Continuation methods of the Summarized

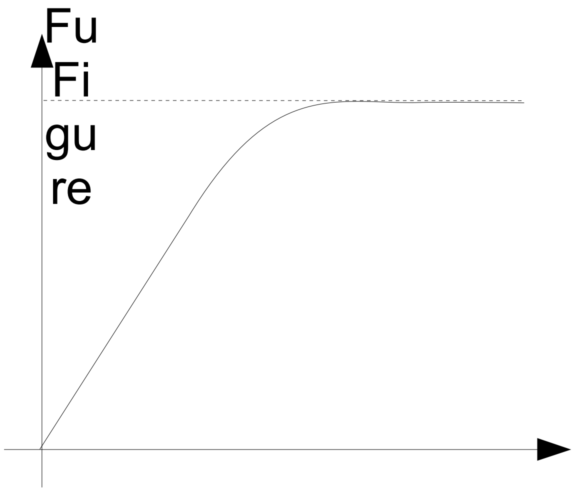
loading:

This document describes the continuation methods of the loading available in *Code_Aster* (by a degree of freedom, length of arc, increment of strain and elastic prediction). They introduce an additional unknown, the intensity on behalf controllable of the loading, and an additional equation, the stress of control. These methods make it possible in particular to as well calculate the response of a structure which would have instabilities, of origins geometrical (buckling) as material (softening).

1 Introduction

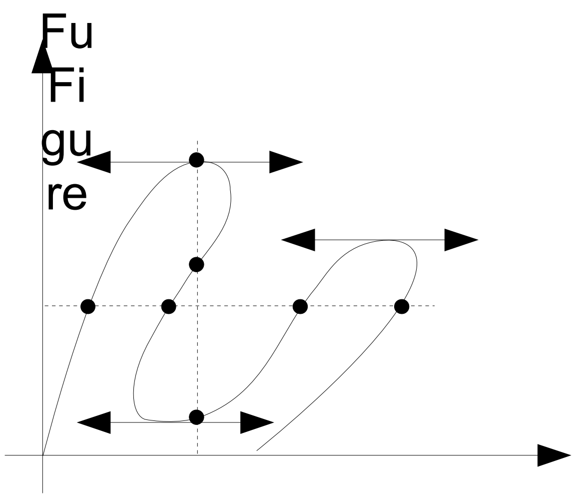
1.1 Non-linearity and failure of Newton

the method of Newton presented in [R5.03.01] fails on certain problems which display a response of structure not strictly monotonous according to displacements or imposed loadings. On the figure 1.1-a, one presents the case where, for a level of loading given, there exist several solutions displacements. This problem is thus not soluble with Newton if the problem in loading is controlled.



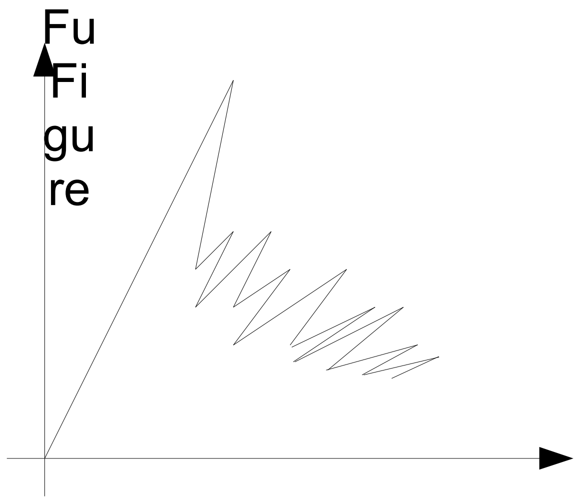
1.1-a: NON-monotonous response according to displacement

a second case (figure 1.1-b) relates to the geometrical nonlinear problems (typically the buckling of the thin shells and the problems of “soft” snap-back). In this case, for a level of loading, there are several solutions in displacement and for a level of displacement there are several possible loadings. This problem is not soluble directly by the method of Newton, whether it is while controlling in loading or displacement.



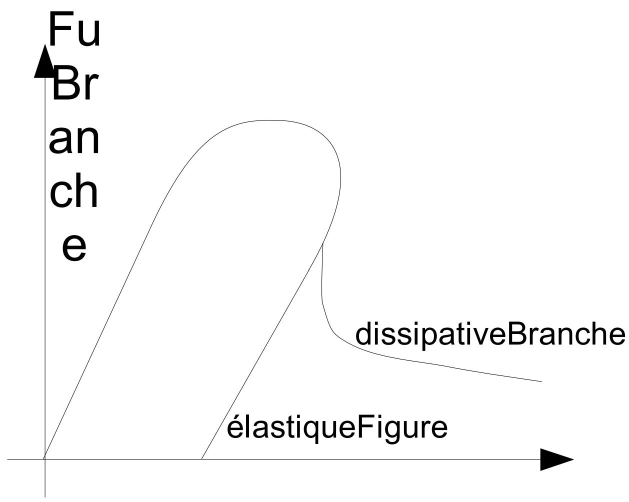
1.1-b: NON-monotonous response in displacement and loading

the third case (figure 1.1-c) described the problems of damages in which abrupt elastic returns (corresponding to the progressive damage of the material) make the curve very irregular (snap-backs “abrupt”)



1.1-c: Response with losses of stiffness

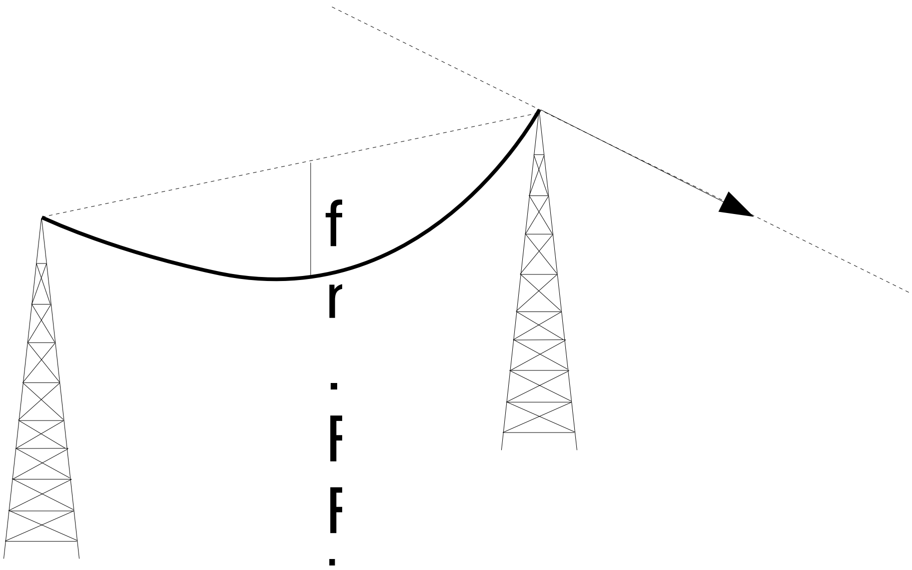
Lastly, there exists finally a whole category of problems in which the loss of ellipticity of the problem creates bifurcations and thus branches of different solutions (see figure 1.1-d). In this case, which interests more the engineer it is generally the dissipative branch and not the elastic branch. We will see that there exist techniques to select “the good” branches.



