Nonlocal modelization with deformation gradient

Summarized

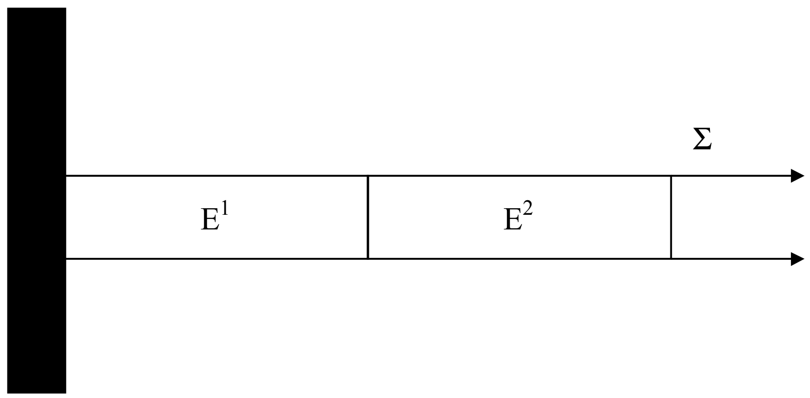
This document presents a model of delocalization of the constitutive laws by regularization of the strain. It introduces an additional nodal variable: regularized strain, dependant on the local strain by an equation of regularization of type least square with penalization of the gradient which one simultaneously solves with the classical balance equation. The regularized strains are used for the computation of the evolution of the local variables (and not for the computation of the stresses!). This method makes it possible to avoid certain problems involved in the digital processing of the local problems like the dependence on the discretization.

1 Nature of the formulation

In the presence of damage (softening), the local constitutive laws lead to a badly posed problem which numerically results in a localization of the strains in a tape of thickness a mesh: with the limit, one breaks without dissipating energy.
There exist several extensions to the local models which make it possible to mitigate this problem of localization (relaxation of potential energy, enrichment of the kinematics, theories with gradient, models nonlocal). This document deals with nonlocal model with deformation gradient, modelization `*_GRAD_EPSI`, drifting of the model with deformation gradient equivalent suggested by Peerlings and al. (1995). One introduces interactions between the material point and his spatial vicinity by regularizing the strains thanks to an operator of delocalization. The regularized strains are then used to evaluate the evolution of the local variable.

It should be noted however that the stresses are calculated starting from the local strains because the use of the strains regularized in the computation of the stresses would return to "too much regularizing" the problem, which would call into question the existence even of solutions. One is convinced some easily thanks to the following example:
Let us consider a bar made up of 2 different materials which have different Young moduli. One exerts on this bar a simple tension. The 2 elements being assembled in series, the stress is equal in the two elements:

$$\sigma = E^1 \varepsilon^1 = E^2 \varepsilon^2 = \sum$$



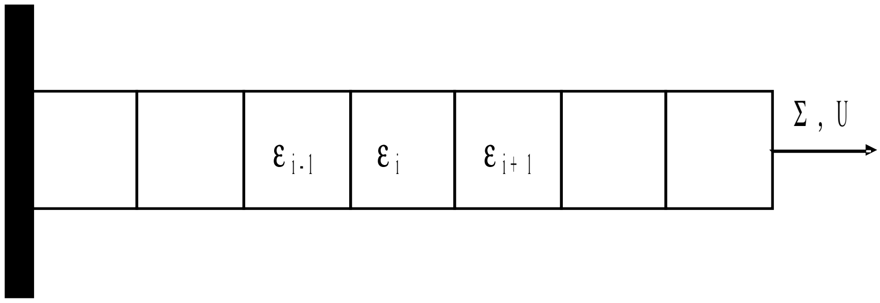
On the interface between the two elements, the discontinuity of modulus Young thus imposes a discontinuity of the strain. Let us consider now either the local strain but a delocalized strain. The classical operators of delocalization cause to return continues the strain in structure, which then generates obligatorily a discontinuity of stress to the interface because of the difference of Young modulus, and this goes against the balance equation.

The regularization of the strains leads us to introduce a characteristic length definite by the operator `DEFI_MATERIAU` under factor key word `NON the LOCAL` which conditions the width of the tapes of localization. The scales thus are not defined any more by the digital processing of the problem but by a material parameter.

2 Limits of the local models

One initially proposes to illustrate the phenomenon of localization in the simple case of a bar subjected to a uniaxial tension.

