Contact in great slidings with X-FEM

Summarized:

This document presents a new approach to deal with the problems of contact in great slidings with the eXtended Finite Element Method (X-FEM) [R7.02.12]. One considers the continuous hybrid formulation of problems of contact between solids [bib2] and the strategy of resolution is similar to that already implemented in Code_Aster for the frame conventional finite elements [bib3]. The processing of contact-friction in small slidings is the object of the document [R5.03.54]. A new type of mixed element of contact is introduced, specific with the frame X-FEM. The geometrical procedure of reactualization and the algorithm of pairing, new elements for the X-FEM, are presented in detail in this document as well as the matric terms resulting from the linearization of the weak formulation of the problem.

The approach is implemented in *Code_Aster* in 2D and 3D, and treats at the same time interfaces completely cut by a crack as well as interfaces with crack tip. It is usable with command `STAT_NON_LINE` [U4.51.03]. The friction of the Coulomb type is taken into account.

Contents

1 Introduction

During the numerical implementation of the X-FEM [bib1, biberon5], the problem of the contact rubbing was treated by the continuous method, called also “continuous hybrid formulation” in the documentation of Code_Aster, under the assumption of the small disturbances (HP). The mathematical formulation of the continuous method [bib2] was thus adapted to the XFEM and one points out here the principal characteristics of the numerical implementation in HP [R5.03.54]:

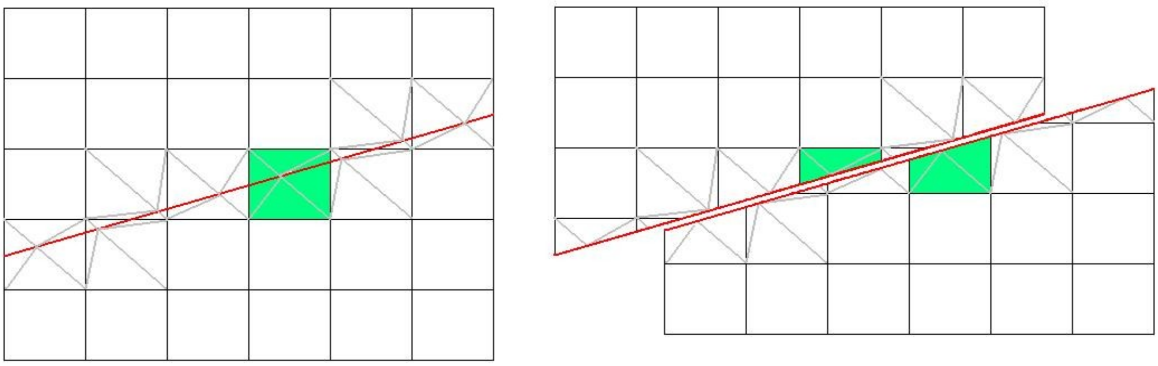
- the lips of crack are treated like only one geometrical surface of discontinuity;
- the geometrical reactualization of contact surface and master-slave pairing are not carried out (the notions of surface slave and main surface do not have a meaning here);
- the jump of displacement is expressed according to the discontinuous degrees of freedom of enrichment introduced by X-FEM.

The computation contributions of contact thus is carried out to the level of the finite elements crossed by crack and one does not pass by mixed elements of contact, supported by the meshes late ones, as it is the case for the classical approach of the continuous method.
In the following chapter one also recalls the weak formulation of the problem of contact with XFEM, solved by the continuous method.

The processing of the contact at the time of the modelization of the great slidings with X-FEM required a new reflection for its implementation. Compared to the case HP (Figure 1a), the main difficulty was to make communicate the piece slave of an element fissured with the main piece of another fissured element (Figure 1b). Indeed, in this case two contact surfaces must be declared and from the points of contact located inside a mesh cut by crack will find themselves in opposite with points belonging to another mesh, so cut by crack.

The difficulty is due to the inexistence of nodes on the interface of contact (the crack generates only points of intersection with edges of meshes). The meshes late ones of contact (segments slave and Master in the case 2D) generated from these nodes for meshes FEM cannot be generated more here as it was the case for the classical formulation [bib3].

The found solution was the creation of news meshes late, of a higher order. These meshes are formed by the degrees of freedom of the mesh slave containing the point of contact and degrees of freedom of the mesh main containing its project. For the case illustrated on the Figure 1b, instead of having meshes late SEG2-SEG2, as it was the case for the classical formulation, in X-FEM great slidings one has meshes late QUAD4-QUAD4. The details on creation and the characteristic of such meshes late, which represent the supports of the new element of contact, will be presented in the third chapter of this document.



a) b)

Figure 1. Mesh X-FEM. a) Processing HP; b) Processing great slidings.

Two other big steps must then be realized before the computation of the contributions of contact: the geometrical reactualization of contact surfaces and the pairing of the points of contact. Their description is the object of the first part of the 4th chapter, named “Strategy of resolution”. In the second part of this chapter one linearizes the mixed variational formulation to extract the discrete formulation from the elementary terms of contact.

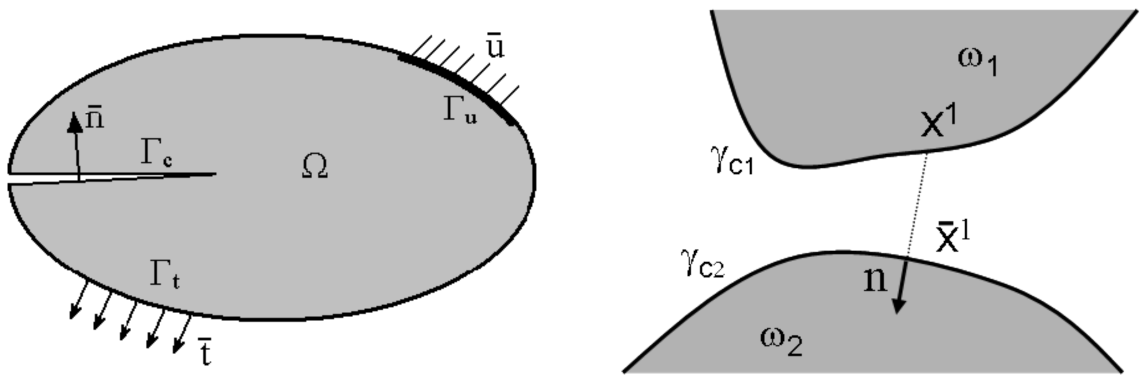
Implementation the data-processing of the approach great slidings with X-FEM is described in the document [D9.05.06], for the actual position of the numerical implementation.

2 Problem of contact with X-FEM

the problem of contact treated in this document relates to cracks modelled by X-FEM: one thus considers only one field Ω for the field displacements. On part of his border Γ_u, one considers a set of conditions of Dirichlet and on another part Γ_t, one considers a set of conditions of Neumann (Figure 2.a). The forces of contact will appear on noted internal discontinuity Γ_c.
One breaks up the density of force of contact r into a normal part λ, which indicates the normal pressure, and another tangential r_τ:

$$r=\lambda n+r_\tau \quad (1)$$

where n the vector of the norm entering represents to ω_2.



a) b)

Figure 2. a) Notation of the problem of contact; b) Evaluating of clearance enters the point of contact and its project.

For a problem of contact, illustrated on Figure 2, let us consider the field u pertaining to all V_0 the kinematically admissible fields of displacements:

$$V_0=\left\{v\in H^1, v \text{ discontinu à travers } \Gamma_c, v=0 \text{ sur } \Gamma_u\right\}.$$

By considering the notations introduced previously, the strong formulation of the local equations of equilibrium defined on the initial configuration, supplemented initial conditions and boundary conditions of the problem considered, is:

$$\begin{array}{ll} \operatorname{div}\Pi+f=0 & \text{dans } \Omega, \\ u=\bar{u} & \text{sur } \Gamma_u \\ \Pi\cdot N=t & \text{sur } \Gamma_t \\ \Pi\cdot N=r & \text{sur } \Gamma_c \end{array} \quad (2)$$

where Π the first tensor of the Piola-Kirchhoff stresses represents, f and t are the densities of the internal forces and surface, respectively, and $\bar{u}$ represents the conditions of Dirichlet.
Let us consider a zoom on the deformed configuration (Figure 2.b) with ω_1 the part slave and ω_2 the part Master and the corresponding borders, γ_{c1} and γ_{c2}, potentially in contact. The principal point for the problem of contact is then the evaluating of clearance between a point of contact x^1 considered on the border slave and its project $\bar{x}^1$:

$$d_n = \left(x^1 - \bar{x}^1\right) \cdot n \quad (3)$$

the project of the point of contact is calculated according to the principle of the minimal distance, representative thus the orthogonal projection of the point of contact x^1 on the border Master γ_{c2} .

For the contact, the models of Signorini are written then:

$$d_n \leq 0 \quad \lambda \leq 0 \ , \ \lambda d_n = 0 \quad (4)$$

to make the equations (4) ready for the weak formulation, one transforms them into only one strictly equivalent equation according to [3] given by the formula (5) :

$$\lambda - \chi(g_n) g_n = 0 \quad (5)$$

In (5),

- χ is the indicating function of $\mathbb{R}^-$ ($\chi = 1$ if contact and $\chi = 0$ if not of contact),
- $g_n = \lambda - \rho_n d_n$ is the multiplier of increased contact, with ρ_n a strictly positive reality.

An alternative consists in adopting a strategy of penalization, in which case the models (4) are written:

$$\lambda + \chi(\lambda - \rho_n d_n) \kappa_n d_n = 0 \quad (6)$$

In (6),

- χ is the indicating function of $\mathbb{R}^-$ ($\chi = 1$ if contact and $\chi = 0$ if not of contact),
- κ_n is a large coefficient of penalization in front of the stiffness of structure.

For the phenomena of friction, one uses the models of Coulomb which are written as follows:

$$\begin{cases} \|r_\tau\| \leq \mu |\lambda| \\ \|r_\tau\| < \mu |\lambda| \Rightarrow v_\tau = 0 \\ \|r_\tau\| = \mu |\lambda| \Rightarrow \exists \alpha \geq 0 \ ; \ v_\tau = -\alpha \cdot r_\tau \end{cases} \quad (7)$$

In (7),

- μ is the coefficient of kinetic friction of Coulomb,
- v_τ is the tangent relative velocity.