1.1-d: Response with branches multiple

1.2 Problems with intensity of the unknown loading

control can also be useful in the case or the problem displays, by its natural setting in data, an additional unknown who is the intensity of the loading applied. Control makes it possible to treat the case where only the direction and the point of application of the loading are known, intensity remaining an unknown of the problem. For example, on the figure 1.2-a, one sees the case of a cable tended between two pylons. It is known that it is necessary to apply a loading which draws the cable (one thus knows the point of application and the direction of the loading) but one is unaware of the intensity η to be applied to obtain a given f deflection.



fr.FFigure

1.2-a: Example of loading partially unknown

2 Principle of the continuation methods of the loading

In a general way, the features of control available in *Code_Aster* make it possible to determine *the intensity of part of the loading* to satisfy a stress relating to displacements. Their employment is restricted with simulations for which time does not play of physical role, which excludes *a priori* the dynamic or viscous problems. It is also incompatible with the problems expressing a condition of unilaterality (contact and friction). One can distinguish several ranges from use which answer as many continuation methods (key word factor `CONTROL`):

- Control physical forces by the displacement of a point of structure: control per imposed degree of freedom (`TYPE = ' DDL_IMPO'`);
- Follow-up of geometrical instabilities (buckling), the response of structure being able to display “soft” snap-back: control by length of arc (`TYPE = ' LONG_ARC'`);
- Follow-up of instabilities materials (in the presence of lenitive constitutive laws), the response of structure being able to display “brutal” snap-back: control by the elastic prediction (`TYPE = “PRED_ELAS”`) or more generally by the increment of strain, (`TYPE = ' DEFORMATION');`
- Computation of the Yield-point loads of structures `(TYPE=' ANA_LIM')`, cf [R7.07.01].

More precisely, the continuation methods available in *Code_Aster* rest on the two following ideas.

On the one hand, one considers that the loading (external forces and displacements given) breaks up additivement into two terms, one known and imposed by the user ($\mathbf{L}_{\text{impo}}^{\text{méca}}$ and $\mathbf{u}_{\text{impo}}^{d}$) and the other ($\mathbf{L}_{\text{pilo}}^{\text{méca}}$ and $\mathbf{u}_{\text{pilo}}^{d}$) whose only direction is known, its intensity η becoming a new unknown of problem:

$$\begin{cases}\mathbf{L}^{\text{méca}}=\mathbf{L}_{\text{impo}}^{\text{méca}}+\eta\,.\,\mathbf{L}_{\text{pilo}}^{\text{méca}}\\ \mathbf{u}^{d}=\mathbf{u}_{\text{impo}}^{d}+\eta\,.\,\mathbf{u}_{\text{pilo}}^{d}\end{cases} \quad (1)$$

In addition, in order to be able to solve the problem, one associates a new equation to him who relates to displacements and which depends on the increment of time: it is the stress of control, which is expressed by:

$$P(\Delta\mathbf{U})=\Delta\tau \quad \text{avec} \quad P(0)=0 \quad (2$$

where $\Delta\tau$ is indirectly an user datum which is expressed via time step running Δt and a coefficient of control (COEF_MULT) such as:

$$\Delta\tau=\frac{\Delta t}{\texttt{COEF\_MULT}} \quad (3)$$

the condition $P(0)=0$ is necessary in order to obtain a all the more small displacement increment as time step is small. Finally, the unknowns of the problem become displacements $\mathbf{u}$, the Lagrange multipliers $\boldsymbol{\lambda}$ associated with the boundary conditions and the intensity with the loading controlled η, baptized ETA_PILOTAGE. The nonlinear system to solve is written henceforth:

$$\begin{cases}\mathbf{L}^{\text{int}}(\mathbf{u})+\mathbf{B}^{T}.\boldsymbol{\lambda} & =\mathbf{L}_{\text{impo}}^{\text{méca}}+\eta\,.\,\mathbf{L}_{\text{pilo}}^{\text{méca}}\\ \mathbf{B}\,.\,\mathbf{u} & =\mathbf{u}_{\text{impo}}^{d}+\eta\,.\,\mathbf{u}_{\text{pilo}}^{d}\\ P(\Delta\mathbf{u}) & =\Delta\tau\end{cases} \quad (4)$$

Note:

At the present time, the following loadings (i.e which depends on displacements) and the conditions of Dirichlet of the type "DIDI ″ are not controllable.
The loading does not depend directly any more on time but results from the resolution of all the nonlinear system (4).4That implies that the controlled share of the loading should not depend on physical time, but corresponds to a force which one adjusts to satisfy an additional kinematical stress. Resolution

3 of the total system

the introduction of a new equation does not disturb in addition to measurement the method of resolution of the nonlinear system. Indeed, one proceeds as in [R5.03.01], i.e. the resolution is incremental. One notes time step in i index and the iteration of Newton while n exposing. The nonlinear problem is then solved in two times:

A phase of prediction which gives a first estimate of displacements and Lagrange multipliers noted; $(\Delta\mathbf{u}_i^0,\Delta\boldsymbol{\lambda}_i^0)$

A phase of correction of Newton which $(\delta\mathbf{u}_i^n,\delta\boldsymbol{\lambda}_i^n)$ comes to correct this first estimate; With

a sufficient number n_{CV} of iterations of Newton, one obtains one result convergé1La[1](5

$$\begin{cases}\mathbf{u}_i^{\text{convergé}}=\mathbf{u}_{i-1}+\Delta\mathbf{u}_i^0+\sum_{j=1}^{n=n_{CV}}\delta\mathbf{u}_i^j\\ \boldsymbol{\lambda}_i^{\text{convergé}}=\boldsymbol{\lambda}_{i-1}+\Delta\boldsymbol{\lambda}_i^0+\sum_{j=1}^{n=n_{CV}}\delta\boldsymbol{\lambda}_i^j\end{cases} \quad)5$$

the principle is thus to linearize the system (4)4 one writes with time step: (i 6

1 notion of result "converged" is more amply detailed in [R5.03.01].:

$$\begin{cases} \mathbf{L}_i^{\text{int}} + \mathbf{B}^T . \boldsymbol{\lambda}_i & = \mathbf{L}_{\text{impo},i}^{\text{méca}} + \eta_i . \mathbf{L}_{\text{pilo},i}^{\text{méca}} \\ \mathbf{B} . \mathbf{u}_i & = \mathbf{u}_{\text{impo},i}^{d} + \eta_i . \mathbf{u}_{\text{pilo},i}^{d} \\ P(\Delta \mathbf{u}_i) & = \Delta \tau_i \end{cases} \quad)6$$

will be no linearization compared to the variable of control. η_i This way, one preserves all the methodology of reactualization of the tangent operator already put in work for computations without control. Moreover, the structure "bandages" tangent matrix is preserved. The diagram of resolution is not thus strictly any more a method of Newton (what is not awkward because we will see that the equation of control is, at most, of degree two). In prediction, if one linearizes the system (6)6 the equation of control, compared to time around, $(\mathbf{u}_{i-1}, \boldsymbol{\lambda}_{i-1})$ one obtains (see all the development in [R5.03.01]): (7

$$\begin{cases} \mathbf{K}_{i-1} . \Delta \mathbf{u}_i^0 + \mathbf{B}^T . \Delta \boldsymbol{\lambda}_i^0 & = \mathbf{L}_{\text{impo},i}^{\text{méca}} + \eta_i . \mathbf{L}_{\text{pilo},i}^{\text{méca}} - \mathbf{Q}_{i-1}^T . \boldsymbol{\sigma}_{i-1} + \Delta \mathbf{L}_i^{\text{varc}} \\ \mathbf{B} . \Delta \mathbf{u}_i^0 & = \mathbf{u}_{\text{impo},i}^{d} + \eta_i . \mathbf{u}_{\text{pilo},i}^{d} - \mathbf{B} . \mathbf{u}_{i-1} \end{cases} \quad)7$$

