# Summarized method

**IMPLEX:**

This document presents a method of resolution of the nonlinear problem, due to *Oliver and al.* [1], substituent with the method of Newton for certain constitutive laws of damage and plasticity (ENDO_FRAGILE [3], ENDO_ISOT_BETON [4] and VMIS_ISOT_LINE [5] at the present time). It is based on an explicit extrapolation of the local variables to determine the degrees of freedom (displacement) from which the behavior is integrated implicitly. The nullity of the residue is not checked. It introduces of this fact an approximation of the resolution but makes it possible to guarantee the robustness of computation.

It thus belongs to the user to have a critical glance on the got results, those being not converged in the classical sense of the term , and potentially being able to be far away from the exact solution; one consequently advises to carry out several computations with different increments of load to make sure that the variation of results obtained is weak.

In cases of brutal expansion of the damaged zone resulting in a fort snap-back total response forces/displacement, the method, if it makes it possible to cross instability, could not be reliable in term of result.

# 1 Introduction

One presents here a method of resolution, robust but approximate, incremental problem of nonlinear quasi-static mechanics, for certain behaviors of damage and plasticity. It is activated by the key word `METHODE = "IMPLEX"` of operator `STAT_NON_LINE`, and replaces the classical method of `NEWTON` (cf [2]).

# 2 Position of the problem

One is placed in the general frame of the resolution of a standard problem of nonlinear mechanics discretized in space $(K \in \Omega)$ and time $(t \in [0, T])$, and written in displacement. Its resolution, at time $t_{n+1}$, consists in determining displacements $\boldsymbol{U}_{n+1}$ (thus strains $\boldsymbol{\varepsilon}(\boldsymbol{U}_{n+1})$ ), the local variables $\alpha_{n+1}$ and the forced $\boldsymbol{\sigma}_{n+1}$ checking:

(I) At the total level, equilibrium of the forces:

$$\boldsymbol{F}_{\mathbf{ext}}(t_{n+1}) - \boldsymbol{F}_{\mathbf{int}}(\boldsymbol{\sigma}_{n+1}(\boldsymbol{U}_{n+1}), t_{n+1}) \quad (1)$$

(II) At the local level, constitutive equations of the constitutive law considered for the material:

State model: $\boldsymbol{\sigma}_{n+1} = \sum (\boldsymbol{\varepsilon}(\boldsymbol{U}_{n+1}), \alpha_{n+1})$

$$\text{Law of evolution:} \begin{cases} f(\alpha_{n+1}, \boldsymbol{\sigma}_{n+1}) \leqslant 0 & \text{Convexe de réversibilité} \\ \dot{\alpha}_{n+1} = \dot{\lambda}_{n+1} \geqslant 0 & \text{Evolution des variables internes} \\ \dot{\lambda}_{n+1} f(\alpha_{n+1}, \boldsymbol{\sigma}_{n+1}) = 0 & \text{Condition de Kuhn-Tucker} \end{cases} \quad (2)$$

with $\boldsymbol{F}_{\mathbf{ext}}$ and the $\boldsymbol{F}_{\mathbf{int}}$ external and internal forces, and $\dot{\lambda}_{n+1}$ the multiplier (of damage or plasticity). One considers in the continuation a behavior independent of physical time (not of viscosity nor of dynamic effect); "pseudo-time" $t_{n+1}$ represents consequently the load factor applied.

In the majority of the cases, each equation (1) and (2) is of relatively easy resolution.

