
Behavior elastoplastic under irradiation of metals: application to the internal reactor vessels

Summarized:

This document presents the writing of a constitutive law under irradiation of the stainless steels 304 and 316, materials of which the structures internal reactor vessels of the nuclear reactors are made up. The formalism of the model is identical for the two materials, only the parameters are different from one material to another. The model takes into account, in addition to thermoelasticity, plasticity, creep under irradiation, as well as a possible swelling under neutron flux.

One details here the placement of the model like his limits.

This constitutive law, if it were developed for specific needs at EDF, can be used to give an account of the behavior of any material presenting of the characteristics of plasticity, creep under irradiation and swelling.

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1 Introduction

This document is describing the development and numeric work the implementation of a constitutive law under irradiation of the materials constituting internal structure of the nuclear reactor reactor vessels: the steels 316 and 316L cold worked for screws and bolts, as well as hyper-hardened steels 304 and 304L for the partitions, the reinforcements and the envelope of heart. This constitutive law is formally the same one for the two types of steel, only the parameters varying from one material to another.

The model developed must be able to give an account (in addition to thermoelasticity) of the plasticity induced by the tightening of the screws, well-known creep under irradiation for the metallic materials, and of a possible swelling induced by the irradiation. The effects of creep under irradiation and swelling are differentiated by the fact that the strain occurs with constant volume for creep, which is not the case for swelling. The various phenomena quoted above are listed in an exhaustive way in document [ref.1] and will thus not be the object of an additional description in what follows.

This document is articulated in the following way: the constitutive law chosen to describe the response of hyper-hardened steels 304/304L and 316/316L hammer-hardened is first of all exposed, as well as the equations allowing its establishment in the form of implicit integration. Each mechanism is detailed in a general way. The dependence of the material parameters to the command variables which are the irradiation and the temperature is clarified. The numerical values of these parameters are exposed in note EDF R & D HT 5/26/045 /A. One presents then the limits of the model and its application in Code_Aster.

It should be noted that the formalism presented here, if it were implemented in the frame of a quite specific application, can be used to describe the behavior of any type of material presenting of the characteristics of plasticity, creep under irradiation and swelling. In this case, it will be important with the user of the Code_Aster to get the coefficients material necessary to the use of the model.

2 Formulation of the constitutive law

One proposes to employ an additive decomposition of the strains. The mechanical strain is then expressed like:

$$\varepsilon_m = \varepsilon_e + \varepsilon_p + \varepsilon_i + \varepsilon_g \quad \text{éq}$$

mechanical strain, ε_m is defined like the total deflection minus the thermal strain: $\varepsilon_m = \varepsilon - \varepsilon_{th}$. The other components are: elastic strain ε_e , plastic strain ε_p , strain of creep of irradiation ε_i and strain of swelling ε_g .

•The elastic strain is connected to the stress by the Hooke's law: $\underline{\sigma} = \underline{E} : \underline{\varepsilon}_e$. The elasticity tensor \underline{E} depends on the temperature T .

•The plastic strain is given by a model of the Von-Settings type. Flow surface is expressed like:

$$f = \sigma_{eq} - \sigma^f \quad \text{éq 2-2}$$

where σ_{eq} is the stress of Von-Settings and σ^f the flow limit. This one depends on the cumulated plastic strain p , the temperature T , and the fluence Φ . Yielding is given by the normality rule so that:

$$\dot{\varepsilon}_p = \dot{p} \underline{n} \quad \text{with} \quad \underline{n} = \frac{3}{2} \frac{\underline{s}}{\sigma_{eq}} \quad \text{éq}$$

where \underline{s} is the deviator of the stresses. \dot{p} is given by the condition of coherence $f=0$ and $\dot{f}=0$. The creep models of irradiation are expressed (for a test with stress and constant flux) [ref.1] :

$$\varepsilon = \max(A_i \cdot \sigma \cdot \Phi - A_0) \quad \text{éq 2-4}$$

coefficient A_0 translates an effect of threshold. In differential form, the preceding model can be rewritten like:

$$\dot{\varepsilon} = A_i \sigma \cdot \varphi \quad \text{so} \quad \eta_i > A_0 / A_i \quad \text{with} \quad \dot{\eta}_i = \sigma \cdot \varphi \quad \text{éq 2-5}$$

