Constitutive law of assembly ASSE_CORN

Summarized:

This document describes the nonlinear behavior of the nonlinear assemblies of angles of pylons modelled by discrete elements DIS_TR. This constitutive law is affected on the discrete elements by means of relation ASSE_CORN called by the operators of resolution of nonlinear problems STAT_NON_LINE [R5.03.01] or DYNA_NON_LINE [R5.05.05].

The model represents at the same time behavior in tension of the assembly and the relation moment-rotation around the axis of the bolts perpendicular to the assembly. The other directions of loading present a linear elastic behavior described by classical characteristics of stiffness.

One distinguishes in the constitutive law two phases associated with two mechanisms: the first representing the friction and the sliding of the bolts until the thrust, and the second representing the plasticization of the assembly until failure. The models of the plastic type describing each one of these phases have even pace and have a concavity to their connection which makes convergence problematic and requires a particular digital processing in the computation options to which the iterative method of Newton appeals.

Contents

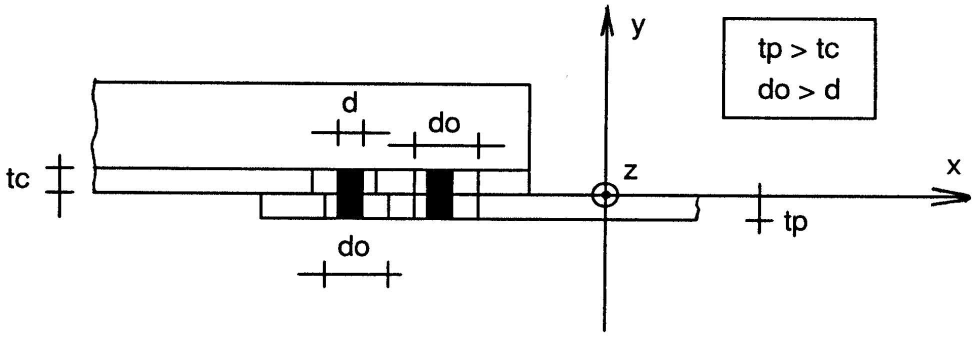
1 Notations

SLF	Surfaces Limit of Friction
M_y	Moment in the assembly around the limiting y
$\overline{N}_1$	axis Force of sliding of the assembly on the limiting x
$\overline{M}_1$	axis Moment of sliding of the assembly on axis y
SLU	Surfaces Ultimate Limit
$\overline{N}_2$	Ultimate limiting force of the assembly on the ultimate x
$\overline{M}_2$	axis limiting Moment of the assembly on the axis y
$\overline{N}$	Force limits
$\overline{M}$	Moment limits
$\overline{U}_1$	limiting Displacement of mechanism 1 on the limiting x
$\overline{\theta}_1$	axis Rotation of mechanism 1 on the limiting y
$\overline{U}_2$	axis Displacement of mechanism 2 on the limiting x
$\overline{\theta}_2$	axis Rotation of mechanism 2 on the axis y
U	Displacement of the assembly on the axis x
θ	Rotation of the assembly on the axis y
n	Force reduces $n=Nx/\overline{N}$
m	Moment reduces $m=My/\overline{M}$
U_r	Displacement reduces $Ur=U/\overline{U}$
θ_r	reduced Rotation $\theta\ r=\theta/\overline{\theta}$
$\overline{U}$	limiting Displacement on the limiting x
$\overline{\theta}$	axis Rotation on the scalar y
$h(x)$	axis Function
a	Parameter of nonConstant
$\overline{d}$	linearity scalar
$\underline{d}$	Vector displacement generalized reduced
$\underline{f}$	Vector reduced force generalized
p	scalar Local variable
feq	Force generalized equivalent reduces scalar
F	Surface of scalar
$R(x)$	loading Function $R(x)=h^{-1}(x)$
$\underline{D}$	Vector displacement generalized
$\underline{F}$	Vector force generalized

$[\overline{D}]$	Matrix generalized displacement limit
$[\overline{F}]$	Stamps force generalized limit
X^+	Value of X at time $t+dt$
X^-	Value of X at time t
e	Eccentricity of loading $e=M_y/N_x$
e_r	reduced Eccentricity of loading $e_r=m/n$
ε	Signs of n
$[\,]$	Matrix
$\{\}$	Vector column
$<\;>$	Vector line
K_o	tangent Operator at time t
K_n	tangent Operator at time $t+dt$
K_{or}, K_{nr}	reduced tangent Operators