At the total level, the method of Newton leads to a succession of linear problems of the type:

$$\boldsymbol{K}^{i}_{n+1} \delta \boldsymbol{U}^{i+1}_{n+1} = \boldsymbol{F}_{\mathbf{ext}}(t_{n+1}) - \boldsymbol{F}_{\mathbf{int}}(\boldsymbol{\sigma}^{i}_{n+1}(\boldsymbol{U}^{i}_{n+1})) \quad (3)$$

where the high index represents the iteration of Newton considered, $\delta \boldsymbol{U}^{i+1}_{n+1}$ is the displacement increment between two successive iterations of Newton and $\boldsymbol{K}^{i}_{n+1}$ is the total tangent matrix. Let us note that this total tangent matrix is written (cf [2])

$$\begin{aligned} \boldsymbol{K}_{n+1} &= \frac{\partial \boldsymbol{F}_{\mathbf{int}}(\boldsymbol{\sigma}_{n+1}(\boldsymbol{U}_{n+1}))}{\partial \boldsymbol{U}_{n+1}} \\ &= \mathbf{A}^{p}_{e=1}\left(\int_{\Omega_e} \nabla N_e . \boldsymbol{C}_{e_{n+1}} . \nabla N_e \, d\Omega\right) \end{aligned} \quad (4)$$

where $\mathbf{A}$ is the operator of assembly, $p$ the number of elements of the mesh, $\boldsymbol{C}_{e_{n+1}}$ the local tangent operator (resulting from the behavior) and the $N_e$ shape functions (for the element considered, noted $e$ ). The quantity $\boldsymbol{F}_{\mathbf{ext}}(t_{n+1}) - \boldsymbol{F}_{\mathbf{int}}(\boldsymbol{\sigma}^{i}_{n+1}(\boldsymbol{U}^{i}_{n+1}))$ represents the residue of equilibrium, which it is necessary in any rigor to cancel. To determine $\delta \boldsymbol{U}^{i+1}_{n+1}$ is thus limited to the inversion of the matrix $\boldsymbol{K}^{i}_{n+1}$. If thus the local elements resulting from the behavior (forced, tangent matrix) are known and fixed, and generate a well $\boldsymbol{K}$ conditioned matrix, the resolution is easy.

Moreover more locally, if displacements $\boldsymbol{U}_{n+1}$ , and consequently the local strains $\boldsymbol{\varepsilon}(\boldsymbol{U}_{n+1})$ , are known, the resolution of the majority of the constitutive laws, i.e. the determination of the stresses $\boldsymbol{\sigma}_{n+1}$ , the local variables $\alpha_{n+1}$ and the local tangent matrix $\boldsymbol{C}_{e_{n+1}}$ , is relatively easy.

It is thus in the simultaneity of these two models that resides the implicit difficulty of resolution. The most classical method of resolution is the iterative algorithm of Newton (cf [2]). This iterative diagram always does not converge. Although that remains prone to discussion (there exist other possible factors of loss of convergence, such as the output of the basin of attraction of Newton or the presence of bifurcations of the solution), the authors allot mainly this loss of robustness to the singular character of the matrixes, in particular for the damage models, whose local variables impact the stiffness matrixes of the behavior: in the cases of important damage, near or reaching the fracture, the softening led to tangent matrixes locally very “weak”, which can to the extreme not be definite positive more. Through the process of assembly (4), when the damage progresses in structure, the total matrix $\boldsymbol{K}$ is deteriorated: it becomes too much “flexible”, and its minimal eigenvalue tends towards zero. It becomes singular then and the algorithm diverges. The robustness of computation is not then assured any more.

To increase the robustness of computation in these situations, Oliver *and al.* [1] propose a special method of resolution, baptized IMPLEX. The total equilibrium is then checked roughly using a tangent matrix (secant in the case of damage models) local explicitly extrapolated and assembled, and the constitutive laws are solved implicitly starting from the field of approximate displacement. The method is presented hereafter.

# 3 Method IMPLEX

the general elements of method IMPLEX are presented here. For more details, one can refer in particular to [1].

The method suggested is based on two successive stages carried out to determine all the unknowns at “pseudo-time” (charges) $t_{n+1}$ .

The first stage consists of *an explicit extrapolation* of the local variables, then stresses, according to the quantities calculated previously (with the load $t_n$ ) and step of load $\Delta t_{n+1}$ . Thanks to this extrapolation, the local tangent matrix is evaluated and solidified; the resolution of the balance equation (3) makes it possible to determine displacements, which one considers right and which are thus fixed in their turn.

