Behavior models of the discrete elements

Summarized:

This document describes the nonlinear behaviors of the discrete elements which are called by the operators of resolution of nonlinear problems `STAT_NON_LINE` or `DYNA_NON_LINE`.

More precisely, the behaviors described in this document are:
- the behavior of the type Von-Put at isotropic hardening used for the modelization of the threaded assemblies, accessible by key keys `DIS_GOUJ2E_PLAS` and `DIS_GOUJ2E_ELAS` from key word `COMP_INCR`,
- the behavior of type contact with shock and friction of Coulomb, accessible by key word `DIS_CHOC` from key word `COMP_INCR`,
- the behavior of the type Von-Put at kinematic hardening linear, accessible by key word `DIS_CINE_LINE` ,
- the viscoelastic behavior linear, accessible by key word `DIS_VISC` ,
- the elastic behavior bilinear, accessible by key word `DIS_BILI_ELAS` ,
- the behavior of type unilateral contact with friction of Coulomb, used to model behavior in translation and rotation of all within the competences of connection roasts - pencil of the fuel assemblies, accessible by key word `DIS_GRICRA`.

The integration of the models of behavior mentioned above is detailed in this document. Other behaviors relating to the discrete elements are available, but nonhere detailed:
- Armament of the lines (Relation `WEAPON`) [R5.03.31],
- Nonlinear assembly of angles of pylons (Relation `ASSE_CORN`) [R5.03.32].

Contents

1 general Principles of the behavior models of the nonlinear

1.1 discrete elements Behavior models (of the discrete elements)

the relations available in *Code_Aster* for the discrete elements are incremental behavior models given under factor key word the COMP_INCR by the key word RELATION in nonlinear operators STAT_NON_LINE and DYNA_NON_LINE. There are:

- the behavior of the type Von-Put at isotropic hardening used for the modelization of the assemblies threaded, implemented in MACR_GOUJ2E_CALC and accessible by key keys DIS_GOUJ2E_PLAS and DIS_GOUJ2E_ELAS,
- the behavior of type contact with shock and friction of Coulomb, accessible by the key word DIS_CHOC,
- the behavior of the type Von Mises to kinematic hardening linear, accessible by key word DIS_CINE_LINE ,
- the viscoelastic behavior linear, accessible by key word DIS_VISC ,
- the elastic behavior bilinear, accessible by key word DIS_BILI_ELAS ,
- the behavior of type unilateral contact with friction of Coulomb, used to model behavior in translation and rotation of all within the competences of connection roasts - pencil of the fuel assemblies, accessible by the key word DIS_GRICRA,

And the behaviors following, which are not here detailed:

- Armament of the lines (relation ARMS) [R5.03.31],
- Nonlinear assembly of angles of pylons (relation ASSE_CORN) [R5.03.32],

the parameters necessary to these relations are provided in operator DEFI_MATERIAU by the key keys:

Behavior in STAT_NON_LINE DYNA_NON_LINE	**Type of element (modelization) in AFFE_MODELE**	**Key words in DEFI_MATERIAU**	**AFFE_CARA_ELEM key words under DISCRET**
DIS_GOUJ2E_ELAS DIS_GOUJ2E_PLAS	2D_DIS_T: discrete element 2D with two nodes in translation	TENSION	CARA: "K_T_D_L"
DIS_ECRO_CINE	DIS_T, D_DIS_T, DIS_TR, 2D_DIS_TR : discrete elements 2D or 3D with one or two nodes in translation/rotation	DIS_ECRO_CINE	CARA: "K_T_D_L" CARA: "K_TR_D_L" CARA: "K_T_D_N" CARA: "K_TR_D_N"
DIS_VISC	DIS_T, 2D_DIS_T, DIS_TR, 2D_DIS_TR : discrete elements 2D or 3D with one or two nodes in translation/rotation	DIS_VISC	CARA: "K_T_D_L" CARA: "K_TR_D_L" CARA: "K_T_D_N" CARA: "K_TR_D_N"
DIS_BILI_ELAS	DIS_T, 2D_DIS_T, DIS_TR, 2D_DIS_TR : discrete elements 2D or 3D with one or two nodes in translation/rotation	DIS_BILI_ELAS	CARA: "K_T_D_L" CARA: "K_TR_D_L" CARA: "K_T_D_N" CARA: "K_TR_D_N"
DIS_CHOC contact and shock with friction of Coulomb	DIS_T, 2D_DIS_T: discrete elements 2D or 3D with two nodes in translation	DIS_CONTACT	CARA: "K_T_D_L" CARA: "K_T_D_N" For the elastic computation of stiffness and eigen modes
DIS_GRICRA	DIS_TR: discrete elements 3D with two nodes in translation/rotation	DIS_GRICRA	CARA: "K_TR_L" For the elastic computation of stiffness and eigen modes

Contrary to the models of behavior 1D [bib3], these relations bind the forces directly and the displacements, instead of being formulated between stresses and strains. They are valid only in small strains. One describes for each behavior model the computation of the field of forces from a given displacement increment (cf algorithm of Newton [R5.03.01]), the computation of the nodal **forces** R and tangent matrix.

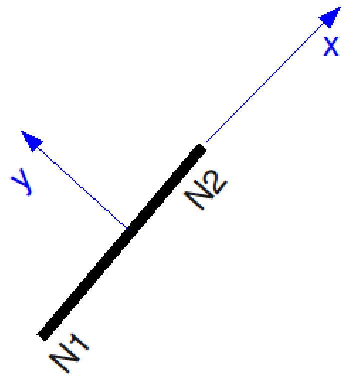
1.2 Computation of the strains (small strains)

For each one of the finite elements of *the Code_Aster*, in STAT_NON_LINE, the total algorithm (Newton) provides to the elementary routine, which integrates the behavior, an increase in the field of displacement [R5.03.01]
For the discrete elements with 2 nodes, one from of deduced the increase in elongation (in translation) or rotation, between nodes 1 and 2 of the element: $\Delta\varepsilon=\Delta u_2-\Delta u_1$.
For the discrete elements with a node, one obtains simply: $\Delta\varepsilon=\Delta u_1$

1.3 Computation of the forces and the nodal forces

After integration of the behavior, it is necessary to provide to the total algorithm (Newton) a vector containing the generalized forces, on the one hand, and on the other hand a vector containing the nodal forces R [R5.03.01] in total reference (X, Y, Z) .

For the discrete elements, the resolution of the nonlinear local problem directly provides the forces in the element (uniforms in the element), in local coordinate system (x, y, z) , which are form:



$$F=\begin{cases}F_1(noeud\,1)\\F_2(noeud\,2)\end{cases}$$ avecen 2D: $F_1=F_2=\begin{cases}F_x\\F_y\end{cases}$

in 3D: $F_1=F_2=\begin{cases}F_x\\F_y\\F_z\end{cases}$ in translation alone,

$F_1=F_2=(F_x\quad F_y\quad F_z\quad M_x\quad M_y\quad M_z)$ translation and rotation.

The vector R of the equivalent nodal forces (which is expressed in the total reference) is deduced from F change from reference:

$$R=P^T R_{loc} P \quad avec \quad R_{loc}=\begin{cases}-F_1(noeud\,1)\\F_2(noeud\,2)\end{cases}$$

where P is the matrix of change of reference, allowing the transition of the total reference towards the local coordinate system, as for the beam elements [R3.08.01].

1.4 General notations

All the quantities evaluated at previous time are subscripted par. $^-$

the quantities evaluated at time $t+\Delta t$ are not subscripted.