As for the models of contact, one can write the friction law (7) as follows in an equivalent way:

$$\begin{gathered} r_\tau = \mu \lambda \Lambda \\ \Lambda - P_{B(0,1)}(g_\tau) = 0 \end{gathered} \quad (8)$$

In (8),

- Λ is the semi-multiplier of friction,
- $P_{B(0,1)}$ is projection on the ball unit,
- $g_\tau = \Lambda + \rho_\tau v_\tau$ is the semi-multiplier (vectorial) of increased friction,
- ρ_τ is a strictly positive reality.

The field of sign is also introduced $S_f = I_{B(0,1)}(g_\tau)$. We have $S_f = 1$ for an adherent point, and $S_f = 0$ a slipping point.

One can also choose a method penalized to write this model:

$$\Lambda - P_{B(0,1)}(\kappa_\tau v_\tau) = 0 \quad (9)$$

In (9),

- κ_τ is the coefficient of penalization in friction.

In 3D, Λ is a vector of the tangent plane on the surface of crack and it is necessary to define a base covariante tangent plane in which it could be expressed as follows:

$$\Lambda=\Lambda^1\tau_1+\Lambda^2\tau_2 \quad (10)$$

There exist an infinity of couples (τ_1,τ_2) being able to form this base.

For the method of contact X-FEM HP, one uses the gradients of the level sets to define this base (see the semi-multiplier part of friction of chapter 4 of [bib1]), which ensures the continuity of the base (τ_1,τ_2) of one node the other of the mesh.

In great slidings one cannot any more define it thus since it must be reactualized with each geometrical iteration (object of chapter 4 of this document) and depends then on the current geometry of the facets of contact Masters.

For the method of contact FEM in great slidings, the selected base is that directed by the current geometry of meshes surface of contact.

One makes in the same way for X-FEM great slidings by replacing the notion of mesh surface by that of a facet of contact. The problem is that the continuity of the base (τ_1,τ_2) is not ensured any more as for the contact X-FEM HP.

For the contact X-FEM in great slidings, one thus decides to reorientate the tangents by means of a fixed vector in the global database, so as to on the way reduce the discontinuity of (τ_1,τ_2) the field of one element to the other.

One first of all calculates the normal vector n which is single, one projects then the fixed vector e_1 of the global database in the plane of norm n to build τ_1 , one builds then τ_2 who is the cross product of n and τ_1 :

$$\tau_1=\frac{P_\tau.e_1}{\|P_\tau.e_1\|};\tau_2=n\wedge\tau_1 \quad (11)$$

In (11),

- n is the directing vector of the tangent plane,
- $P_\tau=(I_d-n\otimes n)$ is the operator of projection on the tangent level with n ,
- $e_1=\begin{pmatrix}1\\0\\0\end{pmatrix}$ is the direction x in the total reference,
- (τ_1,τ_2) is the base chosen to write the equation (8)

Attention, the case $\|P_\tau.e_1\|=0$ is not treated. It causes an error. It is necessary thus to pay attention so that the plane of contact is not directed perpendicular to e_1 .

Spaces of the unknowns of contact are the following:

$$H=\{\lambda\in H^{-1/2}(\Gamma_c),\lambda\le 0 \text{ sur } \Gamma_c\}$$
$$\mathbf{H}=\{r_\tau\in H^{-1/2}(\Gamma_c),\|r_\tau\|\le\mu\lambda_n \text{ sur } \Gamma_c\},$$

the weak formulation at three fields exit of [2] is written then:

To find $(u,\lambda,\Lambda)\in V_0\times H\times\mathbf{H}$ $\forall(u^*,\lambda^*,\Lambda^*)\in V_0\times H\times\mathbf{H}$ such as (12-14) are checked:

Balance equation:

$$\int_M tr(\Pi(\nabla_p(u))(\nabla_p(u^*)))\,dM-\int_{\Gamma_c}\chi(g_n)g_n n\cdot[u^*]\,d\Gamma$$
$$-\int_{\Gamma_c}\mu\chi(g_n)\lambda P_{B(0,1)}(g_\tau)(I_d-n\otimes n)[u^*]\,d\Gamma=L_{meca}(u^*)$$

(12)

Model of contact:

$$\int_{\Gamma_c} -\frac{1}{\rho_n}\left(\lambda-\chi(g_n)g_n\right)\lambda^* d\Gamma=0 \tag{13}$$

Friction law:

$$\int_{\Gamma_c} \frac{\mu\lambda\chi(g_n)}{\rho_\tau}\left(\Lambda-P_{B(0,1)}(g_\tau)\right)\Lambda^* d\Gamma+\int_{\Gamma_c}\left(1-\chi(g_n)\right)\Lambda\Lambda^* d\Gamma=0 \tag{14}$$

Where:

- $tr(\cdot)$ is the operator traces of a tensor,
- Π is the first stress tensor of Piola-Kirchoff,
- $\nabla_p(u^*)$ is the gradient of u^* compared to the coordinates p ,
- $\otimes$ is the operator of the tensor product,
- Id is the second tensor identity,
- $L_{meca}(u^*)$ is the virtual wor of the external forces.

In the frame of a penalized formulation, contact pressures and the shearing stresses due to friction are explicit according to displacement. However and as explained in documentation [R7.02.12], it is necessary to utilize λ in the balance equation for a rigorous satisfaction of condition LBB. On the other hand, the equation of friction does not intervene in the resolution, it has only one role of postprocessing. The three preceding equations (12-14) become then:
Balance equation:

$$\begin{aligned}&\int_\Omega tr(\Pi(\nabla_p(u))(\nabla_p(u^*)))d\Omega-\int_{\Gamma_c}\chi\lambda n\cdot[u^*]d\Gamma\\&-\int_{\Gamma_c}\mu\chi\lambda P_{B(0,1)}(\kappa_\tau v_\tau)(I_d-n\otimes n)[u^*]d\Gamma=L_{meca}(u^*)\end{aligned} \tag{15}$$

Model of contact:

$$\int_{\Gamma_c} -\frac{1}{\kappa_n}\left(\lambda+\chi\kappa_n d_n\right)\lambda^* d\Gamma=0 \tag{16}$$

Friction law:

$$\int_{\Gamma_c}\chi\left(\Lambda-P_{B(0,1)}(\kappa_\tau v_\tau)\right)\Lambda^* d\Gamma+\int_{\Gamma_c}(1-\chi)\Lambda\Lambda^* d\Gamma=0 \tag{17}$$

the linearization as well as the discretization of this formulation are presented in the fourth chapter for the hybrid elements of contact developed in order to implement in Code_Aster the approach great slidings with XFEM. Previously, we will introduce into the chapter according to the new hybrid elements of contact.

3 Hybrid element X-FEM of contact for the approach great slidings

the notion of element hybrid of contact, also called mixed element of contact, was for the first time introduced into Code_Aster for the formulation of hybrid contact continuous [bib3]. It is about a couple formed by a point of integration of contact, located on surface slave, and of the main element containing the project of the point of contact on main surface. The support of such an element, which has geometrical degrees of freedom and of degrees of freedom of contact (multiplying of contact-friction) is named late mesh (denomination used in Code_Aster). A late mesh is not part of the mesh of the starting model. It is formed by association of the mesh slave containing the point of integration of contact and of the mesh main nearest containing its project. Taking into account the definition of the element of contact, recalled above, for each point of integration of contact a new element of contact will be generated. On Figure 3, one shows an example 2D typical of training of a hybrid element of contact.

Following pairing, one finds for the point of contact PC which belongs to the mesh slave $N_1 N_2$, the main mesh $N_3 N_4$. The projection of the point of contact on this mesh will give the project what is called PR. The late mesh thus created will be of type SEG2-SEG2 and one will name it $N_1 N_2 N_3 N_4$ for this example. The degrees of freedom of displacement will be stored in these 4 nodes, which are also nodes constitutive for meshes the QUAD4 of the model. The degrees of freedom of contact-friction are stored, by convention, only with the nodes of the mesh slave $N_1 N_2$.

For the approach great slidings with X-FEM, the approach described above cannot be applied any more because the interface of contact, resulting from a crack X-FEM, does not have any more nodes but only points of intersection which cannot store the degrees of freedom of displacements or contact-friction.

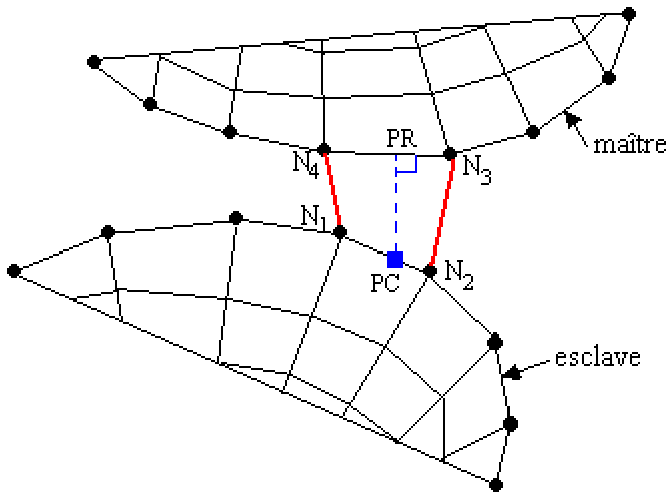


Figure 3. Illustration of the creation of a classical late mesh for a model 2D.

Since it is not possible to define meshes on the potential border in contact (segments for the modelization 2D or meshes surface for that 3D), the found solution was to use meshes crossed by crack to create the meshes late ones of a new type. One makes use of it then to define the hybrid element of contact X-FEM great slidings by considering the degrees of freedom of displacement and contact-friction as well as fields of numerical integration to compute: the contributions of rubbing contact. In addition for the method of contact X-FEM, the choice was to make carry the degrees of freedom of contact by the mesh slave, as for the method of classical contact FEM.