2 Models physical one-way behavior of the assembly

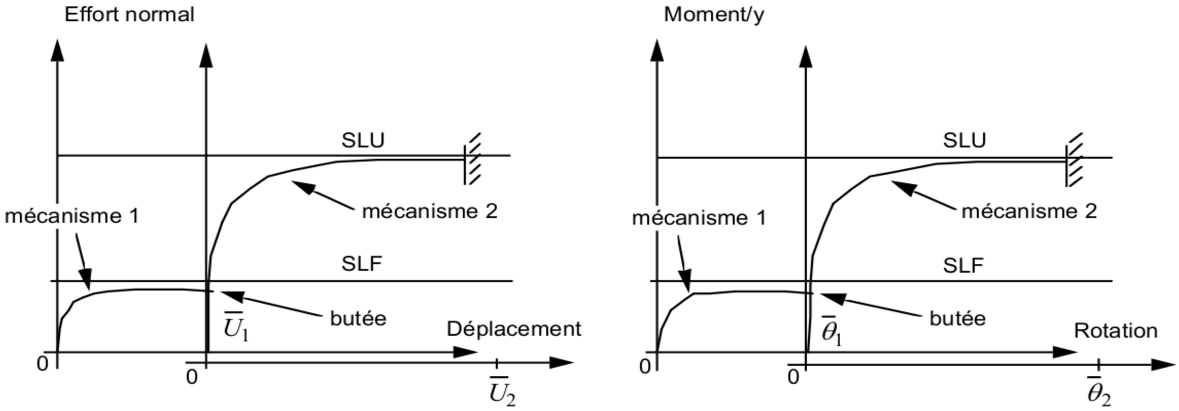
the assembly of an angle on the wing of another or on a plate (gusset or splice plate) by bolts is schematized by [Figure 2-a].



Appear 2-a: local coordinate system of connection; the axis x is confused with the axis of the bar and the axis y is confused with the axis of the bolts

the one-way behavior of the assembly is modelled for the loading in tension or bending.

The modelization retained one-way behavior in loading of the assembly subjected to a normal force or a moment around y is represented by [Figure 2-b].



Appear 2-b: mechanisms of assembly in normal force and moment

One distinguishes two phases of the behavior associated with two mechanisms:

mechanism 1: friction and sliding until the thrust (beginning of the shears of the bolts).
mechanism 2: plasticization of the assembly until failure by shears of the bolts or tearing of the grips.

The limiting surface of friction (SLF) is the curve corresponding to the appearance of the sliding in spaces $(\boldsymbol{N}_x - \boldsymbol{U}_x)$ and $(\boldsymbol{M}_y - \theta_y)$. Friction is described by the model of Coulomb.

Limiting surface ultimate (SLU) is the curve corresponding to the failure of the assembly in spaces $(\boldsymbol{N}_x - \boldsymbol{U}_x)$ and $(\boldsymbol{M}_y - \theta_y)$. Failure can be due, according to the design of the assembly, with the shears of the bolts or the tearing of the grips.

The tests on the same geometry but with tightening torques of the different bolts show that the tangent stiffness of mechanism 2 at the point of thrust decreases when the SLF approaches the SLU.

This justifies the physical modelization retained for the assembly of the two mechanisms [Figure 2-b].

3 Behavior model of the mechanisms

the behavior of mechanisms 1 and 2 is similar. It is nonlinear between a rigid tangent initial behavior and an asymptotic limiting behavior.

It is described by two essential parameters: the parameter of nonlinearity and the parameter surface limit.

The thrust (mechanism 1) or ruins it 2) (mechanism are described by an associated kinematical criterion.

3.1 Behavior one-way

We said to [§2] that the one-way behaviors in normal force and moment around are similar there [Figure 2-b].