The increments are indicated par. Δ One has as follows:

$$Q=Q(t+\Delta t)=Q^{-}(t)+\Delta Q(t)=Q^{-}+\Delta Q$$

2 Behavior model of the threaded assemblies

2.1 Equations of model DIS_GOUJ2E_PLAS

They are deduced from behavior 3D VMIS_ISOT_TRAC [R5.03.02]: one represents there behavior model of an elastoplastic type to isotropic hardening, binding the forces in the discrete element unlike displacement of the two nodes in the local y direction.
In the local x direction, the behavior is elastic, linear, and the coefficient connecting the force Fx to displacement Dx is the stiffness Kx provided via AFFE_CARA_ELEM.
The nonlinear behavior relates to only the local y direction.
By noting $\Delta\varepsilon=\Delta u_y^1-\Delta u_y^2$ and the $\sigma=F_y^1=F_y^2$
relations are written in the same form as the relations of Von-Put 1D [R5.03.09]:

$$
\begin{aligned}
&\dot{e}^p=\dot{p}\frac{s}{|s|}\\
&s=E\left(e-e^p\right)\\
&s_{eq}-R(p)=|s|-R(p)\le 0\\
&s_{eq}-R(p)<0\Rightarrow\dot{p}=0\\
&s_{eq}-R(p)=0\Rightarrow\dot{p}^3 0
\end{aligned}
$$

In these statements, p represents a “cumulated plastic displacement”, and the isotropic function of hardening $R(p)$ is closely connected per pieces, data in the form of a curved force - displacement defined point by point, provided under the key word factor TENSION of operator DEFI_MATERIAU [U4.43.01].
The first point corresponds at the end of the linear field, and is thus used to define at the same time the limit of linearity (similar to the elastic limit), and E which is the slope of this linear part (E is independent of the temperature). The function $R(p)$ is deduced from a curve characteristic of the assembly (modelization of some nets) expressing the force on the pin according to the difference in average displacement between the pin and the flange [bib1]: $F=f(u-v)$.

2.2 Integration of relation DIS_GOUJ2E_PLAS

By direct implicit discretization of the behavior models, in a way similar to integration 1D [R5.03.09] one obtains:

$$
\begin{aligned}
&E\Delta\varepsilon-\Delta\sigma=E\Delta p\frac{\sigma^-+\Delta\sigma}{|\sigma^-+\Delta\sigma|}\\
&|\sigma^-+\Delta\sigma|-R\left(p^-+\Delta p\right)\le 0\\
&|\sigma^-+\Delta\sigma|-R\left(p^-+\Delta p\right)<0\Rightarrow\Delta p=0\\
&|\sigma^-+\Delta\sigma|-R\left(p^-+\Delta p\right)=0\Rightarrow\Delta p\ge 0
\end{aligned}
$$

Two cases arise:

- $|\sigma^-+\Delta\sigma|<R(p^-+\Delta p)$ then $\Delta p=0$ is $\Delta\sigma=E\Delta\varepsilon$ thus $|\sigma^-+E\Delta\varepsilon|<R(p^-)$
- $|\sigma^-+\Delta\sigma|=R(p^-+\Delta p)$ then $\Delta p\ge 0$ thus $|\sigma^-+E\Delta\varepsilon|\ge R(p^-)$

One from of deduced the algorithm from resolution:
let us pose $\sigma^e=\sigma^-+E\Delta\varepsilon$
so $|\sigma^e|\le R(p^-)$ then $\Delta p=0$ and $\Delta\sigma=E\Delta\varepsilon$
so $|\sigma^e|>R(p^-)$ then it is necessary to solve:

$$\sigma^e=\sigma^-+\Delta\sigma+E\Delta p\frac{\sigma^-+\Delta\sigma}{|\sigma^-+\Delta\sigma|}$$

$$\sigma^e=(\sigma^- +\Delta\sigma)\left(1+\frac{E\Delta p}{|\sigma^- +\Delta\sigma|}\right)$$

thus by taking the absolute value:

$$|\sigma^e|=|\sigma^- +\Delta\sigma|\left(1+\frac{E\Delta p}{|\sigma^- +\Delta\sigma|}\right)$$

maybe, by means of $|\sigma^- +\Delta\sigma|=R(p^- +\Delta p)$
$|\sigma^e|=R(p^- +\Delta p)+E\Delta p$

By taking account of the linearity per pieces of $R(p)$, one can explicitly solve this equation to find Δp

One from of deduced: $\frac{\sigma^e}{|\sigma^e|}=\frac{\sigma}{R(p^- +\Delta p)}$

then: $\sigma=\sigma^- +\Delta\sigma=\frac{\sigma^e}{|\sigma^e|}R(p)=\frac{\sigma^e}{1+\frac{E\Delta p}{R(p)}}$

Moreover, option FULL_MECA makes it possible to calculate the tangent matrix $\mathbf{K}_i^n$ with each iteration. The tangent operator who is used for building it is calculated directly on the preceding discretized system. One obtains directly:

- So $|\sigma^e|>R(p^-)$ then $\frac{\delta\sigma}{\delta\epsilon}=E_t=\frac{E R'(p)}{E+R'(p)}$
- if not $\frac{\delta\sigma}{\delta\epsilon}=E$

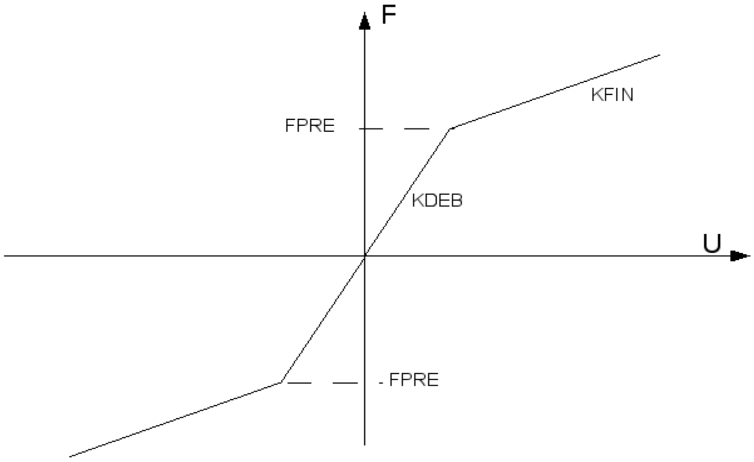
2.3 Local variables

behavior model DIS_GOUJ2E_PLAS produces two local variables: “cumulated plastic displacement” p , and a being worth indicator 1 if the increase in plastic strain is non-zero and 0 in the contrary case.

3 Behavior elastic bilinear: DIS_BILI_ELAS

3.1 Definition

behavior DIS_BILI_ELAS is used to model a bilinear elastic behavior in translation, in each direction. The constitutive law was conceived to be used with all the discrete elements. The behavior is characterized by 2 slopes (KDEB and KFIN) and by force (FPRE) which defines the change of incline.



Appear : Behavior elastic bilinear

For this model and that some either degree of freedom considered, the behavior of discrete is or elastic or elastic-bilinear. So in one of the directions the bilinear behavior is not defined, the behavior in this direction is then elastic and in fact the values given in the command AFFE_CARA_ELEM are taken. The model DIS_BILI_ELAS relates to only the degrees of translation, that thus implies that the behavior is elastic for the degrees of freedom of rotation which exist for this discrete. This local coordinate system is defined in a classical way in the command AFFE_CARA_ELEM under the key key factor ORIENTATION.