correction, one always linearizes the system (66 without the equation of control, compared to time, but around, $(\mathbf{u}_i^n, \boldsymbol{\lambda}_i^n)$ one a: (8

$$\begin{cases} \mathbf{K}_i^{n-1} . \delta \mathbf{u}_i^n + \mathbf{B}^T . \delta \boldsymbol{\lambda}_i^n & = \mathbf{L}_{\text{impo},i}^{\text{méca}} + \eta_i . \mathbf{L}_{\text{pilo},i}^{\text{méca}} - \mathbf{L}_i^{\text{int},n-1} - \mathbf{B}^T . \boldsymbol{\lambda}_i^{n-1} \\ \mathbf{B} . \delta \mathbf{u}_i^n & = \eta_i . \mathbf{u}_{\text{pilo},i}^{d} \end{cases} \quad)8$$

will join together the two systems in a common writing, in order to simplify the talk. The system to be solved is written finally: (9

$$\begin{bmatrix} \mathbf{K}_i^{n-1} & \mathbf{B}^T \\ \mathbf{B} & \mathbf{0} \end{bmatrix} . \begin{Bmatrix} \delta \mathbf{u}_i^n \\ \delta \boldsymbol{\lambda}_i^n \end{Bmatrix} = \begin{Bmatrix} \mathbf{L}_{\text{impo},i}^{n-1} \\ \mathbf{u}_{\text{impo},i} \end{Bmatrix} + \eta_i . \begin{Bmatrix} \mathbf{L}_{\text{pilo},i} \\ \mathbf{u}_{\text{pilo},i} \end{Bmatrix} \quad)9$$

can pass several note:

The matrix depends $\mathbf{K}_i^{n-1}$ at the same time on time step running and possibly on the preceding iteration of Newton. The various ways build it (quasi-Newton, elastic, secant, coherent, tangent of velocity, etc) are described in [R5.03.01]; It

was supposed that the external loadings were linear (they depend only on time step). One thus does not consider the following loadings such as the pressure or the centrifugal force although from the theoretical point of view, that does not raise difficulties. On the other hand, the material can be described with a nonlinear behavior, which implies that depends $\mathbf{L}_i^{\text{int},n-1}$ on the iteration of Newton (result of the linearization of the internal forces).

The limiting conditions of Dirichlet are always linear, which makes it possible the matrix $\mathbf{B}$ to be constant on all the transient. With

the good choice of the tangent matrix and the second member, formally, there is equivalence enters and $(\delta \mathbf{u}_i^{n=0}, \delta \boldsymbol{\lambda}_i^{n=0})$ the increment in prediction. $(\Delta \mathbf{u}_i^0, \Delta \boldsymbol{\lambda}_i^0)$ One

can now express the corrections of displacements and $\delta \mathbf{u}_i^n$ Lagrange multipliers according to $\delta \boldsymbol{\lambda}_i^n$ with the help of η_i the resolution of the linear system (9)9 each of the two second members. I.e. one separates the two solutions: (10

$$\begin{Bmatrix} \delta \mathbf{u}_i^n \\ \delta \boldsymbol{\lambda}_i^n \end{Bmatrix} = \begin{Bmatrix} \delta \mathbf{u}_{\text{impo},i}^n \\ \delta \boldsymbol{\lambda}_{\text{impo},i}^n \end{Bmatrix} + \eta_i . \begin{Bmatrix} \delta \mathbf{u}_{\text{pilo},i}^n \\ \delta \boldsymbol{\lambda}_{\text{pilo},i}^n \end{Bmatrix} \quad)10$$

two solutions correspond to the decoupling of the two loadings: (11

$$\begin{Bmatrix} \mathbf{L}_i^{n-1} \\ \mathbf{u}_i \end{Bmatrix} = \begin{Bmatrix} \mathbf{L}_{\text{impo},i}^{n-1} \\ \mathbf{u}_{\text{impo},i} \end{Bmatrix} + \eta_i . \begin{Bmatrix} \mathbf{L}_{\text{pilo},i} \\ \mathbf{u}_{\text{pilo},i} \end{Bmatrix} \quad)11$$

solves indépendamment2En[2]12

2 practical, the factorization of the matrix is made only once and one solves simultaneously the two systems.: (

$$\begin{bmatrix} \mathbf{K}_i^{n-1} & \mathbf{B}^T \\ \mathbf{B} & \mathbf{0} \end{bmatrix} . \begin{Bmatrix} \delta \mathbf{u}_{\mathrm{impo},i}^n \\ \delta \boldsymbol{\lambda}_{\mathrm{impo},i}^n \end{Bmatrix} = \begin{Bmatrix} \mathbf{L}_{\mathrm{cst},i}^{n-1} \\ \mathbf{u}_{\mathrm{cst},i} \end{Bmatrix}$$)12

: (13

$$\begin{bmatrix} \mathbf{K}_i^{n-1} & \mathbf{B}^T \\ \mathbf{B} & \mathbf{0} \end{bmatrix} . \begin{Bmatrix} \delta \mathbf{u}_{\mathrm{pilo},i}^n \\ \delta \boldsymbol{\lambda}_{\mathrm{pilo},i}^n \end{Bmatrix} = \begin{Bmatrix} \mathbf{L}_{\mathrm{pilo},i}^{n-1} \\ \mathbf{u}_{\mathrm{pilo},i} \end{Bmatrix}$$)13

can now substitute the correction of displacement in $\Delta \mathbf{u}_i^n$ the equation of control of the control of the system; it results a scalar equation from it in: η_i (14

$$\tilde{P}(\eta_i) \underset{\text{déf.}}{=} P\left(\Delta \mathbf{u}_i^{n-1} + \delta \mathbf{u}_{\mathrm{impo},i}^n + \eta_i . \delta \mathbf{u}_{\mathrm{pilo},i}^n\right) = \Delta \tau_i$$)14

the method of solution of this equation depends on nature on control on control adopted cf [§7].7Finally, it any more but does not remain to reactualize the unknowns displacements and Lagrange multipliers : (15

$$\begin{cases} \Delta \mathbf{u}_i^n = \Delta \mathbf{u}_i^{n-1} + \delta \mathbf{u}_{\mathrm{impo},i}^n + \eta_i . \delta \mathbf{u}_{\mathrm{pilo},i}^n \\ \boldsymbol{\lambda}_i^n = \boldsymbol{\lambda}_i^{n-1} + \delta \boldsymbol{\lambda}_{\mathrm{impo},i}^n + \eta_i . \delta \boldsymbol{\lambda}_{\mathrm{pilo},i}^n \end{cases}$$)15

