Damage model of a Summarized brittle elastic

material:

This document the model describes elastic behavior brittle ENDO_FRAGILE available in static and dynamics. The damage is modelled in a scalar way; the loadings in compression and tension are not distinguished. In addition to the model local, the nonlocal formulation with regularized strain is also supported to control the phenomena of localization. The nonlocal formulation with gradient of damage is replaced by model ENDO_SCALAIRE [R5.03.25]

1 Domaine d'application

model ENDO_FRAGILE aims at modelling of the way a simplest possible brittle elastic behavior. The material is elastic isotropic. Its stiffness can decrease in an irreversible way when strain energy becomes important, without distinguishing the tension from compression. This loss of stiffness is measured by a scalar local variable called damage which evolves of 0 (operational material) to 1 (completely damaged material, i.e. without stiffness). Moreover, the stress cannot exceed a threshold which also decrease with the level of damage to reach 0 when the material is completely damaged. One will refer to [bib1] for a description of this kind of phenomenology.

The property of the reduction of the threshold in stress with the level of damage is called softening and generally involves a loss of ellipticity of the equations of the problem. It results from it a localization from the strains and damage in tapes of which the thickness is directly controlled by the size of the finite elements. To mitigate this deficiency of the model a nonlocal formulation is proposed, it leans on a regularization of the strains and activated by modelization *_GRAD_EPSI [R5.04.02]. The width of the tapes of localization is controlled by a material parameter, indicated in operator DEFI_MATERIAU under key word LONG_CARA of factor key word the NON_LOCAL [U4.43.01]. However, obtaining a physical problem posed again well is obtained only at the cost of one important overcost in time computation. In addition, it should well be noticed that only the behavior models are deteriorated and not balance equations. Consequently, the stresses preserve their usual meaning. For the modelization based on the introduction of the gradient of the damage and activated by modelization *_GRAD_VARI [R5.04.01], to refer to model ENDO_SCALAIRE [R5.03.25].

Lastly, that one activates or not these nonlocal formulations, the softening character of the behavior also involves the appearance of instabilities, physics or parasites, which result in snap - backs on the total response and return the control of the essential loading in static. The control of the type PRED_ELAS [R5.03.80] then seems the mode of control of the level of the most suitable loading.

2 Local constitutive law

2.1 Behavior models

the state of the material is characterized by the strain $\boldsymbol{\varepsilon}$ and the damage d understood enters 0 and 1. The relation stress-strain is elastic, the stiffness is affected in a linear way by the damage:

$$\boldsymbol{\sigma}=(1-d)\ \boldsymbol{E}:\boldsymbol{\varepsilon} \qquad \textbf{éq 2.1-1}$$

with E the tensor of Hooke. In addition, the evolution of the damage, always increasing, is controlled by the following function threshold:

$$f(\boldsymbol{\varepsilon},d)=\frac{1}{2}\boldsymbol{\varepsilon}:\boldsymbol{E}:\boldsymbol{\varepsilon}-k(d) \text{ where } k(d)=w^y\left(\frac{1+\gamma}{1+\gamma-d}\right)^2 \qquad \textbf{the 2.1-2}$$

coefficients w^y and γ, both positive, are parameters of the model. The condition of coherence then determines completely the rate of damage $\dot{d}$:

$$f(\boldsymbol{\varepsilon},d)\le 0 \quad \dot{d}\ge 0 \quad \dot{d}\ f(\boldsymbol{\varepsilon},d)=0 \qquad \textbf{éq the 2.1-3}$$

equations [éq 2.1-1] with [éq 2.1-3] are enough to describe entirely constitutive law ENDO_FRAGILE, indeed very simple. One can also notice that it forms part of the formalism suggested by Marigo [bib2].

2.2 Identification of the parameters of the model

the parameters of this constitutive law are four. On the one hand, the Young's modulus E and the Poisson's ratio ν which determine the tensor of Hooke by:

$$\mathbf{E}^{-1}\cdot\boldsymbol{\sigma}=\frac{1+\nu}{E}\boldsymbol{\sigma}-\frac{\nu}{E}(\operatorname{tr}\boldsymbol{\sigma})\,\mathbf{Id} \qquad \text{éq 2.2-1}$$

In addition, w^y and γ which defines the lenitive behavior. They are determined by a simple traction test, cf [Figure 2.2-a]. To simplify the entry of the data of the model, one informs not w^y and γ but directly the tangent modulus E^T and the stress with the peak σ^y under factor key word `the ECRO_LINE` or `ECRO_LINE_FO` of operator `DEFI_MATERIAU`. As for E and ν, they are given classically under factor key word `the ELAS` or `ELAS_FO`.
A all useful ends, here also statements of the strain with fracture ε^R in this simple traction test, as well as voluminal energy k^0 consumed to completely damage a material point, this last statement being valid whatever the load history:

$$\varepsilon^R=\left(\frac{1}{E}-\frac{1}{E^T}\right)\sigma^y \qquad k^0=\frac{1}{2}\left(\frac{1}{E}-\frac{1}{E^T}\right){\sigma^y}^2=\frac{1}{2}\varepsilon^R\sigma^y=w^y\,\frac{1+\gamma}{\gamma} \qquad \text{éq 2.2-2}$$