3 characteristics (KDEB, KFIN, FPRE) are obligatorily given in the local coordinate system of the element, it is thus necessary in the command AFFE_CARA_ELEM under the key key DISCRETE FACTOR to specify REPERE=' LOCAL' . A fatal error is emitted by *Code_Aster* if this condition is not observed.

The quantities KDEB and KFIN are functions which depend on the temperature and can be defined in the form of function, of three-dimensions function or formula.

The incremental equations of the constitutive law are simply:

If $|F| \leq FPRE \Rightarrow dF = \mathrm{KDEB}.\, dU$

If not $dF = \mathrm{KFIN}.\, dU$

F being force at the time t considered in one of the directions of translation, and dU the displacement increment of translation in this direction.

3.2 Local variables

There is 1 local variable per degree of freedom of translation. It can take 3 values:

- $V1=0$, the discrete one was never requested in this direction.
- $V1=1$, one is if $|F| \leq FPRE$
- $V1=2$, one is if $|F| > FPRE$

4 Behavior DIS_ECRO_CINE

4.1 Definition

behavior DIS_ECRO_CINE is elastoplastic constitutive law has nonlinear kinematic hardening for discrete elements. This model is defined for each component of the torsor of the resulting forces. The coefficient material are provided under key word DIS_ECRO_CINE.
For example for the direction X :

- LIMY_DX : F_e Force yield stress along the local axis x of the element
- KCIN_DX : k_r Stiffness along the local axis Rx of the element
- PUIS_DX : n Coefficient of non-linearity along the local axis x of the element (higher than 1)
- LIMU_DX : F_u Force limits along the local axis x of the element.
  - Threshold: $f=|F-X|-F_e$
  - Si $f\leq 0$ the behavior is elastic: $\dot{F}=K_e.\dot{U}$
  - Sinon :

$$f=0 \text{ et } \dot{f}=0$$
$$\dot{U}^{an}=\dot{\lambda}\frac{\partial f}{\partial F} \quad -\dot{\alpha}=\dot{\lambda}\frac{\partial f}{\partial X}$$
$$\dot{F}=K_e(\dot{U}-\dot{U}^{an})$$
$$X=\frac{k_{r.}\alpha}{\left(1+\left(\frac{k_{r.}\alpha}{F_u}\right)^n\right)^{\frac{1}{n}}}$$

4.2 Integration of the behavior

the resolution is obtained after discretization in time in the following way:
$\Delta F=K_e(\Delta U-\Delta U^{an})$ where $\Delta F, \Delta U$ can represent either of the forces and the translations, or of the moments and rotations.

While calculating: $F^{-}=K_e(U^{-}-U^{-}_{an})$, an elastic test is carried out:
So $|F^{-}+K_e(\Delta U)-X^{-}|\leq 0$ then $\Delta F=K_e.\Delta U$

If not, the system to be solved is:
$$f=0\Rightarrow|F^{-}+\Delta F-X^{-}-\Delta X|=F_e$$
$$\Delta U^{an}=\Delta\alpha=\Delta\lambda\frac{F^{-}+\Delta F-X^{-}-\Delta X}{F_e}$$
$$\Delta F=K_e(\Delta U-\Delta U^{an})$$

The 3 unknowns are: $\Delta F ; \Delta U^{an}=\Delta\alpha ; \Delta\lambda$ because of being $\Delta X=\frac{k(\alpha^{-}+\Delta\alpha)}{\left(1+\left(k(\alpha^{-}+\Delta\alpha)\right)^n\right)^{1/n}}-X^{-}$ an analytical function of $\Delta\alpha$.

One can simplify this system in the following way:
$$|F^{-}+\Delta F-X^{-}-\Delta X|=F_e$$
$$\Delta U^{an}=\Delta\lambda\frac{F^{-}+\Delta F-X^{-}-\Delta X}{F_e}$$
$$\Delta F+F^{-}-X^{-}-\Delta X=K_e(\Delta U-\Delta U^{an})+F^{-}-X^{-}-\Delta X$$

thus:

$$|F^- + \Delta F - X^- - \Delta X| = F_e$$

$$\Delta U^{an} = \Delta\lambda \frac{F^- + \Delta F - X^- - \Delta X}{F_e}$$

$$\Delta F + F^- - X^- - \Delta X = K_e \Delta U + F^- - X^- - \Delta X - K_e \Delta\lambda \frac{F^- + \Delta F - X^- - \Delta X}{F_e}$$

The last equation provides:

$(\Delta F + F^- - X^- - \Delta X)(1 + K_e \Delta \frac{\lambda}{F_e}) = K_e \Delta U + F^- - X^- - \Delta X$ where ΔX function of $\Delta\alpha$ thus of $\Delta\lambda$ (and of the sign of $F - X$). By taking the norm on the right and on the left one obtains: $F_e + K_e \Delta\lambda = |K_e \Delta U + F^- - X^- - \Delta X|$.

This single scalar equation can be solved by a classical method of search for zero of function (Newton, secant, etc… to see for example [R5.03.04]). If kinematic hardening is linear, (only thermodynamically justified case [feeding-bottle 8] one obtains the solution analytically $\Delta\lambda$:

$$F_e + (K_e + k)\Delta\lambda = |K_e \Delta U + F^- - X^-|$$

One calculates then the other unknowns by:

$\Delta F = \dfrac{K_e \Delta U + F^- - X^- - \Delta X}{(1 + K_e \dfrac{\Delta\lambda}{F_e})} - F^- + X^- + \Delta X$ and $\Delta U^{an} = \Delta\lambda \dfrac{F^- + \Delta F - X^- - \Delta X}{F_e}$

the current programming solves the system in a simplified way:
$f = |F - X| - F_e$ is discretized explicitly: $f = |F - X^-| - F_e$

then: $F_e + K_e \Delta\lambda = |K_e \Delta U + F^- - X^-|$
One calculates then $\alpha = \alpha^- + \Delta U^{an}$, which makes it possible to bring up to date:

$$X = \frac{k_{r.}\alpha}{\left(1 + \left(\frac{k_{r.}\alpha}{F_u}\right)^n\right)^{\frac{1}{n}}}$$

The tangent stiffness in this direction is approached by:

$$K_{tgt} = \frac{\Delta F}{\Delta U}$$

4.3 Local variables

There are 3 local variables per degree of freedom:

- $V1$ contains U^{an} at every moment
- $V2$ contains α at every moment
- $V3$ contains at every moment reactualized total energy.

5 Behavior DIS_VISC

5.1 Definition

behavior DIS_VISC is a linear viscoelastic behavior, applicable to each degree of freedom of the discrete elements (see an example in test SSND101).

The coefficients material are provided, in by the key word DIS_VISC . For each direction, 2 coefficients are provided: for example for the direction X :

- PUIS_DX: α Power in the local direction Dx of the element.
- COEF_DX : C Coefficient in the local direction Dx of the element.

The force in the direction concerned is: $F=-C\,V^{\alpha}$

5.2 Integration of the behavior

the resolution is immediate, after discretization in time:
So in the direction considered, the coefficients C and α are defined, then:

$$V=\frac{\Delta U}{\Delta t}$$

$$F=-C\,|V|^{\alpha}.signe(V)$$

The tangent stiffness in this direction is approached by:

$K_{tgt}=\frac{\Delta F}{\Delta U}$ what requires to store F with each time step.