3.1 Hybrid element of contact X-FEM

the degrees of freedom of contact are introduced only with the nodes carrying the geometrical degrees of freedom already, in accordance with the algorithm of satisfaction of condition LBB suggested in [bib9]. Thus on Figure 4, defining mesh late (it is considered that the mesh slave is that located in lower part and the master mesh that above), the mesh slave is a QUAD4 ($N_1 N_2 N_3 N_4$) of which all the nodes store at the same time the degrees of freedom of contact and the geometrical degrees of freedom. The master mesh is also a QUAD4 ($N_9 N_{10} N_{11} N_{12}$): its nodes store only geometrical degrees of freedom.

It is then a question of pairing two meshes crossed by crack, that slave containing the point of contact PC (on the segment $P_1^e P_2^e$) and that main containing the project of the point of contact PR (on the segment $P_1^m P_2^m$). For the hybrid element of contact shown on Figure 4, the segment of contact is $P_1^e P_2^e$, formed starting from the 2 points of intersection of the lip slave. The area of reference for the shape functions is illustrated on the right of the image.

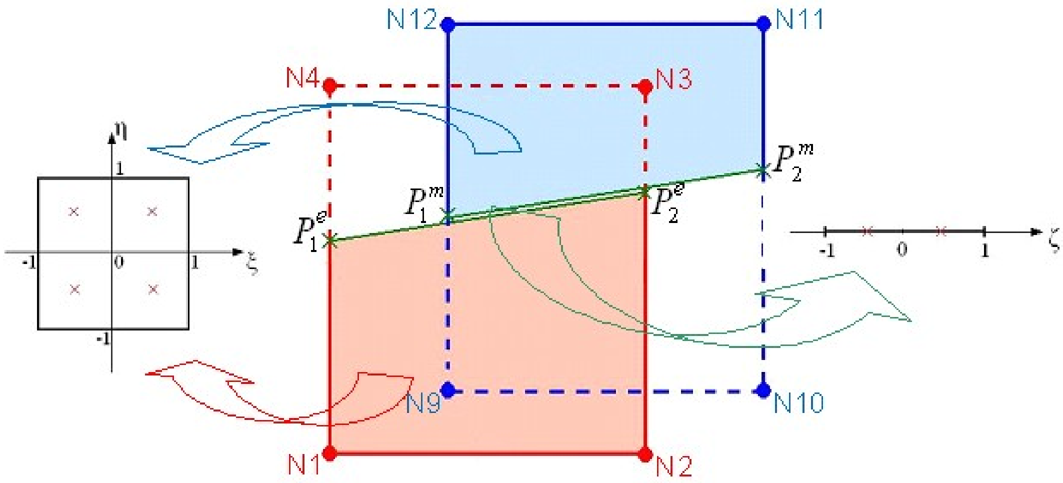


Figure 4. Hybrid example of element of contact great slidings X-FEM.

Being given the characteristic of the method X-FEMs, which is to consider the geometrical degrees of freedom of two types, classical a_i and nouveau riches b_i (for more details on the characteristics of the elements X-FEM one can refer to [bib1]), it results a hybrid element from it X-FEM of the type QUAD4-QUAD4 of which the degrees of freedom for each node are presented in Table 2.

Node	*Degree of freedom*													
N1	DX	DY	H1X	H1Y	H2X	H2Y	H3X	H3Y	H4X	H4Y	E1X	E1Y	λ	Λ
N2	DX	DY	H1X	H1Y	H2X	H2Y	H3X	H3Y	H4X	H4Y	E1X	E1Y	λ	Λ
N3	DX	DY	H1X	H1Y	H2X	H2Y	H3X	H3Y	H4X	H4Y	E1X	E1Y	λ	Λ
N4	DX	DY	H1X	H1Y	H2X	H2Y	H3X	H3Y	H4X	H4Y	E1X	E1Y	λ	Λ
N9	DX	DY	H1X	H1Y	H2X	H2Y	H3X	H3Y	H4X	H4Y	E1X	E1Y		
N10	DX	DY	H1X	H1Y	H2X	H2Y	H3X	H3Y	H4X	H4Y	E1X	E1Y		
N11	DX	DY	H1X	H1Y	H2X	H2Y	H3X	H3Y	H4X	H4Y	E1X	E1Y		
N12	DX	DY	H1X	H1Y	H2X	H2Y	H3X	H3Y	H4X	H4Y	E1X	E1Y		

Table 2. The table of the degrees of freedom for a hybrid element of contact, QUAD4-QUAD4.

One is interested now in the approximation of displacements for the points located on the lips of crack. For a point located on the segment of contact slave, therefore pertaining to the part full of the mesh slave, this approximation is written:

$$u^e(x)=\sum_{i=1}^{nnes} a_i \phi_i^e(x)+\sum_{i=1}^{nnes}\sum_{j=1}^{nfhe} He_i^{j,e} b_i^j \phi_i^e(x)+\sum_{i=1}^{nnes} He_i^{1,e}\sqrt{r_e} c_i^1 \phi_i^e(x) \ , \tag{18}$$

Where:

- $nnes$ indicates the number of nodes slaves tops,
- $nfhe$ indicates the number of degrees of freedom Heaviside present in the element slave (1 in the classical case, 4 at the most in meshes the muti-Heaviside),
- a_i and b_i c_i^1 is respectively the classical degrees of freedom, nouveau riches Heaviside and Ace-tip (the first corresponding to non-zero shape functions on the lip of crack),

- $He_i^{j,e}$ corresponds to the value of the function Heaviside side slave, associated with the node i for the Heaviside degree of freedom j. Let us note that in the classical case (with $nfhe=1$), one has $\forall i \in [1, nnes],\ He_i^{1,e} = -1$;
- $r_e = \sum_{i=1}^{nnes} |lst_i| \phi_i^e(x)$ the distance from the point slave to the crack tip,
- the formula $\sum_{i=1}^{nnes} \sqrt{r_e}\, c_i^1\, \phi_i^e(x)$ is present only if the element slave has degrees of freedom Ace Tip i.e the element slave is of type Heaviside Ace-tip, (see § 3.2 for element types considered), formula
- ϕ_i^e the shape functions of the element relative (quadrilateral with 4 nodes for the example set on Figure 4 where one shows on the left image the area of reference).

Same way, for a point located on the master mesh, one will have: formulate

$$u^m(x) = \sum_{i=1}^{nnm} a_i \phi_i^m(x) + \sum_{i=1}^{nnm} \sum_{j=1}^{nfhm} He_i^{j,m} b_i^j \phi_i^m(x) + \sum_{i=1}^{nnm} He_i^{1,m} \sqrt{r_m}\, c_i^1 \phi_i^m(x) \quad (19)$$

Where

: formula

- nnm the number of main nodes, formula
- $nfhm$ the number of degrees of freedom Heaviside present in the main element (1 in the classical case, 4 at the most in meshes the muti-Heaviside). formula
- $He_i^{j,m}$ to the value of the function Heaviside main side, associated with the node formulates i the Heaviside degree of freedom formulates j that in the classical case (with formula $nfhm=1$ one has formula $\forall i \in [1, nnm],\ He_i^{1,m} = +1$
- $r_m = \sum_{i=1}^{nnm} |lst_i| \phi_i^m(x)$ the distance from the main point to the crack tip,
- the formula term $\sum_{i=1}^{nnes} \sqrt{r_m}\, c_i^1\, \phi_i^e(x)$ is present only if the main element has degrees of freedom Ace Tip i.e the main element is of type Heaviside Ace-tip (see § 3.2 for element types considered), formula
- ϕ_i^m the shape functions of the element relative. By means of

(18) and (19), one can write the discretized relation of clearance between the point of contact (formula PC project (formula PR

$$d_n = \left[\sum_{i=1}^{nnes} (a_i + \sum_{j=1}^{nfhe} (He_i^{j,e} b_i) + He_i^{1,e} \sqrt{r_e} c_i^1) \phi_i^e(x_{CP}) - \sum_{i=1}^{nnm} (a_i + \sum_{j=1}^{nfhm} (He_i^{j,m} b_i) + He_i^{1,m} \sqrt{r_m} c_i^1) \phi_i^m(x_{PR}) \right] \cdot n_{PR}$$

the approximation

of the unknowns of contact utilizes them (shape functions ϕ_i^e of the element relative slave trained by the nodes $nnes$ slaves tops), so that: , (21

$$\lambda(x) = \sum_{k=1}^{nnes} \lambda_k \phi_k(x) \ \text{(a)} \quad \Lambda(x) = \sum_{k=1}^{nnes} \Lambda_k \phi_k(x)$$

) In other words

, the shape functions of contact are the same ones as those of displacements in the element slave. Another

innovation is the possibility of taking into account crack tips with contact in great slidings: therefore one represented in table 2 besides the geometrical degrees of freedom classical (D) and Heaviside (H1), the degrees of freedom ace-tips (E1 only because it is only whose singular function is non-zero on discontinuity and who intervenes for the contact). Nevertheless these degrees of freedom ace-tips are optional, whether it is for the main element or the element slave. Thus in the table, the degrees of freedom in blue appear only if the element slave is Heaviside-ace-tip, and the degrees of freedom in

green appear only if the main element is Heaviside-ace-tip. It is then possible to have 4 element types late different for the same types of meshes: Heaviside
•– Heaviside, Heaviside
•– Heaviside Ace-tip, Heaviside
•Ace-tip – Heaviside, Heaviside
•Ace-tip – Heaviside Ace-tip, On

the element containing the point (Ace-tip), the interface does not undergo great slidings, It is thus not considered to be useful to consider a D-pairing for this mesh. One thus does not take counts the following elements of them: Heaviside
•Ace-tip – Ace-tip, Ace
•- tip – Heaviside Ace-tip, Ace
•- tip – Ace-tip. The integration

of the terms of contact on the element Ace-tip will be treated same way as in small slidings. The taking

into account of the multi-Heaviside elements is carried out with the addition of the additional Heaviside degrees of freedom (). Thus $H2, H3, H4$ in the same way as for the taking into account of the degrees of freedom ace-tips, these additional degrees of freedom are optional, than it is for the main element or the element slave. Thus in table 2, the degrees of freedom in red can appear only if the element slave is multi-Heaviside and the degrees of freedom in magenta can appear only if the main element is multi-Heaviside. It is then possible to have 15 element types late additional for each type of meshes:
• $H1-H2$ $H1-H3$ $H1-H4$
• $H2-H1$ $H2-H2$ $H2-H3$ $H2-H4$
• $H3-H1$ $H3-H2$ $H3-H3$ $H3-H4$
• $H4-H1$ $H4-H2$, $H4-H3$ For example $H4-H4$

for late mesh QUAD4 - QUAD4, the element have $H4-H2$ the d.o.f. of table 3. Node

