Regularized damage model ENDO_SCALAIRE

Summarized:

This document the model describes elastic behavior brittle ENDO_SCALAIRE available only for the nonlocal modelization to gradient of damage GRAD_VARI. The damage is modelled in a scalar way; the loadings in compression and tension are not distinguished. Unlike the other damage models introduced previously, the latter behaves in a regular way (not snap-back, lengthening finished with the fracture) at least in the unidimensional cases.

1 Domaine d'application

model ENDO_SCALAIRE returns in a broad family of the damage models (see for example [R5.03.18]). It aims in particular at modelling a brittle elastic behavior in nonlocal version (GRAD_VARI [R5.04.01]) so that its behavior at least in the unidimensional cases is regular. The parameters of the model were selected to ensure at the same time the absence of snap-back in the response force-displacement, as well as the finished lengthening of the bar 1D to the fracture. This last property distinguishes it from model ENDO_FRAGILE [R5.03.18] in nonlocal version, ENDO_SCALAIRE is more regular. The local version of the model is not implemented, because it is equivalent to that of model ENDO_FRAGILE to a change close to parameters. The modelled material is elastic isotropic. Its stiffness can decrease in an irreversible way when strain energy becomes important, without distinguishing the tension from compression. The width of the tapes of localization is controlled by a material parameter, indicated in operator DEFI_MATERIAU under key word C_GRAD_VARI of factor key word the NON_LOCAL [U4.43.01].
The control of the type PRED_ELAS [R5.03.80] seems the mode of control of the level of the most suitable loading.

2 Variational formulation of the problem of damage

2.1 Case of a generic model

Two equivalent approaches can be used to describe the process of damage of a brittle isotropic material. On a side it is possible to derive the damage model in the frame from generalized standard description. In this case it is necessary to define a free energy of the system, as well as the potential of dissipation. The flow rule then established the evolution of the local variables.
As for the description of damage one needs only a scalar variable, preceding description is simplified and been able to be brought back towards a variational problem under stress of increase in damage [bib2].
To define a constitutive law in gradient of damage [R5.04.01] it is thus enough to express the density of total free energy (elastique+dissipation) according to strain tensor $\boldsymbol{\varepsilon}$ and of variable to damage $0 \leqslant a \leqslant 1$. The spatial distribution of the damage is given then by a field $a(x)$. The density of free energy arises in general in the following form:

$$\Phi(\boldsymbol{\varepsilon}, a) = A(a) w(\boldsymbol{\varepsilon}) + \omega(a) + c/2 (\nabla a)^2 \qquad \textbf{éq 2.1-1}$$

Here c is the parameter of nonlocality (C_GRAD_VARI) $w(\boldsymbol{\varepsilon})$ elastic strain energy, $\omega(a)$ the energy of dissipation and $A(a)$ the function of stiffness. $a=0$ corresponds to the operational material and $a=1$ corresponds to the material completely damaged: $A(1)=0, A(0)=1$. The problem of evolution is from now on a simple problem of minimization of free energy of Helmholtz $F \equiv \int \Phi(\boldsymbol{\varepsilon}, a) d\,\Omega$ under stress $\dot{a} \geq 0$ [1].

$$min_{(\boldsymbol{\varepsilon},a)} F(\boldsymbol{\varepsilon}, a), \text{ où } \qquad F(\boldsymbol{\varepsilon}, a) = \int \left[A(a) \boldsymbol{\varepsilon} : \boldsymbol{E} : \boldsymbol{\varepsilon} + \omega(a) + c/2 (\nabla a)^2 \right] d\,\Omega$$

where one replaced $w(\boldsymbol{\varepsilon}) = \boldsymbol{\varepsilon} : \boldsymbol{E} : \boldsymbol{\varepsilon}/2$ by means of the definition of the tensor of Hooke. Two equations derive from the variational problem of minimization: $\delta F(\boldsymbol{\varepsilon}, a)/\delta \boldsymbol{\varepsilon} = 0$ [2] and $\delta F(\boldsymbol{\varepsilon}, a)/\delta a \geq 0$. The inequality in the second equation is related to the presence of imposed stress. These two equations must be satisfied everywhere in the field with integration Ω . They are supplemented by an equation of coherence of Kuhn-Tucker $\dot{a} \cdot \delta F(\boldsymbol{\varepsilon}, a)/\delta a = 0$. On edges $\partial \Omega$ we

