Modelization of damping in linear dynamics

Summarized:

The linear dynamic analyzes of structures subjected to imposed forces or motions require to add characteristics of mechanical cushioning to the characteristics of stiffness and mass of the model.

One lays out of several classical modelizations, applicable to all element types the finished available ones:

- the model of viscous damping,
- the model of hysteretic damping (known as also “structural damping”)
  for the harmonic analysis of the viscoelastic materials.

For the analyses using the methods of dynamic response by modal recombination, with a modal base of real eigen modes, it is possible to introduce modal damping coefficients.

Contents

1 Notion of mechanical cushioning

1.1 Models of damping

motion of structures subjected to forces or motions imposed, variable in the course of time, depends, in particular of the properties of damping, i.e. of dissipation of energy in the materials constitutive of structure and connections of the various structural elements between them and with the surrounding medium.

The physical phenomena intervening in this dissipation of energy are many frictions, interaction fluid-structure in a fluid blade, shocks, viscosity and plasticity, vibratory radiation with the bearings.

The models of behavior representing these phenomena are often badly known and it is difficult to explicitly describe them at the elementary level. This is why the most used models are the simple models which make it possible to reproduce on a macroscopic scale the principal effects on the structures [bib1] [bib2]. Those currently available in *Code_Aster* are:

- viscous damping: dissipated energy proportional to the speed of motion,
- hysteretic damping (known as also “structural damping”): dissipated energy proportional to displacement such as the damping force of sign is opposed to that the velocity.

Let us note that the damping of Coulomb, which corresponds to a damping of friction for which dissipated energy is proportional by the strength of normal reaction to the direction of displacement requires to model the contact, which leaves the strictly linear frame. The nonlinear operators can take it into account in all its generality [R5.03.50 & R5.03, 52] while the transitory operator of resolution into modal can model the friction of Coulomb in the frame of specific contacts [R5.06.03].

The values of the parameters of these models are deduced, when they are available, of results experimental. At the stage of the design, one limits oneself to the use of guiding values.

1.2 General standards to characterize damping [bib1]

1.2.1 loss Ratio

the loss ratio η is an adimensional coefficient characteristic of the damper effect defined as the ratio of the energy dissipated during a cycle in the maximum potential energy multiplied by 2π :

$$\eta=\frac{E_{d\,\text{par cycle}}}{2\pi\,E_{p\max}}$$ éq 1.2-1

1.2.2 Reduced damping

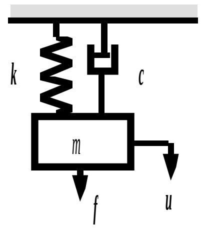
By definition reduced damping is equal to half of the loss ratio

$$\xi=\frac{\eta}{2}$$ éq 1.2-2

2 Models physical

2.1 viscous damping Definition of viscous damping

the classical cushioning devices (for example, by rolling of a viscous fluid through the openings of a piston pulled by vibratory motion) deliver forces proportional to the speed of motion and opposite sign. During a cycle, the work of these forces is positive: it is viscous damping.



For a simple oscillator of stiffness k , mass m and viscous damping c , the external force applied equilibrium three components: elastic recoil force ku , damping force $c\dot{u}$ and inertia force $m\ddot{u}$ from where the dynamic equation moving absolute motion:

$$m\ddot{u}+c\dot{u}+ku=f \qquad \text{éq 2.1-1}$$

For this model of viscous damping the energy dissipated during a cycle of pulsation ω is proportional to the vibratory speed $-\omega u_0 \sin(\omega t)$ associated with displacement $u_0 \cos(\omega t)$:

$$E_{d\,\text{par cycle}}=\int_0^{2\pi} -c\,\omega\, u_0 \sin\omega t d\left(u_0\cos\omega t\right)=\pi c\,\omega u_0^2$$

and potential energy for a sinusoidal displacement $u_0 \cos\omega t$ is:

$$E_{p\max}=\int_{\pi/2}^{0} ku_0\cos\omega t\, d\left(u_0\cos\omega t\right)=\frac{1}{2}ku_0^2$$

For a cycle of pulsation ω and sinusoidal displacement $u_0\cos\omega t$, the loss ratio is proportional to the frequency of motion:

$$\eta=\frac{c\,\omega}{k} \qquad \text{éq 2.1-2}$$

2.2 harmonic Oscillator with viscous damping

classical analysis of the undamped model associated with the equation [éq. 2.1-1], put in the form $\left(k-m\omega^2\right)u=0$ gives us $\omega_0=\sqrt{\frac{k}{m}}$ the own pulsation.