Degree of freedom	DX DY										
$N1$	H1 X	H1 Y	H2X	H2Y	H3X	H3Y	H4X	H4X	DX	DY	λ	Λ
$N2$	H1 X	H1 Y	H2X	H2Y	H3X	H3Y	H4X	H4X	DX	DY	λ	Λ
$N3$	H1 X	H1 Y	H2X	H2Y	H3X	H3Y	H4X	H4X	DX	DY	λ	Λ
$N4$	H1 X	H1 Y	H2X	H2Y	H3X	H3Y	H4X	H4X	DX	DY	λ	Λ
$N9$	H1 X	H1 Y	H2X	H2Y	DX	DY						
$N10$	H1 X	H1 Y	H2X	H2Y	DX	DY						
$N11$	H1 X	H1 Y	H2X	H2Y	DX	DY						
$N12$	H1 X	H1 Y	H2X	H2Y	Table							

3. The table of the degrees of freedom for the hybrid element of contact H4-H2. For the moment

an element X-FEM cannot be multi-Heaviside and ace-tip at the same time. That supposes that a crack tip must be relatively distant from a junction (spacing from at least two meshes). One thus did not consider pairing between a multi-Heaviside element and an element ace-tip. Once the characteristics of L" element X-FEM hybrid of contact defined, one can pass to the presentation of the strategy of resolution of the problem of contact in great slidings, strategy which will be the object of the following chapter. Strategy

4 of resolution the main steps

of the resolution of a problem of contact in great slidings with the continuous hybrid formulation can be presented as 4 loops imbricated for each time step as it follows: reactualization

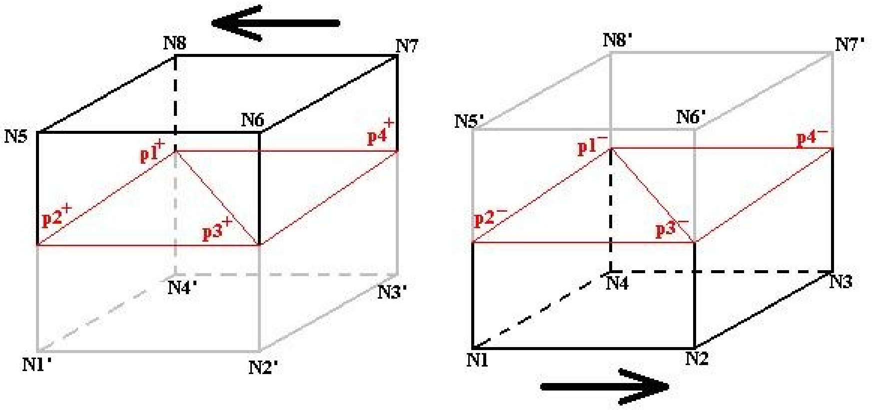
- of the geometry of contact surfaces and launching of the algorithm of pairing; buckle
- on the thresholds of friction (method of POINT-fixes); buckle
- on the statutes of contact (method of the active stresses); buckle
- Newton generalized. In method

of contact XFEM small slidings, the first stage is not present for the approach HP: she was added for the approach great slidings. Its presentation is the object of the first part of this chapter. For

the great slidings with XFEM, one decides to free oneself from the second stage. Rather than a fixed point on the threshold of friction, one prefers process non-linearity by implementing the terms linearized in the tangent matrix. This been part of one of the many changes which appeared during the linearization of the variational mixed formulation following the introduction of the new hybrid element of contact. Thus the second part of this chapter is dedicated to the determination of the new discrete statements for the terms corresponding to the contributions of contact. Geometrical

4.1 reactualization and pairing For

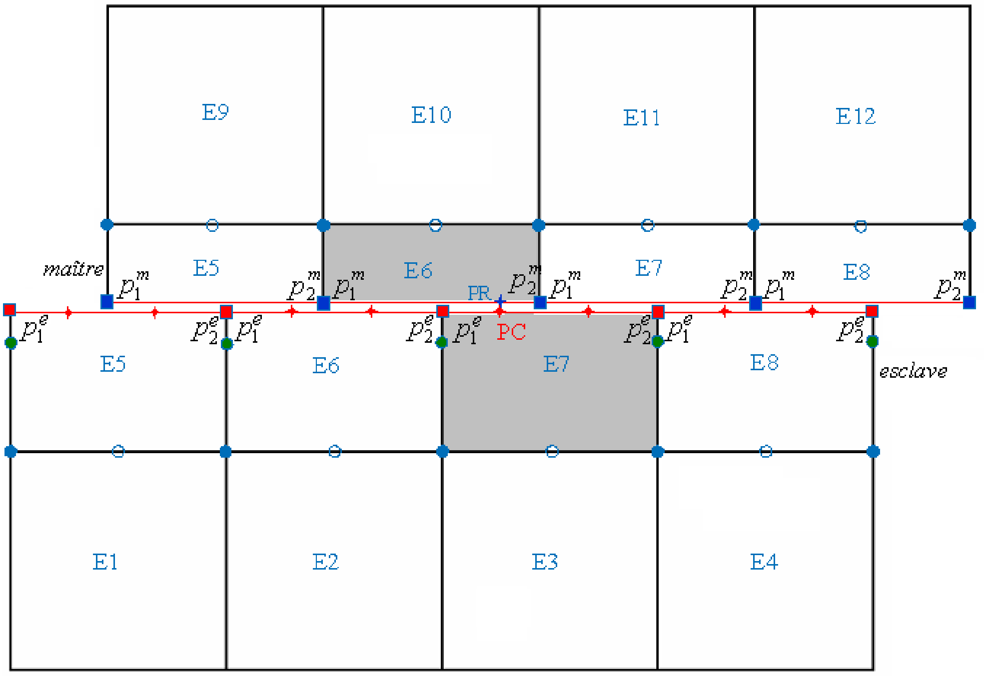
the geometrical reactualization of contact surfaces resulting from a crack modelled by the X-FEM the first operation to make is the duplication of the lips of crack. This operation makes then possible, at the beginning of each time step, the pairing of the points of contact located on the surface designated as slave. In a preoccupation with a generality, one presents the principles of the duplication of the lips of crack using an example 3D shown on Figure 5. Figure



5. Illustration of the duplication of the facets of contact. Following

the duplication of the points of intersection between the edges of the mesh and one fissures it obtains, for each fissured element, two series of facets of contact (facets which are formed with these points): facets slaves which will be attached to the part of the mesh located below the crack (Heaviside function associated with crack which generates the facet), and $H(x,y)=-1$ main facets, located above the crack (Heaviside function associated with crack which generates the facet). For example $H(x,y)=+1$, for a mesh HEXA8 fissured in great slidings (Figure 5), there will be the facets slaves and, $p1^- \, p2^- \, p3^-$ as well as $p1^- \, p3^- \, p4^-$ the main facets and. $p1^+ \, p2^+ \, p3^+$ $p1^+ \, p3^+ \, p4^+$ The geometrical reactualization relates to the computation of the geometry of these facets before each new pairing. For each facet of contact, the geometrical coordinates of the points of intersection are calculated by adding with the initial coordinates the displacements carried out since the initial configuration. The algorithm

of pairing which follows the geometrical reactualization has like purpose to find for each point of contact considered on surface slave the facet in opposite located on main surface. To illustrate this algorithm, which is based on the principles of the similar algorithm developed for the formulation continues classical, one uses the example shown on Figure 6. Figure



6. Illustration of the algorithm of pairing. Thus

let us consider the situation illustrated on Figure 6, a 2D mesh containing meshes QUAD4, with 4 meshes crossed by a crack X-FEM, meshes which can also be made quadratic for the moment in their adding nodes mediums carrying of the Lagrange multipliers of contact-friction. After the geometrical reactualization, one seeks of the mesh to build the late mesh corresponding to the point of contact PC located on the segment of contact $p_1^e p_2^e$ slave. With this intention $E7$, one buckles on all meshes contact (meshes fissured) and then, inside each mesh, one buckles on the points of intersection with the crack (main side). The point of intersection nearest to the point of contact is retained (of on P_2^m $E6$ the figure in our case). One makes then the projection of the point of contact on the facets Masters of meshes which are connected to the point of intersection selected. In the example, one thus projects on the facets of meshes and $E6$ one $E7$ checks on whom projection is interior. One finds thus that the master mesh $E6$ will be paired with the mesh slave for $E7$ the point of contact. Below PC

one presents the principal stages of this algorithm of pairing: •

buckle on meshes contact slaves ○ buckles
on the facets of contact slaves ■ buckles
on the points of integration of contact slaves - computation
of the real coordinates of the point of contact □ buckles
on meshes contact Masters ◘ buckles
on the main points of intersection - computation
of the distance compared to the points of intersection - choice
of the fine point of intersection main ◘
of loop on the main points of intersection □ fine
of loop on meshes of contact Masters - determination

of meshes connected to the main point of intersection □boucle
on meshes of contact connected to the main point of intersection ◘ buckles
on the facets of contact Masters - projection
of the point of contact on the facet Master - computation
of clearance between the point of contact slave and the facet Master - if
clearance is smallest: information storage on fine pairing ◘
buckles on the facets of fine contact Masters ■
of loop on the points of fine contact ○
of loop on the facets of contact slaves •
end of loop on meshes of contact Following

pairing, a card of contact is filled for each point of contact. This card contains the necessary information for the computation of the contributions of contact at the elementary level. The details on the composition of this card are given in [bib8]. Let us note

well that the geometrical algorithm of reactualization fixes a maximum value at the coefficients of penalization of contact and friction. To understand it, let us consider elements parents of characteristic size, h a contact pressure whose order of magnitude is, σ a coefficient of penalization of contact leading κ_n to a typical interpenetration. $\delta=\sigma/\kappa_n$ Convergence is regarded as attack when the relative residue is lower than. η D