1 note by ∇a spatial derivative of the field of damage and by $\dot{a}$ that related to the temporal evolution
2 $\delta F(\boldsymbol{\varepsilon}, a)/\delta \boldsymbol{\varepsilon}$ is the variational derivative partial according to the direction of the spatial field $\boldsymbol{\varepsilon}(x)$, built-in $a(x)$ remaining field.

obtain an additional condition of normality $\nabla a \cdot \boldsymbol{n}=0$, where $\boldsymbol{n}$ is vector-normal. Finally the variable of damage and its gradient must be continuous at interior of the field of integration to carry out the minimum of functional calculus in question (see [bib2,4] for more details).

2.2 Behavior models

the restrain between the variational formulation and the usual laws of evolution is direct. The state of the material is characterized by the strain $\boldsymbol{\varepsilon}$ and the damage a, ranging between 0 and 1. The relation stress-strain is defined, which remains elastic, and the stiffness is affected by the damage:

$$\boldsymbol{\sigma}=\delta F(\boldsymbol{\varepsilon}, a)/\delta\boldsymbol{\varepsilon}=A(a)\boldsymbol{E}:\boldsymbol{\varepsilon} \quad \text{éq 2.2-1}$$

with $\boldsymbol{E}$ the tensor of Hooke. The evolution of the damage, always increasing, is controlled by the following function threshold:

$$f(\boldsymbol{\varepsilon}, a)=-\delta\Phi(\varepsilon, a)/\delta a=-\frac{1}{2}A'(a)\boldsymbol{\varepsilon}:\boldsymbol{E}:\boldsymbol{\varepsilon}-\omega'(a)+c\,\Delta a \quad \text{éq 2.2-2}$$

the condition of coherence takes its usual form then:

$$f(\boldsymbol{\varepsilon}, a)\leq 0 \quad \dot{a}\geq 0 \quad \dot{a}\,f(\boldsymbol{\varepsilon},\ a)=0 \quad \text{éq 2.2-3}$$

One notes two characteristics of this formulation. Firstly, the function threshold is NON-local because of the presence of the Laplacian of damage. Then, the absence of flow condition is justified by the double role of the damage a, on a side it is presented in the form of an internal variable of evolution, other side it fulfills the mission of the parameter of Lagrange $\lambda\equiv a$.
One sees also the advantage of presentation of the damage models in their variational form. It is enough to describe the density of total free energy (éq.2.1-1), which includes dissipation, to define the law of evolution completely.

2.3 Identification of the parameters for model ENDO_SCALAIRE

In model ENDO_SCALAIRE the functions of stiffness and dissipation are selected as follows:

$$\omega(a)=ka, \quad A(a)=\left(\frac{1-a}{1+\gamma a}\right)^2$$

The parameters of this constitutive law are then five. On the one hand, the Young's modulus E and the Poisson's ratio ν which determine the tensor of Hooke by:

$$\mathbf{E^{-1}}\cdot\boldsymbol{\sigma}=\frac{1+\nu}{E}\boldsymbol{\sigma}-\frac{\nu}{E}(\mathrm{tr}\,\boldsymbol{\sigma})\,\mathbf{Id} \quad \text{éq 2.2-1}$$