The damping criticizes from which the differential equation [éq 2.1-1] does not have any more an oscillating solution is given by the formulas $c_{critique}=2\sqrt{km}=2\text{m}\,\omega_0=\frac{2\text{k}}{\omega_0}$ what makes it possible to give a numerical interpretation of the reduced damping, which is often expressed expressed as a percentage critical damping:

$$\xi=\frac{\eta}{2}=\frac{c}{c_{critique}}=\frac{c}{2\text{m}\,\omega_0} \qquad \text{éq 2.1-3}$$

2.2.1 Response with releasing excitation

From a static strain $u_{st}=\frac{f_0}{k}$, to release (release of the system) produced a free oscillatory motion $u_l(t)=u_0 e^{-\xi\omega_0 t}\cos\omega_0' t$ which reveals the own pulsation of the damped system $\omega'_0=\omega_0\sqrt{(1-\xi^2)}$.

In the course of time, the extreme amplitude (u_1, u_2) decreases at each period of $e^{-\xi\omega_0 T}=e^{-2\pi\xi}=e^{-\delta}$ where δ is the decrement logarithmic curve: $\delta=2\pi\xi$

2.2.2 Response with a harmonic excitation

the response with a harmonic excitation of the form $f(t)=f_0 e^{j\omega t}$ is written with a forced response permanent particular solution $u(t)=u_0 e^{(j\omega t-\varphi)}$ which is written with the reduced pulsation $\lambda=\frac{\omega}{\omega_0}$

$\frac{ku_0}{f_0}=\frac{1}{1-\lambda^2+j2\xi\lambda}=H_v(j\omega)$ where $H_v(j\omega)$ is the complex transfer transfer function of a simple oscillator with viscous damping.

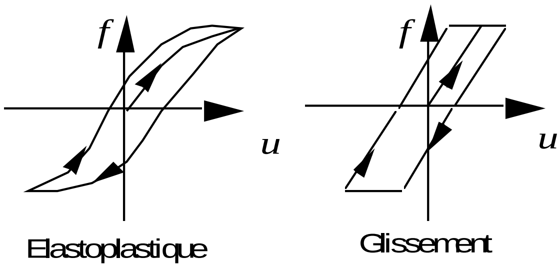
The modulus of the response $\frac{u_0}{u_{st}}=\frac{ku_0}{f_0}=|H_v(j\omega)|=\frac{1}{\sqrt{(1-\lambda^2)^2+(2\xi\lambda)^2}}$ reveals a dynamic amplification compared to the static response u_{st} .

This amplification is maximum for $\lambda=\frac{\omega_0'}{\omega_0}=\sqrt{(1-\xi^2)}$ and gives the value of maximum displacement $\frac{u_{0\max}}{u_{st}}=\frac{1}{2\xi\sqrt{(1-\xi^2)}}$. If the vibratory velocity is observed $\dot{u}(t)=j\omega u(t)$, the amplification vibratory velocity is maximum for $\lambda=\frac{\omega_0}{\omega_0}=1$ and the maximum amplitude velocity is $\dot{u}_{0\max}=\frac{1}{2\xi}=Q$, where Q is the mechanical analogy of the factor of overpressure of the electricians. These properties are at the origin of the methods of measurement of the characteristics of damping of mechanical structures.

3 Model physical

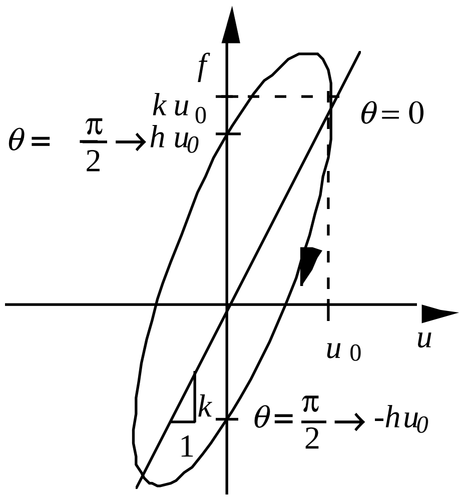
3.1 hysteretic damping Definition of hysteretic damping

For a sinewave excitation applied to an elastoplastic structure or with an elastic structure with friction, curved force-displacement reveals a positive work of the external force which corresponds to an energy dissipated in structure, that one can at first approximation represent like below:



In both cases the loss ratio believes, in general with the amplitude of the cycle. For low values of the loss ratio (< 0.2), the form of the cycle does not have an appreciable effect on motion and one can compare it to an ellipse [bib1].