They can be described by the same relation if the adimensional quantities are used:

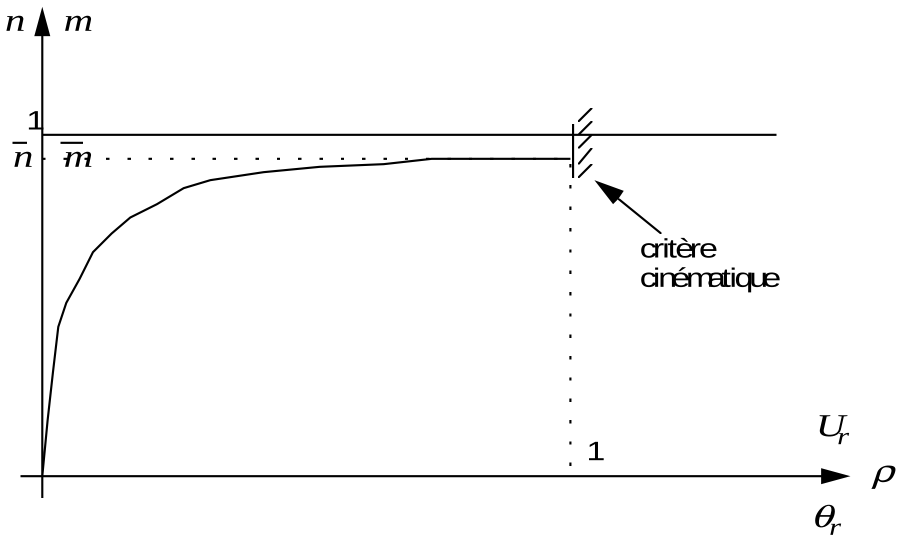
1) reduced forces: $n = \frac{N_x}{\overline{N}}$ et $m = \frac{M_y}{\overline{M}}$

2) reduced displacements: $U_r = \frac{U}{\overline{U}}$ et $\theta_r = \frac{\theta}{\overline{\theta}}$

[Figure 3.1-a] represents in adimensional form the one-way behavior. Analytically, it can be written (it is a choice):

$$U_r = h(n) \text{ ou } \theta_r = h(m)$$
$$\text{avec } h(x) = \frac{1}{\overline{d}} \frac{x^{a+1}}{1 - x^a}$$
$$\overline{d} = \frac{\overline{n}^{a+1}}{1 - \overline{n}^a}$$

a is the scalar parameter of nonlinearity. $\overline{n}$ and a are identified on the one-way tests. $\overline{n}$ who takes into account the variability of the tests generally takes the value 0.95.



$$n = N_x / \overline{N}$$
$$m = M_y / \overline{M}$$
$$U_r = U / \overline{U}$$
$$\theta_r = \theta / \overline{\theta}$$

Appear 3.1-a: behavior model of assembly

One notices that $h(\overline{n})=1$ or $h(\overline{m})=1$, i.e.: $U_r=1$ or $\theta_r=1$, or: $U=\overline{U}$ or $\theta=\overline{\theta}$.

The one-way kinematical criterion is thus checked for $n=\overline{n}$ or $m=\overline{m}$.

3.2 Behavior two-dimensional incremental

the coupling in extreme cases is defined by limiting surface:

$$\left(\frac{N_x}{\overline{N}}\right)^2+\left(\frac{M_y}{\overline{M}}\right)^2=1$$

The one-way behavior in reduced variables is described by the relation of [§3.1]:

$$\underline{d}=h(\underline{f})$$

where $\underline{d}$ is the vector reduced displacements $\begin{pmatrix}U_r\\ \theta_r\end{pmatrix}$

$\underline{f}$ is the vector forces reduced $\begin{pmatrix}n\\ m\end{pmatrix}$

In two-dimensional behavior, the isotropy is translated by a model with a scalar **local variable** p such as:

$$p=h(feq) \text{ en chargement}$$

where feq is the equivalent reduced force (scalar).

feq is defined such as:

$$\underline{F}=feq*\overline{\underline{F}}*$$

where $\underline{F}$ is the point running of loading $\begin{pmatrix} N_x \\ M_y \end{pmatrix}$

$\overline{\underline{F}}*$ is the limiting loading associated with $\underline{F}$ $\begin{pmatrix} \overline{N}_x^* \\ \overline{M}_y^* \end{pmatrix}$

the statement of feq results from the statement of limiting surface. The belonging of $\overline{\underline{F}}*$ on the limiting surface is written:

$$\left(\frac{\overline{N}_x^*}{\overline{N}}\right)^2+\left(\frac{\overline{M}_y^*}{\overline{M}}\right)^2=1$$