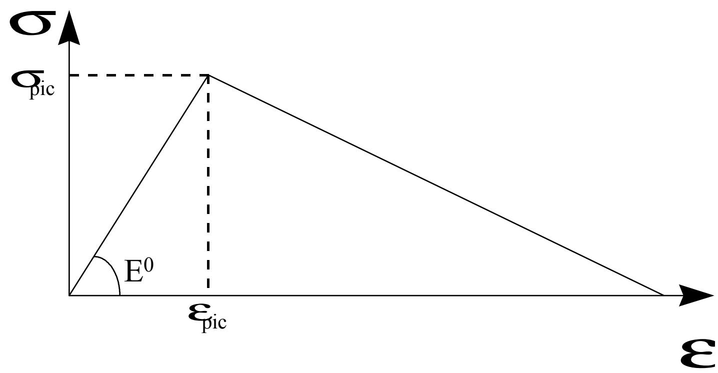
One thus regards an assembly of identical elements assembled in series subjected to a tension as represented on [Figure 2-a].



Appear 2-a: Assembly of identical elements assembled in series subjected to a traction test

Each element obeys the same constitutive law of the elastic type endommageable with softening [Figure 2-b]. The state of the material is described by two variables which are the strain ε and the damage characterized by the scalar variable d . This variable is worth 0 when the material is operational and grows up to 1 when it is completely damaged.

We will not enter here in detail of the equations governing such a behavior of the material. Let us specify simply that these equations make it possible to describe the behavior of the material completely. They indeed give us access to the stresses and the damage according to strain rate, to see for example [R5.03.18].



Appear 2-b: Constitutive law of the material in simple tension uniaxial

the elements of the studied bar are assembled in series, which implies, because of the balance equation of structure, the equality of the stress in all the elements:

$$\sigma_i = \Sigma$$

One can consequently study the total response of the assembly with a simple traction test. This response breaks up into two phases. In a first phase, the behavior of all the elements is elastic and the damage remains null. The response of structure thus exists and is single. The strain is identical for all the elements and is worth:

$$\varepsilon_i = \frac{\Sigma}{E^0}$$

This phase continues as much as the peak of stress is not reached. Microphone-heterogeneities of the material imply light fluctuations of the field of elasticity between the various elements, which will involve the damage of a fastener before the others. The second phase starts when one of the elements which one notes A damages. The stress in the group of structure reached its maximum. While continuing the tension, the stress supported by structure will decrease. The element A having passed the peak, it is in the lenitive phase of the behavior of the material, which means that it will continue to damage itself during the tension. The other elements did not reach the critical point, they thus simply will undergo an elastic discharge during the decrease of the stress. This phase finishes when the element A is completely damaged. Finally, the damage as well as the strain thus concentrated in only one element.

One then understands easily the numerical consequences of the localization. The phenomenon describes previously on a simple sample will occur whatever the structure with a grid by finite elements. For reasons of stability, the localised solution tends to being selected. The damage and the strain will concentrate in a tape of thickness an element and any refinement of the mesh then will modify the total response of structure. One understands then well that it is impossible to describe the scale of the tapes of localization, the length of the damaged tape coming from the mesh and not from a physical principle. Moreover, one result obtains one physically inadmissible from an energy point of view. Indeed, the energy dissipated at the time of the damage will depend on the refinement of the mesh, and one can even imagine the total fracture of a structure without energy expense if an extremely fine mesh is considered.

3 Formulation with regularized strains

3.1 Principle

One considers the state of the material defined locally by the strain ε and of the local variables α . The data of the free potential energy $\varphi(\varepsilon,\alpha)$ makes it possible to define the stress σ .
In a general way, the constitutive law is given by the statement of the stress and the law of evolution of the local variables:

$$\begin{aligned}&\sigma(\varepsilon,\alpha)\\&\dot{\alpha}=g(\dot{\varepsilon},\varepsilon,\alpha)\end{aligned}$$

The principle of the method of delocalization of the strains is to use the strains regularized in the model devolution of the local variables:

$$\begin{aligned}&\sigma(\varepsilon,\alpha)\\&\dot{\alpha}=g(\dot{\bar{\varepsilon}},\bar{\varepsilon},\alpha)\end{aligned}$$

One thus understands the generality of the method which does not force to reconsider the integration of the constitutive law. It is indeed the same one as for the model local but by replacing ε par. $\bar{\varepsilon}$ It is necessary nevertheless well to distinguish the computation from the local variables, which utilizes regularized strains, of that of the stresses, which utilizes only the local strains.