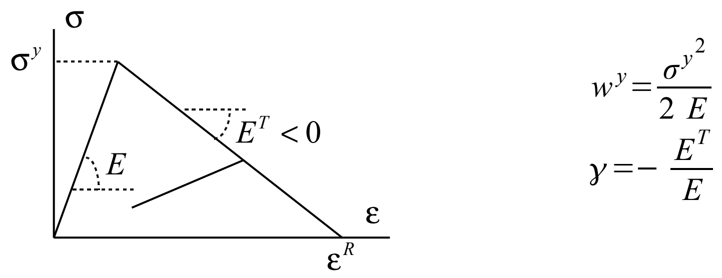

Figure 2.2-a: Simulation of a traction test simple

2.3 Integration of the constitutive law

temporal discretization of the equations [éq 2.1-1] with [éq 2.1-3] on time step $[t^- \; t]$ is realized by a diagram of implicit Eulerian. For any function of time q, one notes $q^-=q(t^-)$ and $q=q(t)$. To integrate in time the constitutive law then means to determine the stress state and of damage solution of the following nonlinear system, where the strain $\boldsymbol{\varepsilon}$ and the state of the material at the beginning of time step $(\boldsymbol{\varepsilon}^-, d^-)$ are given:

$$\boldsymbol{\sigma}=(1-d)\boldsymbol{E}\,.\boldsymbol{\varepsilon} \qquad \text{éq 2.3-1}$$

$$f(\varepsilon,\, d)\leq 0 \quad d-d^-\geq 0 \quad (d-d^-)\, f(\varepsilon,\, d)=0 \qquad \text{éq 2.3-2}$$

a method of resolution was proposed by [bib3]. It starts by examining the solution without evolution of the damage (also called elastic test) then, if necessary, carries out a correction to check the condition of coherence. In this case, the existence and the unicity of the solution guarantee the good performance of the method. Let us consider the elastic test:

$d=d^{-}$ solution if $f^{\mathrm{el}}(\varepsilon)=f(\varepsilon,\ d^{-})\leq 0$ **éq 2.3-3**

In the contrary case, the damage is obtained while solving $f(\varepsilon,\ d)=0$:

$$d=(1+\gamma)\left(1-\sqrt{\frac{w^{y}}{w}}\right) \text{ where } w=\frac{1}{2}\,\boldsymbol{\epsilon}\cdot E\cdot\boldsymbol{\epsilon} \quad \textbf{éq 2.3-4}$$

As for the stress, it is given by [éq 2.3-1] in all the cases.
It still remains to be made sure that the damage does not exceed value 1. In fact, when $d=1$, the stiffness of the material point considered is cancelled. Insofar as no technique of suppression of the finite elements "broken" is put in work (technical possibly delicate when the finite elements have several Gauss points), of the null pivots can appear in the stiffness matrix. This is why a numerical threshold is introduced d_c beyond which one considers an elastic residual stiffness for the tangent matrix, equations of behavior remaining unchanged.

To preserve a reasonable conditioning of the stiffness matrix, one chooses

$$d_c=1-10^{-5} \quad \textbf{éq 2.3-5}$$

an indicator χ, arranged in the second local variable, then specifies the behavior during time step running:

- $\chi=0$ elastic behavior (strain energy lower than the threshold)
- $\chi=1$ evolution of the damage
- $\chi=2$ (saturated damage) ($d=1$).

2.4 Description of the local variables

the local variables are two:

- $VI(1)$ indicating d
- $VI(2)$ damage χ

3 Formulation with gradient of damage

the formulation in gradient of damage is not available any more (from version 10.2 of Aster), to refer to the constitutive law `ENDO_SCALAIRE` [R5.03.25], which replaces `ENDO_FRAGILE` for modelization `GRAD_VARI` and this mainly for reasons of robustness of computations.

4 Formulation with regularized strain

4.1 Formulation continues in time

the approach with regularized strain [R5.04.02] also makes it possible to control the phenomena of localization and for this reason seems an alternative to the formulation with gradient of damage. But unlike the latter, this formulation has the advantage of resorting to the standard algorithms for the nonlinear problems. Indeed, the only difference compared to the local constitutive law lies in the data of two strains instead of one, the local strain $\boldsymbol{\varepsilon}$ which intervene in the relation stress-strain and the regularized strain $\bar{\boldsymbol{\varepsilon}}$ which controls the evolution of the damage. This one results from the local strain by resolution of the system of equations with partial derivatives according to:

$$\begin{cases} \bar{\boldsymbol{\varepsilon}} - L_c^2 \, \Delta \bar{\boldsymbol{\varepsilon}} = \boldsymbol{\varepsilon} & \text{dans la structure} \\ \nabla \bar{\boldsymbol{\varepsilon}} \cdot \boldsymbol{n} = 0 & \text{sur le bord de normale } \boldsymbol{n} \end{cases} \quad \text{éq 4.1-1}$$