In the typical case of a relation force-displacement whose cycle is of form elliptic, the statement of the loss ratio is simple. For an applied force F and a displacement $u = u_0 \cos q$ the recoil force is $k u_0 \cos\theta$, k being the "classical" stiffness of the mechanical system, and the damping force $-h u_0 \sin\theta$, h being the stiffness out of phase of $90°$, which leads to the relation of equilibrium $F = k u_0 \cos\theta - h u_0 \sin\theta$.



Energies, dissipated during a maximum cycle and potential, are:

$E_{d\,\text{par cycle}} = \int_0^{2\pi} -h u_0 \sin\theta \, d\left(u_0 \cos\theta\right) = \pi h u_0^2$ and $E_{pmax} = \int_{\pi/2}^{0} k u_0 \cos\theta \, d\left(u_0 \cos\theta\right) = \frac{k u_0^2}{2}$

from where the loss ratio

$$\eta = \frac{\pi h u_0^2}{2\pi \frac{k u_0^2}{2}} = \frac{h}{k} \qquad \text{éq 3.1-1}$$

For a sinusoidal cycle $\theta = \omega t$, the hysteretic damping coefficient $\eta = \frac{h}{k}$ is independent of ω . It can be given from a test under harmonic cyclic loading.

3.2 Harmonic oscillator with hysteretic damping

The model of hysteretic damping is usable to treat the harmonic structure responses with viscoelastic materials.

The energy dissipated by cycle in the form $E_{d\,\text{par cycle}}=\int_0^{2\pi}\sigma\,d\varepsilon$ makes it possible to highlight a complex Young modulus E^* from the relation stress-strain of a viscoelastic material $\sigma=\sigma_0 e^{j\omega t}$ and $\varepsilon=\varepsilon_0 e^{j(\omega t-\varphi)}$ where σ_0 and ε_0 are the amplitudes and φ the phase:

$$E^*=\frac{\sigma}{\varepsilon}=\left(\frac{\sigma_0}{\varepsilon_0}\right)e^{j\varphi}=\left(\frac{\sigma_0}{\varepsilon_0}\right)(\cos\varphi+j\sin\varphi)$$

By noting $E_1=\left(\frac{\sigma_0}{\varepsilon_0}\right)\cos\varphi$ the real part and $E_2=\left(\frac{\sigma_0}{\varepsilon_0}\right)\sin\varphi$ the imaginary part one obtains $E^*=E_1+jE_2=E_1(1+j\eta)\,\text{avec}\,\eta=\frac{E_1}{E_2}=\text{tg}\,\varphi$, where j is also called loss angle.

The classical analysis of the equation [éq 2.1-1] has meaning, with a model of hysteretic damping, only for one harmonic excitation $f(t)=f_0 e^{j\omega t}$ which leads to the equation

$$m\ddot{u}+k(1+j\eta)u=m\ddot{u}+(k+jh)u=f_0 e^{j\omega t} \qquad \text{éq 3.2-1}$$

where the real part of displacement u represents the displacement of the mass and $h=k\eta$.

As previously cf [§ 2.2], the harmonic response can be written, with the reduced pulsation $\lambda=\frac{\omega}{\omega_0}$, in the form $\frac{ku_0}{f_0}=\frac{1}{1-\lambda^2+j\eta}=H_h(j\omega)$ where $H_h(j\omega)$ is the complex transfer transfer function of a simple oscillator with hysteretic damping.

The modulus of the response $\frac{u_0}{u_{st}}=\frac{ku_0}{f_0}=|H_h(j\omega)|=\frac{1}{\sqrt{(1-\lambda^2)^2+\eta^2}}$ reveals a dynamic amplification compared to the static response, amplification which is maximum for $\lambda=1$ and gives the value of maximum displacement $\frac{u_{0\,\max}}{u_{st}}=\frac{1}{\eta}=\frac{1}{2\xi}$.

In conclusion, reduced damping associated with hysteretic damping is:

$$\xi=\frac{\eta}{2}=\frac{h}{2\text{k}}=\frac{h}{2\text{m}\,\omega_0^2} \qquad \text{éq 3.2-2}$$

4 Other models of damping

One does not treat models here representing the damping “added” by the confined motionless fluids or the fluids moving. One will refer to the documents [R4.07.xx], treating fluid coupling - structure.

5 Structure with damping analyzes

the modelizations presented are not easily generalizable with the various structure analyses cf [§1].

