Nonlocal modelization with gradients of nodal damage GVNO

Summarized

One presents here the nonlocal modelization to gradient of nodal damage entitled GVNO in Code_Aster, resulting from works of thesis of J. Beaurain. This modelization can be seen like a simplification of the nonlocal modelization with gradient of local variable GRAD_VARI.

The nonlocal modelizations of type GVNO are available in 3D (3D_GVNO), axisymmetric (AXIS_GVNO) and plane strains (D_PLAN_GVNO).

The use of GVNO is very simple, since it is enough to specify modelization X_GVNO in AFFE_MODELE, to specify a characteristic length under key word NON_LOCAL in DEFI_MATERIAU, and to check that the constitutive law which one wishes to use is quite available in nonlocal version.

One presents the writing and the digital processing of this model.

Contents

1 Recall on the theory of the models with gradient

the models with gradient presented here were developed by E. Lorentz [bib1] in order to be able to describe the behavior materials requested by strong gradients of the mechanical fields which appear in the damaged zones or in the vicinity of geometrical singularities. Indeed, in the case of strong gradients, the behavior of a material point is not independent any more of its entourage but depends on the behavior on its vicinity, from where the introduction of gradients into the models.
From a numerical point of view, the computation of a structure with a classical local damage model shows that the damaged zone is always located on only one layer of finite elements and thus that the response of structure depends on the adopted mesh: the models with gradient mitigate this problem.
In what follows, we make a short recall of this theory.

1.1 Construction of the models with gradient

This formulation is restricted with the generalized standard materials (cf [Feeding-bottle 2]).

The models with gradient of local variables consist in introducing the gradient of local variables into a generalized standard formulation (cf [Feeding-bottle 2]).
Either a a local variable and A its associated thermodynamic force, and or $\Delta(\dot{a})$ potential of dissipation. If it is considered that Δ also depends on the gradient on $\dot{a}$ $\Delta=\Delta(\dot{a},\nabla\dot{a})$, one cannot then write the principle of normality locally:
$A\in\partial\Delta(\dot{a},\nabla\dot{a})$

Indeed, such a writing would require the introduction of 2 variables locally independent a and a_∇, with which one would associate 2 thermodynamic forces $A=-\frac{\partial\Phi}{\partial a}$, $A_\nabla=-\frac{\partial\Phi}{\partial a_\nabla}$ such as:

$(A,A_\nabla)\in\partial\Delta(\dot{a},\dot{a}_\nabla)$

If one calls F the threshold of elasticity associated with the potential $\Delta(\dot{a},\dot{a}_\nabla)$, the preceding equation is equivalent to:

$$\Delta(\dot{a},\dot{a}_\nabla)=\underset{(A,A_\nabla)\ /\ F(A,A_\nabla)\le 0}{Sup}\left[\dot{a}\ A+\dot{a}_\nabla A_\nabla\right]$$

And one a:
$\dot{a}=\dot{\lambda}\frac{\partial F}{\partial A}$, $\dot{a}_\nabla=\dot{\lambda}\frac{\partial F}{\partial A_\nabla}$

the problem here is that the variables are not independent and are bound by the nonlocal stress $a_\nabla=\nabla a$ so that one is not sure to check:
$\dot{a}_\nabla=\dot{\lambda}\frac{\partial F}{\partial A_\nabla}=\nabla\dot{a}$

One then proposes to forget the normal flow assumption in each point of structure while preserving the formalism of the generalized standard materials but at the level of structure, where the variables of state are now the strain field $\boldsymbol{\varepsilon}$ and the field of local variables a. The total free energy and the total potential of dissipation are thus defined:

$$F_\Phi(\boldsymbol{\varepsilon},a)=\int_\Omega \Phi(\boldsymbol{\varepsilon}(x),a(x),\nabla a(x))dx$$

$$D(\dot{a})=\int_{\Omega}\Delta(\dot{a}(x),\nabla\dot{a}(x))dx$$

The total potential of dissipation D is now a function of the field $\dot{a}$, and the writing $A\in\partial D(\dot{a})$ takes again a meaning.