The second stage consists in carrying out *the implicit integration* of the behavior, according to the degrees of freedom evaluated at the preceding stage.

The first stage is of explicit type, whereas second is of implicit type, from where the name of IMPLEX. At the conclusion of these two stages, the balance equation is not checked (it is it in term of the fields extrapolated at the stage (1) only); however, more the increment of load is small, more the made mistake must be weak; one advises of this fact of carrying out computations with several different steps of loads in order to check that the difference between the responses obtained is weak (response converged in term of step of load). In the continuation, the two stages are described. An assessment of the key points is made.

## 3.1 A method in 2 stages

We here will successively present the 2 fundamental stages of method IMPLEX, synthesized in Table 1.

### 3.1.1 Extrapolation clarifies and determination of the degrees of freedom

This first stage relates to in particular the local variable $\alpha$, whose evolution is governed by a model of the type (2).

With the beginning of the step of load $t_{n+1}$, one has all information resulting from the step from load $t_n$ and the former steps of load. It is then possible to write the following developments of Taylor (the evolution of the local variable is considered sufficiently regular to be able to do it):

$$\begin{cases} \alpha_{n+1} & = & \alpha_n+\Delta t_{n+1}\dot{\alpha}_n+O(\Delta t_{n+1}^2) \\ \alpha_n & = & \alpha_{n-1}+\Delta t_n\dot{\alpha}_{n-1}+O(\Delta t_n^2) \\ \Delta t_n\dot{\alpha}_n & = & \Delta t_n\dot{\alpha}_{n-1}+\Delta t_n(\Delta t_n\ddot{\alpha}_{n-1})+\Delta t_n O(\Delta t_n^2) \quad \Rightarrow \Delta t_n\dot{\alpha}_n=\Delta t_n\dot{\alpha}_{n-1}+O(\Delta t_n^2) \end{cases} \quad (5)$$

$$\Rightarrow \begin{cases} \dot{\alpha}_n & = & \dfrac{\Delta\alpha_n}{\Delta t_n}-O(\Delta t_n^2) \\ \alpha_{n+1} & = & \alpha_n+\dfrac{\Delta t_{n+1}}{\Delta t_n}\Delta\alpha_n-\Delta t_{n+1}O(\Delta t_n^2)+O(\Delta t_{n+1}^2) \end{cases}$$

with $\Delta X_i=X_i-X_{i-1}$. By truncation, by neglecting the terms of order two, one obtains the following prediction $\tilde{\alpha}_{n+1}$ for the variable $\alpha_{n+1}$:

$$\tilde{\alpha}_{n+1}=\alpha_n+\frac{\Delta t_{n+1}}{\Delta t_n}\Delta\alpha_n \quad (6)$$

Through (6), it is noted that $\tilde{\alpha}_{n+1}$ is well obtained explicitly, with the load $t_{n+1}$, according to the quantities obtained implicitly with the load $t_n$ . Figure 1 schematizes this process of extrapolation.

The error of extrapolation can be defined like the difference between the value of the extrapolated local variable and its real end value; subject to a sufficiently regular evolution of the local variable to carry out of them the developments of Taylor to the sufficient order, one can evaluate this error with:

$$e_{\alpha_{n+1}}=|\tilde{\alpha}_{n+1}-\alpha_{n+1}|\approx|\ddot{\alpha}_n|\Delta t_{n+1}^2 \quad (7)$$

This shows that in the case of sufficiently regular evolution of the local variable, the error decreases in a quadratic way according to the step of load.

Once the extrapolated local variable, one can determine the stress $\tilde{\boldsymbol{\sigma}}_{n+1}(\boldsymbol{\varepsilon}_{n+1},\tilde{\alpha}_{n+1})$ and the local tangent operator $\tilde{\boldsymbol{C}}_{e_{n+1}}$ .

