 4.2.1 implicit Discretization of the equations of the model

Considering that an increment of time characterizes a new state of the system [éq 4.2-1] and [éq 4.2-2] solved by an algorithm of Newton, one choosesto identify the state of one quantity at previous time by exhibitor $^{T-1}$ whereas its current state is noted without exhibitor. Thus the variation of a quantity for the increment of time considered arises by $U = U^{t-1} + \Delta U = U^{t-1} + \Delta t \ \dot{U}(\theta \Delta t)$
For $\theta = 0$ , one obtains an explicit diagram and for $\theta = 1$ , one obtains a purely implicit diagram.

With these notations, the discretized form of the vectorial system is written:

$$\underline{Rb} \equiv \underline{\beta} - \left(1 - \left(D^{t-1} + \Delta t \ \dot{D}\right)\right) \underline{\underline{\Lambda}} \left(\underline{\varepsilon} - \underline{\varepsilon}_{th} - \left(\underline{\varepsilon}_{vp}^{t-1} + \Delta t \ \frac{3}{2} \ \dot{r} \ \frac{\underline{\sigma}'(\underline{\beta})}{\left(1 - \left(D^{t-1} + \Delta t \ \dot{D}\right)\right) \sigma_{eq}(\underline{\beta})}\right)\right) = \underline{0}$$

**éq 4.2.1-1**

or more simply $\underline{Rb} \equiv \underline{\beta} - \left(1 - \left(D^{t-1} + \Delta t \ \dot{D}\right)\right) \underline{\underline{\Lambda}} \ \underline{\varepsilon}_{el} = \underline{0}$

$\underline{\Omega} \equiv \underline{\sigma} = \underline{\beta}$

where $\underline{\beta}$ is the vector 6 components from the tensor of the stresses $\underline{\underline{\sigma}}$ .

$$\underline{Rp} \equiv \begin{cases} \dot{r} - \left\langle \dfrac{\sigma_{eq}(\underline{\beta}) - \sigma_y \left(1 - \left(D^{t-1} + \Delta t\ \dot{D}\right)\right)}{\left(1 - \left(D^{t-1} + \Delta t\ \dot{D}\right)\right)\ K\ \left(r^{t-1} + \Delta t\ \dot{r}\right)^{1/M}} \right\rangle^N = 0 \\ \dot{D} - \left\langle \dfrac{\chi(\underline{\beta})}{A} \right\rangle^R \left(1 - \left(D^{t-1} + \Delta t\ \dot{D}\right)\right)^{-k(\chi(\underline{\beta}))} = 0 \end{cases} \quad \text{éq 4.2.1-2}$$

the evolution of the variables of state is described by the system of equations $\underline{\Theta}$ :

$$\underline{\Theta} \equiv \begin{cases} D = D^{t-1} + \Delta t\ \dot{D} \\ \underline{\varepsilon}_{vp} = \underline{\varepsilon}_{vp}^{t-1} + \Delta t\ \dfrac{3}{2} \dfrac{\dot{r}}{\left(1 - \left(D^{t-1} + \Delta t\ \dot{D}\right)\right)} \dfrac{\underline{\sigma}'}{\sigma_{eq}} = \underline{\varepsilon}_{vp}^{t-1} + \Delta t\ \dfrac{\dot{r}}{\left(1 - \left(D^{t-1} + \Delta t\ \dot{D}\right)\right)} \dot{\sigma}_{eq} \\ r = r^{t-1} + \Delta t\ \dot{r} \end{cases} \quad \text{éq 4.2.1-3}$$

where $D$ , $\underline{\varepsilon}_{vp}$ and $r$ is the variables of state whose history is preserved.

The strains $\underline{\underline{\varepsilon}}$ and the variable of states are not unknowns of the problem. These quantities will be filed with each increment of time converged to be re-used with the following increment.

### 4.2.2 Numerical resolution

the resolution of the nonlinear system $\begin{cases} \underline{\mathbf{Rb}} = \underline{0} \\ \underline{\mathbf{Rp}} = \underline{0} \end{cases}$ uses the method of Newton-Raphson associated with a technique with tangential approximation in order to seek the solutions in a field where the functions are correctly conditioned.