By the definition of feq, one can write:

$$\left(\frac{N_x}{feq\,\overline{N}}\right)^2+\left(\frac{M_y}{feq\,\overline{M}}\right)^2=1$$

i.e. according to the reduced forces n and m :

$$\left(\frac{n}{feq}\right)^2+\left(\frac{m}{feq}\right)^2=1$$

from where $$feq=\sqrt{n^2+m^2}$$

One defines then the surface of loading F, homothetic on limiting surface, by:

$$F: \quad feq-R(p)=0$$
$$\text{où } R(p)=h^{-1}(p)$$

For a formalism similar to that of plasticity with isotropic hardening [bib2], one obtains the behavior model continues expressed in reduced quantities:

$$\dot{\underline{d}} = \dot{p}\frac{\partial F}{\partial \underline{f}} = \dot{p}\frac{\underline{f}}{feq}$$

$$\dot{p} = 0 \quad \text{si} \quad feq - R(p) < 0$$

$$\dot{p} = h'(feq)\,\dot{f}\ eq \quad \text{si} \quad feq - R(p) = 0$$

Behavior model of the rigid type - plastic without elasticity is written finally:

$$\dot{\underline{D}} = \frac{\dot{p}}{feq}[\overline{D}][\overline{F}]^{-1}\ \underline{F}$$

$$\text{où} \quad \underline{D} = \begin{pmatrix} U \\ \theta \end{pmatrix} \quad et\ \underline{F} = \begin{pmatrix} N_x \\ M_y \end{pmatrix}$$

$$[\overline{D}] = \begin{bmatrix} \overline{U} & 0 \\ 0 & \overline{\theta} \end{bmatrix} \ et\ [\overline{F}] = \begin{bmatrix} \overline{N} & 0 \\ 0 & \overline{M} \end{bmatrix}$$

The behavior model incremental in reduced quantities is obtained by integration of the continuous relation between t (variables -) and $t+dt$ (variables +).

In loading, Δp check F = 0 with $t + dt$:

$$feq^{+} = R\ (p^{-} + \Delta p) \qquad \text{éq 2.2-1}$$

By introducing the behavior model,

$$\Delta\,\underline{d} = \Delta p\,\frac{\underline{f}^{+}}{feq^{+}} \qquad \text{éq 2.2-2}$$

one deduces the value from Δp ,

$$\Delta p = \|\Delta\underline{d}\ .\ \Delta\underline{d}\| = \sqrt{\Delta U_r^2 + \Delta\theta_r^2}$$

and one calculates the value from feq^{+} [éq 2.2-1]. The behavior model [éq 2.2-2] gives the reduced forces:

$$n^{+} = \frac{\Delta U_r}{\Delta p}\ R(p^{-} + \Delta p)$$

$$m^{+} = \frac{\Delta\theta_r}{\Delta p}\ R(p^{-} + \Delta p)$$

In unloading, Δp = 0 and one has by [éq 2.2-2]:

$$\Delta\,\underline{d} = 0$$

4 Establishment in *Code_Aster*

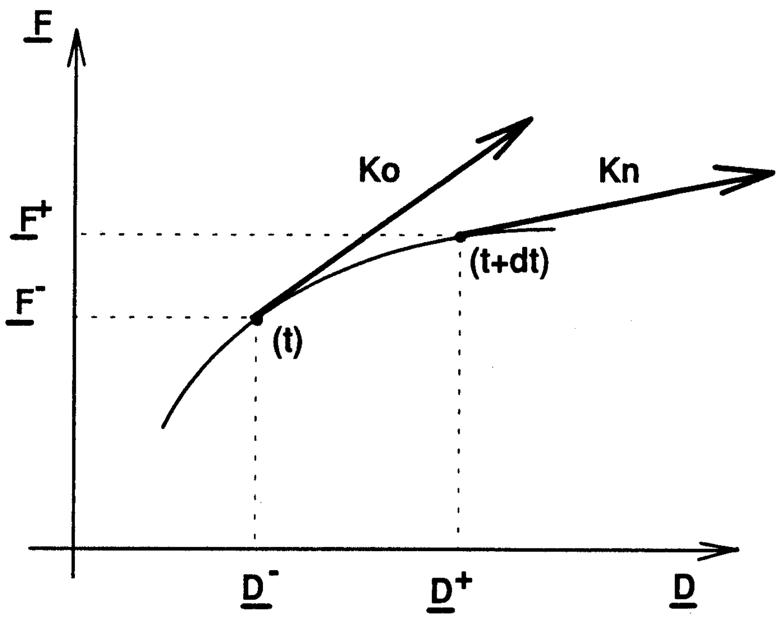
behavior model ASSE_CORN is assigned to discrete elements of modelization DIS_TR to 2 confused nodes. This relation is called by the operators of resolution of nonlinear problems STAT_NON_LINE [R5.03.01] or DYNA_NON_LINE [R5.05.05].