5.3 Local variables

There are 2 local variables per degree of freedom:

- $V1$ the force contains F at every moment
- $V2$ contains at every moment reactualized total energy: $V2=-\sum F.\Delta U$

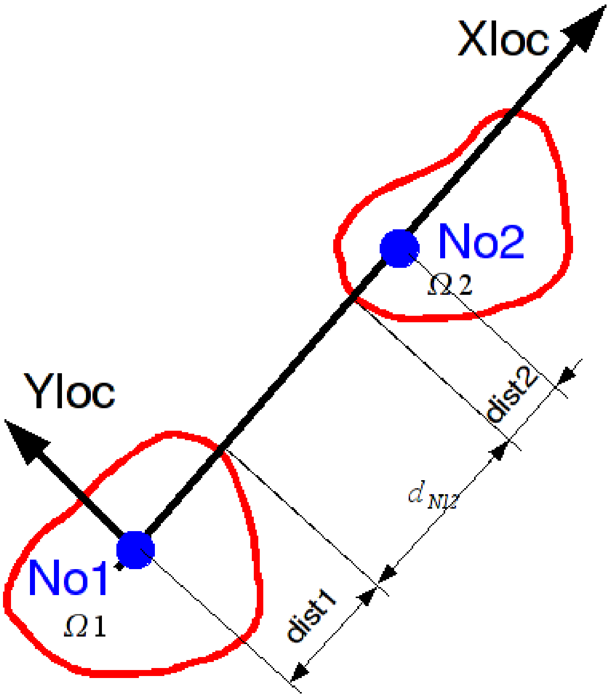
6 Modelization of the shocks and friction: `DIS_CHOC`

behavior `DIS_CHOC` translates the contact with shock and friction between two structures, via two types of relations:

- the relation of unilateral contact which expresses to it not inter-penetrability between the solid bodies,
- the relation of friction which governs the variation of the tangential stresses in the contact. One will retain for these developments a simple relation: the friction law of Coulomb.

6.1 Relation of unilateral contact

Are two structures Ω_1 and Ω_2. One notes $d_N^{1/2}$ the normal distance between structures, $F_N^{1/2}$ the normal reaction force of Ω_1 on Ω_2.



Appear 6.1-a : Definition of the distances for relation DIS_CHOC.

In the local coordinate system with the element, the normal distance d_N to for statement:

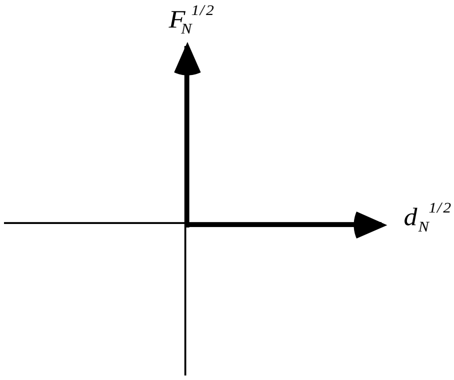
$$d_N=((X^0_{loc2}+u_2)-(X^0_{loc1}+u_1))-dist1-dist2$$

The model of the action and the reaction imposes:

$$F_N^{2/1}=-F_N^{1/2} \quad \text{[éq 6.1-1]}$$

the conditions of unilateral contact, still called conditions of Signorini [bib5], are expressed in the following way:

$$d^{1/2}\geq 0,F_N^{1/2}\geq 0,d_N^{1/2}\cdot F_N^{1/2}=0 \text{ and } F_N^{2/1}=-F_N^{1/2} \quad \text{6.1-2]}$$



6.1-b 6.1-b of the relation of unilateral contact.

This graph translates a relation force-displacement which is not differentiable. It is thus not usable in a simple way in an algorithm of dynamic computation.

6.2 Friction law of Coulomb

the model of Coulomb expresses a tangential limitation the effort of $\mathbf{F}_\mathbf{T}^{1/2}$ tangential reaction of Ω_1 on Ω_2 . Either $\dot{\mathbf{u}}_\mathbf{T}^{1/2}$ the relative velocity from Ω_1 ratio with Ω_2 in a point of contact and or μ the coefficient of kinetic friction of Coulomb, one has [bib5]:

$$s=\|\mathbf{F}_\mathbf{T}^{1/2}\|-\mu\cdot F_{N^{1/2}}\leq 0,\ \ \exists\lambda\ \ \dot{\mathbf{u}}_\mathbf{T}^{1/2}=\lambda\ \mathbf{F}_\mathbf{T}^{1/2},\ \lambda\leq 0,\ \lambda.s=0 \qquad 6.2\text{-}1$$

and the model of the action and the reaction:

$$\mathbf{F}_\mathbf{T}^{2/1}=-\mathbf{F}_\mathbf{T}^{1/2} \qquad 6.2\text{-}2\ ]$$

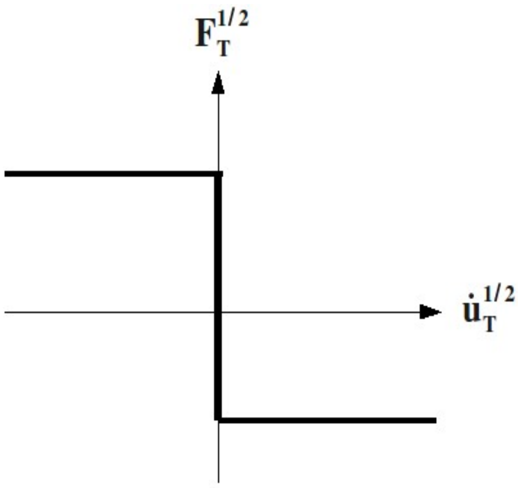


Figure 6.2-a : Graph of the friction law of Coulomb.

The graph of the model of Coulomb is also nondifferentiable and is thus not simple to use in a dynamic algorithm.

6.3 Approximate modelization of the relations of contact by Model

6.3.1 penalization of normal force of contact

the principle of the penalization applied to the graph of [Figure 5.3.1-a] consists in introducing a univocal relation $F_N^{1/2}=f_\epsilon\left(d_N^{1/2}\right)$ by means of a parameter ϵ. The graph of f_ϵ must tend towards the graph of Signorini when ϵ tends towards zero [bib6].

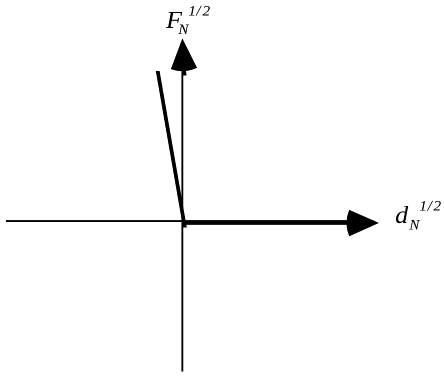
One of the possibilities consists in proposing a linear relation enters $d_N^{1/2}$ and $F_N^{1/2}$:

$$F_N^{1/2}=-\frac{1}{\epsilon}d_N^{1/2} \text{ if } d_N^{1/2}\le 0 \;\; ; \;\; F_N^{1/2}=0 \text{ not} \qquad 6.3.1\text{-}1$$

If one notes $K_N=\frac{1}{\epsilon}$ called commonly "**stiffness of shock**", one finds the classical relation, modelling an elastic shock:

$$F_N^{1/2}=-K_N\cdot d_N^{1/2} \qquad \text{[éq6.3.1-2]}$$

the approximate graph of the model of contact with penalization is the following:



Appear Graph of the relation of unilateral contact approached by penalization.