- geometrical actualization implies the computation of shape functions of the elements Master and slaves at the points of contact in the present configuration. The geometrical reactualization makes it possible to calculate with δ an accuracy. In addition $\epsilon_{\text{machine}} h$, when convergence is reached, the accuracy on this same quantity must be lower than. We $\eta \dfrac{\sigma}{\kappa_n}$ thus have: that is to say

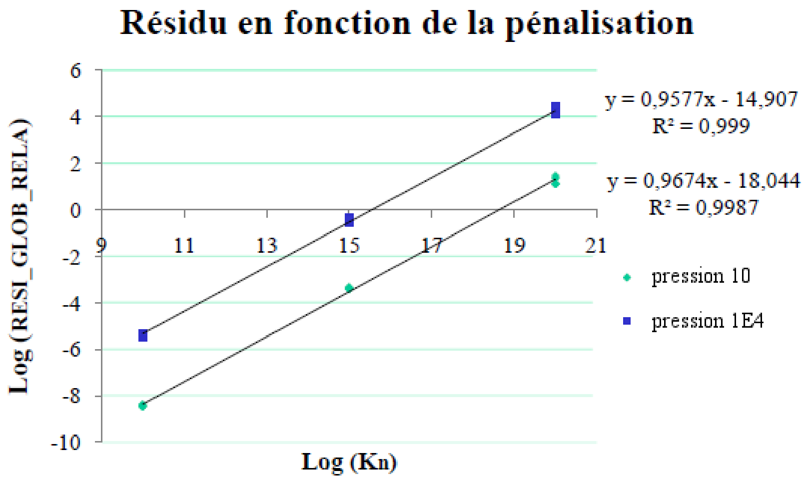
$\epsilon_{\text{machine}} h<\eta \dfrac{\sigma}{\kappa_n}$ the validity $\kappa_n<\dfrac{\eta\sigma}{\epsilon_{\text{machine}} h}$

of this higher limit can be illustrated by numerical tests. For that, we consider a uniform test of compression SSNV182G adapted in great slidings penalized, for which we inform in the method of Newton an excessively small η' tolerance, for example. We $\eta'=1.10^{-20}$ note whereas residue RESI_GLOB_RELA "`reaches a maximum`", i.e. reached a minimal value in on this side which it cannot go down because of accuracy machine. We trace this minimal value then atteignable relative residue, which we call according to $\eta_{\min}$ the coefficient of penalization, for κ_n two values different from the contact pressure. Taking into account the study carried out in the preceding paragraph we expect to find: The fact

$$\kappa_n=\frac{\eta_{\min}\sigma}{\epsilon_{\text{machine}} h}$$

that the coefficients associated directors with the representation of according to $\log(\eta_{\min})$ are $\log(\kappa_n)$ very close to 1 confirms the proportionality enters and. η The machine accuracy κ_n of the machine used being and, 10^{-14} one $h\sim 1$ expects according to the analytical formula to find Y-coordinates in the beginning: , which

$\log\left(\dfrac{\epsilon_{\text{machine}} h}{\sigma}\right)=\begin{vmatrix}-15 \text{ si } \sigma=10\\ -18 \text{ si } \sigma=10^4\end{vmatrix}$ is close to the actual values numerically (respectively and) $-14{,}90$ and $-18{,}04$ confirms the analytical reasoning. Linearization



4.2 of the mixed variational formulation Let us consider

an iteration of Newton for which the fields, and u λ Λ are initially given. One notes, and δu their $\delta\lambda$ $\delta\Lambda$ variations so that the new values at the end of the iteration, and u^f λ^f Λ^f are determined by: , (

$$u^f = u + \delta u$$ 22)
$$\lambda^f = \lambda + \delta\lambda \cdot$$
$$\Lambda^f = \Lambda + \delta\Lambda$$

Because of

dependence of the threshold of sliding to the contact pressure, the model of Coulomb is a nonassociated model. This point is largely developed in [R5,03,50], §2,3 to which one will be able to refer for more precise details. We can rewrite the condition of admissibility of the stresses of the model of Coulomb (7) in (7) the form: The flow

$$f(\boldsymbol{r}, \mu) \overset{\text{déf}}{=} \|\boldsymbol{r}_\tau\| - \mu|\lambda| \leq 0$$

model is written when with it: with

$$[\![\dot{\boldsymbol{u}}]\!] = -\alpha\, \boldsymbol{r}_\tau$$ We $\alpha f(\boldsymbol{r}, \mu) = 0$

let us see whereas flow direction, in other words the direction relative velocity, is not colinéaire with the norm on the surface threshold. This nonassociativeness is source of non-linearity (known as not linearity of threshold). One can deal with the problem with a fixed threshold of friction, which one reactualizes in an external loop. One then finds a pseudopotential (known as criterion of Tresca) which is associated. This
strategy was put in work in the contact X-FEM in small slidings (see [R7,02,12]). We

benefit here from recent developments in the method of contact continues (see [R5,03,52]) to adopt a direct linearization. With this intention, we linearize by considering the variable threshold λ , put except for when one is on the surface of the cone of Coulomb, because that would thus return as we saw to recommend a nontangent relative velocity, and incorrect. In short, the threshold is linearized for the adherent points, not for the slipping points. This method in particular makes it possible to bring back in the cone of dependancy of the points considered slipping wrongly. It was already implemented in continuous formulation, with conclusive results. The linearization

of the formulation (12-14), while developing and, δg_n gives δg_τ: (23)

$$\begin{aligned}
&\int_{\Omega} tr(\Pi(\nabla_p(\delta u)):\nabla_p(u^*))\,d\Omega-\int_{\Gamma_c}\chi\delta\lambda[u^*]\cdot n\,d\Gamma+\int_{\Gamma_c}\chi\rho_n[\delta u]\cdot n[u^*]\cdot n\,d\Gamma\\
&-\int_{\Gamma_c}\chi S_f\,\mu\,\delta\lambda\,g_\tau[u^*]_\tau d\Gamma\\
&-\int_{\Gamma_c}\chi\mu\lambda K(g_\tau)\delta\Lambda[u^*]_\tau d\Gamma-\int_{\Gamma_c}\chi\mu\lambda\rho_\tau K(g_\tau)[\delta u]_\tau[u^*]_\tau d\Gamma\\
&=-\int_{\Omega} tr(\Pi(\nabla_p(u)):\nabla_p(u^*))\,d\Omega+L_{meca}(u^*)\\
&+\int_{\Gamma_c}\chi g_n[u^*]\cdot n\,d\Gamma+\int_{\Gamma_c}\chi\mu\lambda P_B(g_\tau)[u^*]_\tau d\Gamma
\end{aligned} \quad (24)$$

$$-\int_{\Gamma^c}\frac{1-\chi}{\rho_n}\delta\lambda\,\lambda^*d\Gamma-\int_{\Gamma_c}\chi[\delta u]\cdot n\,\lambda^*d\Gamma=+\int_{\Gamma_c}\left(\frac{1-\chi}{\rho_n}\lambda\lambda^*+\chi[u_e-u_m]\cdot n\,\lambda^*\right)d\Gamma \quad (25)$$

$$\begin{aligned}
&-\int_{\Gamma_c}\chi\mu\lambda K(g_\tau)[\delta u]_\tau\Lambda^*d\Gamma+\int_{\Gamma_c}\frac{\chi\mu\lambda}{\rho_\tau}(I_d-K(g_\tau))\delta\Lambda\Lambda^*d\Gamma+\int_{\Gamma_c}(1-\chi)\delta\Lambda\Lambda^*d\Gamma\\
&-\int_{\Gamma_c}\chi S_f\,\mu\,\delta\lambda\,\nu_\tau[\delta u]_\tau\Lambda^*d\Gamma\\
&=-\int_{\Gamma_c}\frac{\mu\lambda\chi}{\rho_\tau}(\Lambda-P_B(g_\tau))\Lambda^*d\Gamma-\int_{\Gamma_c}(1-\chi)\Lambda\Lambda^*d\Gamma
\end{aligned}$$

With

: if so

$K(g_\tau)=Id$ sliding dependancy

$K(g_\tau)=\frac{1}{\|g_\tau\|}\left(Id-\frac{g_\tau\cdot g_\tau^T}{\|g_\tau\|^2}\right)$ Note

that one solves the geometric nonlinearity by a fixed problem of point, by supposing that is fixed n during the variation of displacement. With regard to

the penalized formulation, an iteration of the method of Newton is written: (26)

$$\begin{aligned}
&\int_M tr(\Pi(\nabla_p(\delta u)):\nabla_p(u^*))\,dM-\int_{\Gamma_c}\chi\delta\lambda[u^*]\cdot n\,d\Gamma\\
&-\int_{\Gamma_c}\chi S_f\,\mu\,\delta\lambda\,\kappa_\tau\nu_\tau[u^*]_\tau d\Gamma-\int_{\Gamma_c}\chi\mu\lambda\kappa_\tau K(\kappa_\tau\nu_\tau)[\delta u]_\tau[u^*]_\tau d\Gamma\\
&=-\int_M tr(\Pi(\nabla_p(u)):\nabla_p(u^*))\,dM+L_{meca}(u^*)\\
&+\int_{\Gamma_c}\chi\lambda[u^*]\cdot n\,d\Gamma+\int_{\Gamma_c}\chi\mu\lambda P_B(\kappa_\tau\nu_\tau)[u^*]_\tau d\Gamma
\end{aligned} \quad (27)$$

$$-\int_{\Gamma^c}\frac{1}{\kappa_n}\delta\lambda\,\lambda^*d\Gamma-\int_{\Gamma_c}\chi[\delta u]\cdot n\,\lambda^*d\Gamma=+\int_{\Gamma_c}\left(\frac{1}{\kappa_n}\lambda\lambda^*+\chi[u]\cdot n\,\lambda^*\right)d\Gamma \quad (28)$$

$$\begin{aligned}
&\int_{\Gamma_c}\delta\Lambda\Lambda^*d\Gamma\\
&=-\int_{\Gamma_c}\chi(\Lambda-P_B(\kappa_\tau\nu_\tau))\Lambda^*d\Gamma-\int_{\Gamma_c}(1-\chi)\Lambda\Lambda^*d\Gamma
\end{aligned}$$