3.2 Choice of the operator of delocalization

the choice of the operator of regularization is purely arbitrary and does not lean on any physical reasoning. One however may find it beneficial to choose an operator who is integrated easily and directly in `STAT_NON_LINE` by the finite element method. Thus, the use of an integral formulation, where the coupling between the finite elements on the level of the integration of the constitutive laws causes to increase considerably the bandwidth of the tangent matrix and to thus increase the number of operations to be carried out, is not judicious. The operator of regularization retained, proposed by Peerlings and al. (1995), employs a delocalization by least squares with penalization of the gradient:

$$\mathrm{R}(\varepsilon)=\min_{\bar{\varepsilon}}\int_{\Omega}\frac{1}{2}(\bar{\varepsilon}-\varepsilon)^2+\frac{1}{2}\left(L_c\nabla\bar{\varepsilon}\right)^2 d\,\Omega$$

The term in gradient introduces the interaction between the material point and its vicinity and makes it possible to limit the strong concentration of deformation gradient. To minimize such an integral amounts solving the following differential equation:

$$\bar{\varepsilon}-L_c^2\Delta\bar{\varepsilon}=\varepsilon$$

One sees appearing a major interest of the choice of this operator of regularization. The differential equation can be integrated classically by the finite element method, and this without introducing new not linearities. It is enough for that to introduce new nodal variables representing the generalized strains.
There exists moreover a tangent matrix of reasonable bandwidth (compared to an integral formulation) but it should be noted that the tangent matrix is not symmetric, as one will further see it by clarifying the tangent matrix.

Note:

The operator of delocalization was modified in the case of the damage models quasi-brittle (`ENDO_FRAGILE, MAZARS, ENDO_ISOT_BETON` and `ENDO_ORTH_BETON`), in order to try to solve the problem of inopportune widening of the tape of damage when the material is ruined (cf [feeding-bottle 5]). An operator of relaxation depending on the damage is introduced into the operator of regularization:

$$\bar{\varepsilon}=\mathbf{R}\,\varepsilon=\arg\min_{\bar{\varepsilon}}\int_{\Omega}\left[\frac{1}{2}\|\mathbf{A}^{relax}:(\bar{\varepsilon}-\varepsilon)\|^2+\frac{1}{2}\|L_c\vec{\nabla}\bar{\varepsilon}\|^2\right]d\,\Omega$$

where $\mathbf{A}^{relax}$ of a nature 4 making it possible to release the strain regularized compared to the local strain in the completely damaged directions is a tensor. The tensor $\mathbf{A}^{relax}$ is worth the identity when the damage is not total. For the isotropic models (`ENDO_FRAGILE, MAZARS, ENDO_ISOT_BETON`), $\mathbf{A}^{relax}$ becomes the null tensor of order 4 when the damage is total (one release in all the directions). In the case of orthotropic model `ENDO_ORTH_BETON` , the tensor $\mathbf{A}^{relax}$ cancels itself only in the completely damaged directions (cf [feeding-bottle 5]).

3.3 Variational formulation

In the model, two equations control the process of strain, on the one hand the classical balance equation and on the other hand the differential equation characterizing the regularization of the strains. The integral formulation of our problem is the following one:

$$\forall v\in V^{ad} \qquad \int_{\Omega}(\nabla v)^C:\sigma\,d\,\Omega=\int_{G}v.T dG+\int_{\Omega}f\,dV$$

V^{ad} : space acceptable displacements
T : forces imposed on edge G

$$\forall \mathrm{e}\in\left[H^1(\Omega)\right]^6 \qquad \int_{\Omega}\left(e\,\bar{\varepsilon}+\nabla e.L_c^2\nabla\bar{\varepsilon}\right)d\,\Omega=\int_{\Omega}e\,\varepsilon\,d\,\Omega$$

the limiting conditions for the generalized strains are the natural conditions rising from the equation of regularization. They are of Neumann type:

$$\nabla \bar{\varepsilon}.n = 0$$

One indeed imposes no particular condition on edge in the equation of regularization.

4 Discretization

4.1 Equations discretized

the balance equation discretized between the external forces and interior is classical form (cf [R5.03.01]):

$$F_{\text{int}}+D^T\lambda=F_{ext}$$

with $F_{\text{int}}=\int_\Omega B^T\sigma\,d\Omega$ and $F_{ext}=\int_\Gamma N^T T\,d\Gamma$
(D^T : cf B^T of [R5.03.01])

where N are the shape functions associated with the field with displacement and the B derived from the shape functions.

The differential equation on the regularized strains is discretized in the same way:

$$K^{\varepsilon\varepsilon}\bar{\varepsilon}=F^{\varepsilon}$$

with $K^{\varepsilon\varepsilon}=\int_\Omega\left(\tilde{N}^T\tilde{N}+L_c^2\tilde{B}^T\tilde{B}\right)d\Omega$ and $F^{\varepsilon}=\int_\Omega\tilde{N}^T\varepsilon\,d\Omega$

where $\tilde{N}$ are the shape functions associated with the strain field generalized and the $\tilde{B}$ derived from the shape functions. It should be noted here that the shape functions associated with the generalized strains are different from the shape functions associated with displacements.