To take account of a possible loss of energy in the shock, one introduces a "damping of shock" C_N the statement of the normal force of contact expresses oneself then by:

$$F_{N1/2}=-K_N\cdot d_{N1/2}-C_N\cdot \dot{u}_{N1/2} \qquad \text{[éq}$$

where $\dot{u}_N^{1/2}$ is the normal velocity relative from Ω_1 ratio to Ω_2. To respect the relation of Signorini (not blocking), one must on the other hand check a posteriori who $F_N^{1/2}$ is positive or null. One will thus take only the positive part $\langle\rangle^+$ of the statement [éq6.3.1-3]:

$$\langle x\rangle^+=x \quad \text{if} \quad x\ge 0$$
$$\langle x\rangle^+=0 \quad\quad x<0$$

the complete relation giving the normal force of contact which is retained for the algorithm is the following one:

$$\left.\begin{array}{l} si \quad d_N^{1/2}\le 0 \quad F_N^{1/2}=\langle -K_N\cdot d_N^{1/2}-C_N\cdot \dot{u}_N^{1/2}\rangle^+; \quad F_N^{2/1}=-F_N^{1/2} \\ sinon \quad F_N^{2/1}=F_N^{1/2}=0 \end{array}\right\} \qquad 6.3.1\text{-}4\ ]$$

6.3.2 of tangential force of contact

the graph describing Models the tangential force with model of Coulomb is NON-differentiable for the phase of dependancy $\dot{\mathbf{u}}_T^{1/2}=0$. One thus introduces a univocal relation binding relative tangential

displacement $\mathbf{d}_\mathbf{T}^{1/2}$ and the tangential force $\mathbf{F}_\mathbf{T}^{1/2}=f_\xi\left(\mathbf{d}_\mathbf{T}^{1/2}\right)$ by means of a parameter ξ. The graph of F_ξ must tend towards the graph of Coulomb when ξ tends towards zero [bib6].
One of the possibilities consists in writing a linear relation enters $\mathbf{d}_\mathbf{T}^{1/2}$ and $\mathbf{F}_\mathbf{T}^{1/2}$:
by noting a^0 the value of a quantity has at the beginning of time step:

$$\mathbf{F}_\mathbf{T}^{1/2}-\mathbf{F}_\mathbf{T}^{1/2\ 0}=-\frac{1}{\xi}\cdot\left(\mathbf{d}_\mathbf{T}^{1/2}-\mathbf{d}_\mathbf{T}^{1/2\ 0}\right) \qquad \text{[éq 6.3.2-1}$$

If one introduces a “tangential stiffness” $K_T=\frac{1}{\xi}$, one obtains the relation:

$$\mathbf{F}_\mathbf{T}^{1/2}=\mathbf{F}_\mathbf{T}^{1/2\ 0}-K_T\cdot\left(\mathbf{d}_\mathbf{T}^{1/2}-\mathbf{d}_\mathbf{T}^{1/2\ 0}\right) \qquad 6.3.2\text{-}2$$

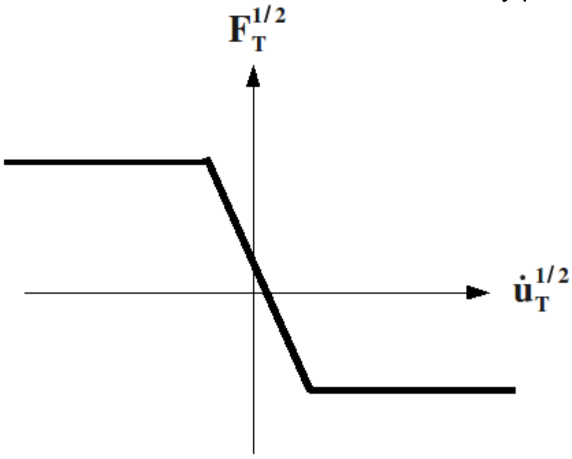
For numerical reasons, related to the dissipation of parasitic vibrations [bib7] in phase of dependancy, one is brought to add a “tangential damping” C_T in the statement of the tangential force. Its final statement is:

$$\mathbf{F}_\mathbf{T}^{1/2}=\mathbf{F}_\mathbf{T}^{1/2\ 0}-K_T\cdot\left(\mathbf{d}_\mathbf{T}^{1/2}-\mathbf{d}_\mathbf{T}^{1/2\ 0}\right)-C_T\cdot\dot{\mathbf{u}}_\mathbf{T}^{1/2}\ ,\ \mathbf{F}_\mathbf{T}^{2/1}=-\mathbf{F}_\mathbf{T}^{1/2} \qquad 6.3.2\text{-}3$$

It is necessary moreover than this force checks the criterion of Coulomb, that is to say:

$$\left\|\mathbf{F}_\mathbf{T}^{1/2}\right\|\leq\mu\cdot F_N^{1/2} \text{ if not one applies } F_T^{1/2}=-\mu\cdot F_N^{1/2}\cdot\frac{\dot{\mathbf{u}}_T^{1/2}}{\left\|\dot{\mathbf{u}}_T^{1/2}\right\|}\ ,\ \mathbf{F}_\mathbf{T}^{2/1}=-\mathbf{F}_\mathbf{T}^{1/2} \qquad \text{[éq]}$$

the approximate graph of the friction law of Coulomb modelled by penalization is the following:



Appear 6.3.2-a : Graph of the friction law approached by penalization.

6.4 Definition of the parameters of contact

One specifies the key words here making it possible to define the parameters of contact, damping and friction [U4.43.01]:
Operand RIGI_NOR is compulsory, it makes it possible to give the value of normal stiffness of shock K_N.
The other operands are optional:

- Operand AMOR_NOR makes it possible to give the value of normal damping of shock C_N.
- Operand RIGI_TAN makes it possible to give the value of tangential stiffness K_T.
- Operand AMOR_TAN makes it possible to give the tangential value of damping of shock C_T.
- Operand COULOMB makes it possible to give the value of the coefficient of Coulomb.
- Operand DIST_1 makes it possible to define the distance characteristic of matter surrounding the first node of shock
- Operand DIST_2 makes it possible to define the distance characteristic of matter surrounding the second node of shock (shock between two mobile structures)

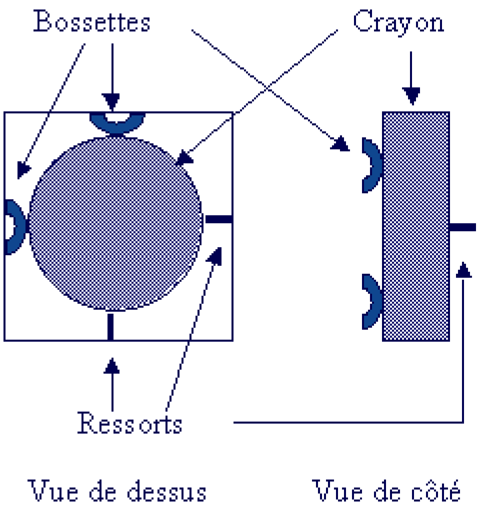
- Operand JEU defines the distance between the node of shock and an obstacle not modelled (case of a shock between a mobile structure and an indeformable and motionless obstacle).

7 Modelization of connection grid-pencil: DIS_GRICRA

behavior DIS_GRICRA is used to model the behavior in translation and in rotation within the competences of connection - pencil of the fuel assemblies roasts. The constitutive law was conceived to be used with the discrete ones with two nodes MECA_DIS_TR_L.

7.1 General presentation

the maintenance of the pencils in the cells of grid is ensured by the system of bosses and springs represented on the following figure:



Appear7.1-a a cell of grid.