According to the algorithm of Newton-Raphson, one solves this system in an iterative way on the following sequence:

1) Initialization of the unknowns
2) Searches of a direction of descent by the resolution of the system [éq 4.2-3]
3) Test of convergence $err = \dfrac{\sum |\Delta x|}{\sum |x|}$

### 4.2.3 Operator of tangent behavior

the tangent operator is obtained by deriving the stresses compared to the total deflections according to the made up derivative rules:

$$\frac{d\,\underline{\sigma}}{d\,\underline{\varepsilon}} = \frac{\partial \Sigma}{\partial \underline{\beta}} \frac{\partial \underline{\beta}}{\partial \underline{\varepsilon}} + \frac{\partial \Sigma}{\partial \underline{p}} \frac{\partial \underline{p}}{\partial \underline{\varepsilon}} + \frac{\partial \Sigma}{\partial \underline{\varepsilon}} \quad \text{éq 4.2.3-1}$$

where the stress function $\Sigma\left(\underline{\beta}, \underline{p}, \underline{\varepsilon}, \underline{v}_{etat}^{t-1}\right) = \underline{\beta}$ . The derivatives of the unknowns compared to the total deflections are obtained by deriving the system [éq 4.2-1] that is to say:

$$\begin{bmatrix} \frac{\partial \underline{Rb}}{\partial \underline{\beta}} & \frac{\partial \underline{Rb}}{\partial \underline{p}} \\ \frac{\partial \underline{Rp}}{\partial \underline{\beta}} & \frac{\partial \underline{Rp}}{\partial \underline{p}} \end{bmatrix} \begin{bmatrix} \frac{\partial \underline{\beta}}{\partial \underline{\varepsilon}} \\ \frac{\partial \underline{p}}{\partial \underline{\varepsilon}} \end{bmatrix} = - \begin{bmatrix} \frac{\partial \underline{Rb}}{\partial \underline{\varepsilon}} \\ \frac{\partial \underline{Rp}}{\partial \underline{\varepsilon}} \end{bmatrix}$$ **éq 4.2.3-2**

### 4.2.4 Cas particulier of the plane stresses

the elements 2D in plane stresses having to be usable for this model of behavior, one carries out an additional processing out of on-layer of the general processing carried out in 3D.
A positive test on the case of the plane stresses means:

1) To the resolution of the system [éq 4.2-1], one adds the additional equations $\sigma_{33} = \sigma_{13} = \sigma_{23} = 0$
2) One modifies the tangent operator to ensure the energy equilibrium.

## 4.3 Optimized implicit integration

If one does not take account of the trace of the stresses (obtained for zero values of the coefficients $\alpha$ *and* $\beta$ *),* the function $\chi(\underline{\underline{\sigma}})=J_2(\underline{\underline{\sigma}})$
So moreover one a: $R=k$ , then the equations of the model are reduced to:

$$
\left\{
\begin{array}{l}
\Delta(\varepsilon^e)=(\boldsymbol{\Delta}\,\varepsilon-\boldsymbol{\Delta}\,\varepsilon^{\mathbf{th}}-\boldsymbol{\Delta}\,\varepsilon^{\mathbf{vp}})=\Delta\left(\dfrac{\Lambda^{-1}\sigma}{1-D}\right)=\left(\dfrac{\Lambda^{-1}\sigma}{1-D}\right)-\left(\dfrac{\Lambda^{-1}\sigma}{1-D^-}\right)^-\\
\Rightarrow \quad tr(\boldsymbol{\Delta}\,\varepsilon-\boldsymbol{\Delta}\,\varepsilon^{\mathbf{th}})+\left(\dfrac{tr\,\sigma^-}{(3\lambda+2\mu)^-(1-D^-)}\right)=\dfrac{tr(\sigma)}{(3\lambda+2\mu)(1-D)}\\
\qquad 2\mu\,(\Delta\varepsilon'-\boldsymbol{\Delta}\,\varepsilon^{\mathbf{vp}})+2\mu\left(\dfrac{\sigma'}{2\mu(1-D^-)}\right)^-=\dfrac{(\sigma')}{(1-D)}\\
\boldsymbol{\Delta}\,\varepsilon^{\mathbf{vp}}=\dfrac{3}{2}\dfrac{\Delta r}{(1-D)}\dfrac{\sigma'}{\sigma_{eq}}\\
\Delta r=\Delta t\left\langle\dfrac{\sigma_{eq}-\sigma_y(1-D)}{(1-D)K\,r^{1/M}}\right\rangle^N\\
\Delta D=\Delta t\left\langle\dfrac{\sigma_{eq}}{A(1-D)}\right\rangle^R
\end{array}
\right.
$$

$\Delta a=a-a^-$ Represent the variation of the quantity has between current time and previous time noted it

chooses a pure implicit discretization here, i.e. that the terms in D, R which appear are selected at the time of current computation.
Note: one can write in an equivalent way the first equation in the form:

$$(\varepsilon^e)=(\varepsilon-\varepsilon^{\mathbf{th}}-\varepsilon^{\mathbf{vp}})=\left(\frac{\Lambda^{-1}\sigma}{1-D}\right)\Rightarrow\left(\frac{\sigma'}{1-D}\right)=2\mu(\varepsilon-\boldsymbol{\Delta}\,\varepsilon^{\mathbf{th}}-\boldsymbol{\Delta}\,\varepsilon^{\mathbf{vp}})=2\mu(\varepsilon^--\varepsilon^{p-}+\Delta\varepsilon'-\boldsymbol{\Delta}\,\varepsilon^{\mathbf{vp}})$$

$$\text{soit }(\varepsilon^e)=2\mu(\varepsilon^{e-}+\Delta\varepsilon'-\boldsymbol{\Delta}\,\varepsilon^{\mathbf{vp}})=2\mu\left(\left(\frac{\sigma'}{(1-D)2\mu}\right)^-+\Delta\varepsilon'-\boldsymbol{\Delta}\,\varepsilon^{\mathbf{vp}}\right)$$