where φ is the flux ($\varphi = \dot{\Phi}$). One thus introduces an additional variable of state η_i , to describe the effect of threshold. This uniaxial model must be wide with the multiaxial case. The creep of irradiation being done without variation of volume, one will describe this mechanism by a viscoplastic model founded under equipotential data by the stress of Von-Settings. It will be also supposed that the evolution of the variable η_i is managed by this same stress. The following models then are obtained:

$$\dot{\eta}_i = \zeta_f \cdot \sigma \cdot \varphi \quad \text{éq}$$

$$\dot{p}_i = A_i \cdot \sigma \cdot \varphi \quad \text{if} \quad \eta_i > \eta_i^s \quad \text{not} \quad \dot{p}_i = 0 \quad \text{éq}$$

$$\dot{\varepsilon}_i = \dot{p}_i \underline{n} \quad \text{éq}$$

where p_i is the strain of creep of equivalent irradiation. A_i and η_i^s are coefficients of the model which can depend on the temperature (one can consider a dependence with respect to the fluence but the model will be more complex to identify). The function ζ_f makes it possible to introduce a dependence with respect to the temperature into the law of evolution of the variable η_i (see equation 4.3-1). It will be noted that the model thus formulated will be able to function for variable temperatures and flux.

Notice n°1:

It will be noticed that the "creep" of irradiation is entirely controlled by the fluence and not by time. If a thermal mechanism of creep would be added, time would play an explicit part of way of course.

Notice n°2:

To make sure of the good crossing of the threshold η_i^s , (from a numerical point of view) it is necessary to define a criterion of error. This criterion "TOLER_ET" corresponds to % of going beyond the threshold which one authorizes during numerical integration. The strain of creep of

irradiation begins as soon as the threshold η_i^s is crossed, but one forces Code_Aster to respect a reasonable evolution of the variable η_i in the vicinity of the threshold η_i^s . So during computation the criterion is not respected Code_Aster subdivides time step, provided that the subdivision of time step is authorized.

Swelling can be described by models of the type:

$$\frac{\Delta V}{V_0} = F_g(\Phi) \quad \text{éq 2-9}$$

where ΔV is the variation of volume and V_0 initial volume. By differentiating this equation and by supposing that the variation of volume remains weak ($\Delta V \ll V_0$) one obtains:

$$\frac{\dot{V}}{V} = \frac{dF_g}{d\Phi} \varphi \quad \text{éq 2-10}$$

This velocity of variation is identified with the trace of the tensor velocities of swelling: $\dot{V}/V = \text{trace}(\dot{\xi}_g)$. It is supposed here that swelling is made in an isotropic way and thus that $\dot{\xi}_g$ can express itself like:

$$\dot{\xi}_g = \dot{g} \mathbf{1} \quad \text{éq 2-11}$$

where $\mathbf{1}$ is the tensor unit. One thus has $\dot{V}/V = 3 \dot{g}$

$$\text{Is: } \dot{g} = A_g \varphi \quad \text{éq 2-12}$$

with $A_g = \frac{1}{3} dF_g / d\Phi$. A_g is a new material parameter. This one depends on the temperature and the fluence. In a problem with variable temperature, it is thus important to employ equation 2-12 and not equation 2-9 (this one being implicitly written for a constant temperature). A possible coupling between stress state and swelling is neglected.

The definition of the model rests on five local variables: p , η_i , p_i , g and ξ_e . The temperature and the fluence are regarded as parameters imposed for a given computation. These values come from computations of neutronics and thermal. The equations of the model are recalled in table 2.1.

Local variables	Equations of evolution
p	coherence \dot{p}
η_i	$\dot{\eta}_i = \zeta_f \cdot \sigma_{eq} \cdot \varphi$
p_i	$\dot{p}_i = A_i \cdot \sigma \cdot \varphi$ if $\eta_i > \eta_i^s$ not $\dot{p}_i = A_i \cdot \sigma \cdot \varphi$ $\dot{p}_i = 0$
G	$\dot{g} = A_g \varphi$
ξ_e	$\dot{\xi}_e = \dot{\xi}_m - \dot{p} \cdot \underline{n} - \dot{p}_i \cdot \underline{n} - \dot{g} \cdot \mathbf{1}$

Table 2.1 Equations of the model.