4 of control of control Control

4.1 by control of a degree of freedom of displacements: DDL_IMPO `For`

this first type of control, the function P is restricted to extract a degree of freedom from the displacement increment. In particular, it is thus about a linear function: (16

$$P(\Delta \mathbf{u}_i^n) = \langle \mathbf{S} \rangle . \{\Delta \mathbf{u}_i^n\} = \Delta \tau_i$$)16

the nodal vector is $\langle \mathbf{S} \rangle$ a vector of selection which is null everywhere except for the degree of freedom being extracted where it is worth one: (17

$$\langle \mathbf{S} \rangle = \langle 0 \ldots 0 \ldots \underset{\text{noeud n, ddl i}}{1} \ldots 0 \ldots 0 \rangle$$)17

the equation (14)14 is reduced then also to a linear equation: (18

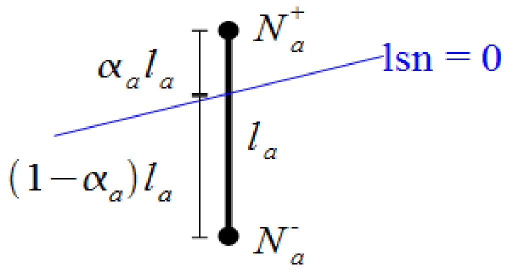
$$\eta_i = \frac{\Delta \tau_i - \langle \mathbf{S} \rangle . \{\Delta \mathbf{u}_i^{n-1}\} - \langle \mathbf{S} \rangle . \{\delta \mathbf{u}_{\mathrm{impo},i}^n\}}{\langle \mathbf{S} \rangle . \{\delta \mathbf{u}_{\mathrm{pilo},i}^n\}}$$)18

will be noted that there is no solution when the correction of controlled displacement $\delta \mathbf{u}_{\mathrm{pilo},i}^n$ does not make it possible to adjust the required degree of freedom, which can arrive if, by error, one blocks the degree of freedom in question. In this case, the code will stop in ECHEC `OF CONTROL. Control`

4.2 in modelization X-FEM by control of the jump of displacement according to a direction: SAUT_IMPO `In the case of`

a classical modelization FEM, one controls the displacement increment of only one node project on only one direction, both being specified by the user. With the finite element method wide (XFEM), one can control the increment of jump of displacement through the interface. This jump is controlled on average, for a certain number of points of intersection of the interface with the edges of the mesh and project on only one direction. $\boldsymbol{n}_a$ He is written using the detailed degrees of freedom enriched $\boldsymbol{b}$ in [R7.02.12]. For an intersected edge, a with the notations defined on the figure (4.2 4.2-a the function of control is: (19

$$P(\Delta \mathbf{u}_i^n) = P([\![\Delta \mathbf{u}]\!]_i^n) = \frac{1}{N} \sum_{aretes\ a}^{N} \boldsymbol{n}_a \cdot \left(\alpha_a \Delta \boldsymbol{b}_i^n (N_a^-) + (1-\alpha_a) \Delta \boldsymbol{b}_i^n (N_a^+)\right) = \Delta \tau_i$$)19

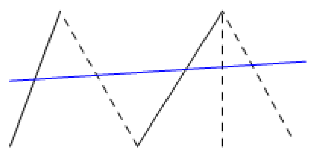


4.2 4.2-a intersected by the interface

to take again the preceding notations, the nodal vector $\langle \mathbf{S} \rangle$ is thus written: (20

$$\langle \mathbf{S} \rangle = \frac{1}{N} \langle 0 \ldots \underbrace{\alpha_1 n_x}_{\substack{noeud\ N_1^+ \\ ddl\ h_x}} \quad \underbrace{\alpha_1 n_y}_{ddl\ h_y} \ldots \underbrace{(1-\alpha_1) n_x}_{noeud\ N_1^-} \quad (1-\alpha_1) n_y \quad \ldots \quad \underbrace{\alpha_N n_x}_{noeud\ N_N^+} \quad \alpha_N n_y \ldots 0 \rangle \qquad)20$$

the edges are a selected like pertaining to a set of independent edges, i.e. that they have no joint node, and which it is impossible to add some such as this property is preserved (figure 4.2 4.2-bSuch a set of edges controlled satisfied condition LBB for the processing of the contact (see [R7.02.12]) and thus for any imposition with condition of control on the interface. Nevertheless, the condition of the type SAUT_IMPO being imposed on average, a control on all the degrees of freedom should be possible, since this one takes place in fine degree of freedom per degree of freedom. The choice of a set of independent edges is justified all the same because it largely facilitates the data-processing storage of the coefficients since one has only one information to store per degree of freedom in the vector (see $\langle \mathbf{S} \rangle$ formula 20 20In addition, the re-use of algorithms of selection explained in [R7.02.12] makes it possible to build such a group easily. Figure



4.2 4.2-b of independent edges Control

4.3 by length of arc: LONG_ARC `Another`

form of control very largely used consists in controlling the norm of the displacement increment (compared to some nodes and certain components): one speaks then about control by cylindrical length of arc, to see Bonnet and Wood [bib1]. More precisely, the function P is expressed by: (21

$$P\left(\Delta \mathbf{u}_i^n\right) = \|\Delta \mathbf{u}_i^n\|_{<S>} = \sqrt{\left(\langle \mathbf{S} \rangle . \{\Delta \mathbf{u}_i^n\}\right) . \left(\langle \mathbf{S} \rangle . \{\Delta \mathbf{u}_i^n\}\right)} = \Delta \tau_i \qquad)21$$

, again, the nodal vector makes it possible $\langle \mathbf{S} \rangle$ to select the degrees of freedom employed for the computation of the norm (it is worth for the selected degrees of freedom and zero elsewhere). In this case there one will notice that $\langle \mathbf{S} \rangle . \{\Delta \mathbf{u}_i^n\}$ is not a scalar, but a vector corresponding to the product of the components from $\langle \mathbf{S} \rangle$ those of. $\{\Delta \mathbf{u}_i^n\}$ In this case, the equation of control is reduced to a quadratic equation: (22

$$\eta_i^2.\left[\left(\langle\mathbf{S}\rangle.\{\delta\,\mathbf{u}^n_{\text{pilo},i}\}\right).\left(\langle\mathbf{S}\rangle.\{\delta\,\mathbf{u}^n_{\text{pilo},i}\}\right)\right]+$$
$$2.\eta_i.\left[\left(\langle\mathbf{S}\rangle.\{\Delta\,\mathbf{u}_i^{n-1}+\delta\,\mathbf{u}^n_{\text{impo},i}\}\right).\left(\langle\mathbf{S}\rangle.\{\delta\,\mathbf{u}^n_{\text{pilo},i}\}\right)\right]+ \qquad)22$$
$$\left[\left(\langle\mathbf{S}\rangle.\{\Delta\,\mathbf{u}_i^{n-1}+\delta\,\mathbf{u}^n_{\text{impo},i}\}\right).\left(\langle\mathbf{S}\rangle.\{\Delta\,\mathbf{u}_i^{n-1}+\delta\,\mathbf{u}^n_{\text{impo},i}\}\right)-\Delta\tau_i^2\right]=0$$

equation can not admit a solution. In this case, one chooses the value which η_i minimizes the polynomial (22)22One checks then well. $\tilde{P}(\eta_i)>\Delta\tau_i$ In the contrary case, she admits two roots (or a double root). One chooses that of both which minimizes: that is to say

the norm of the displacement increment; $\|\Delta\,\mathbf{u}_i^n\|$ that is to say

the residue of equilibrium; $\mathbf{R}(\mathbf{u}_i,t_i)=\mathbf{L}_i^{\text{int}}-\mathbf{L}_i^{\text{méca}}$ that is to say

the angle formed by and $\Delta\,\mathbf{u}_{i-1}$ (where $\Delta\,\mathbf{u}_i^n$ is $\Delta\,\mathbf{u}_{i-1}$ the displacement increment solution of time step preceding), i.e. that which maximizes the cosine of this angle whose statement is: (23

$$\cos(\Delta\,\mathbf{u}_{i-1},\Delta\,\mathbf{u}_i^n)=\frac{\langle\Delta\,\mathbf{u}_i^{n-1}+\delta\,\mathbf{u}^n_{\text{impo},i}+\eta_i.\delta\,\mathbf{u}^n_{\text{pilo},i}\rangle.\{\Delta\,\mathbf{u}_{i-1}\}}{\|\Delta\,\mathbf{u}_i^{n-1}+\delta\,\mathbf{u}^n_{\text{impo},i}+\eta_i.\delta\,\mathbf{u}^n_{\text{pilo},i}\|.\|\Delta\,\mathbf{u}_{i-1}\|} \qquad)23$$

the angle, one uses only the unknowns corresponding to displacements, other than all the others (in particular rotations in the structural elements). It

is noticed that this last angular criterion selects the bad solution when one passes from a phase of load to a phase of discharge or vice versa, i.e. when the coefficient of control changes C_i sign. The value C_{i-1} thus is put in memory and is brought up to date with each time step converged, its initialization for each call to STAT_NON_LINE `being` transparent for the user: C_{i-1} is thus initialized during the reading of the initial state, C_i having been included in the parameters of computation to file in the result concept. If , $C_i C_{i-1}>0$ one maximizes the criterion (23)23 In the contrary case, one minimizes it in order to select the displacement increment in opposite meaning of the preceding increment. Control