The nodal residues associated with these two equations are the following ones:

$$F^u=F_{\text{int}}+D^T\lambda-F_{ext}$$
$$F^{\bar{\varepsilon}}=K^{\varepsilon\varepsilon}\bar{\varepsilon}-F^{\varepsilon}$$

The tangent matrix associated with the resolution of this system by the method of Newton is the following one:

$$K=\begin{pmatrix}\dfrac{\partial F^u}{\partial u} & \dfrac{\partial F^u}{\partial\bar{\varepsilon}}\\ \dfrac{\partial F^{\bar{\varepsilon}}}{\partial u} & \dfrac{\partial F^{\bar{\varepsilon}}}{\partial\bar{\varepsilon}}\end{pmatrix}$$

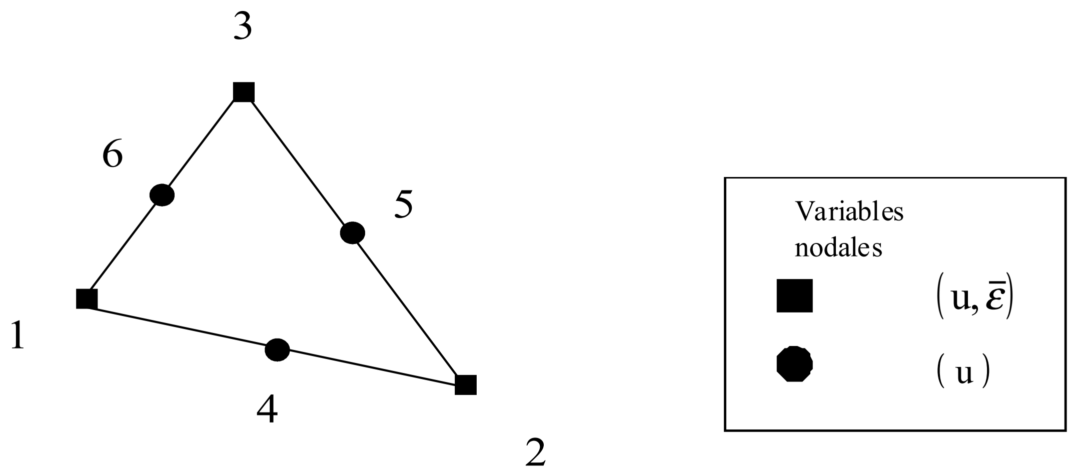
The various blocks of the tangent matrix are the following:

$$\frac{\partial F^u}{\partial u}\Big|_{i-1}=\int_\Omega B^T\frac{\partial\sigma}{\partial\varepsilon}B\,d\Omega$$
$$\frac{\partial F^u}{\partial\bar{\varepsilon}}\Big|_{i-1}=\int_\Omega B^T\frac{\partial\sigma}{\partial\bar{\varepsilon}}\tilde{N}\,d\Omega$$
$$\frac{\partial F^{\bar{\varepsilon}}}{\partial\bar{\varepsilon}}\Big|_{i-1}=\int_\Omega\left(\tilde{N}^T\tilde{N}+L_c^2\tilde{B}^T\tilde{B}\right)d\Omega$$
$$\frac{\partial F^{\bar{\varepsilon}}}{\partial u}\Big|_{i-1}=\int_\Omega-\tilde{N}^T B\,d\Omega$$

It should be noted that the tangent matrix is NON-symmetric.

4.2 Choice of the finite elements

the introduction of new nodal variables forces to use new elements compatible with the new formulation. One is in the presence of two nodal unknowns: displacements and the regularized strains. The strain being the spatial derivative of a displacement, if one uses shape functions P^2 for displacement, it is preferable to use shape functions P^1 for the strains regularized for reasons of homogeneity. The quadratic elements, TRIA6 and QUAD8 for 2D, TETRA10, PENTA15 and HEXA20 for 3D, were developed. The components of displacement are assigned to all the nodes of the element whereas the components of the regularized strains are affected only with the nodes tops. For more clearness, element TRIA6 is represented below:



4.3 Modelizations available

These various elements are used in three types of modelizations:

Computation 2D in plane strains:	`D_PLAN_GRAD_EPSI` (cf [U3.13.06])
Computation 2D in plane stresses:	`C_PLAN_GRAD_EPSI` (cf [U3.13.06])
Computation 3D:	`3D_GRAD_EPSI` (cf [U3.14.11])

axisymmetric mode is not available yet.

5 Interface with the constitutive laws

the use of this method of delocalization requires the computation of the following terms on the level of the constitutive law:

$$(\varepsilon, \bar{\varepsilon}) \Rightarrow \sigma, \alpha, \frac{\partial \sigma}{\partial \varepsilon}, \frac{\partial \sigma}{\partial \bar{\varepsilon}}$$

The last two terms are necessary only for the computation of the tangent matrix.

6 Bibliography

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7 of the document Version Aster Author

(S) Organization	**(S) Description of the modifications**	**7e.Lorentz EDF-R&D**
7	initial Text 8,4 V	. Godard EDF-R&
8.4		