In addition, k, γ, c which defines the lenitive behavior, as well as the width characteristic of the tape of damage. The latter can be readjusted with the macroscopic parameters starting from the unidimensional model, which admits an semi-analytical solution (feeding-bottle 6,7). By noting the stress with the peak by σ_y, the energy of the fracture of Griffith by G_f and cuts it zone damaged with the fracture by D one obtains:

$$k=\frac{3G_f}{4D}, \quad c=\frac{3}{8}DG_f, \quad \gamma=\frac{3EG_f}{4\sigma_y^2 D}-1$$

The numerical tests showed that to avoid the presence of snap-back in the response force-displacement in 1D, it would be necessary to have $\gamma\geq 2.8$. For this reason the choice was made to simplify the entry of the data of the model, one not informs the complete clearance of macroscopic parameters σ_y, G_f, D, but directly the parameters of the model γ, c and the stress with the peak σ_y, given under the key words factors ENDO_SCALAIRE (GAMMA, SY) and NON_LOCAL

(C_GRAD_VARI) of operator DEFI_MATERIAU. As for E and ν, they are given classically under factor key word the ELAS or ELAS_FO. The reasoning which follows, is valid with the strict meaning only for the modelization 1D, but can be useful for the not informed users. If the parameters E, ν, G_f, σ_y are a priori defined, the user can vary the parameter D in order to satisfy the condition with absence of snap-backs local $\gamma \geq 2.8$. He must make sure thereafter that the size of the system considered is higher than the bandwidth of damage D.

Example of the concrete in tension	$E=30\,\text{GPa},\ \nu=0.2$ $G_f=100\,\text{N/m}$ $\sigma_y=3\,\text{MPa}$	$\text{ELAS}(E=3e10, NU=0.2)$ $\text{ENDO\_SCALAIRE}(GAMMA=1/(4D)-1, SY=3e6)$ $\text{NON\_LOCAL}(C\_GRAD\_VARI=37.5D)$

the bandwidth of damage is to be chosen by respecting $\gamma \geq 2.8$ <=> $D \leq 66\,\text{mm}$

2.4 Integration of the constitutive law locally

We present here the integration method of model ENDO_SCALAIRE in its local version ($c=0$), so that the user can make a generalization for the case NON-room, which it is generic and rests entirely on the algorithm presented in Doc. [R5.04.01].

Temporal discretization of the equations [éq 2.2-1] with [éq 2.1-3] on time step $[t^- \ t]$ is realized by a diagram of implicit Eulerian. To integrate in time the constitutive law consists in determining the stress state and of damage of the solution of the following nonlinear system:

$$\boldsymbol{\sigma}=A(a)\boldsymbol{E}:\boldsymbol{\varepsilon} \qquad \text{éq 2.4-1}$$

$$f_{\text{loc}}(\varepsilon, a)\leq 0 \quad a-a^{-}\geq 0 \quad \left(a-a^{-}\right)\cdot f_{loc}(\varepsilon, a)=0 \qquad \text{éq 2.4-2}$$

where the variables without indices correspond to time step final t, such as for example the strain $\boldsymbol{\varepsilon}$; the state of the material at the beginning of time step $(\boldsymbol{\varepsilon}^{-}, a^{-})$ is indicated by the index "-". The local function threshold is given by (éq. 2.2-2): $f_{\text{loc}}(\boldsymbol{\varepsilon}, a)=(1+\gamma)\frac{(1-a)}{(1+\gamma a)^3}\boldsymbol{\varepsilon}:\boldsymbol{E}:\boldsymbol{\varepsilon}-k$

A method of resolution was proposed by [bib3]. It starts by examining the solution without evolution of the damage (also called elastic test) then, if necessary, carries out a correction to check the condition of coherence. In this case, the existence and the unicity of the solution guarantee the good performance of the method. Let us consider the elastic test:

$$a=a^{-} \text{ solution if } f^{\text{el}}(\boldsymbol{\varepsilon})\equiv f_{\text{loc}}\left(\boldsymbol{\varepsilon},\ a^{-}\right)\leq 0 \qquad \text{éq 2.4-3}$$

In the contrary case, the damage is obtained while solving $f_{\text{loc}}(\boldsymbol{\varepsilon},\ a)=0$ (polynomial of order 3).

$$(1-a)(1+\gamma)\boldsymbol{\varepsilon}:\boldsymbol{E}:\boldsymbol{\varepsilon}/k=(1+\gamma a)^3 \qquad \text{éq 2.4-4}$$

It is the largest root which is selected among the three existing.