The pencil is thus maintained in the cell of grid by 6 points of contact (cf appears7.1-a): 4 bosses and 2 springs. It is possible to model each element of contact (bosses and springs) thanks to discrete elements, to which one affects a friction law of the type DIS_CHOC. Such a modelization makes it possible satisfactorily to represent the behavior of a cell of grid in translation (extraction of the pencil) and in rotation. Such a method has the following disadvantages however:

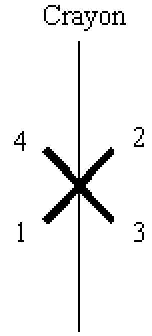
- Complexity of the mesh to be produced, because one needs 6 discrete per cell of grid (or grid in a homogenized model), with different heights for the bosses and the springs.
- Complexity in the definition of the characteristics of the discrete ones, because the bosses should be differentiated and the springs
- Dissymmetry of the modelization (boss on a side and arises in opposition), whereas the behavior homogenized on a grid can be regarded as symmetric, taking into account the alternation of the directional sense of the cells of grid in the grids.
- Difficulties to identify the parameters of the behavior of discrete at the level of a fuel assembly.

A called equivalent modelization DIS_GRICRA was proposed, which makes it possible to find the same behavior as the system of 6 discrete in translation and rotation, while avoiding the disadvantages quoted above:

- Connection grid-pencil is modelled by 4 discrete elements out of system in the same plane, which simplifies the mesh and makes it possible to symmetrize the problem.
- The 4 elements are affected same parameters for constitutive law DIS_GRICRA.
- Behaviors in translation and rotation are treated separately, which facilitates an identification of the parameters.

Note:

In order to correctly represent the behavior of connection in all the directions, especially in rotation, it is necessary to use 4 discrete laid out out of system, the pencil being related to discrete in the middle of the device out of system.

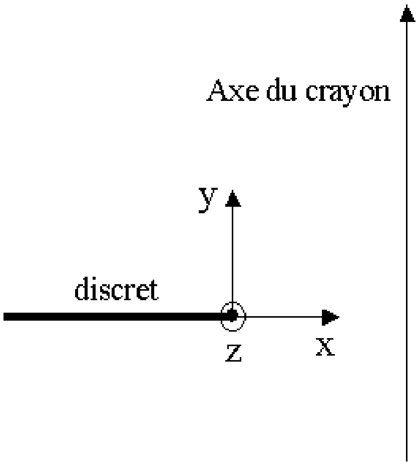


Appear of a cell of grid.

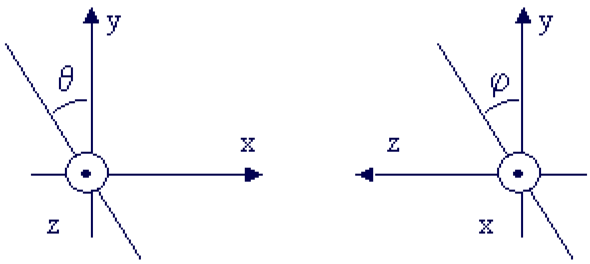
7.2 Definition of the local coordinate system

the equations of the constitutive law are written in the local coordinate system of the discrete. Taking into account the orthotropic character of the behavior in the tangential directions with discrete, one adopts following convention for the local definition of the reference of the discrete one: the axis x represents the axis of the discrete one, the axis y corresponds to the direction of the pencil, and it axis z is the orthogonal axis with x and y. For the swing angles, one will note Φ the swing angle around the axis x (DRX) and θ the swing angle around axis z (DRZ). One is not interested in the swing angle around there (DRY), because it is necessary to block this rotation (condition limits) in the command file.

With this definition of the local coordinate system, the direction y is common to all discrete system out of system represented on the figure7.2-a. The direction x from discrete the 1 and 2 corresponds to the direction z from discrete the 3 and 4 (and vice versa). The angle Φ of 1 and 2 corresponds to the angle θ of 3 and 4 (and vice versa).



Appear : Definition of the local coordinate system.

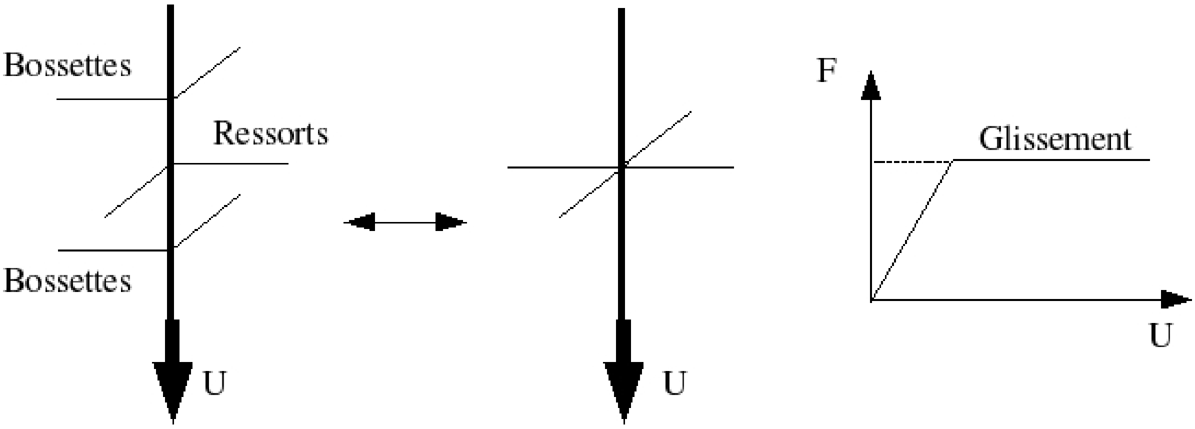


7.2-b.

7.3 Behavior in translation

7.3.1 Presentation of the behavior

the behavior in translation is modelled by a model of type friction of Coulomb [Figure 7.3.1-a].



7.3.1-a Constitutive law in translation.

Types of different behaviors are affected according to the directions:

Direction x

One considers an elastic behavior, with an initial force with null displacement equal contrary to the gripping force (the force exerted by the pencil on the discrete one compresses the discrete one):

$$F_x = K_N \Delta x - F_N^0$$

Direction y

One considers a quasi-perfect elastoplastic behavior to model the friction of Coulomb and the possibility of sliding. The force is expressed in the following way:

$$F_y = K_T \left[\Delta y - U_y^p \right]$$

where Δy is the strain of discrete according to y and U_y^p represents the sliding

the criterion of sliding is the following:

$$\|F_y\| \le F^S + \kappa \lambda \text{ with } F^s = -\mu F_N^0$$

where κ is a parameter of the model, λ is the hardening parameter and μ is the coefficient of Coulomb. The flow model is the following one: $\dot{U}_y^p = \dot{\lambda} \frac{\mathbf{F_y}}{\|\mathbf{F_y}\|}$

1D see document [R5.03.09] on the integration of the nonlinear models for the numerical integration of this elastoplastic model of type Von Mises with isotropic hardening.

This force is identical for the 4 discrete system out of system because they have the same direction y.

Direction z

One recalls that the constitutive law grid-pencil DIS_GRICRA must be used with a configuration of discrete out of system. It is consequently useless to define a force in the direction z of the discrete one, because the stiffness “is taken again” by the discrete orthogonal ones (cf preceding section).

7.3.2 Introduced parameters

the behavior in translation requires the introduction of 5 parameters:

- KN_AX : axial rigidity of the discrete one

- KT_AX : tangencial stiffness (in the direction of sliding) of the discrete one
- F_SER or F_SER_FO : force tightening of the pencil in the grid, the axial direction of the discrete one
- COUL_AX : coefficient of Coulomb for the friction law
- ET_AX : hardening parameters allowing to make converge the friction law. This parameter is optional, a value of 10^{-7} is proposed by default in Code_Aster.