3 Establishment: implicit integration

implicit integration consists in finding the increments of the local variables $\Delta V_i = (\Delta P, \Delta \eta_i, \Delta P_i, \Delta g, \Delta \varepsilon_e)$ on a discrete increment of time Δt [ref.2]. Discretization of the equations of table 3.1 conduit to the following system of equations nonlinear:

$$\begin{aligned} R_e &= \Delta \varepsilon_e + \Delta p \underline{n} + \Delta p_i \underline{n} + \Delta g \underline{1} - \Delta \varepsilon_m = 0 \\ R_p &= \sigma_{eq} - \sigma^f = 0 \\ R_\eta &= \Delta \eta_i - \zeta_f \sigma_{eq} \Delta \varphi = 0 \\ R_i &= \Delta P_i - H(\eta_i - \eta_i^s) A_i \sigma_{eq} \Delta \varphi = 0 \\ R_g &= \Delta g - A_g \Delta \Phi = 0 \end{aligned}$$

Table 3.1 system of equations discretized

Where H is the function of Heavyside and $\Delta \Phi = \phi \Delta t$ is the increment of fluence. While posing $R = (R_e, R_p, R_\eta, R_i, R_g)$, one thus seeks to solve the system: $R(\Delta V_i) = 0$. The local variables appearing directly or indirectly (for example σ_{eq} can express itself according to ε_e) are expressed like: $v_i = v_i^0 + \theta \Delta v_i$, where v_i^0 represents the values of the variables at the beginning of increment. θ is a variable parameter between 0 and 1. For $\theta = 0$, one obtains an explicit diagram of Eulerian (to be avoided); for $\theta = 1$ one obtains a completely implicit diagram.

The system of equations of table 3.1 is solved by employing a method of Newton-Raphson which requires the computation of the Jacobian: $J = \partial R / \partial \Delta v_i$. This one can be calculated block per block (the null terms are omitted). One notes $\underline{N} = \partial \underline{n} / \partial \underline{\sigma}$ and $\sigma_{,p}^f = \partial \sigma^f / \partial p$.

Derived from R_e :

$$\begin{aligned} \frac{\partial R_e}{\partial \Delta \varepsilon_e} &= \underline{1} + \theta \cdot \Delta p \cdot \underline{N} : \underline{E} + \theta \cdot \Delta p_i \cdot \underline{N} : \underline{E} & \frac{\partial R_e}{\partial \Delta p} &= \underline{n} \\ \frac{\partial R_e}{\partial \Delta p_i} &= \underline{n} & \frac{\partial R_e}{\partial \Delta g} &= \underline{1} \end{aligned}$$

Derived from R_p :

$$\frac{\partial R_p}{\partial \Delta \varepsilon_e} = \theta \underline{n} : \underline{E} \qquad \frac{\partial R_p}{\partial \Delta p} = -\theta \sigma_{,p}^f$$

Derived from R_η :

$$\frac{\partial R_\eta}{\partial \Delta \varepsilon_e} = -\zeta_f \theta \cdot \Delta \varphi \cdot \underline{n} : \underline{E} \qquad \frac{\partial R_\eta}{\partial \Delta \eta_i} = 1$$

Derived from R_i :

$$\frac{\partial R_i}{\partial \Delta \varepsilon_e} = -A_i \cdot \theta \cdot \Delta \varphi \cdot \underline{n} : \underline{E} \qquad \frac{\partial R_i}{\partial \Delta p_i} = 1$$

Derived from R_g :

$$\frac{\partial R_g}{\partial \Delta g} = 1$$

4 Coefficients materials

In the continuation, one presents the forms of evolution of the parameters of the equations according to the command variables which are the temperature and the fluence. The values of these parameters for the materials 304 and 316 are consigned in ratio HT 5/26/045 /A.

4.1 Thermoelasticity

the modulus Young [ref.3] is given by:

$$E = C_0^E + C_1^E \cdot T \quad \text{éq 4.1-1}$$

the Poisson's ratio is given by:

$$\nu = C_0^V + C_1^V \cdot T \quad \text{éq}$$

the thermal tangent coefficient of thermal expansion is given by:

$$\alpha = C_0^\alpha + C_1^\alpha \cdot T + C_2^\alpha \cdot T^2 \quad \text{éq}$$

the secant coefficient of thermal expansion is then given, by taking a reference temperature null, by:

$$\alpha^{sec} = C_0^\alpha + \frac{1}{2} C_1^\alpha \cdot T + \frac{1}{3} C_2^\alpha \cdot T^2 \quad \text{éq}$$

4.2 Plasticity

This part to compute: exploits ratio [3 3] the curves of hardening of steels constitutive of the structures internal reactor vessels after irradiation for various temperatures. One uses the statements of the elastic limit with 0,2% of plastic strain $R_{0,2}$, of the ultimate stress R_m and of the lengthening distributed e_u according to the temperature T , the irradiation Φ (ratio [ref.3] uses the term d to describe the irradiation) and of the rate of cold hardening c . Ratio [ref.3] also provides lengthening with fracture but this data is not exploitable in practice because it depends on the type of studied test-tube.