The local axes of these elements X, there, Z are defined as on [Figure 2-a].

The integration of this behavior model of the assemblies in operator STAT_NON_LINE of *Code_Aster* requires the formulation of the tangent operators K_o and K_n [bib3].

1) K_o is the tangent stiffness at the beginning of time step, time t.
2) K_n is the tangent stiffness at the end of time step, time $t+dt$.

The illustration of the operators K_o and K_n is given by [Figure 4-a].



Appear 4-a: definition of the operators K_o and K_n

4.1 Formulation in quantities reduced in loading

4.1.1 Operator K_{nr}

We saw with [§3.2] that the behavior model is written:

$$\underline{f}^{+}=\frac{\Delta \underline{d}}{\Delta p} R\left(p^{-}+\Delta p\right)$$
$$\text{avec } \Delta p=\|\Delta \underline{d}\ .\ \Delta \underline{d}\|=\sqrt{\Delta U_r^2+\Delta\theta_r^2}$$

The operator K_{nr} is defined by:

$$K_{nr}=\left[\frac{\partial f_i}{\partial d_j}\right]\ 1\leq i,\ j\leq 2$$

He is written:

$$K_{nr}=\frac{\Delta p[Id]-\{\Delta \underline{d}\}.<\frac{\partial \Delta p}{\partial \Delta d_j}>^{+}}{\Delta p^2} R\left(p^{+}\right)+\{\Delta \underline{d}\}.<\frac{\partial \Delta p}{\partial \Delta d_j}>^{+}\frac{R'\left(p^{+}\right)}{\Delta p}$$

The computation gives then:

$$<\frac{\partial \Delta p}{\partial \Delta d_j}>^{+}=\left(\frac{\Delta U_r}{\Delta p}\ ;\ \frac{\Delta\theta_r}{\Delta p}\right)$$
$$\{\Delta \underline{d}\}\ .<\frac{\partial \Delta p}{\partial \Delta d_j}>^{+}=\begin{bmatrix}\frac{\Delta U_r^2}{\Delta p} & \frac{\Delta U_r\ \Delta\theta_r}{\Delta p}\\ \frac{\Delta U_r\ \Delta\theta_r}{\Delta p} & \frac{\Delta\theta_r^2}{\Delta p}\end{bmatrix}$$

and with $\underline{a=1}$ (only case currently treated), one a: $h(x)=\frac{1}{\bar{d}}\ \frac{x^2}{1-x}$

$$R(p)=h^{-1}(p)=\frac{1}{2}\left(-\bar{d}\ p+\sqrt{\bar{d}^2\ p^2+4\ \bar{d}\ p}\right)$$
$$R'(p)=\frac{1}{h'[R(p)]}=\frac{\bar{d}\ [1-R(p)]^2}{R(p)[2-R(p)]}$$

4.1.2 Operator K_{or}

For the elastoplastic behaviors, the operator K_o with $t=0$ is equal to the stiffness of elastic structure. In our case, the tangent initial behavior is rigid. The operator K_{or} is defined then by the transition in the limit when p tends towards 0 of the operator K_{nr}. One obtains:

$$R'(p) \underset{p\to 0}{=} \frac{R(p)}{p}$$
$$\text{d'où } K_{or} \underset{p\to 0}{=} \frac{R(p)}{p}[Id]$$

However $R(p)<1 \ \forall p$ and if one supposes that the user gives, for the first step of loading, of the values such as $\Delta p>10^{-4}$, one can retain in practice:

$$K_{or_{t=0}}=\begin{bmatrix}10^4 & 0\\ 0 & 10^4\end{bmatrix}$$

This value by default is modifiable (cf §5).
These remarks are illustrated by [Figure 4.1.2-a].