While introducing

(20-21) into the linearized formulation of the problem (23-25), one obtains a system who can be put in the following matric form: (29)

$$\begin{bmatrix} K_{meca}+A^u+B^u & A^T+D^T & B_r^T\\ A & C & 0\\ B_r & E & F\end{bmatrix}\begin{pmatrix}\delta u\\ \delta\lambda\\ \delta\Lambda\end{pmatrix}=\begin{pmatrix}L_{meca}+L^1_{cont}+L^1_{frott}\\ L^2_{cont}\\ L^3_{frott}\end{pmatrix}$$

where

: is

- K_{meca} the mechanical stiffness matrix, is
- A^u the increased stiffness matrix due to the contact, is
- B^u the increased stiffness matrix due to friction, is
- A the matrix binding the terms of displacement to those of contact (matrix of the model of contact), is
- B_r the matrix binding the terms of displacement to those of friction (matrix of the friction law), is
- C the matrix allowing to determine contact pressures in the lack of contact case, is
- D a matrix binding the terms of displacement and the contact pressure. It comes from the linearization of the threshold of friction for the adherent points. is
- E a matrix binding and multiplier contact pressure of friction. It comes from the linearization of the threshold of friction for the adherent points. is
- F the matrix allowing to determine the multipliers of friction in the cases not contacting, NON-rubbing, or contacting rubbing slipping, is
- L_{meca} the second member representing the internal forces and the increments of loadings, and are
- L^1_{cont} L^2_{cont} the second members due to the contact, and are
- L^1_{frott} L^3_{frott} the second members related to friction. It is noted

that the choice of a direct linearization of the threshold of friction led to an asymmetric matrix. One discretizes
by keeping only the contributions of contact and one obtains: (30)

$$\begin{bmatrix} \begin{bmatrix} A+B_{aa} & A+B_{ab} & A+B_{ac} \\ A+B_{ba} & A+B_{bb} & A+B_{bc} \\ A+B_{ca} & A+B_{cb} & A+B_{cc} \end{bmatrix}^{u}_{es:es} & \begin{bmatrix} A_a+D_a \\ A_b+D_b \\ A_c+D_c \end{bmatrix}_{es:cont} & \begin{bmatrix} B_a \\ B_b \\ B_c \end{bmatrix}_{es:cont} & \begin{bmatrix} A+B_{aa} & A+B_{ab} & A+B_{ac} \\ A+B_{ba} & A+B_{bb} & A+B_{bc} \\ A+B_{ca} & A+B_{cb} & A+B_{cc} \end{bmatrix}^{u}_{es:ma} \\ [A_a \quad A_b \quad A_c]_{cont:es} & C & 0 & [A_a \quad A_b \quad A_c]_{cont:ma} \\ [B_a \quad B_b \quad B_c]_{cont:es} & E & F & [B_a \quad B_b \quad B_c]_{cont:ma} \\ \begin{bmatrix} A+B_{aa} & A+B_{ab} & A+B_{ac} \\ A+B_{ba} & A+B_{bb} & A+B_{bc} \\ A+B_{ca} & A+B_{cb} & A+B_{cc} \end{bmatrix}^{u}_{ma:es} & \begin{bmatrix} A_a+D_a \\ A_b+D_b \\ A_c+D_c \end{bmatrix}_{ma:cont} & \begin{bmatrix} B_a \\ B_b \\ B_c \end{bmatrix}_{ma:cont} & \begin{bmatrix} A+B_{aa} & A+B_{ab} & A+B_{ac} \\ A+B_{ba} & A+B_{bb} & A+B_{bc} \\ A+B_{ca} & A+B_{cb} & A+B_{cc} \end{bmatrix}^{u}_{ma:ma} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \delta a \\ \delta b \\ \delta c \end{bmatrix}_{es} \\ \delta\lambda \\ \delta\Lambda \\ \begin{bmatrix} \delta a \\ \delta b \\ \delta c \end{bmatrix}_{ma} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} L_a \\ L_b \\ L_c \end{bmatrix}^{1\ cont\ frot}_{es} \\ L^2 \\ L^3 \\ \begin{bmatrix} L_a \\ L_b \\ L_c \end{bmatrix}^{1\ cont\ frot}_{ma} \end{bmatrix}$$

the indices

and indicate es ma the contributions slave and Master, respectively, while, and a indicate b c the parts classic, enriched Heaviside and enriched Tip Ace, respectively. If

one treats one element of type exclusively Tip Ace, one regains the shape HP and the matrix becomes: The statements

$$\begin{bmatrix} 4[A_{c:c}+B_{c:c}]^u & 2[A_{c:\lambda}+D_{c:\lambda}] & 2[B_{c:\Lambda}] \\ 2[A_{\lambda:c}] & C & 0 \\ 2[B_{\Lambda:c}] & E & F \end{bmatrix} \begin{bmatrix} \delta c \\ \delta\lambda \\ \delta\Lambda \end{bmatrix} = \begin{bmatrix} 2L_c^{1\ cont}+2L_c^{1\ frot} \\ L^2 \\ L^3 \end{bmatrix}$$

of the terms present in the system are: and,

- A^u block B^u slave – slave: where:

$$[A^u_{es:es}]_{ij}=\int_{\Gamma_c}\rho_n\chi\,\phi_i^{es}\phi_j^{es}\,n\otimes n[MM]\,d\Gamma$$

$$[B^u_{es:es}]_{ij}=\int_{\Gamma_c}-\rho_\tau\chi\mu\lambda\,\phi_i^{es}\phi_j^{es}[P_\tau]^T\cdot[K(g_\tau)]\cdot[P_\tau][MM]\,d\Gamma$$

if ace

$$[MM]=\begin{bmatrix}1 & -1 & -\sqrt{r_e}\\ & 1 & \sqrt{r_e}\\ sym & & r_e\end{bmatrix}$$ - tip,

$[MM]_{1,1}=1$ and $[MM]_{1,1+l}=He_j^{l,e}$ if $[MM]_{1+k,1}=He_i^{k,e}$ multi $[MM]_{1+k,1+l}=He_i^{k,e}He_j^{l,e}$ - Heaviside One distinguishes

the cases ace-tip and multi-Heaviside bus currently, an element cannot be at the same time ace-tip and multi-Heaviside. Let us note that if one would have such an element, the forms of the elementary matrixes would be obtained easily by combination of the preceding terms. In other words there would not be great a deal to make in the TE of assembly of these terms, as long as there is only one enrichment ace-tip associated with the one with cracks with the element. and,

- A^u block B^u slave – Master and master-slave: where:

$$[A^u_{es:ma}]_{ij}=\int_{\Gamma_c}\rho_n\chi\phi_i^{es}\phi_j^{ma}n\otimes n[MM]d\Gamma$$
$$[A^u_{ma:es}]_{ij}=\int_{\Gamma_c}\rho_n\chi\phi_i^{ma}\phi_j^{es}n\otimes n[MM]^T d\Gamma=[A^u_{es:ma}]^T_{ji}$$
$$[B^u_{es:ma}]_{ij}=\int_{\Gamma_c}-\rho_\tau\chi\mu\lambda\phi_i^{es}\phi_j^{ma}[P_\tau]^T\cdot[K(g_\tau)]\cdot[P_\tau][MM]d\Gamma$$
$$[B^u_{ma:es}]_{ij}=\int_{\Gamma_c}-\rho_\tau\chi\mu\lambda\phi_i^{ma}\phi_j^{es}[P_\tau]^T\cdot[K(g_\tau)]\cdot[P_\tau][MM]^T d\Gamma$$

if ace

$$[MM]=\begin{bmatrix}-1 & -1 & -\sqrt{r_m}\\ 1 & 1 & \sqrt{r_m}\\ \sqrt{r_e} & \sqrt{r_e} & \sqrt{r_e}\cdot\sqrt{r_m}\end{bmatrix}$$ - tip,

$[MM]_{1,1}=-1$ and $[MM]_{1,1+l}=-He_j^{l,m}$ if $[MM]_{1+k,1}=-He_i^{k,e}$ multi $[MM]_{1+k,1+l}=-He_i^{k,e}He_j^{l,m}$ - Heaviside. and,

- A^u main B^u block – Master where:

$$[A^u_{ma:ma}]_{ij}=\int_{\Gamma_c}\rho_n\chi\phi_i^{ma}\phi_j^{ma}n\otimes n[MM]d\Gamma$$
$$[B^u_{ma:ma}]_{ij}=\int_{\Gamma_c}-\rho_\tau\chi\mu\lambda\phi_i^{ma}\phi_j^{ma}[P_\tau]^T\cdot[K(g_\tau)]\cdot[P_\tau][MM]d\Gamma$$

if ace

$$[MM]=\begin{bmatrix}1 & 1 & \sqrt{r_m}\\ & 1 & \sqrt{r_m}\\ sym & & r_m\end{bmatrix}$$ - tip,

$[MM]_{1,1}=1$ and $[MM]_{1,1+l}=He_j^{l,m}$ if $[MM]_{1+k,1}=He_i^{k,m}$ multi $[MM]_{1+k,1+l}=He_i^{k,m}He_j^{l,m}$ - Heaviside. Note

the term

> *being $n\otimes n$ symmetric, of this fact is symmetric $[A^u]$. The term is also $[P_\tau]^T\cdot[K(g_\tau)]\cdot[P_\tau]$ symmetric, of this fact is $[B^u]$ symmetric . , and*