The generalized behavior model is written now:

$$\sigma=\frac{\partial F_{\Phi}}{\partial\varepsilon}\quad A=-\frac{\partial F_{\Phi}}{\partial a}\ ,\ A\in\partial D(\dot{a})$$

1.2 Discretization in time

One will suppose subsequently that the energy of the model regularized is the sum of the energy of the local model and an additional term depending only on the gradient of the local variable which one regularizes:

$$F_{\Phi}(\boldsymbol{\varepsilon},a)=\int_{\Omega}\Phi^{loc}(\boldsymbol{\varepsilon}(x),a(x))+\Phi^{grad}(\nabla a(x))dx$$

This separation corresponds to the cases which we treat in Code_Aster, but it is possible to build models with gradient which do not correspond to this case. Contrary to what is made for modelization GRAD_VARI (nonlocal modelization with gradient of local variables), also available in the code, one considers that the damage with Gauss points is the interpolation of the damage to the nodes. One thus avoids bringing back oneself to a mixed formulation.

While leaning on the assumption of convexity compared to (u,α) potential F_{Φ} and while adopting an implicit diagram of Eulerian, the resolution of the balance equations is brought back to a problem of minimization relating to the increments $(\Delta u,\Delta\alpha)$. The discretized problem is written then:

$$\min_{\Delta u}\min_{\Delta\alpha}\left(F_{\Phi}(\boldsymbol{\varepsilon}^{-}+\Delta\boldsymbol{\varepsilon},\alpha^{-}+\Delta\alpha)-W^{ext}\right)$$

where W^{ext} the work of the external mechanical forces represents.

1.3 Spatial discretization by finite elements

to solve the problem of search for extrema, one carries out a spatial discretization by finite elements of the following quantities:

Displacement: $\boldsymbol{\varepsilon}(x)=\sum_{k\ noeuds}B_{k^u}(x)\,u_k$
Regularized damage: $\alpha(x)=\sum_{k\ noeuds}N_{k^{\alpha}}(x)\,\alpha_k$.
The gradient is expressed then thanks to the gradient of the shape functions: $\nabla\alpha(x)=\sum_{k\ noeuds}\nabla N_{k^{\alpha}}(x)\alpha_k$

where $N_{k^{\alpha}}$ are the shape functions associated with the node k for the field α, B_{k^u} are the shape functions of the calculated strains starting from derivatives of the shape functions associated with displacements with the node k. One differentiates the shape functions according to the field insofar as the degree of interpolation used can be different according to the quantity considered.

In the model finite elements with integration with Gauss points, the integral on the volume of structure is evaluated by summation on Gauss points. Energy F_{Φ} is thus written for the problem spatially discretized:

$$F_{\Phi}(\boldsymbol{\varepsilon}, \alpha)=\sum_{g} \omega_{g} \underbrace{\left[\Phi^{loc}(\boldsymbol{\varepsilon_g}, \alpha_{g})+\Phi^{grad}((\nabla \alpha)_{g})\right]}_{\Phi_{g}^{nonlocal}(\boldsymbol{\varepsilon_g}, \alpha_{g})}$$

where ω_g corresponds to the weight of Gauss points, and the index g indicates that the field is evaluated with Gauss points starting from the values with the nodes:

$$\varepsilon_{g}=\sum_{\text{k noeuds}} B_{k}^{u}(x_{g}) u_{k}$$

$$\alpha_{g}=\sum_{k \ noeuds} N_{k^{\alpha}}(x_{g})\, \alpha_{k}$$

$$(\nabla \alpha)_{g}=\sum_{k \ noeuds} \nabla N_{k^{\alpha}}(x_{g})\, \alpha_{k}$$

1.4 Computation of the internal forces

the mechanical balance equation as well as the equation of regularization are solved with the local nodes by search for extremum:

$$\min_{\Delta u} \min_{\Delta \alpha}\left(F_{\Phi}(\varepsilon^{-}+\Delta \varepsilon, \alpha^{-}+\Delta \alpha)-W^{ext}\right)$$

One does not solve the constitutive law locally, like that is done with the modelization in gradient of local variables. The criterion admits from now on a nodal resolution. The objects with which one works are the state of equilibrium and the criterion of the selected formulation defined by derivatives first of energy compared to displacements and the damage.