where the characteristic length L_c is again indicated under key word LONG_CARA of DEFI_MATERIAU. Finally, the behavior model is written in the following way, where the function threshold F was already defined in [éq 2.1-2]:

$$\boldsymbol{\sigma} = (1-d) \, \boldsymbol{E} \cdot \boldsymbol{\varepsilon} \quad \text{éq 4.1-2}$$

$$f(\bar{\boldsymbol{\varepsilon}}, d) \le 0 \quad \dot{d} \ge 0 \quad \dot{d} \, f(\bar{\boldsymbol{\varepsilon}}, d) = 0 \quad \text{éq 4.1-3}$$

4.2 Integration of the constitutive law

One of the advanced advantages for the nonlocal formulation with regularized strain is the little of modifications which it involves in the construction of the constitutive law. Indeed, the integration of the local variables is completely controlled by the regularized strain $\bar{\boldsymbol{\varepsilon}}$. The statements of the local law thus are found:

$$\begin{cases} \text{si} \quad f^{\text{el}}(\bar{\boldsymbol{\varepsilon}}) = f(\bar{\boldsymbol{\varepsilon}}, d^-) \le 0 & d = d^- \\ \text{si} \quad f^{\text{el}}(\bar{\boldsymbol{\varepsilon}}) = f(\bar{\boldsymbol{\varepsilon}}, d^-) > 0 & d = (1+\gamma) \left(1 - \sqrt{\frac{w^y}{\bar{w}}}\right) \end{cases} \quad \text{with } \bar{w} = \frac{1}{2} \, \bar{\boldsymbol{\varepsilon}} \cdot \boldsymbol{E} \cdot \bar{\boldsymbol{\varepsilon}} \quad \text{éq 4.2-1}$$

the stress is then obtained directly by the relation [éq 4.1-2]. Moreover, one introduces a critical damage [éq 2.3-5], as in the local case, to preserve a residual stiffness.

4.3 Local variables

They is the same local variables as for the local law:

$VI(1)$ indicating d
$VI(2)$ damage χ

5 Control by elastic prediction

the control of the type PRED_ELAS standard controls the intensity of the loading to satisfy a certain equation related to the value with the function threshold f^{el} during the elastic test [bib5]. Consequently, only the points where the damage is not saturated are taken into account. The algorithm which deals with this mode of control, cf [R5.03.80], requires the resolution of each one of these Gauss points of the following scalar equation in which $\Delta\tau$ is a data and η the unknown:

$$f^{el}(\boldsymbol{\varepsilon}_{\text{impo}} + \eta \, \boldsymbol{\varepsilon}_{\text{pilo}}, a^-) = \Delta\tau \quad \text{éq 3-1}$$

Let us note that this equation is modified for control PRED_ELAS in ENDO_SCALAIRE in order to have the parameter $\Delta\tau$ which corresponds to the increment of damage that one seeks to obtain for at least a point of structure. One then does not seek any more one parameter of control η which makes time step leave the criterion $\Delta\tau$ a value with the damage resulting from preceding (cf Eq 3-1), but a parameter η which brings back for us on the criterion with a damage increased by $\Delta\tau$:

$$f^{el}(\boldsymbol{\varepsilon}_{\text{impo}} + \eta \, \boldsymbol{\varepsilon}_{\text{pilo}}, a^-) = \Delta\tau \quad \Rightarrow \quad f^{el}(\boldsymbol{\varepsilon}_{\text{impo}} + \eta \, \boldsymbol{\varepsilon}_{\text{pilo}}, a^- + \Delta\tau) = 0 \quad \text{éq 3-2}$$

where Δt corresponds to the increment of time defined in the list of times of computation and COEF_MULT is the coefficient specified by the key word COEF_MULT of the option CONTROL in operator STAT_NON_LINE [U4.51.03].

6 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
5.0	E. LORENTZ EDF-R&D/AMA	initial Text
10.0	K. KAZYMYRENKO EDF-R&D/AMA	Taken into account of model ENDO_SCALAIRE

7 Bibliography

1. LEMAITRE J., CHABOCHE J.L.: Mechanics of the solid materials. Dunod: Paris, 1988.
2. MARIGO J.J.: Formulation of a damage model of an elastic material. Count - returned Academy of Science, Paris 1981; series II, 292(19): 1309-1312.
3. SIMO J.C., TAYLOR R.L.: Consist tangent operators for failure-independent elastoplasticity. Methods computer in Applied Mechanics and Engineering 1985; 48: 101-118.
4. LORENTZ E., ANDRIEUX S.: A variational formulation for nonlocal ramming models. International Newspaper of Plasticity 1999; 15: 119-138.
5. LORENTZ E., BADEL P.: A load control method for ramming finite element simulations. International Newspaper for Numerical Methods in Engineering, submitted 2002.