It still remains to be made sure that the damage does not exceed value 1. In fact, when $a=1$, the stiffness of the material point considered is cancelled $A(1)=0$. Insofar as no technique of suppression of the finite elements "broken" is put in work (technical possibly delicate when the finite elements have several Gauss points), of the null pivots can appear in the stiffness matrix. This is why one introduces a numerical threshold of elastic residual stiffness for the tangent matrix, which can be indicated under factor key word the COEF_RIGI_MINI of operator DEFI_MATERIAU. This value without dimension is a multiplying coefficient of the elastic modulus of an isotropic linear model. To preserve a reasonable conditioning of the stiffness matrix, the value by default is chosen $min\, A(a)=10^{-5}$.

An indicator χ , arranged in the second local variable, then specifies the behavior during time step running:

$\chi=0$ elastic behavior (strain energy lower than the threshold)
$\chi=1$ evolution of the damage
$\chi=2$ saturated damage ($a=1$).

2.5 Integration of the constitutive law in nonlocal

We present here only the integration method of model ENDO_SCALAIRE in its local version ($c=0$), because generalization for the case NON-room is generic and rests entirely on the algorithm presented in Doc. [R5.04.01]. It is noted that for the nonlocal version the function threshold is shifted, we thus obtain a polynomial of order 4 to solve. As for the stress, it is given by [éq 2.4-1] in all the cases.

2.6 Description of the local variables

the local variables are three:

$VI(1)$ indicating a
$VI(2)$ damage χ
$VI(3)$ residual stiffness $1-A(a)$

3 Control by elastic prediction

the control of the type PRED_ELAS standard controls the intensity of the loading to satisfy a certain equation related to the value with the function threshold f^{el} during the elastic test [bib5]. Consequently, only the points where the damage is not saturated are taken into account. The algorithm which deals with this mode of control, cf [R5.03.80], requires the resolution of each one of these Gauss points of the following scalar equation in which $\Delta\tau$ is a data and η the unknown:

$$f^{el}(\boldsymbol{\varepsilon}_{\mathrm{impo}}+\eta\,\boldsymbol{\varepsilon}_{\mathrm{pilo}},a^{-})=\Delta\tau \qquad \textbf{éq 3-1}$$

Let us note that this equation is modified for control PRED_ELAS in ENDO_SCALAIRE in order to have the parameter $\Delta\tau$ which corresponds to the increment of damage that one seeks to obtain for at least a point of structure. One then does not seek any more one parameter of control η which makes time step leave the criterion $\Delta\tau$ a value with the damage resulting from preceding (cf Eq 3-1), but a parameter η which brings back for us on the criterion with a damage increased by $\Delta\tau$:

$$f^{el}(\boldsymbol{\varepsilon}_{\mathrm{impo}}+\eta\,\boldsymbol{\varepsilon}_{\mathrm{pilo}},a^{-})=\Delta\tau \quad\Rightarrow\quad f^{el}(\boldsymbol{\varepsilon}_{\mathrm{impo}}+\eta\,\boldsymbol{\varepsilon}_{\mathrm{pilo}},a^{-}+\Delta\tau)=0 \qquad \textbf{éq 3-2}$$

4 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
10.0	K.KAZYMYRENKO, E.LORENTZ, S.CUVILLIEZ EDF-R&D/AMA	initial Text
10.2	K.KAZYMYRENKO, EDF-R&D/AMA	minor Corrections of the notations

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