In the typical case of the isotropic damage models, it is the local secant operator who is used; the extrapolated stresses and this operator are written then:

$$\begin{cases} \tilde{\boldsymbol{\sigma}}_{n+1}(\boldsymbol{\varepsilon}_{n+1},\tilde{\alpha}_{n+1}) & = & (1-\alpha_{n+1})\boldsymbol{C}^{\mathbf{elas}}:\boldsymbol{\varepsilon}_{n+1} \\ \tilde{\boldsymbol{C}}_{e_{n+1}}=\dfrac{\partial\tilde{\boldsymbol{\sigma}}_{n+1}}{\partial\tilde{\boldsymbol{\varepsilon}}_{n+1}} & = & (1-\tilde{\alpha}_{n+1})\boldsymbol{C}^{\mathbf{elas}} \end{cases} \quad (8)$$

These extrapolated local quantities are used for the assembly of the total tangent matrix $\tilde{\boldsymbol{K}}_{\boldsymbol{n+1}}$ *via* (4), and to the determination of the internal force.

The balance equation in term of extrapolated field $\boldsymbol{F}_{\mathbf{ext}}(t_{n+1})-\boldsymbol{F}_{\mathbf{int}}(\boldsymbol{U}_{\boldsymbol{n+1}},\tilde{\boldsymbol{\sigma}}_{n+1},t_{n+1})=\mathbf{0}$ is then solved with this extrapolated matrix and the field of displacement $\tilde{\boldsymbol{U}}_{n+1}$ obtained. It is solved by a method of Newton-Raphson classical; it is thus linearized par. $\tilde{\boldsymbol{K}}_{\boldsymbol{n+1}}\delta\tilde{\boldsymbol{U}}_{\boldsymbol{n+1}}=\boldsymbol{F}_{\mathbf{ext}}(t_{n+1})-\boldsymbol{F}_{\mathbf{int}}\left(\tilde{\sigma}_{n+1},t_{n+1}\right)$ In the case of the constitutive laws treated here (isotropic damage with secant operator and linear isotropic plasticity), the operator of behavior is constant during one time step (i.e. independent of the strain state running $\boldsymbol{\varepsilon}_{n+1}$ ); the process of resolution of Newton-Raphson becomes linear by time step and thus converges in an iteration. This stage is however iterative in certain cases (nonlinear plasticity for example), because of the nonconstant character of the tangent operator. With the first iteration, one would take then $\boldsymbol{\varepsilon}_{n+1}^{0}=\boldsymbol{\varepsilon}_{n}$ .

At the end of this first stage, the field of displacement $\boldsymbol{U}_{n+1}$ is regarded as equal to the field obtained by the resolution of the balance equation written in term of the extrapolated fields: $\boldsymbol{U}_{n+1}=\tilde{\boldsymbol{U}}_{n+1}$ . This field of displacement will not be modified any more thereafter.

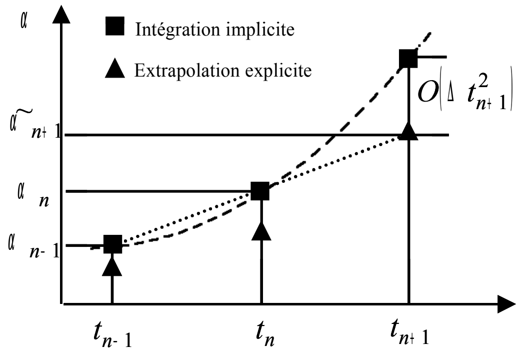


***Figure 1: Schematization of the method of implicit***

### 3.1.2 extrapolation Determination of the elements of the constitutive law

Following the first phase, the field of displacement is known. It is fixed for the step of load and will thus not be modified.

One determines the strain then $\boldsymbol{\varepsilon}_{n+1}(\boldsymbol{U}_{n+1})$ , then the equations (2) of the constitutive law are implicitly solved in order to obtain the stress fields $\boldsymbol{\sigma}_{n+1}$ and of local variable $\alpha_{n+1}$ .