Then by eliminating the viscoplastic strain, and by gathering the unknown terms in the equation in stresses:

$$
\begin{cases}
tr(\sigma)=(3\lambda+2\mu)\left[\left(\dfrac{tr\,\sigma}{(3\lambda+2\mu)(1-D)}\right)^{-}+tr\left(\boldsymbol{\Delta\varepsilon}-\boldsymbol{\Delta\varepsilon}^{th}\right)\right]\\
2\mu\left(\boldsymbol{\Delta\varepsilon}^{\mathbf{vp}}\right)=2\mu\left(\dfrac{\sigma'}{2\mu(1-D)}\right)^{-}+2\mu\left(\Delta\varepsilon'\right)-\dfrac{\sigma'}{(1-D)}=s^{e}-\dfrac{\sigma'}{(1-D)}\\
\text{avec }\ s^{e}=2\mu\left(\dfrac{\sigma'}{2\mu(1-D)}\right)^{-}+2\mu\left(\Delta\varepsilon'\right)\\
2\mu\,\boldsymbol{\Delta\varepsilon}^{\mathbf{vp}}=\dfrac{3}{2}2\mu\dfrac{\Delta r}{(1-D)}\dfrac{\sigma'}{\sigma_{eq}}=s^{e}-\dfrac{\sigma'}{(1-D)}\\
\dfrac{\Delta r}{\Delta t}=\left\langle\dfrac{\dfrac{\sigma_{eq}}{(1-D)}-\sigma_{y}}{K\,r^{1/M}}\right\rangle^{N}\Leftrightarrow K\left(\dfrac{\Delta r}{\Delta t}\right)^{1/N}r^{1/M}+\sigma_{y}=\dfrac{\sigma_{eq}}{(1-D)}\quad\text{si }\dfrac{\sigma_{eq}}{(1-D)}\geq\sigma_{y},\quad\Delta r=0\ \text{ sinon}\\
\Delta D=\Delta t\left\langle\dfrac{\sigma_{eq}}{A(1-D)}\right\rangle^{R}\Leftrightarrow\Delta D=\Delta t\left\langle\dfrac{1}{A}\left(K\left(\dfrac{\Delta r}{\Delta t}\right)^{1/N}r^{1/M}+\sigma_{y}\right)\right\rangle^{R}
\end{cases}
$$

In this case one can simplify this system of equations while bringing back oneself to only one equation in $\Delta r$, in the following way:

$$
\Delta r\ \text{ solution of: }\begin{cases}K\left(\dfrac{\Delta r}{\Delta t}\right)^{1/N}r^{1/M}+\sigma_{Y}+\dfrac{3\mu\Delta r}{1-D}=s_{eq}^{e} & \text{si }\dfrac{\sigma_{eq}}{1-D}\geq\sigma_{Y},\\ \Delta r=0 & \text{sinon}\end{cases}
$$

$$
\frac{\sigma'}{1-D}\left(1+\frac{3\,\mu\,\Delta\,r}{\sigma_{eq}}\right)=s^{e}\Rightarrow\sigma_{eq}+3\,\mu\,\Delta\frac{r}{1-D}=s_{\mathrm{eq}}^{e}
$$

where $\Delta D$ is a function of $\Delta r$ defined by $\Delta D=\Delta t\left\langle\dfrac{1}{A}\left(K\left(\dfrac{\Delta r}{\Delta t}\right)^{1/N}r^{1/M}+\sigma_{Y}\right)\right\rangle^{R}$

Numerically, one multiplies this equation by the term in $\Delta t^{1/N}$ (to avoid the concern when time step tends towards 0), and by $1-D$ :

$$
\Delta r\ \text{ solution de :}
$$

$$
f(\Delta r)=(1-D)\left(K(\Delta r)^{1/N}r^{1/M}+\sigma_{y}(\Delta t)^{1/N}\right)+3\,\mu\,\Delta r(\Delta t)^{1/N}-(1-D)\,s^{e_{eq}}\,(\Delta t)^{1/N}=0
$$

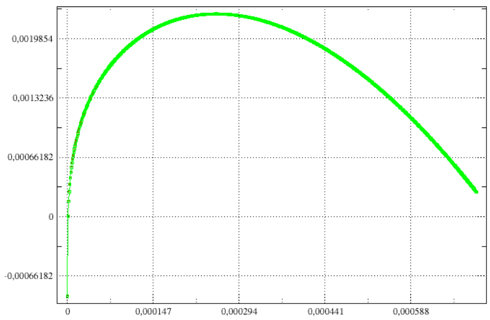
In fact, $D$ cannot increase up to 1, but is restricted with $0.99$, which makes it possible to avoid singular tangent matrixes.