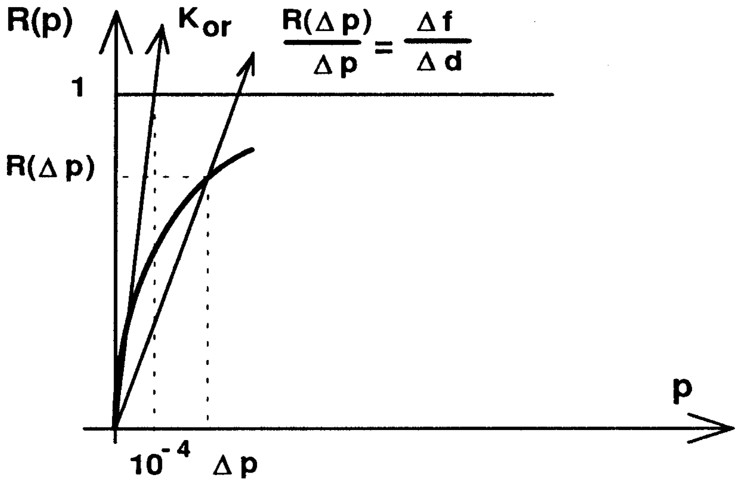


Figure 4.1.2-a : operator KB with T = 0

At time t running, the operator K_{or} is equal to the operator K_{nr} of the preceding step defined by [§4.1.1].

4.2 Formulation in quantities reduced in unloading

to avoid numerical problems, one describes the behavior (rigid) in unloading by:

$$K_{or}=K_{nr}=K_{or_{t=0}}$$

4.3 Tangent operators K_n and K_o

the tangent operator K_n is written, from the tangent operator reduced calculated to the §4.1

$$K_n=\left[\frac{\partial F_i}{\partial D_j}\right]\ 1\leq i,\ j\leq 6$$

with

$$\frac{\partial F_1}{\partial D_1}=\frac{\partial N_x}{\partial U}=\frac{\partial n}{\partial U_r}\times\frac{\overline{N}}{\overline{U}}$$

$$\frac{\partial F_1}{\partial D_5}=\frac{\partial N_x}{\partial \theta}=\frac{\partial n}{\partial \theta_r}\times\frac{\overline{N}}{\overline{\theta}}$$

$$\frac{\partial F_5}{\partial D_1}=\frac{\partial M_y}{\partial U}=\frac{\partial m}{\partial U_r}\times\frac{\overline{M}}{\overline{U}}$$

$$\frac{\partial F_5}{\partial D_5}=\frac{\partial M_y}{\partial \theta}=\frac{\partial m}{\partial \theta_r}\times\frac{\overline{M}}{\overline{\theta}}$$

$$\frac{\partial F_2}{\partial D_2}=K_y$$

$$\frac{\partial F_3}{\partial D_3}=K_z$$

$$\frac{\partial F_4}{\partial D_4}=KR_x$$

$$\frac{\partial F_6}{\partial D_6}=KR_z$$

the other values are null.
The tangent operator K_o, to t = 0, is written:

$$K_o=\begin{bmatrix} 10^4\frac{\overline{N}}{\overline{U}} & & & & O & \\ & K_y & & & & \\ & & K_z & & & \\ & & & K_{Rx} & & \\ O & & & & 10^4\frac{\overline{M}}{\overline{\theta}} & \\ & & & & & K_{Rz} \end{bmatrix}$$

4.4 Digital processing of connection between the mechanisms of the model of assembly

During the resolution of each step of loading by the iterative method of Newton, one must calculate with each iteration the tangent with the curve of equilibrium force-displacement of the constitutive law. The problem is that connection between the mechanisms of the model of assembly, on the constitutive law, has a positive concavity (cf [Figure 2-b]) which makes convergence problematic when, during a step of loading, one passes from one mechanism to the other.