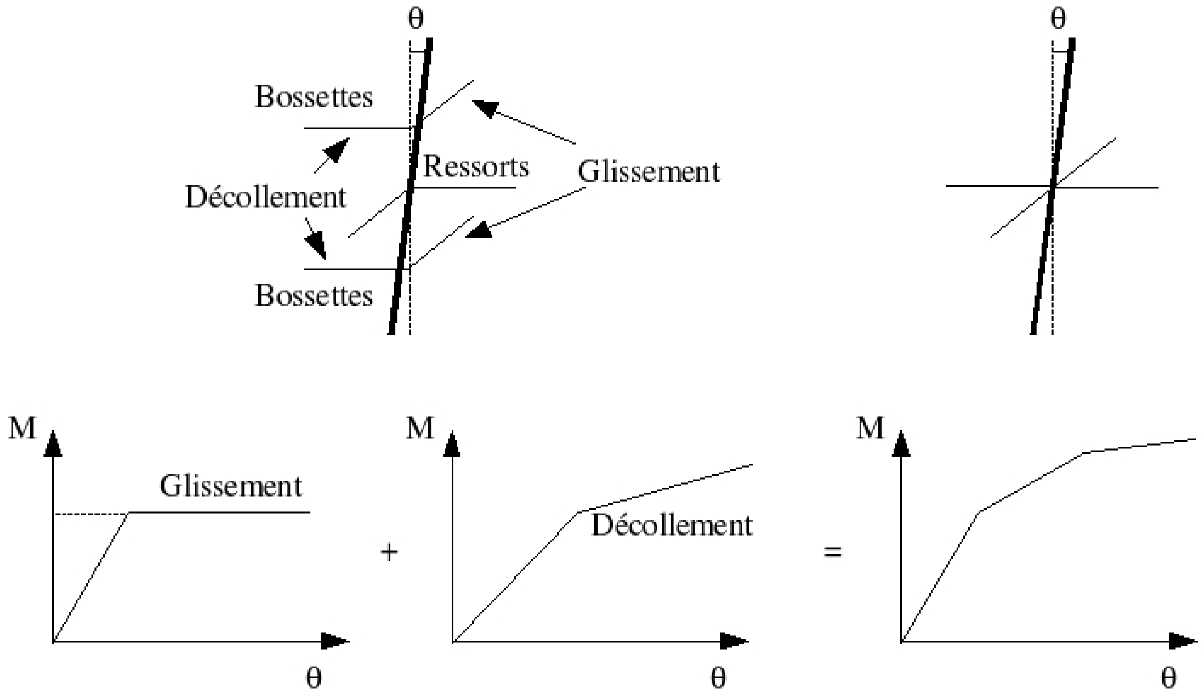
The force necessary to extract a pencil from a grid is equal to F_SER*COUL_AX (force of extraction).

7.4 Behavior in rotation

7.4.1 Presentation of the behavior

the study of connection grid-pencil in rotation with the system boss-springs showed that one could describe the behavior by superimposing two simple behaviors:

- a phenomenon of sliding of the orthogonal elements to the plane of rotation, modelled by a model of Coulomb similar to that used in translation [Figure 7.4.1-a].
- a bilinear elastic behavior in the plane of rotation, induced by the possibility of separation of the pencil compared to a boss from a certain angle [Figure 7.4.1-a].



Appear of the behavior of connection grid-pencil in rotation.

It is to ensure this superposition of an elastoplastic behavior and with a bilinear elastic behavior that the configuration of 4 discrete out of system is necessary. If one considers a rotation of an angle θ in the total reference corresponding to the local coordinate system from discrete the 1 and 2, this angle corresponds to the angle Φ from discrete the 3 and 4. It is then enough to impose on discrete the elastoplastic model on the angle θ (of the local coordinate system) and the bilinear elastic model on the angle Φ (of the local coordinate system) to have a superposition of the two behaviors thanks to the system the discrete ones out of system.

Types of different behaviors are affected according to the angles:

Rotation around the axis x

One considers an elastoplastic behavior of type Von Mises with isotropic hardening to model friction in rotation. The moment is expressed according to the angle and of a "plastic" angle or angle of repose:

$$M_x = K_\phi \left[\phi - \phi^p\right]$$

where Φ is the swing angle around x and Φ^p represents the angle of repose

the criterion of sliding is the following:

$\|M_\phi\| \leq M^S + \kappa\lambda$ where κ is a parameter of the model, λ is the hardening parameter and M^s is the moment-threshold defined by parameters of the model.

The flow model is the following one:

$$\dot{M}_\phi^p = \dot{\lambda} \frac{M_\phi}{\|M_\phi\|}$$

1D see document [R5.03.09] on the integration of the nonlinear models for the numerical integration of this elastoplastic model of type Von Mises with isotropic hardening, by replacing the force by the moment and the strain by the difference in angle.

Rotation around the axis z

the behavior around the axis z is considered elastic bilinear. The moment around the axis z is expressed according to the swing angle around the axis z in the following way:

$$M_\theta = K^{1\theta}\theta \text{ if } \theta \in \left[-\theta^s, \theta^s\right]$$

$$M_\theta = K^{2\theta}\left[\theta - \theta^s\right] + K^{1\theta}\,\theta^s$$

where θ^s is the angle of elastic break of slope.

Rotation around the axis y

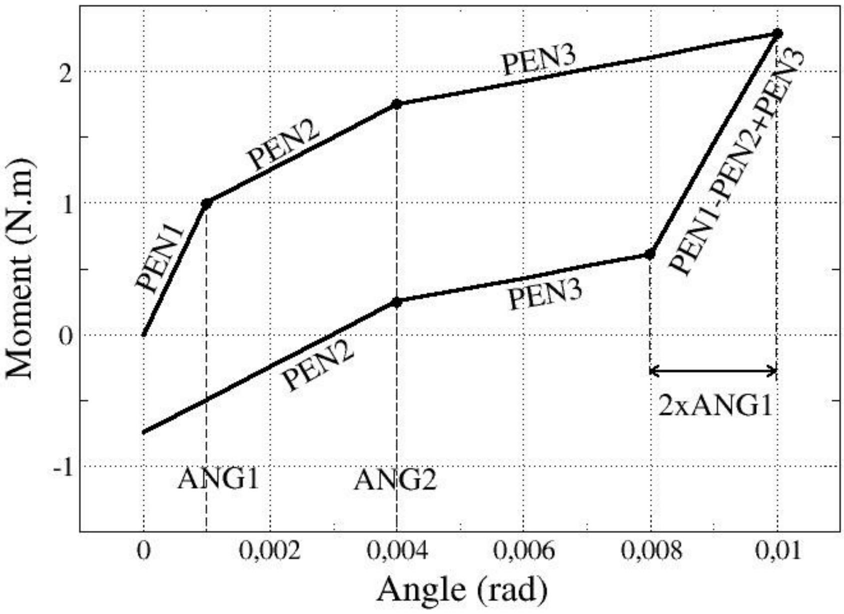
With regard to rotation around the axis y, it is imperative to block this rotation because no model was identified for this degree of freedom, the not turning pencil. However, in order to limit the bad conditioning of the stiffness matrix, a stiffness is introduced into the code, starting from the other stiffness in rotation. This stiffness is invisible for the user and does not have an influence on computations, the degree of freedom corresponding being blocked.