This stage is identical to the standard resolution of the equations of the constitutive laws. The only difference is the need for storing $\dfrac{\Delta\alpha_{n+1}}{\Delta t_{n+1}}$ to carry out the extrapolation of the local variable to the following step.

At the end of this stage, the field of displacement $\boldsymbol{U}_{n+1}$ and the stress fields $\boldsymbol{\sigma}_{n+1}$ and local variable $\alpha_{n+1}$ are thus known. A major difference compared to an implicit classical computation is the fact that the real balance equation is not checked; it is it only in term of the extrapolated fields.

| **Stage 1: Extrapolation Clarifies** | **Stage 2: Implicit integration** |
|---|---|
| Extrapolation of the local variables: $\tilde{\alpha}_{n+1}=\alpha_n+\dfrac{\Delta t_{n+1}}{\Delta t_n}\Delta\alpha_n$ | Fixed displacements: $\dot{\boldsymbol{U}}_{n+1}=\boldsymbol{0}$ |
| Computation of the extrapolated stress: $\tilde{\boldsymbol{\sigma}}_{n+1}(\boldsymbol{\varepsilon}_{n+1},\tilde{\alpha}_{n+1})$ | Implicit resolution of the constitutive laws: |

| | |
|---|---|
| Resolution of the balance equation in extrapolated fields: $\boldsymbol{F}_{\text{ext}}(t_{n+1})-\boldsymbol{F}_{\text{int}}(\boldsymbol{U}_{n+1},\tilde{\boldsymbol{\sigma}}_{n+1},t_{n+1})=\mathbf{0}$ | State model: $\boldsymbol{\sigma}_{n+1}=\sum(\boldsymbol{\varepsilon}(\boldsymbol{U}_{n+1}),\alpha_{n+1})$<br>Law of evolution: $\begin{cases} f(\alpha_{n+1},\boldsymbol{\sigma}_{n+1})\leqslant 0 \\ \dot{\alpha}_{n+1}=\dot{\lambda}_{n+1}\geqslant 0 \\ \dot{\lambda}_{n+1} f(\alpha_{n+1},\boldsymbol{\sigma}_{n+1})=0 \end{cases}$ |
| **Output of stage 1:** $\boldsymbol{U}_{n+1}$ | **Output of stage 2:** $\boldsymbol{U}_{n+1}$ $\boldsymbol{\sigma}_{n+1}$ , $\alpha_{n+1}$ |

**Table 1: Summary of automatic method**

## 3.2 IMPLEX Management of time step

introduced `method` IMPLEX, like all the explicit methods, an intrinsic error which must decrease in a quasi-quadratic way with the step of load. The solution can thus depend slightly on the step of load selected by the user.
This one can, if it wishes it, use an automatic management of the step of step of load *via* command `DEFI_LIST_INST`, by specifying `METHODE= "AUTO"` and `MODE_CALCUL_TPLUS=' IMPLEX'` (cf [9]). This method makes it possible to control the error while optimizing the computing time provided the user chose a first time step gauged well.
The goal is thus to minimize the error defined by the equation (7). For that one will maximize the increment of variable extrapolated by a noted quantity $Tol$ :

$$\Delta\tilde{\alpha}_{n+1}=\frac{\Delta t_{n+1}}{\Delta t_n}\Delta\alpha_n\leqslant Tol \quad (9)$$

From where:

$$\Delta t_{n+1}\leqslant\frac{Tol.\Delta t_n}{\Delta\alpha_n} \quad (10)$$

As the increments of local variable depend on the point considered, time step will be selected like the minimal value of the statement (10) on the group of structure, that is to say finally:

$$\Delta t_{n+1}=Tol.\Delta t_n MIN_{x\in\Omega}\left(\frac{1}{\Delta\alpha_n(x)}\right) \quad (11)$$