In the subroutine `TE0041` which calculates, for each increment of load, the elementary matrix of tangencial stiffness of a discrete finite element with 2 nodes having of the degrees of freedom in translation and rotation, it proved to be necessary to converge, to calculate a secant stiffness directed of the initial state of force and null displacement towards the state, at the end of the step of loading, constituted by the imposed force and displacement corresponding on the curve of equilibrium of the constitutive law. It was necessary for that, which was unusual on the level of this option, to know the internal number of iteration of the numerical process calculating the step of loading, then to consider the force imposed on the element at the end of this step.

Indeed, if one notes F^+ the force imposed on the level of an element (a priori unknown since only the assembled forces are known), U^+ displacement corresponding on the curve of equilibrium, and for the iteration i, the respective values $U(i), F(i), K s(i)$ of displacement, the force and the secant matrix – serving as tangent matrix – calculated at the end of the iteration, one knows only as starter above mentioned subroutine U^i , and the values at the beginning of the step of load $F(0)$ and $U(0)$, because one did not store the values with the preceding iteration $i-1$. In the statement of the residue calculated at the end of the iteration $i-1$: $F^+ - F(i-1) = K_s(i-1) \,.\, (U(i) - U(i-1))$, one thus does not know any more but $U(i)$ with the iteration i, except in the typical case $i=1$ where one a:

$$F^+ - F(0) = K_s(0) \,.\, (U(1) - U(0))$$

F^+ is there the only unknown value at the beginning and results from the others. One from of also deduced displacement U^+ at the end of the step according to the relation of equilibrium: $\dot{p} \,.\, [n,\ m] = R(p) \,.\, \left[\dot{U}_r, \dot{\Theta}_r\right]$, from where secant stiffness $K_s(1) = F^+ / U^+$.

The problem is that in this first iteration, imposed $U(1)$ displacement is different from final displacement to calculating U^+ in equilibrium with F^+ from now on known (with the test of equilibrium close to the step of preceding loading). The force calculated at the end of this iteration $F(1)$ must thus be also different from F^+ and such as $F(1) = K_s(1) \,.\, U(1)$ so that starting from the couple $U(1)$ and $F(1)$, one points with the secant $K_s(1)$ on the couple U^+ and F^+ . One thus obtains at the beginning of iteration 2 a displacement $U(2)$ very close to U^+ and one can then calculate by the relation of equilibrium $F(2)$ very near also to F^+ as well as the secant stiffness $K_s(2) = F(2) / U(2)$.

If one converged exactly with the preceding step of load, 2 internal iterations are enough to converge exactly, if not one needs some additional iterations to satisfy the test with equilibrium on the residue.

The method known as of “directed secant” is schematized on [Figure 4.3-a] where one has the following correspondences:

$$U^i = U(i)$$
$$K_t(U^i) = K_s(i)$$

for a constitutive law $LC(U^i) = F(i)$.