Lengthening distributed is expressed like:

$$e_u = e_u^0(T) \eta_3(c) \xi_3(d) \quad \text{éq 4.2-1}$$

the elastic limit with 0,2% is expressed like:

$$R_{0,2} = R_{0,2}^0(T) \eta_1(c) \xi_1(d) \quad \text{éq}$$

the ultimate stress is not expressed directly. The difference $\Delta R = R_m - R_{0,2}$ is first of all represented by a function:

$$\Delta R = (R_m^0(T) - R_{0,2}^0(T)) \eta_2(c) \xi_2(d) \quad \text{éq 4.2-3}$$

To adjust ΔR instead of R_m makes it possible to ensure that: $R_m > R_{0,2}$. R_m is thus obtained like:

$$R_m(T, c, d) = R_{0,2}(T, c, d) + \Delta R(T, c, d)$$

Let us note that the functions e_u^0 , $R_{0,2}^0$ and R_m^0 are valid for the two materials (304 and 316) constitutive of the internal reactor vessels. The functions $\eta_{1,2,3}$ and $\xi_{1,2,3}$ depend on the material. One proposes to represent the curve of hardening of the materials for values of T , d and c given by a model power of the type:

$$\sigma_f(p) = K (p + p_0)^N \quad \text{éq}$$

where p is the equivalent plastic strain of Von-Settings. K , p_0 and N are parameters with calculating in order to obtain the values of $R_{0,2}^0$, R_m^0 and e_u .

The value of the strain corresponding to lengthening distributed (noted $\varepsilon_u = \log(1 + e_u)$) is obtained by the condition of Considère (by neglecting the elastic strain):

$$\frac{d\sigma^f}{dp} = \sigma^f \quad \text{éq 4.2-5}$$

is:

$$n K (p + p_0)^{n-1} = K (p + p_0)^n \Rightarrow p^0 = n - \varepsilon_u \quad \text{éq 4.2-6}$$

the ultimate stress is equal to:

$$R_m = \sigma^f(\varepsilon_u) \exp(-\varepsilon_u) = K n^n \exp(-\varepsilon_u) \quad \text{éq}$$

is:

$$K = \frac{R_m}{n^n} \exp(\varepsilon_u) \quad \text{éq}$$

the elastic limit is given by (one neglects the variation of section here):

$$R_{0,2} = K (p_e + p_0)^n \quad \text{with } p_e = 0,002 \quad \text{éq 4.2-9}$$

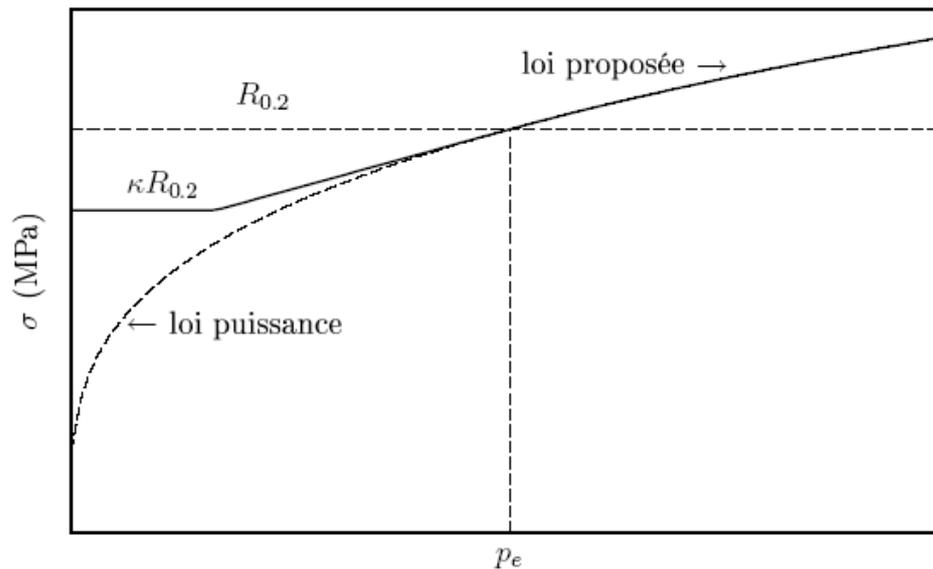
It thus remains to solve a single nonlinear equation compared to N :

$$S = R_{0,2} - \frac{R_m}{n^n} \exp(\varepsilon_u) (p_e + n - \varepsilon_u)^n = 0 \quad \text{éq}$$

the search for the solution is made by dichotomy while taking ε_u for initial value of n . One calculates then n and K by means of equations 4.2-8 and 4.2-10.