7.4.2 Introduced parameters

behavior in rotation requires the introduction of 5 parameters, either in the form of constants (ANG1, ANG2, PEN1, PEN2, PEN3), or in the form of functions of the temperature and irradiation (ANG1_FO, ANG2_FO, PEN1_FO, PEN2_FO, PEN3_FO). An optional additional parameter ET_ROT (equal to $10E\text{-}7$ by default) makes it possible to make converge the behavior of sliding. The role of each parameter is explained easily over the curved angle-moment represented on the figure7.4.2-a :

- At the beginning there is no sliding and not separation, the stiffness in rotation is equal to PEN1 (or PEN1_FO).
- From angle ANG1 (or ANG1_FO), the phenomenon of sliding on the discrete orthogonal ones with the plane of rotation is activated, these elements thus do not take part any more in the stiffness in rotation and the slope decreases and becomes PEN2 (or PEN2_FO).
- From angle ANG2 (or ANG2_FO), there is always the sliding, to which the separation of the pencil compared to certain bosses is added, from where a reduction in the stiffness. This one becomes equal to PEN3 (or PEN3_FO).
- When then one discharges, there is always separation, but there is no more sliding, the slope becomes PEN1-PEN2+PEN3 (or PEN1_FO-PEN2_FO+PEN3_FO).
- When one twice reaches ANG1 (or ANG1_FO) starting from the beginning of the discharge, the sliding is reactivated, and one always has separation, from where a slope equal to PEN3 (or PEN3_FO).
- From ANG2 (or ANG2_FO), there is resticking of the pencil on the bosses, from where a slope equal to PEN2 (or PEN2_FO).

The values of parameters used for the figure7.4.2-a are the following: $ANG1 = 0.001\text{rad}$ $ANG2 = 0.004\text{rad}$ $PEN1 = 1000\ N.m.rad^{-1}$ $PEN2 = 250\ N.m.rad^{-1}$ $PEN3 = 90\ N.m.rad^{-1}$.



Appear angle-moment on a bearing test and discharge in rotation for model `DIS_GRICRA`.

7.5 Notice

the forces and moments depend on displacements and relative angles swing between the two nodes on the discrete one. One writes in this section the forces and moments with node 2, and one takes as convention of writing $\Delta\alpha = \Delta\alpha(n2) - \Delta\alpha(n1)$, which represents the "strain" associated with the degree of freedom α for the discrete one expressed with the node $n2$. The force with the node $n1$ is equal contrary to the force to the node $n2$. It results from it that the tangent matrix is following form:

$$\underline{\underline{K}}^{\tan} = \begin{pmatrix} \underline{\underline{K}} & -\underline{\underline{K}} \\ -\underline{\underline{K}} & \underline{\underline{K}} \end{pmatrix}$$

The behavior of connection roasts pencil is different for each degree of freedom, and independent of the other degrees of freedom. It results from it that each one of the components of the force depends only on the degree of freedom with which it is associated, and which the blocks $\underline{\underline{K}}$ of the tangent matrix $\underline{\underline{K}}^{\tan}$ are diagonal.

7.6 Standard

7.6.1 use of elements

the following characteristics of the discrete elements must be affected in the command AFFE_CARA_ELEM to be able to use `DIS_GRICRA` (cf document [V6.02.131]):

```
AFFE_CARA_ELEM (
    MODELE=modele,
    DISCRET=_F (   GROUP_MA= (“meshes”),
                CARA = “K_TR_L”,
                VALE = “list of 78 terms”,
                REPERE=' LOCAL')
```

behavior DIS_GRICRA for connection grid-pencil can be used only with discrete elements with two nodes and 6 degrees of freedom MECA_DIS_TR_L. One must thus specify "K_TR_L" under CARA.
Contrary to the other behaviors for the discrete ones, DIS_GRICRA does not use the stiffness of discrete the data in AFFE_CARA_ELEM under the key word VALE. Since the key word VALE must be filled all the same, it is advised to affect a list of 78 null terms. In the case of a computation of eigen mode, constitutive law DIS_GRICRA is not requested, therefore it is then necessary to return the stiffness of discrete which corresponds to the elastic mode of DIS_GRICRA.

One works in the local coordinate system.

```
ORIENTATION=_F (   GROUP_MA= ("meshes",),
                   CARA=' VECT_Y', GOES LE = vect_crayon
```

vect_crayon is a directing vector of the pencil. The direction of the pencil must imperatively be given under key word ORIENTATION.
In addition, the exposure field is obtained by office plurality of the lived history (stored in local variable) and of the increment of the field introduced by the command variables between time step running and previous time.

7.6.2 Definition of the characteristics of the material

behavior DIS_GRICRA requires the introduction of 10 parameters. These data must be provided in DEFI_MATERIAU. The parameters of entry of this model are the following:

Behavior in axial sliding: 5 parameters (of which an arbitrary, purely numerical parameter):

- normal stiffness of discrete the KN_AX ;
- tangencial stiffness (in the direction of the sliding) KT_AX ;
- coefficient of kinetic friction of Coulomb COUL_AX ;
- force tightening F_SER (limit of sliding = COUL_AX*F_SER);
- hardening parameter ET_AX (the constitutive law can be comparable to perfect plasticity. The hardening parameter is only used to ensure the convergence of computation; a value by default of $10E\text{-}7$ him is affected);

Behavior in rotation: 6 parameters (of which a purely numerical parameter)

- successive slopes PEN1, PEN2 and PEN3 of the curve Moment = F (angle);
- angles ANG1 and ANG2 of the points of inflection of the curve;
- hardening parameter ET_ROT (parameter being used only to ensure the convergence of computation; a value by default of $10E\text{-}7$ him is affected).

The gripping forces can vary according to the temperature and from the irradiation. These dependences are affected on slopes PEN1 and PEN2 for behavior in rotation and on the gripping force F_SER for the behavior in axial sliding. The functions of dependence are directly defined in the form of a FORMULA in the command file.

The names of the followed parameters by suffix _FO make it possible to inform the value in the form of a function.
A certain number of parameters additional, available for this behavior but which do not appear in this document, are clarified in [V6.04.131]. The parameters of dependence in temperature and irradiation of the gripping force are defined in the document [V6.02.131].

7.6.3 Local variables

They are 5:

- $V1$: cumulated plastic displacement (axial direction)
- $V2$: indicator of contact/friction (1 so sliding, 0 so not sliding)
- $V3$: indicator of separation in rotation,
- $V4$: plastic angle (sliding),
- $V5$: cumulated plastic angle,
- $V6$: memorizing of the history of irradiation (fluence).

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9 Doc. Version Code_Aster Author (S) contributor

(S	**) organization *(S) Description***	**of modifications A5 J.M.PROIX, B.QUINNEZ**	**EDF/R&DMMN initial Text, constitutive laws**
		DIS_GOUJ2E_ PLAS ,	DIS_GOUJ2E_ELAS and DIS_CONTACT C 4/7/10 G.DEVESA EDF/R & D /AMA Addition of model DIS_CHOC
	04/07/10	V.GODARD EDF/R & D /AMA	Addition of model DIS_GRICRA
	E 9,4	F.VOLDOIRE , J.L.FLEJOU EDF/R & D	/AMA Addition of DIS_BILI_ELAS
,	9.4	, DIS_VISC, resorption of	DIS_CONTACT 11 J.L.FLEJOU EDF/R & D /AMA Corrections formulas, updates compared to the
	11	version 11.	