4.4 in modelization X-FEM by control of the norm of the jump of displacement: SAUT_LONG_ARC `In the case of`

a modelization XFEM, control by length of arc consists in controlling the norm of the increment of the jump of displacement for a certain number of points of intersection of the mesh with the interface, which are selected on a set of independent edges in the same way that into 4.2 4.2The function of control is written then: (24

$$P(\Delta\,\mathbf{u}_i^n)=P([\Delta\,\mathbf{u}]_i^n)=\sqrt{\frac{1}{N}\sum_{aretes\,a}^{N}\left[\alpha_a\Delta\,\boldsymbol{b}_i^n(N_a^-)+(1-\alpha_a)\Delta\,\boldsymbol{b}_i^n(N_a^+)\right]^2}=\Delta\tau_i \qquad)24$$

we develop this last equation, we have: (25

$$P([\Delta\,\mathbf{u}]_i^n)^2=$$
$$\underbrace{\frac{1}{N}\sum_{aretes\,a}^{N}\left(\alpha_a\Delta\,\boldsymbol{b}_i^n(N_a^-)\right)^2+\left((1-\alpha_a)\Delta\,\boldsymbol{b}_i^n(N_a^+)\right)^2}_{N\|\langle\mathbf{S}\rangle.\{\Delta u_i^n\}\|^2}+\underbrace{\frac{1}{N}\sum_{aretes\,a}^{N}2\alpha_a(1-\alpha_a)\Delta\,\boldsymbol{b}_i^n(N_a^+)\Delta\,\boldsymbol{b}_i^n(N_a^-)}_{N([M]\langle\mathbf{S}\rangle.\{\Delta u_i^n\})\cdot(\langle\mathbf{S}\rangle.\{\Delta u_i^n\})} \qquad)25$$

is $\langle\mathbf{S}\rangle$ the nodal vector defined by the formula 20 20 will be noticed that $\langle\mathbf{S}\rangle.\{\Delta\,\mathbf{u}_i^n\}$ is not a scalar, but a vector corresponding to the product of the components from $\langle\mathbf{S}\rangle$ those of), $\{\Delta\,\mathbf{u}_i^n\}$ and stamps $[M]$ it which with the degrees of freedom of a node makes correspond the degrees of freedom of the node located on the same edge independent, at the other end, i.e.: (26

$$\forall \text{arête}\, a\,, \forall\, \text{direction}\, i\,, [M]\langle \mathbf{S^i}(N_a^+)\rangle = \langle \mathbf{S^i}(N_a^-)\rangle \quad et \quad [M]\langle \mathbf{S^i}(N_a^-)\rangle = \langle \mathbf{S^i}(N_a^+)\rangle$$
$$\text{où}\, \langle \mathbf{S^i}(N)\rangle = \langle 0\ldots \underset{\substack{\text{noeud } N\\ \text{direction } i}}{1} \ldots 0\rangle \qquad)26$$

, the notation in terms of vectors is: (27

$$P\left(\Delta \mathbf{u}_i^n\right) = \sqrt{N \|\langle \mathbf{S}\rangle.\{\Delta u_i^n\}\|^2 + N\left([M]\langle \mathbf{S}\rangle.\{\Delta u_i^n\}\right)\cdot\left(\langle \mathbf{S}\rangle.\{\Delta u_i^n\}\right)} = \Delta\tau_i \qquad)27$$

of the initialization of control, it is thus necessary to initialize besides the nodal vector $\langle \mathbf{S}\rangle$ a structure which $[M]$ locates which points belong to the same edge. While breaking up, $\{\Delta \mathbf{u}_i^n\}$ one finds an equation similar to (22)22 takes account of the additional term. For

the modelization XFEM, the methods of selection of the solution remain strictly identical, put except for the replacement in the statements of the criteria of norm and angle of displacement by the jump of displacement through the interface. Thus, the angular criterion amounts for example maximizing. $\cos\left(⟦\Delta \mathbf{u}⟧_{i-1}, ⟦\Delta \mathbf{u}⟧_i^n\right)$ Control

4.5 by the increment of strain: DEFORMATION

control by increment of strain consists in requiring that the increment of strain of the step running remain close in the direction of the strain at the beginning to time step, and this for at least a Gauss point of structure. One requires thus qualitatively whom minimum **a point** of structure has preserves the mode of strain that it had as a preliminary (for example, tension in a given direction). Case

4.5.1 of the small strains For

the small strains, one can give an account of this requirement with the help of the choice of the following function of control: (28

$$P\left(\Delta \mathbf{u}_i^n\right) = \underset{g}{\text{Max}} \frac{\boldsymbol{\varepsilon}_{g,i-1}}{\|\boldsymbol{\varepsilon}_{g,i-1}\|}.\Delta \boldsymbol{\varepsilon}_{g,i} = \Delta\tau_i \qquad)28$$

the index sweeps g Gauss points structure (or only meshes specified by keyword GROUP_MA `in` CONTROL `) and` where the strain in a Gauss point results from the nodal vector of displacements via the symmetric use of the matrixes “left the gradient of the shape functions” ($\mathbf{B}_g$ not to be confused with the matrix of the conditions of Dirichlet): and

$$\boldsymbol{\varepsilon}_{g,i-1} = \mathbf{B}_g \cdot \mathbf{u}_{i-1} \;\; (29 \;\; \Delta \boldsymbol{\varepsilon}_{g,i} = \mathbf{B}_g \cdot \Delta \mathbf{u}_i^n \qquad)29$$

the control of control according to η_i is written then: (30

$$P\left(\Delta \mathbf{u}_i^n\right) = \underset{g}{\text{Max}}\left(A_g^{(0)} + \eta_i \cdot A_g^{(1)}\right) = \underset{g}{\text{Max}}\left(L_g(\eta_i)\right) = \Delta\tau_i \qquad)30$$

the two terms: (31

$$A_g^{(0)} = \frac{\boldsymbol{\varepsilon}_{g,i-1}}{\|\boldsymbol{\varepsilon}_{g,i-1}\|} \cdot \mathbf{B}_g\left(\Delta \mathbf{u}_i^{n-1} + \delta\, \mathbf{u}_{\text{impo},i}^n\right) \qquad)31$$

: (32

$$A_g^{(1)} = \frac{\boldsymbol{\varepsilon}_{g,i-1}}{\|\boldsymbol{\varepsilon}_{g,i-1}\|} \cdot \mathbf{B}_g\left(\delta\, \mathbf{u}_{\text{pilo},i}^n\right) \qquad)32$$

control does not depend on the constitutive law, provided which it uses the tensor of the small strains (option DEFORMATION = ' PETIT' or DEFORMATION = ' PETIT_REAC'). Case