If the material presents little hardening (i.e strong irradiation or very important hardening cold), it is not possible to find a solution with equation 4.2 4.2-4 this case one will use the model power of the form with $\sigma^f(p) = K p^n$ and $n = \varepsilon_u$ is $\sigma^f(p) = K p^{nK} = R_m \exp(\varepsilon_u) / n^n$. $p_0 = 0$

The use of the preceding models for the low values of p can lead to not very realistic results. One will be able to consider that the yield stress cannot be lower than (with $\kappa R_{0,2}$ close relation κ of 1). The effect of the choice of on κ the response in creep of a structure is negligible for values ranging between 0,8 and 1. One recommends, for a computation in Code_Aster, to retain the value. $\kappa = 0,8$ One will use moreover a linear extrapolation enters and $p = 0$ obtained $p = p_e$ starting from the values in p_e stress and hardening. Figure 4.2 4.2-1 in a schematic way the pace of the model of hardening proposed. Figure



4.2 4.2-1of hardening proposed. Creep

4.3 of irradiation

the data of creep of irradiation collected in [ref. 1]1 to determine the values of and $A_i \cdot \eta_i^s$. These values are a priori constant with the temperature. It is however probable that at low temperature, there is no creep of irradiation. To model this evolution, it is possible to make depend the temperature coefficient A_i : éq

$$A_i = A_i^0 \zeta_f(T) \quad 4.3 \ 4.3-1$$

is A_i^0 the value of the parameter for the high temperatures. In this equation is ζ_f a function which can make it possible to stop the phenomenon of creep below a threshold of temperature. It can be

written in the form, $\zeta_f(T) = \frac{1}{2} \left(1 + \tanh \left(\mu_T (T - T_c) \right) \right)$ where allows T_c to regulate the temperature

for which the creep of irradiation begins and where allows μ_T to regulate the width of the transition between temperature ranges with and without creep from irradiation. Swelling

4.4 One

will use a bilinear model of Foster which makes it possible to represent a time of incubation then a linear swelling [ref. 1, ref.1 4].4One has then: éq

$$\frac{\Delta V}{V_0} = F_g(\Phi) = R \cdot \left(\Phi + \frac{1}{\alpha} \text{Log} \left(\frac{1 + \exp(\alpha(\Phi_0 - \Phi))}{1 + \exp(\alpha\Phi_0)} \right) \right) \quad 4.4 \ 4.4-1$$

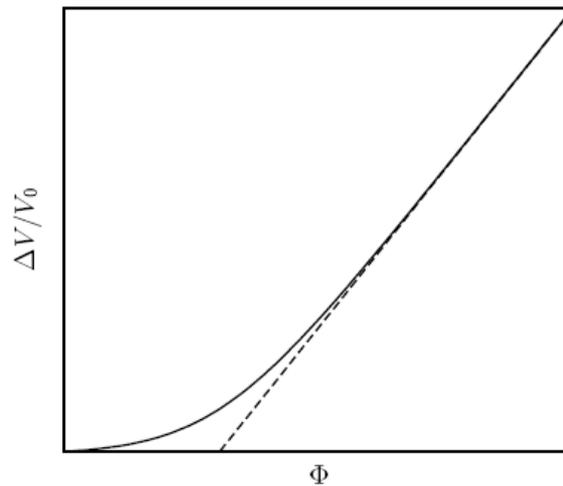
one $\Phi \rightarrow \infty$ obtains: éq

$$\frac{\Delta V}{V_0} = R \cdot \Phi - \frac{R}{\alpha} \text{Log} \left(1 + \exp(\alpha\Phi_0) \right) \quad 4.4 \ 4.4-2$$

the fluence of incubation is thus worth: (n.b. $\text{Log} \left(1 + \exp(\alpha\Phi_0) \right) / \alpha$: if this one $\alpha\Phi_0 \gg 1$ is worth).