For more simplicity, we leave side the external mechanical forces which are treated except for in Code_Aster. The internal forces associated with the nodal variables with the node n (u_{n}, α_{n}) have as statements:

$$F^{u}|_{n}=\frac{\partial F_{\Phi}}{\partial u_{n}}=\sum_{g} \omega_{g} \frac{\partial \Phi^{loc}}{\partial u_{n}}=\sum_{g} \omega_{g} \frac{\partial \Phi^{loc}}{\partial \varepsilon_{g}} : \frac{\partial \varepsilon_{g}}{\partial u_{n}}=\underbrace{\sum_{g} \omega_{g} \sigma_{g} : B_{n}^{u}}_{\int_{\Omega} B^{T} \sigma d \Omega}$$

$$F^{\alpha}|_{n}=\frac{\partial F_{\Phi}}{\partial \alpha_{n}}=\sum_{g} \omega_{g}\left[\frac{\partial \Phi^{grad}}{\partial(\nabla \alpha)_{g}}(\nabla N)_{n}^{\alpha}+\frac{\partial \Phi^{loc}}{\partial \alpha_{g}} N_{n}^{\alpha}\right]$$

The tangent matrix is written in the following way:

$$K=\begin{pmatrix} \frac{\partial F^u}{\partial u} & \frac{\partial F^u}{\partial \alpha} \\ \frac{\partial F^\alpha}{\partial u} & \frac{\partial F^\alpha}{\partial \alpha} \end{pmatrix}$$

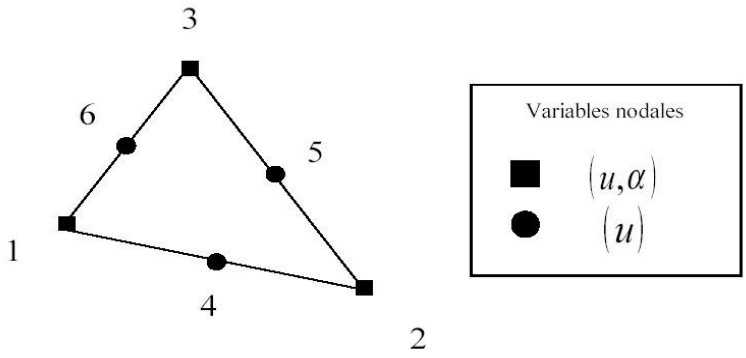
The frame of the generalized standard materials in which our modelization fits ensures the symmetry of the tangent matrix. It is thus enough to calculate the lower triangular matrix:

$$\frac{\partial F^u|_n}{\partial u_m}=\sum_g \omega_g B^u_m : \frac{\partial \sigma_g}{\partial \varepsilon_g} : B^u_n$$
$$\frac{\partial F^u|_n}{\partial \alpha_m}=\sum_g \omega_g N^\alpha_m \frac{\partial \sigma_g}{\partial \alpha_g} B^u_n$$
$$\frac{\partial F^\alpha|_n}{\partial \alpha_m}=\sum_g \omega_g \left[(\nabla N)^\alpha_m \frac{\partial^2 \Phi^{grad}}{\partial^2 (\nabla \alpha)_g} (\nabla N)^\alpha_n + N^\alpha_m \frac{\partial^2 \Phi^{loc}}{\partial^2 \alpha_g} N^\alpha_n \right]$$

2 Choice of the finite elements

One is in the presence of two nodal unknowns: displacements u and the damage α.