- A , B blocks D slave – contact and contact – slave: where:

$$[(A+D)_{es:cont}]_{ij}=\int_{\Gamma_c}\chi\phi_i^{es}\phi_j^{es}(n+S_f\mu g_\tau\cdot[P_\tau])[V]d\Gamma$$
$$[A_{cont:es}]_{ij}=\int_{\Gamma_c}\chi\phi_i^{es}\phi_j^{es}n^T[V]^T d\Gamma$$
$$[B_{es:cont}]_{ij}=\int_{\Gamma_c}\chi\mu\lambda\phi_i^{es}\phi_j^{es}[P_\tau]^T\cdot[K(g_\tau)]\cdot[\tau_1,\tau_2][V]d\Gamma$$
$$[B_{cont:es}]_{ij}=\int_{\Gamma_c}\chi\mu\lambda\phi_i^{es}\phi_j^{es}[\tau_1,\tau_2]^T[K(g_\tau)]\cdot[P_\tau][V]^T d\Gamma$$

if ace

$$[V]=\begin{bmatrix}-1\\1\\\sqrt{r_e}\end{bmatrix}$$ - tip, and

$[V]_1=-1$ if $[V]_{1+k}=-He_i^{k,e}$ multi - Heaviside and,

- A blocks B Master – contact and contact – Master: where:

$$[(A+D)_{ma:cont}]_{ij}=\int_{\Gamma_c}\chi\,\phi_i^{ma}\,\phi_j^{es}(n+S_f\,\mu\,g_\tau\cdot[P_\tau])[V]\,d\Gamma$$
$$[A_{cont:ma}]_{ij}=\int_{\Gamma_c}\chi\,\phi_i^{ma}\,\phi_j^{es}\,n^T[V]^T\,d\Gamma$$
$$[B_{ma:cont}]_{ij}=\int_{\Gamma_c}\chi\,\mu\,\lambda\,\phi_i^{ma}\,\phi_j^{es}[P_\tau]^T\cdot[K(g_\tau)]\cdot[\tau_1,\tau_2][V]\,d\Gamma$$
$$[B_{cont:ma}]_{ij}=\int_{\Gamma_c}\chi\,\mu\,\lambda\,\phi_i^{ma}\,\phi_j^{es}[\tau_1,\tau_2]^T[K(g_\tau)]\cdot[P_\tau][V]^T\,d\Gamma$$

if ace

$$[V]=\begin{bmatrix}1\\1\\\sqrt{r_m}\end{bmatrix}$$ - tip, if

$[V]_1=1$ multi $[V]_{1+k}=He_i^{k,m}$ - Heaviside Notes

If

> *the contact were applied, is A symmetric only if it there does not have friction and is symmetric B if and only if I. : : $\mu=1$:*

- C Note

$$[C]_{ij}=-\int_{\Gamma_c}\frac{1-\chi}{\rho_n}\phi_i^{es}\,\phi_j^{es}\,d\Gamma$$

- E

$$[E]_{ij}=-\int_{\Gamma_c}\chi\,S_f\,\mu\,\phi_i^{es}\,\phi_j^{es}(\nu_\tau\cdot[\tau_1,\tau_2]^T)[\tau_1,\tau_2]\,d\Gamma$$

- F

$$[F]_{ij}=\int_{\Gamma_c}(1-\chi)\phi_i^{es}\,\phi_j^{es}[\tau_1,\tau_2]^T\cdot[\tau_1,\tau_2]\,d\Gamma+\int_{\Gamma_c}\frac{\chi\,\mu\,\lambda}{\rho_\tau}\phi_i^{es}\,\phi_j^{es}[\tau_1,\tau_2]^T\cdot[I_d-K(g_\tau)]\cdot[\tau_1,\tau_2]\,d\Gamma$$

In the absence of

> *friction or when, $\mu=0$ the matrix is symmetric because the terms, and D E are not assembled. With friction, it is not it any more. , block*

- L^1 slave: where:

$$[L_{es}^{1,cont}]_i=-\int_{\Gamma_c}\chi(\lambda-\rho_n d_n)\phi_i^{es}\,n[V]\,d\Gamma$$
$$[L_{es}^{1,frot}]_i=-\int_{\Gamma_c}\chi\mu\,\lambda[P_\tau]^T\cdot P_B(\Lambda+\rho_\tau[u_e-u_m]_\tau)\phi_i^{es}[V]\,d\Gamma$$

if ace

$$[V]=\begin{bmatrix}-1\\1\\\sqrt{r_e}\end{bmatrix}$$ - tip, if

$[V]_1=-1$ multi $[V]_{1+k}=-He_i^{k,e}$ - Heaviside, main

• L^1 block: where:

$$[L_{ma}^{1,cont}]_i = -\int_{\Gamma_c} \chi(\lambda - \rho_n d_n) \phi_i^{ma} n [V] d\Gamma$$

$$[L_{es}^{1,frot}]_i = -\int_{\Gamma_c} \chi\mu\lambda [P_\tau]^T \cdot P_B(\Lambda + \rho_\tau [u_e - u_m]_\tau) \phi_i^{ma} [V] d\Gamma$$

if ace

$$[V] = \begin{bmatrix} 1 \\ 1 \\ \sqrt{r_m} \end{bmatrix}$$ - tip, if

$[V]_1 = 1$ multi $[V]_{1+k} = He_i^{k,m}$ - Heaviside: : Note

• L^2

$$[L^2]_i = \int_{\Gamma_c} (\frac{1-\chi}{\rho_n} \lambda + \chi [u_e - u_m] \cdot n) \phi_i^{es} d\Gamma$$

• L^3

$$[L^3]_i = -\int_{\Gamma_c} (1-\chi) \phi_i^{es} [\tau_1, \tau_2]^T \cdot [\tau_1, \tau_2] \Lambda d\Gamma - \int_{\Gamma_c} \frac{\chi\mu\lambda}{\rho_\tau} \phi_i^{es} [\tau_1, \tau_2]^T \cdot [\tau_1, \tau_2] (\Lambda - P_B(\Lambda + \rho_\tau [u_e - u_m]_\tau)) d\Gamma$$

Attention

, in routines FORTRAN calculating the second members, all the terms expressed L above are multiplied by because -1 in Code_Aster one considers, historically, that the second member is located in the left part of the system. In

the Augmented Lagrangian cases as in penalization, the matrix is singular if and. $\chi = 1$ It $\lambda = 0$ is thus considered, in the equation of friction only, that if. $\chi = 0$ $\lambda = 0$ The initial linear system then takes the shape Improvement

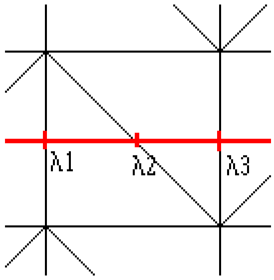
$$\begin{bmatrix} K_{meca} + A^u + B^u & A^T & 0 \\ A & C & 0 \\ 0 & 0 & F \end{bmatrix} \begin{pmatrix} \delta u \\ \delta \lambda \\ \delta \Lambda \end{pmatrix} = \begin{pmatrix} L_{meca} \\ 0 \\ 0 \end{pmatrix}$$

5 of integration for the contact Conflict

5.1 between the relations imposed by condition LBB and the changes of status of contact Relation

5.1.1 linear on the way contacting/not contacting We

had noted that the linear relations introduced on the degrees of freedom of contact-friction, in order to satisfy condition LBB in X-FEM, posed a problem in great slidings. Indeed during the contacting transition/not contacting, they introduced an useless relation which was at the origin of oscillations on the profile of pressure [bib10]. This generated difficulties to converge on the statutes of contact. To illustrate this phenomenon, let us defer on figure 7. Figure



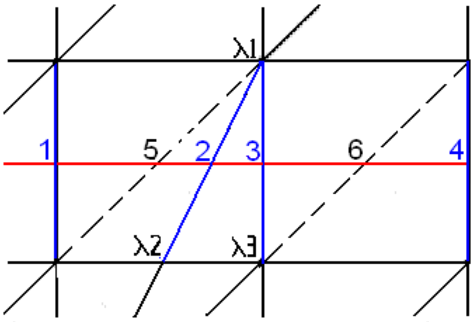
7 - the 3 nodes are seen imposing the linear relation by L $\lambda_1-2\lambda_2+\lambda_3=0$ "algorithm of stabilization of the LBB. If

items 1 and 2 are not contacting and item 3 is contacting on this figure, there is the linear relation besides $\lambda_1-2\lambda_2+\lambda_3=0$, which $\begin{cases}\lambda_1=0\\ \lambda_2=0\end{cases}$ imposes. However $\lambda_3=0$

the contribution to the node 3 N" is normally not null. It is determined by the weak formulation of the Principle of the Virtual wors. The lost
pressure is then compensated by a peak on the Lagrange of contact of the node contacting according to. This peak is followed of oscillations on the profile of pressure. These oscillations cause difficulties of convergence on the statutes of contact.
To solve this problem, one should not take account of the linear relation when one finds oneself in such a case, which is difficult given that it was introduced previously via another operator. The made choice is thus not to assemble in the equation of contact the contribution of a point which "is taken" in a linear relation and which is not contacting. On
the example of figure 7, that amounts not imposing more because $\lambda_2=0$ item 2 "is then taken" in the linear relation. One thus has only, which $\begin{cases}\lambda_1-2\lambda_2+\lambda_3=0\\ \lambda_1=0\end{cases}$ does not impose any more. With $\lambda_3=0$
the new formulation [bib9] which consists in storing the degrees of freedom of contact only to nodes having degrees of freedom of displacement already, that amounts not assembling on the level of the model of contact a point of integration not contacting which is not on a vital edge: one then assembles neither the contribution in the elementary matrix, nor the contribution to the second member of this point of integration (but the assembly is made for the equilibrium). For
more details on the definition of a vital edge in Code_Aster, one can refer to part LBB condition of chapter 4 of [bib1]. Relation

5.1.2 of equality on the way contacting/not contacting Just

as for the linear relations, and the same reasons, we had noted that the relations of equality introduced on the degrees of freedom of contact-friction in order to satisfy condition LBB in X-FEM posed also a problem in great slidings. They during introduced also an useless relation the contacting transition/not contacting, the final consequence being the difficulty in converging on the statutes of contact. To illustrate this phenomenon, let us defer on Figure 8. Figure