4.5.2 of the large deformations In the presence of

large deformations, one can generalize the function of control (28)28 employing strains of Green-Lagrange (Lagrangian measurement of the strains in the initial configuration): (33

$$P\left(\Delta \mathbf{u}_i^n\right)=\underset{g}{\operatorname{Max}} \frac{\mathbf{E}_{g,i-1}}{\left\|\mathbf{E}_{g,i-1}\right\|} . \Delta \mathbf{E}_{g,i} \qquad)33$$

the measurement of the strains $\mathbf{E}$ is written according to the tensor gradient of transformation: $\mathbf{F}$ with

$$\mathbf{E}=\frac{1}{2}\cdot\left(\mathbf{F}^T . \mathbf{F}-\mathbf{I}\right) \quad (34 \quad \mathbf{F}=\mathbf{I}+\nabla \mathbf{u} \qquad)34$$

, one would not lead any more like previously to a function closely connected per pieces. To cure it, one decides to carry out a linearization from $\Delta \mathbf{E}_{g,i}$ ratio with. $\Delta \mathbf{u}_i^{n-1}$ P a statement similar has then to (30 30 with: (35

$$A_g^{(0)}=\frac{\mathbf{E}_{g,i-1}}{\left\|\mathbf{E}_{g,i-1}\right\|}\cdot \operatorname{sym}\left[\mathbf{F}_{g,i}^T . \nabla_g\left(\Delta \mathbf{u}_i^{n-1}+\delta \mathbf{u}_{\text{impo},i}^n\right)\right] \qquad)35$$

: (36

$$A_g^{(1)}=\frac{\mathbf{E}_{g,i-1}}{\left\|\mathbf{E}_{g,i-1}\right\|}\cdot \operatorname{sym}\left[\mathbf{F}_{g,i}^T . \delta \mathbf{u}_{\text{pilo},i}^n\right] \qquad)36$$

$\nabla_g\left(\mathbf{u}\right)$ the gradient (not symmetrized) of displacements evaluated at $\mathbf{u}$ the Gauss point of index indicates. g Just like in the preceding case, this control does not depend on the constitutive law, provided which it uses the tensor of the large deformations. Resolution

4.5.3 of the nonlinear equation of control

the function is $P\left(\Delta \mathbf{u}_i^n\right)$ convex and linear per pieces. She generally admits no, one or two solutions, cf appears (4.6.4-a4.6.4-aWhen she does not admit solutions, the algorithm of STAT_NON_LINE (failure of control) is stopped: if the user uses the automatic subdivision of the time step, this last will then be activated. If she admits two solutions, three opportunities are given: one chooses that of both which minimizes: that is to say

the norm of the displacement increment; $\left\|\Delta \mathbf{u}_i^n\right\|$ that is to say
the residue of equilibrium; $\mathbf{R}\left(\mathbf{u}_i, t_i\right)=\mathbf{L}_i^{\text{int}}-\mathbf{L}_i^{\text{méca}}$ that is to say
the angle formed by and $\Delta \mathbf{u}_{i-1}$; $\Delta \mathbf{u}_i^n$

To solve the equation (30)30 one proposes the algorithm presented below. It is based on the construction of encased intervals: the limits of the last of them are the solutions of the equation and, as announced previously, one chooses that which leads to $\tilde{\mathbf{u}}\left(\eta\right)$ nearest to. $\mathbf{u}_{i-1}$ This algorithm, rapid, lean on the solution of linear G scalar equations, where G the nombre total indicates of Gauss points. The algorithm can end prematurely when one of the intervals is empty, which means that the equation (30)30 does not admit solutions. (1

) Initialization of the interval (2) — $I_0=]-\infty,+\infty[$
Loop on Gauss points (2.0 g
) the slope is null (2.0 — Si $A_g^{(1)}=0$
.1) If failure $A_g^{(0)}>\Delta\tau$ If not
one continues (2.1
) Root of the linear function activates (2.2 — η_g tel que $L_g(\eta_g)=\Delta\tau$
) Construction of the following interval (2.2

.1) If (2.2 the active linear function is increasing $\text{Si } A_g^{(1)}>0 \Rightarrow I_g = I_{g\text{-}1} \cap]-\infty, \eta_g]$

.2) If (2.3 the active linear function is decreasing $\text{Si } A_g^{(1)}<0 \Rightarrow I_g = I_{g\text{-}1} \cap [\eta_g, +\infty[$

) To stop if the interval is empty (3

) the solutions are the limits of the intervalle3On[3] $\eta \in \mathrm{Fr}(I_G) \Rightarrow \max_g L_g(\eta) = \Delta\tau$

of resolution of the equation closely connected per pieces Control

4.6 by the elastic prediction: PRED_ELAS Algorithm

4.6.1 If

control by the increment of strain proves to be sufficient to follow dissipative solutions in most instabilities materials, the existence of solutions nevertheless is not proven. One then prefers a continuation method founded to him on the elastic prediction for which the existence of solutions is shown but which, on the other hand, is specific to each constitutive law (established only for models ENDO_SCALAIRE , ENDO_FRAGILE , ENDO_ISOT_BETON , ENDO_ORTH_BETON , BETON_DOUBLE_DP , VMIS _ISOT_* and those relating to the elements of joints [R 5.06.09]). More precisely, when the constitutive law is controlled by a threshold, one on all the defines P as the maximum Gauss points of the value of the function threshold in the case of an elastic test (elastic incremental response of the material). Thus

, let us consider that the state of the material is described by the strain and $\boldsymbol{\varepsilon}$ a set of local variables. a Respectively and the let us call $\boldsymbol{\sigma}(\boldsymbol{\varepsilon}, a)$ $A(\boldsymbol{\varepsilon}, a)$ forced and the thermodynamic forces associated with. a Let us suppose moreover that the laws of evolution of a are controlled by a threshold: $f(A, \boldsymbol{\varepsilon}, a)$ (37

$$f(A, \boldsymbol{\varepsilon}, a) \leq 0 \text{ et } \lambda \cdot f(A, \boldsymbol{\varepsilon}, a) = 0 \quad)37$$

a flow function: $G(A, \boldsymbol{\varepsilon}, a)$ (38

$$\dot{a} = \lambda\, G \cdot (A, \boldsymbol{\varepsilon}, a) \quad)38$$

a formulation includes most models of behavior dissipative and independent the rate loading. The function threshold is worth then for an elastic test: (39

$$f^{\mathrm{el}}(\boldsymbol{\varepsilon}) = f\left(A(\boldsymbol{\varepsilon}, a_{i-1}), \boldsymbol{\varepsilon}, a_{i-1}\right) \quad)39$$

simplifies the problem while linearizing compared to f^{el} in $\boldsymbol{\varepsilon}$ the vicinity of the point such as $\boldsymbol{\varepsilon}^*$: $f^{\mathrm{el}} = \Delta\tau$ (40

$$f_L^{\mathrm{el}}(\boldsymbol{\varepsilon}) \underset{\mathrm{def}}{=} f^{\mathrm{el}}(\boldsymbol{\varepsilon}^*) + \left(\frac{\partial f}{\partial A} \cdot \frac{\partial A}{\partial \boldsymbol{\varepsilon}} + \frac{\partial f}{\partial \boldsymbol{\varepsilon}}\right)\Bigg|_{\boldsymbol{\varepsilon}^*} \cdot (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^*) \quad)40$$

the choice of makes it possible $\boldsymbol{\varepsilon}^*$ to solve the exact problem, as shown in [bib3]. It thus requires to solve a local problem nonlinear for each Gauss point, then the total algorithm is based on lines, which ensures a great effectiveness to him. There exists zero, one or two solutions with the local problem. $f^{\mathrm{el}} = \Delta\tau$ In the case of absence of solution to the local problem, the total problem cannot have solution: one stops. If not (case with one or two solutions), one linearizes around each solution. Finally