One considers shape functions P^2 for u and P^1 for α.
The quadratic elements, TRIA6 and QUAD8 for 2D, TETRA10, PENTA15 and HEXA20 for 3D, were developed. The components of displacement are assigned to all the nodes of the element whereas the components of damage are affected only with the nodes tops. For more clearness, element TRIA6 is represented below:



One uses the families of Gauss points corresponding elements linear, which results in an under-integration on displacements. The use of the families of Gauss points of the quadratic elements would imply an on-integration for α causing inopportune oscillations.

3 Methods of resolution

If one excludes the unilateral condition from irreversibility of the damage, the algorithm of Newton such as it is currently programmed in the code is enough with the resolution. In our case, one must take account of the degradation of the material. Concerning modelization GRAD_VARI, this stress is

managed locally, in each Gauss point, by projection on the old value if it is about a behavior of discharge. The resolution of the criterion not being local with GVNO, it is not possible any more to carry out this kind of projection and one must impose the irreversibility at the total level.

Two methods of resolution can then be under consideration with this modelization. It is possible to work with a classical Newton, by ensuring the projection of the negative increments of damage. Maybe with an algorithm “secant” which consists in putting at 0 the extra-diagonal blocks and thus to consider the variables of independent displacement and damage. What is equivalent to the algorithm of minimization alternated in the case of quadratic constitutive laws in displacement and damage. One ensures this time the irreversibility by projection of the negative increments of damage.

In both cases, the resolution is carried out in three stages:

1) One is interested initially in the increments of damage to each iteration. When the increment is negative, one projects it to 0, of kind to respect the irreversibility.
2) One builds the stiffness matrix and the (only diagonal blocks in the case of the secant) internal forces.
3) One definite residues of damage by the relation of complementarity:

$$R_n^{\alpha} = F^{\alpha}|_n \, \Delta\alpha_n$$

With respectively: $F^{\alpha}|_n \geq 0$ and $\Delta\alpha_n \geq 0$

That implies to look at the sign of the internal forces associated with the degrees of freedom of damage projected at stage 1. Indeed, if that Ci is negative one does not respect all the imposed conditions and one must keep the internal force like residue. If that Ci is positive the residue is then null since the increment is null.

More precisely, the method of resolution, in the case of the secant (diagonal matrix per blocks), can arise in the following way:

Itération n :

$\mathbf{u}^-, \boldsymbol{\alpha}^-, \Delta\mathbf{u}^{\mathbf{n}}, \Delta\boldsymbol{\alpha}^{\mathbf{n}}, \mathbf{Vect} = \mathbf{0}$ (vecteur indicateur)

CALCUL DE LA NOUVELLE DIRECTION DE DESCENTE

IF ($\Delta\alpha_i^n \leq 0$) THEN (i-ème terme)

$\mathbf{Vect}(\alpha_i) = 1$

$\Delta\alpha_i^n = 0$ (projection de l'incrément d'endommagement)

ENDIF

DEBUT BOUCLE SUR LES ELEMENTS

$$\mathbf{K} = \begin{bmatrix} \mathbf{K_{uu}}(\boldsymbol{\alpha}^- + \Delta\boldsymbol{\alpha}^{\mathbf{n}}) & 0 \\ 0 & \mathbf{K_{\alpha\alpha}}(\mathbf{u}^- + \Delta\mathbf{u}^{\mathbf{n}}) \end{bmatrix}, \mathbf{F} = \begin{pmatrix} \mathbf{F_u} \\ \mathbf{F_\alpha} \end{pmatrix}$$

IF ($\Delta\alpha_i^n = 0$) THEN

$\mathbf{K}^{\mathbf{n}}_{(\alpha_i, \alpha_k)} = \delta_{i,k}$ (nécessaire pour l'inversion dans la résolution)