By Φ_0 deriving equation 4.4-1 4.4-1 : éq

$$A_s = \frac{1}{3} R \left(1 - \frac{\exp(\alpha(\Phi_0 - \Phi))}{1 + \exp(\alpha(\Phi_0 - \Phi))} \right) \quad 4.4-3 \ 4.4-3$$



4.4-1 4.4-1of Foster [ref. 4]. 4The material parameters

of the model of swelling are thus R , and α . As Φ_0
in the case of the creep of irradiation, it can be necessary to introduce a dependence with respect to the temperature not to impose a swelling at low temperature. One will be able to make by means of depend R on the temperature a function similar ζ_g to that employed for the creep of irradiation: with

$$R = R^0 \zeta_g(T) \quad \text{éq} \quad \zeta_g(T) = \frac{1}{2} \left(1 + \tanh \left(\mu_g (T - T_c^g) \right) \right)$$

the 4.4-4 4.4-4

5 of the model The model

suggested was established from an experimental base in which the tests are carried out for monotonous requests: (I) traction tests on materials preradiated in isothermal condition; creep tests of irradiation (and swelling) in isothermal condition for a constant flux. This model must however be employed to simulate the behavior of the internal reactor vessels in real conditions; i.e. with variable flux and temperatures. Thermal

cycling induced a cyclic mechanical loading. The equations of the model, written in differential form, make it possible to manage the temperatures and variable flux but certain physical effects are not taken into account. It is for example the case of the kinematic hardening which could intervene during a thermal cycling. It does not exist, to our knowledge, of traction tests/compression cyclic on irradiated materials. Other questions can also arise: (I) which is behavior in creep of irradiation of a preradiated material (with a different temperature and/or under stress)? (II) exist does a coupling between creep of irradiation and plasticity? Swelling

is treated like an irreversible voluminal strain. It is however the appearance of cavities which can grow under tensile stress (to even fill itself under that but compressive stress remains very hypothetical). This adverse effect, since the rate of vacuum increases more quickly, is however not taken into account. The field of application

of the model in temperature, fluence and stress creep is specified in note HT 5/26/045 /A for the materials 304 and 316. However, it is possible to use such a model in the frame of a computation with Code_Aster for any other material that those. In this case, it is responsibility for the user of the code to make sure that the coefficients material which it uses are in agreement with the field of validity of its computation. Application

6 of the model in Code_Aster the definition

of the model in a data file of Code_Aster is done in the following way: ACIER

```
=DEFI_MATERIAU (ELAS_
  FO=_F (E=fonction
                                     (T) NU=fonction
                                     (T), ALPHA
    =fonction                          (T), TEMP_
    DEF_ALPHA=réel                      ), IRRAD

3M=_F (R02=fonction
                                     (T, dpa) (cf eq          4.2-2 4.2-2
    U=fonction                          (T, dpa) (cf eq          4.2-1 4.2-1
                                     (T, dpa) (cf eq          4.2-3 4.2-3
    (                                     eq 4.3-1                4.3-1
    F=fonction                          (T) (cf eq          2-6, 2-6 4.3-1
    S=réel                               ( cf eq                2-7) 2-7
                                     (T) (cf eq          4.4-1 4.4-1
    =réel                               ( cf eq                4.4-1 4.4-1
    PHI0=                               ( cf eq                4.4-1 4.4-1
    =réel                               ( cf figure           4.2-1 4.2-1
    G=fonction                          (T) (cf eq          4.4-4 4.4-4
    _ET=réel                            ( cf notice           2 §2)),)
```

the local variables

are the following ones: V1: p.v.

2: V3

: V4 η_i

: G p_i

V5: indicator

of plasticization (0 if not from plasticization, 1 if plasticized) V6: V7

irradiation: Temperature

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Besson5, S.Leclercq, Constitutive law for A304 steels and A316 of the internal reactor vessels under irradiation, Note EDF/R & D - HT 5/26/045 /A, 2005. History

8 of the versions of the document Version

Aster Author	(S) or contributor organization (S) Description	(S) of the modifications 8.4 S
LECLERCQ	J.BESSON EDF/R & D /MMC, ENSMP-CDM initial	Text, formulation of model IRRAD3M 9.1 J
L.	FLEJOU EDF/R & D /AMA Improvement	of the numerical integration of the models: analytical for swelling, and criterion of crossing of the threshold for creep. 10.0
J.L.	FLEJOU EDF/R & D /AMA Modification	of the model for the computation of the sensitivity to the IASCC.