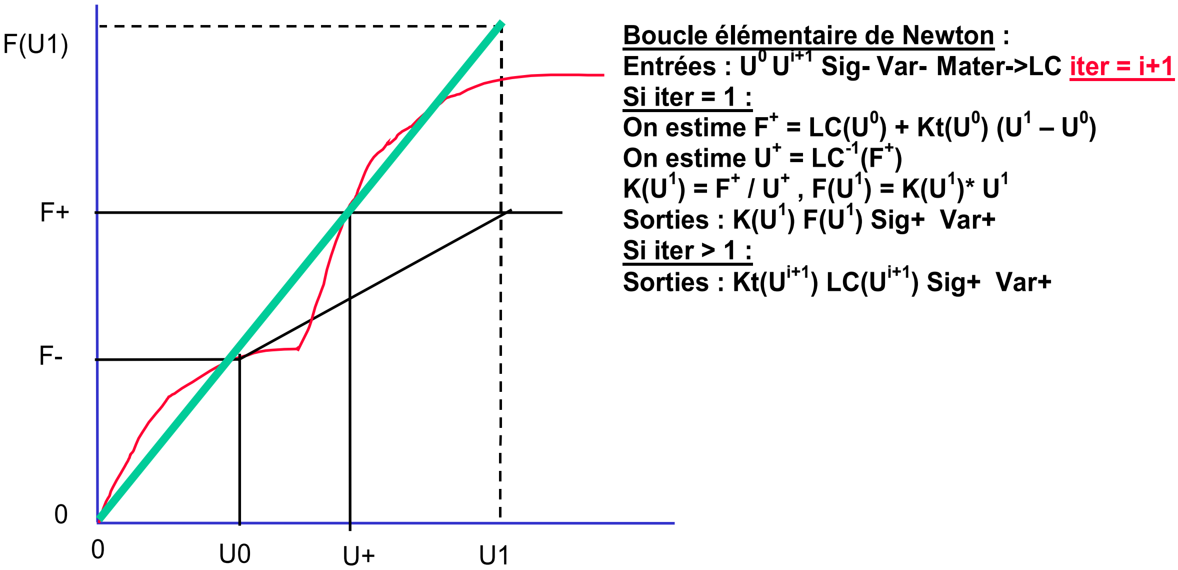


Figure 4.3-a : method of directed secant

One thus sees now why it was necessary in the option calculated by the above mentioned subroutine to know the number of iteration interns i in order to distinguish the typical case $i=1$.
Variables and Variable parameters of

4.5 the constitutive law of the model

the constitutive law comprises 7 local variables per point of computation:

1) $V1$ is displacement reduces equivalent *p* maximum reached out of mechanism 1,
2) $V2$ is displacement reduces equivalent *p* maximum reached out of mechanism 2,
3) $V3$ is an indicator which is worth 1 or 2 according to whether one is respectively on the limiting surface of mechanism 1 or 2, and 0 if one is under this limiting surface (after discharge for example),
4) $V4$ and $V5$ is respectively the maximum force and the moment reached out of mechanism 2 before discharge,
5) $V6$ and $V7$ is respectively displacement and the rotation origins of mechanism 1, which can be non-zero when after discharge out of mechanism 2, the loading changes sign to pass by again out of mechanism 1.

4.6 parameters of the model

the parameters of the model of behavior entered like data under key word `ASSE_CORN` of the command `DEFI_MATERIAU` [U4.43.01]:

- `NU_1` : one enters behind this key word the value of the parameter $\overline{N}_1$ of mechanism 1,
- `MU_1` : one enters behind this key word the value of the parameter $\overline{M}_1$ of mechanism 1,
- `DXU_1` : one enters behind this key word the value of the parameter $\overline{U}_1$ of mechanism 1,

- `DRYU_1` : one enters behind this key word the value of the parameter $\overline{\theta}_1$ of mechanism 1,
- `C_1` : one enters behind this key word the value common to the parameters $\overline{n}$ and $\overline{m}$ of mechanism 1,
- `NU_2` : one enters behind this key word the value of the parameter $\overline{N}_2$ of mechanism 2,
- `MU_2` : one enters behind this key word the value of the parameter $\overline{M}_2$ of mechanism 2,
- `DXU_2` : one enters behind this key word the value of the parameter $\overline{U}_2$ of mechanism 2,
- `DRYU_2` : one enters behind this key word the value of the parameter $\overline{\theta}_2$ of mechanism 2,
- `C_2` : one enters behind this key word the value common to the parameters $\overline{n}$ and $\overline{m}$ of mechanism 2,
- `KY, KZ, KRX, KRZ` respectively take the values of the characteristics of linear behavior in the local *directions* ",
- `RP_0` : one enters behind this key word a possible value of K_{or} (10 ^{4} by defaults).

Note: The parameter a is not accessible to the user. It is fixed at the value $a=1$.

5 Bibliographical references

1) P. PENSERINI: "Modelization of the assemblies bolted in the webmasts" Notes EDF/R & D HM-77/93/287
2) P. PENSERINI: "Characterization and modelization of the behavior of connections structure metal-foundation" Thesis of doctorate of the University Paris 6,1991
3) J.P. LEFEBVRE, P. MIALON: "Quasi-static nonlinear Algorithm of *the Code_Aster* " Notes EDF/R & D HI-75/7832

6 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
6	J.M.PROIX-R&D/AMA initial Text	8,4 G. DEVESA
8.4	*, J.L. FLEJOU, P. PENSERINI EDF-R&D/* AMA EDF/LME	