ENDIF

FIN BOUCLE SUR LES ELEMENTS

ASSEMBLAGE DE LA MATRICE GLOBALE ET DU VECTEUR RESIDU

IF ($\mathbf{Vect}(\alpha_i) \# 0$) THEN

IF ($\mathbf{F}^{\mathbf{n}}_{\alpha_i} \geq 0$) THEN (l'endommagement cherche encore à diminuer)

$\mathbf{F}^{\mathbf{n}}_{\alpha_i} = 0$ (Afin de converger, comme $\dot{\alpha}_i = 0$)

ELSE (il faut laisser l'endommagement évoluer)

ENDIF

ENDIF

RESOLUTION

IF (RESIDU < CRITERE) THEN (on a convergé, fin du pas de temps)

$\mathbf{u}^- = \mathbf{u}^- + \Delta\mathbf{u}^{\mathbf{n}}$

$\boldsymbol{\alpha}^- = \boldsymbol{\alpha}^- + \Delta\boldsymbol{\alpha}^{\mathbf{n}}$

ELSE (on calcule les incréments pour l'itération suivante)

n = n +1, $\Delta\mathbf{u}^{\mathbf{n}} = \Delta\mathbf{u}^{\mathbf{n-1}} + \delta\mathbf{u}^{\mathbf{n-1}}$

$\Delta\boldsymbol{\alpha}^{\mathbf{n}} = \Delta\boldsymbol{\alpha}^{\mathbf{n-1}} + \delta\boldsymbol{\alpha}^{\mathbf{n-1}}$

ENDIF

4 Modelizations available

These various elements are used in three types of modelizations:

Computation 2D in plane strains: D_PLAN_GVNO
Computation 2D into axisymmetric: AXIS_GVNO
Computation 3D: 3D_GVNO

the plane stresses mode is not available yet.

5 Constitutive laws available with modelizations GVNO

the constitutive law currently available in the nonlocal version to gradient of nodal damage is the following one:

ENDO_CARRE Constitutive law of damage regularized quadratic

ENDO_CARRE (cf [R5.03.26])

6 the Councils/Procedure for the implementation of a new constitutive law with gradients of damage

the addition of a new constitutive law is very simple. It is just enough to inform calculated derivatives first and seconds locally and independently of the part in gradient which is managed in a generic way.

7 The Councils on the use of GVNO

It is preferable to use modelization GVNO with the traditional method of Newton in the case of computations of damage which do not have strong instabilities (snap-back). Indeed, the method of Newton is faster but it does not make it possible to cross the snap-backs without the use of an additional algorithm, standard control, which was not developed yet here. In the contrary case, it is preferable to use the "secant" method, by starting a PAS_MINI_ELAS sufficiently large to make sure that one checks the condition with each time step. This method allowing to cross instabilities and thus always to converge towards a solution. However, the nombre of iterations necessary to convergence can be rather important (ex: 800 iterations). It is thus necessary to authorize a maximum number of iterations significant in the command file (approximately 1200). The speed of convergence depends on the model used and also on the temporal discretization. Time step the too large ones can lead to a well too difficult convergence. This method is very effective for the quadratic constitutive laws in damage and displacements since one works then with an algorithm of alternate minimization robust.

8 Features and checks

Inventory of the tests in support to the development:

Case test	Description
SSNP307a	test of D_PLAN_GVNO (cf [V6.03.307])
SSNA119a	test of AXIS_GVNO (cf [V6.01.119])
SSNV220a	test of 3D_GVNO (cf [V6.04.220])

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[1] Bibliography E.: "Constitutive laws with gradients of local variables: construction, variational formulation and numeric work", Doctorate implementation of the university Paris 6, April 27th, 1999.

[2] HALPHEN B., NGUYEN Q.S.: "On the generalized standard materials", Newspaper of Mechanics, vol. 14, No 1,1975.

10 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of modifications
10.	J.BEAURAIN, K.KAZYMYRENKO EDF-R&D/AMA	initial Text