8 - edges 5 and 6 (in dotted lines) are nonvital, edges 1,2,3 and 4 (in blue) are vital. On edges 2 and 3, there is the relations $\lambda 1 = \lambda 2$ and $\lambda 1 = \lambda 3$ If

the point of contact of edge 2 is not contacting, and the point of edge 3 contacting on this figure, one has the relations, which $\begin{cases}\lambda_1=\lambda_2=0\\ \lambda_1=\lambda_3\end{cases}$ imposes on the level $\lambda_3=0$ of the model of contact. However

the contribution on the edge 3 N" is normally not null. It is determined by the weak formulation of the Principle of the Virtual wors. It
acts by way of same problem as into 5.1.1. The choice
made to solve this problem is the same one as for the problem with the linear relations, i.e not to assemble in the equation of contact the contribution of a point which "is taken" in a relation of equality and which is not contacting, if another point "taken" in this relation of equality is contacting. In the example
of Figure 8, that amounts not imposing more because the point $\lambda_2=0$ of integration not contacting of the vital edge 2 "is taken" in the relation between equality and the contacting point of the vital edge 3. One thus has only, which $\lambda_1=\lambda_2=\lambda_3$ does not impose any more. More generally $\lambda_3=0$
, one should not assemble in the equation of contact the contributions of contact-friction of the points of integrations not contacting when a relation of equality connects them to a contacting point. To do
that, in formulation with the nodes tops [bib9], one defines as connecting a node which connects several vital edges, i.e the score of this node at the end of the algorithm to satisfy the condition LBB is strictly higher than 1 (on Figure 8, node 1 which connects edges 2 and 3 is regarded as connecting). Then, each
connecting node defines a group of edges connected between them. One numbers the groups of edges of 1 with and in each n_{group} group one numbers the edges of 1 with. One then $n_{arête}$
defines in each group a determinant edge. It will be only group which one will assemble the contributions in the equation of contact if no point of integration of the group is contacting. In addition so for the group there exist then contacting points of integration one assembles in the equation of contact all the contacting contributions and one does not assemble the contributions not contacting. One describes
the choice of the determinant edge now. For the first iteration of contact of the first time step, all the statutes are identical, therefore one can choose the determinant edge arbitrarily. Then, each reactualization of the statutes of contact, one carries out in the wake the reactualization of the determinant edges. One presents
below the principal stages of the algorithm which makes this reactualization and which is launched just after having reactualized the statutes of contact: • Buckle
on the points of integration ○ If the point
of integration in progress belongs to a group of connected vital edges, that it becomes contacting and that it is not on the determinant edge: ■ Loop
on the points of integration of the group One searches
the statute of the points on the determinant edge □ If the statute
of the points of the determinant edge is not contacting: the statute of
the edge in progress becomes determining instead of the other □ Fine if
■ Fine of
loop on the points of integration of the Fine ○ group if
• End of
loop on the points of integration Summarized

5.1.3 the algorithm

of selection detailed in the two preceding paragraphs consists in integrating the equation of contact on all the contacting edges, and integrating the contribution for the edges not contacting only if the group of vital edges does not comprise any contacting edge. Only one edge of the group, known as determinant, is enough then. This is explained

by the fact why the force of contact in the balance equation is implicit in the case contacting and explicit in the lack of contact case. In this last case, the multipliers of contact do not intervene nowhere in the equilibrium: the force is directly put at 0 via the statutes of contact. Since in the equation of contact one cannot assemble terms of different statutes within the same group, one preferentially assembles the contributions due to the contact where they describe a reaction force intervening in the equilibrium, i.e. on the contacting edges. Conflict between

5.2 the relations of equalities imposed by condition LBB and the changes of status of dependancy If

the example of figure 7 is taken again, in the case where, this time, the 3 points is contacting, but where items 1 and 2 are slipping and item 3 is adherent. One has, in 2D for example, the linear relation: besides $\Lambda_1-2\Lambda_2+\Lambda_3=0$ what imposes $\Lambda_1=1;\Lambda_2=1$. However the contribution $\Lambda_3=1$

to node 3 is normally strictly lower than 1 since the point is adherent (inside the cone of friction), it is determined by the weak formulation of the Principle of the Virtual wors. One finds a risk of NON-convergence similar to the contact. It is necessary to remove in the equation of friction at least the assembly of the contribution of a point which “is taken” in a linear relation and which is slipping into the friction law. On the example of figure 7, that amounts not imposing more because item $\Lambda_2=1$ 2 “is then taken” in the linear relation. One thus has only

, which $\begin{cases}\Lambda_1-2\Lambda_2+\Lambda_3=0\\ \Lambda_1=1\end{cases}$ does not impose any more. With the new $\Lambda_3=1$

formulation [bib9] which consists in storing the degrees of freedom of contact only into cubes nodes having degrees of freedom of displacement already, that amounts not assembling on the level of the friction law a slipping point of integration which is not on a vital edge: one then assembles neither the contribution in the elementary matrix, nor the contribution to the second member of this point of integration (but the assembly is made for the equilibrium). The major difference with the case of the statute in contact, is that while thus making for friction, the matrix becomes asymmetrical. It is not a problem because that joined the choice already made in the chapter 4 precedent. Another difference with the contact comes owing to the fact that item 2 not contacting does not intervene in the equilibrium because of being worth 0 in X this case: if one does not assemble his contribution in the model of contact, and that its value normally null is not put at zero by the following iteration of contact then it will be worth because by $\lambda_2=\lambda_3/2$ the linear $\lambda_1=0$ relation but without $\lambda_1-2\lambda_2+\lambda_3=0$ that not modifying the equilibrium. For friction

, slipping item 2 intervenes in the equilibrium: if one does not assemble his contribution in the friction law, one does not impose that the value of is equal Λ to 1 with the following iteration of Newton (it will be worth because). One $\Lambda_2=(\Lambda_1+\Lambda_3)/2=(1+\Lambda_3)/2<1$ is then likely $\Lambda_3<1$ to modify the equilibrium contrary to the case of the contact. Knowing that the friction law does not impose either, two cases $v_{\tau 2}=0$ are then possible: if is rather

- large ρ_τ so that. The statute $g_{\tau 2}=\Lambda_2+\rho_\tau v_{\tau 2}>1$ of item 2 remains slipping and balances it is not modified since one uses in the dissipative $P_B(g_{\tau 2})$ term associated with friction; if is not
  - large ρ_τ enough and that the statute of $g_{\tau 2}<1$ item 2 changes and becomes adherent. One assembles with the following iteration the friction law which imposes and which consolidates $v_{\tau 2}=0$ the adherent statute. In the same way

and to take again the example of figure 8, in the case where, this time, the 2 points are contacting, but where the point of contact of edge 2 is slipping and the point of contact of edge 3 is adherent, one has the relations, which imposes $\Lambda_1=\Lambda_2=1 ; \Lambda_1=\Lambda_3$ on the level of $\Lambda_3=1$ the friction law. However the contribution on edge 3 is normally strictly lower than 1 since the point is adherent (inside the cone of friction). The same risk of NON-convergence that previously is present. As for

the contact pressure, the semi-multiplier of friction is implicit in the adherent and explicit case in the slipping case where we can express it in the form: (31) Like,

$$\Lambda=-\frac{v_\tau}{\|v_\tau\|}$$ differential

from ratio Λ with data by v_τ : associated with

$$\frac{\partial \Lambda}{\partial v_\tau}=\frac{1}{\|v_\tau\|}\left(Id-\frac{v_\tau \otimes v_\tau}{\|v_\tau\|^2}\right)\underset{v_\tau \to 0}{\to}\infty$$

the choice of (31) is not limited when the velocity of sliding becomes small, the purely explicit formulation of the sliding is not retained, and one retains the following shape of the semi-multiplier to correct this default: Consequently

$$\Lambda=P_{B(0,1)}(\Lambda+\rho_\tau v_\tau)$$

and contrary to the contact pressure which intervenes only in the contacting case, the semi-multiplier of friction intervenes at the same time in dependancy or sliding. If one assembles selectively the adherent edges in the equation of friction – in the same way that what was made for the edges contacting in the equation of contact – one takes the chance to observe oscillations of the statute of dependancy preventing convergence because the semi-multiplier determined by the equilibrium contains the contributions of the slipping terms which are not taken into account in the equation of friction (comparatively, in the case of the contact, all the contributions not contacting do not intervene in the balance equation). One could observe such oscillations on the test ssnv209j by implementing this technique. This is why

it is necessary to fix a priori a determinant edge by group of vital edge. Finally and whatever its statute, the equation of friction is assembled on this only edge only (what means that the determinant edge gives to the group its slipping or adherent characteristic and that within the same group one cannot have the presence of two distinct statutes, the value resulting from semi-mulitplicateur being realized). Finally one recapitulates

here the call to the algorithms presented higher: • Not

fixes on the statutes of contact ○ Iterations
of Newton - Computation of
the contributions of Fine contact-friction ○ of
the iteration of Newton Computation of
statutes of contact Algorithm
of reactualization of the determinant edge according to the statutes of contact • Fine fixed
point on the statutes of contact Prospects

6 for evolution of the approach great slidings with XFEM the development

of the approach great slidings with X-FEM as its numerical implementation in Code_Aster were extended to friction in 2D (plane stresses and plane strains for element types finished the TRI3 and QUAD4 , SORT 6 and QUAD 8) and in 3D (for finite elements HEXA8, PENTA 6, and TETRA4). The approach was also extended to the crack tips. The fundamental scientists were posed and described in this document. It is also

possible to treat the contact in the elements multi-fissured (junctions in particular) in 2D and 3D, but it remains to implement the terms of friction. Forces must also be made to guarantee the robustness of the method in this case. It remains for example to implement the recutting of the facets in the elements containing the junction. For the moment the contributions of contact are not taken into account on these facets. Let us note that this work must also be carried out for the small slidings. Generalization with the great slidings should not then pose problem. Bibliography

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8 of the versions of the document Index Doc.

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	EDF	/R & D /AMA initial Text	B 9.7.4 I.Nistor
	, P	. Massin EDF/R & D /AMA Mr. SIAVELIS , Mr. GUITON IFP Card-indexes 12608	: elements triangles, diagram of Simpson and Newton-Dimensions, computation of clearance. C 10.1.1 P.
Massin	EDF	/R & D /AMA Mr. SIAVELIS , Mr. GUITON IFP Card-indexes 14123	: introduction of the crack tips.