One adds conditions then limiting acceleration and deceleration, as well as limits minimal and maximum for the increment of time. The conditions of acceleration $(\mu)$ and deceleration are not modifiable and were gauged on practical cases, whereas the limits minimal and maximum can be modified by the user (one gives the values by default here).

$$\begin{gathered}\Delta t_{n+1}\leqslant\mu\Delta t_n=1{,}2\Delta t_n\\ \Delta t_{n+1}\geqslant 0.5\Delta t_n\\ \Delta t_{n+1}\leqslant 10\Delta t_0\\ \Delta t_{n+1}\geqslant 0.001\Delta t_0\end{gathered} \quad (12)$$

Thus, the first time step $\Delta t_0$ , provided by the user, defines the limits of the increment of time. To choose it, one advises with the user to have carried out preliminary computations with the method of Newton and to know the yield stress of structure; it would seem that the choice of a first time step equal to half of the yield stress allows a good effectiveness of the method.

Let us show whereas the made mistake is well controlled. For that one makes the assumption that equation 9 is checked. One has thus $\frac{\Delta t_{n+1}}{\Delta t_n}\Delta\alpha_n\leqslant Tol$ , and if it is considered that the factors limits acceleration and deceleration make it possible to write $\frac{\Delta t_{n+1}}{\Delta t_n}\approx 1$ , it has $\Delta\alpha_n\leqslant Tol$ ; and gradually:

$$\forall n\in\mathbb{N},\Delta\alpha_n\leqslant Tol \quad (13)$$

What finally makes it possible to write:

$$e_{\alpha_{n+1}}\approx|\ddot{\alpha}_n|\Delta t_{n+1}^2\leqslant Tol\,\mu^2(1+\mu) \quad (14)$$

the error is thus controlled.

## 3.3 Key points of the method

After this summary presentation, some remarks are made here to understand the interest of the method and also its limitations.

First of all, it is based on an extrapolation of the local variables, determined from development of Taylor. The developments of Taylor being valid only for sufficiently regular functions, the method will be it too. Thus, the crossing of the yield stress, or the transition of a state of load to discharge, are points for which the method is not in any not adapted rigor: the derivative of the damage is null side of the discharge or yield stress, and non-zero on the side of the load. However, if the steps of load are sufficiently small, the approximation can be made, insofar as the implicit correction takes place and thus that the errors of extrapolations are partly gummed. In situations unstable however, for example when the damaged zone grows brutally (what is characterized in general by an important snap-back of the total response forces/displacement), the method, although robust, cannot guarantee a reliable response, whatever the increment of load used: it is not adapted to this kind of situation.

According to the equation (8), in the case of isotropic damage models, because of limitation of $\alpha$ to 1 and the symmetric and definite nature positive of the local elastic matrix $\boldsymbol{C}^{\mathbf{elas}}$ , the local secant matrix $\tilde{\boldsymbol{C}}_{e_{n+1}}$ is always symmetric definite positive. So by assembly, the total tangent matrix $\tilde{\boldsymbol{K}}_{\boldsymbol{n+1}}$ remains conditioned well: the problems of robustness mentioned in introduction, dependant on the increasingly singular character of $\boldsymbol{K}$ , must thus be eliminated.

Moreover, for a step of load given and the constitutive laws developed here (`ENDO_FRAGILE` confer [3], `ENDO_ISOT_BETON` confer [4] and `VMIS_ISOT_LINE` confer [5]), the local tangent matrix is known by extrapolation clarifies and remains constant during all the step of load. The linearization of the balance equation (1) by a method of Newton-Raphson leads to a constant total tangent matrix $\tilde{\boldsymbol{K}}_{\boldsymbol{n+1}}$ ; in other words, the balance equation becomes linear with each step of load, and its resolution requires only one iteration.