, the function of control of control is defined like the maximum of compared to f_L^{el} all Gauss points, g function which depends only on: $\Delta\mathbf{U}$ (41

$$P(\Delta\mathbf{u}_i^n) = \max_g f_L^{\mathrm{el}}\left(\boldsymbol{\varepsilon}_{g,i-1} + \mathbf{B}_g \cdot \Delta\mathbf{u}_i^n\right) = \Delta\tau_i \quad)41$$

, the equation of control of control is written: (42

$$P(\Delta\mathbf{u}_i^n) = \max_g\left(A_g^{(0)} + \eta_i \cdot A_g^{(1)}\right) = \max_g\left(L_g(\eta_i)\right) = \Delta\tau_i \quad)42$$

: (43

3 note $\eta \in \mathrm{Fr}(I_G)$ the fact that belongs η at the boundaries of the interval Algorithm I_G

$$A^{(0)}=f^{el}(\boldsymbol{\varepsilon}^*)+\left(\frac{\partial f}{\partial A}\cdot\frac{\partial A}{\partial \boldsymbol{\varepsilon}}+\frac{\partial f}{\partial \boldsymbol{\varepsilon}}\right)\Bigg|_{\boldsymbol{\varepsilon}^*}\cdot\left(\mathbf{B}_g\left(\mathbf{u}_{i-1}+\Delta\mathbf{u}_i^{n-1}+\delta\mathbf{u}_{\text{impo},i}^n\right)-\boldsymbol{\varepsilon}^*\right)$$)43

: (44

$$A^{(1)}=\left(\frac{\partial f}{\partial A}\cdot\frac{\partial A}{\partial \boldsymbol{\varepsilon}}+\frac{\partial f}{\partial \boldsymbol{\varepsilon}}\right)\Bigg|_{\boldsymbol{\varepsilon}^*}\cdot\left(\mathbf{B}_g\cdot\delta\mathbf{u}_{\text{pilo},i}^n\right)$$)44

4.6.2 according to the constitutive laws One

is thus brought back to a problem identical to that of control by the increment of the strain. One of course employs the same algorithms of resolution as those presented in the preceding paragraph (see §11)11

The characteristics of the equation of control depend on the constitutive law because the statement of the elastic function threshold is f^{el} itself a nonlinear fact of the case of behavior: For

- the elements with internal discontinuity PLAN_ELDI and AXIS_ELDI , the constitutive law is of type CZM_EXP and the form of the model of control will be in documentation [R7.02.14]. The elastic function threshold is written; $f_{el}(\sigma_n,\sigma_t)=\sqrt{<\sigma_n>_+^2+\sigma_t^2}-R(\kappa)$ For
- the elements of joint * _ JOINED, the constitutive laws are regularized models CZM CZM_EXP_REG and CZM_LIN_REG (see [R7.02.11]), and for the elements of the type interfaces * _ INTERFACE, the constitutive laws are models CZM NON-regularized like CZM_OUV_MIX and CZM_TAC_MIX . The form of the model of control will be in documentation [R7.02.11]. The elastic function threshold is written; $f_{el}\simeq \max_{pt\ gauss}\left\{\frac{\|\delta^0+\eta\,\delta^1\|-\kappa^-}{G_c/\sigma_c+\kappa^-}\right\}=\frac{\Delta\tau}{C}$ For
- model ENDO_ISOT_BETON ([R7.01.04]), there is a characteristic. Instead of seeking a parameter of control which η makes time step leave the criterion a value $\Delta\tau$ with the damage resulting from preceding, one seeks a parameter which η brings back for us on the criterion with a damage increased by. $\Delta\tau$ The elastic function threshold is thus written; $\tilde{f}^{el}(\eta,d^-)=\Delta\tau \;\Rightarrow\; \tilde{f}^{el}(\eta,d^-+\Delta\tau)=0$ For
- model ENDO_ORTH_BETON , to see [R7.01.09]; For
- the other damage models of the elastic type like ENDO_SCALAIRE or ENDO_FRAGILE (with local formulation or with gradients), just like for the model ENDO_ISOT_BETON , the function elastic threshold is selected so as to seek a parameter of control controls by the value of damage and not by the output of the criterion. The elastic function threshold is written: (45

$$\tilde{f}^{\text{el}}(\eta)=\frac{1}{2}\left(\boldsymbol{e}_0+\eta\,\boldsymbol{e}_1\right)\cdot\boldsymbol{E}\cdot\left(\boldsymbol{e}_0+\eta\,\boldsymbol{e}_1\right)-s$$

$$\begin{cases}\text{loi locale} & \boldsymbol{e}_0=\boldsymbol{\epsilon}_0 \quad e_1=\boldsymbol{\epsilon}_1 \quad s=k(d^-)\\ \text{gradient d'endommagement} & \boldsymbol{e}_0=\boldsymbol{\epsilon}_0 \quad \boldsymbol{e}_1=\boldsymbol{\epsilon}_1 \quad s=k(d^-)-c\,\Delta d^-\\ \text{déformation régularisée} & \boldsymbol{e}_0=\bar{\boldsymbol{\epsilon}}_0 \quad \boldsymbol{e}_1=\bar{\boldsymbol{\epsilon}}_1 \quad s=k(d^-)\end{cases}$$)45

4.6.3 of the algorithm of maximization In order to

illustrate our matter and in particular to include why such a choice of makes it possible $\boldsymbol{\varepsilon}^*$ to solve the exact problem, one takes the example of a cohesive constitutive law CZM_EXP_REG or CZM_LIN_REG (cf [R7.02.11]) implemented on an element of joint (cf [R3.06.09]). The jump of displacement is $\boldsymbol{\delta}$ the primal variable (noted generically in $\boldsymbol{\epsilon}$ what precedes), and κ the local variable (noted generically). A The function threshold is written then: (46

$$f^{\grave{e}l}(\boldsymbol{\delta})=\frac{\|\boldsymbol{\delta}\|_{\grave{e}q}-\kappa_{i-1}}{\kappa_{REF}} \text{ avec } \kappa_{i-1}=\max_{v\in[0,t_{i-1}]}\|\boldsymbol{\delta}(v)\|_{\grave{e}q} \;;\; \|\boldsymbol{\delta}\|_{\grave{e}q}=\sqrt{\langle\delta_n\rangle_+^2+\delta_\tau^2} \quad)46$$

detailing the demonstration, one can show that checks $\|\boldsymbol{\delta}\|_{\grave{e}q}$ a triangular inequality. This enables us to prove the convexity of the function: (47

$$f^{\grave{e}l}(\eta)=f^{\grave{e}l}(\boldsymbol{\delta}(\eta))=\frac{\|\boldsymbol{\delta}_{i-1}+\Delta\boldsymbol{\delta}_i^n+d\,\boldsymbol{\delta}^{cte}+\eta\,d\,\boldsymbol{\delta}^{pilo}\|_{\grave{e}q}-\kappa_{i-1}}{\kappa_{REF}} \quad)47$$