To find the solution of this equation, one uses a method of secant, after having obtained a framing. The function $f$ is negative into 0 by construction. It is a question of seeking a higher limit for which $f$ is positive.

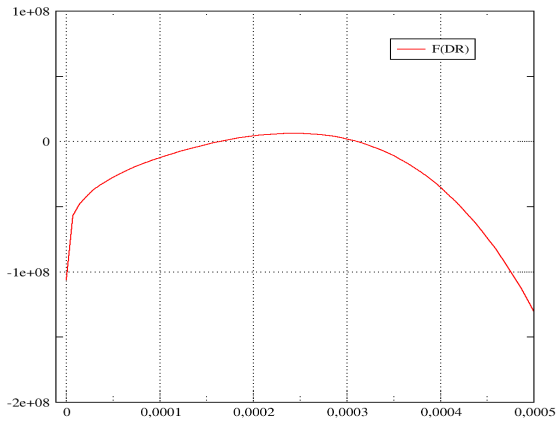
$$\left\{\Delta r=-\frac{(1-D)}{3\mu}\left(K\left(\frac{\Delta r}{\Delta t}\right)^{1/N} r^{1/M}+\sigma_y\right)+\frac{(1-D)}{3\mu}s^{e}{}_{eq} \leq \frac{(1-D)}{3\mu}s^{e}{}_{eq} \leq \frac{1}{3\mu}s^{e}{}_{eq}\right.$$

If $f\left(\frac{1}{3\mu}s^{e}{}_{eq}\right)>0$, then the framing is well defined. The solution is sought in an interval defined by the limits: $[0,\frac{1}{3\mu}s^{e}{}_{eq}]$.

If not, it is necessary to seek a higher limit making the function positive. However this function admits two solutions in general: on the following curve the function in $f(\Delta r)$ two typical cases is represented: for a situation where $D$ is small, and for a situation where $D$ is close to 1. It is noted that the function crosses the x-axis twice. The solution which one seeks is that which corresponds to $\Delta r$ minimum.




Allure de f(dR) dans le cas ou D est proche de 1




With this intention, since $f\left(\frac{1}{3\mu}s^{e}{}_{eq}\right)<0$, one calculates derivative of $f$. If east is positive one increases the higher limit until finding a value such as $f$ is positive. If the derivative is negative, one decreases the value of the limit to obtain $f>0$. One applies then the method of rope.

The accuracy chosen for the resolution is of $\mathrm{RESI\_INTE\_RELA} * f(0)$.

Once the value of $\Delta r$ obtained, one has immediately that of $\Delta D$, then the deviator of the stresses is calculated by:

$$\sigma' = (1-D)\left(1+\frac{3\mu\Delta r}{\sigma_{eq}}\right)^{-1} s^{e}$$

and traces it by:

$$tr(\Delta\varepsilon-\Delta\varepsilon^{\mathrm{th}})+\left(\frac{tr\,\sigma^{-}}{(3\lambda+2\mu)^{-}(1-D^{-})}\right)=\frac{tr(\sigma)}{(3\lambda+2\mu)(1-D)}$$

# 5 Meaning of the local variables

the local variables of the model to Gauss points (key word `VARI_ELGA`) are accessible by:

1) $V1=\varepsilon_{vp}^{11}$
2) $V2=\varepsilon_{vp}^{22}$
3) $V3=\varepsilon_{vp}^{33}$
4) $V4=\varepsilon_{vp}^{12}$
5) $V5=\varepsilon_{vp}^{13}$
6) $V6=\varepsilon_{vp}^{23}$
7) $V7=p$ , cumulated plastic strain
8) $V8=r$ , the variable of isotropic hardening viscoplastic
9) $V9=D$ , the variable of damage
10) $V10=ind$ , indicator being worth 0 if the current point remained elastic with the current step, 1 if not.

# 6 Bibliography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# 7 of the versions of the document Version

| **Aster Author** | **(S) notes Organization (S)** | **Description of the modifications** |
|---|---|---|
| 6.4 | P.DUPAS | initial Text |
| 8.4 | J.M.PROIX, 0.DIARD Modifications | into implicit 9.4 J.M.PROIX Modification |
| | of | the key words and local variables 10,1 J.M.PROIX Improvement |
| 10.1 | the search | for the solution for the cases where the damage is strong: new algorithm of framing and method of secant. |