Then, and to the risk to be redundant, the method leads to external forces and interns not balanced at the end of each step of load; they are it only at the end of the first phase, i.e. only in term of extrapolated fields:

$$\begin{cases}\boldsymbol{F}_{\mathbf{ext}}(t_{n+1})-\boldsymbol{F}_{\mathbf{int}}(\boldsymbol{\sigma}_{\boldsymbol{n+1}}(\boldsymbol{U}_{\boldsymbol{n+1}}),t_{n+1}) & \neq & \mathbf{0}\\ \boldsymbol{F}_{\mathbf{ext}}(t_{n+1})-\boldsymbol{F}_{\mathbf{int}}(\boldsymbol{U}_{\boldsymbol{n+1}},\tilde{\boldsymbol{\sigma}}_{n+1},t_{n+1}) & = & \mathbf{0}\end{cases} \quad (15)$$

Consequently, during computation, one should not require of the algorithm to check the residue except for a tolerance, as it is usual to do it (into implicit). From this point of view still, the robustness is guaranteed, since the classical criterion of stop is without object.

To finish, this method has the role only to increase the robustness of computation, and not the quality of the response obtained. Thus, it introduces an intrinsic error by the means of extrapolation. At best, one can get only results almost as good as those obtained by an implicit classical method of local resolution. This error, must decrease in a quasi-quadratic way according to the step of load imposed, subject to a sufficiently regular evolution of the local variable (what excludes in fact the unstable propagations of damaged zones).

The use of this method thus requires a certain critical glance. In order to secure strong errors, one recommends to carry out computations with increments of loads different and small, in order to check that the solution does not differ too much from one increment to another (in other words, that the solution is close to convergence in term of increments of load). Moreover, in the case of crossing of unstable situations, the got results should be considered only for their qualitative aspect.

# 4 Practical aspects of use

This method of resolution is activated while specifying some, under key word simple METHODE of operator STAT_NON_LINE, METHODE= "IMPLEX" and by specifying the constitutive law used under COMP_INCR.
Only certain constitutive laws are currently available with method IMPLEX. Table 2 recapitulates the constitutive laws available according to the type of elements considered.

| **Surface elements or voluminal** | **Elements of bar** |
|---|---|
| ELAS | ELAS |
| VMIS_ISOT_LINE | VMIS_ISOT_LINE |
| ENDO_ISOT_BETON | |
| ENDO_FRAGILE | |

**TABLEAU 2: constitutive laws available with method IMPLEX**

For each constitutive law, the last local variable is modified and corresponding in this case to the ratio $\frac{\Delta\alpha}{\Delta t}$.

The method imposes a reactualization of the matrix on each increment (REAC_INCR = 1) and only one iteration. The residue of equilibrium is calculated, but no criterion is associated for him. One will be able to realize of possible an important error by seeing that the relative residue is high.

For more information on the practical aspects, one will refer to [6] and [7]. Method IMPLEX is illustrated through the case test SSNP140 [8].

# 5 Bibliography

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[2] Manual of reference R5.03.01 Aster, quasi-static nonlinear Algorithm

[3] Manual of reference R5.03.18 Aster, Damage model of a brittle elastic material

[4] Manual of reference Aster R7.01.04-C, Constitutive law ENDO_ISOT_BETON

[5] Manual of reference R5.03.02 Aster, elastoplastic Integration of the behavior models of Von Mises

[6] Instruction manual U4.51.03 Aster, Operator STAT_NON_LINE

[7] Instruction manual U4.51.11 Aster, nonlinear Behaviors

[8] Manual of validation V06.03.140 Aster, Plate perforated in tension with method IMPLEX

[9] Instruction manual U4.34.03 Aster, Operator DEFI_LIST_INST

# 6 Description of the versions of the document

| **Version Aster** | **Author (S) Organization (S)** | **Description of the modifications** |
|---|---|---|
| 10.2 | R.BARGELLINI-S.FAYOLLE R & D /AMA | initial Text |
| 10 4 | R.BARGELLINI | Addition of automatic management of time step |