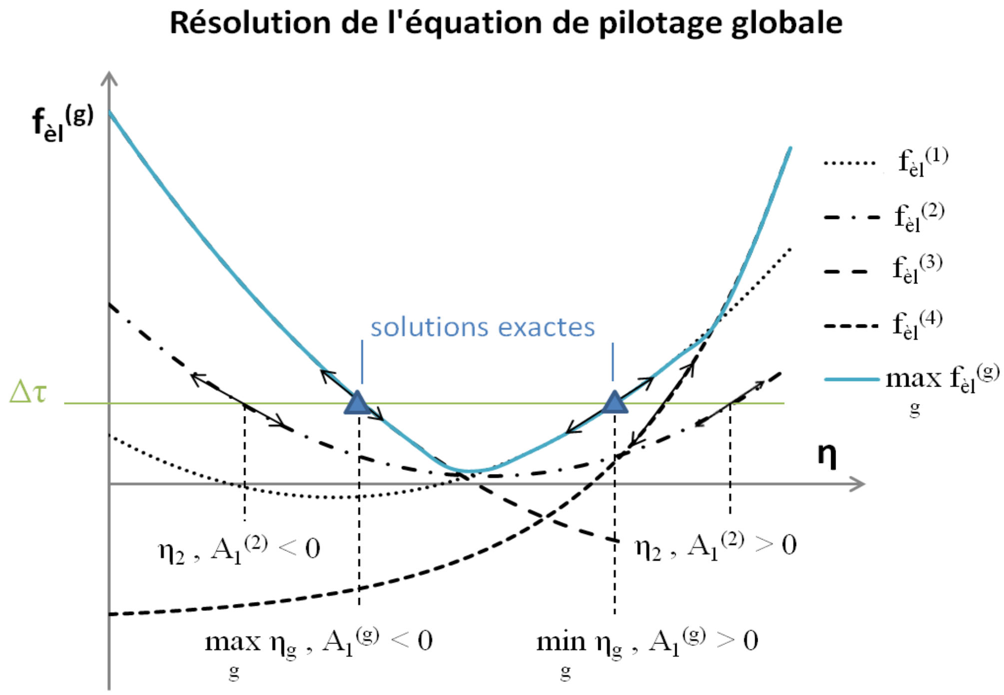
traced in figure 4.6 4.6.3-a a qualitative chart **of** the functions for $f^{\grave{e}l}$ the different ones Gauss points from the elements of joint, as well as the solutions of the exact problem of maximization total on the group of Gauss points: It

$$\max_g f_g^{\grave{e}l}(\eta)\overset{d\acute{e}f}{=}\max_g f^{\grave{e}l}(\boldsymbol{\delta}_g(\eta))=\Delta\tau_i$$

appears whereas the solutions of η this exact problem are identical to the solutions obtained with the algorithm of resolution of the equation closely connected per pieces, i.e. the solutions of the equation (42)42, $\max_g A_1^{(g)}\eta+A_0^{(g)}=\Delta\tau_i$ with in our example: where

$A_1^{(g)}=\frac{\partial f_g^{\grave{e}l}}{\partial\eta}(\eta_g^*) \;;\; A_0^{(g)}=\Delta\tau-A_1^{(g)}\eta_g^*$ is η_g^* solution of the local equation We $f_g^{\grave{e}l}(\eta)=\Delta\tau_i$

once more return the reader to [3] for a rigorous demonstration. Figure

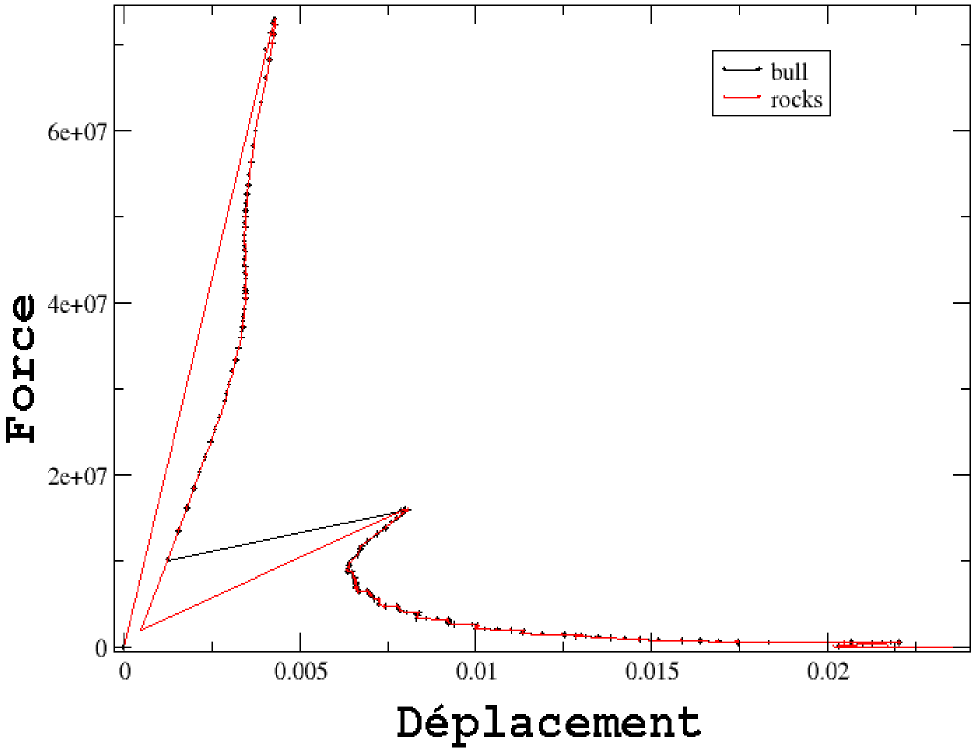


4.6 4.6.3-a of the equation of control, qualitative chart Choice

4.6.4 of solution If

the total problem has two solutions, the algorithms of choices described into 4.3 4.3 are re-used. In general, the selection by residue is retained because of its robustness. However, the fact of choosing the solution which minimizes the residue does not guarantee us the convergence of the method of Newton . Certain cases could be displayed for which the solution of maximum initial residue converges while it is not the case of the solution of minimal initial residue. In general

for computations controlled time step filled more the role of loading, but just a parameter of follow-up of the solution. Although the mechanical solutions are identical, one can thus obtain a significant difference with the same step in control for the problems with important instabilities, by changing for example the object computer. What is illustrated on figure 4.6 4.6.4-a during the snap-back in the total response force-displacement one observes the shift of time of computation between the two curves. Figure



4.6 4.6.4-a of shift of the time of control on a curved force-displacement for a strongly unstable problem according to the machine accuracy Bibliography

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1) J. and Wood R.D. [1997]. Nonlinear continuum mechanics for finite element analysis. Cambridge university near. Bonnans

2) J.F., Gilbert J. - C., Lemaréchal C. and Sagastizabal C. [1997]. Numerical optimization: theoretical and practical aspects. Mathematics and Applications, 27, ED. Springer . Lorentz

3) E. and Badel P. [2001]. A new path-following constraint for strain-softening finite element simulations. International newspaper for numerical methods in engineering. 2004, vol. 60, No 2, pp. 499 – 526. Shi

4) J. and Crisfield Mr. A. [1995]. Combining arc-length control and line searches in path following. Com. Num. Meth. Eng. 11, pp. 793-803. Description

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Mr.	Abbas EDF - R&D/AMA Put	in conformity of the notations with [R5.03.01], addition of an explanatory introduction of the utility of control 10.5
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