Note:

> *The two modelizations do not have the same field of linear analysis:*
> - *viscous damping is usable in transient analysis or harmonic,*
> - *hysteretic damping is usable only in harmonic analysis.*

The options of modelizations in *Code_Aster* allow the definition:

- of a total damping for structure,
- of depreciation located on mesh or on group of elements.

5.1 Total damping of structure

In the absence of sufficient information on the components and connections creating a dissipation of energy, a current modelization consists in building a damping matrix “total”.

5.1.1 Viscous damping proportional “total”

One is placed in the frame of the classical equations of the dynamics of linear structures:

$$\boldsymbol{M}\,\ddot{\boldsymbol{U}}+\boldsymbol{C}\,\dot{\boldsymbol{U}}+\boldsymbol{K}\boldsymbol{U}=\boldsymbol{F}(t) \quad \text{éq 5.1.1-1}$$

the notion of damping of RAYLEIGH makes it possible to define the damping matrix C as linear combination of the stiffness matrixes and mass:

$$\boldsymbol{C}=\alpha\,\boldsymbol{K}+\beta\,\boldsymbol{M} \quad \text{éq 5.1.1-2}$$

Advantages:

- easy to implement by means of the operators `DEFI_MATERIAU` [U4.43.01 §3.1] and `ASSEMBLY (OPTION=' AMOR_MECA')` [U4.61.21]. One can also use operator `COMB_MATR_ASSE` [U4.53.01], after having assembled the stiffness matrixes and of mass with real coefficients.;
- useful for the validation of algorithms of resolution;
- historically, its success is attached to the methods of analysis transient by modal recombination from a base of real eigen modes.
  The properties of orthogonality of the real eigen modes solution of the problem to the eigenvalues $(\boldsymbol{K}-\omega^2\boldsymbol{M})\varphi=0$ result in the simultaneous diagonalization in the transition into modal coordinates generalized of $\varphi^T\boldsymbol{K}\varphi$ and $\varphi^T\boldsymbol{M}\varphi$.
  The damping of RAYLEIGH is a condition sufficient for diagonalizing $\varphi^T\boldsymbol{C}\varphi$.
  The system of equations modal $\ddot{\boldsymbol{q}}+\frac{\varphi^T\boldsymbol{C}\varphi}{\varphi^T\boldsymbol{M}\varphi}\dot{\boldsymbol{q}}+\omega^2\boldsymbol{q}=\frac{\varphi^T}{\varphi^T\boldsymbol{M}\varphi}\boldsymbol{F}(t)$ becomes diagonal then.

$$\ddot{q}+2\xi\omega\dot{q}+\omega^2 q=\frac{\varphi^T}{\varphi^T M\varphi}F(t)$$ éq 5.1.1-3

Disadvantages:

- This modelization does not make it possible to represent the heterogeneity of structure compared to damping.
- The damping actually introduced into the model strongly depends on the identification on the coefficients α and β cf [§ 5.1.2].

5.1.2 Influence damping coefficients proportional

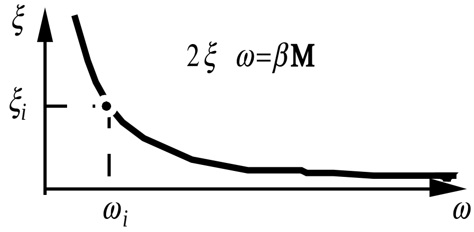
Three simple cases of identification are presented here to illustrate, the effects induced by this modelization:

- proportional damping with the characteristics of inertia: $\alpha=0,\beta$

  This case was very much used of direct transitory resolution: if the mass matrix is diagonal, that of damping is still and the saving space memory is obvious in it.

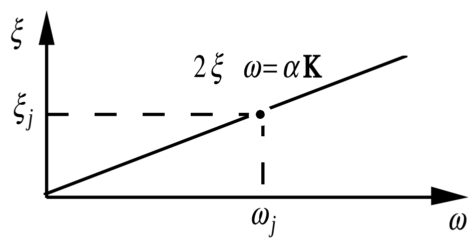
  The coefficient β can be identified with the experimental reduced damping ξ_1 of the eigen mode (φ_1,ω_1) which takes part more in the response cf [éq. 2.1-1] from where $\beta=2\xi_1\omega_1$. For any other pulsation one obtains a reduced modal damping $\xi=\beta\frac{\omega_1}{\omega}$.

  The high modes $\omega>>\omega_i$ will be far from damped and the too damped $\omega<\omega_1$ low frequency modes.



- proportional damping with the characteristics of stiffness: α,β .

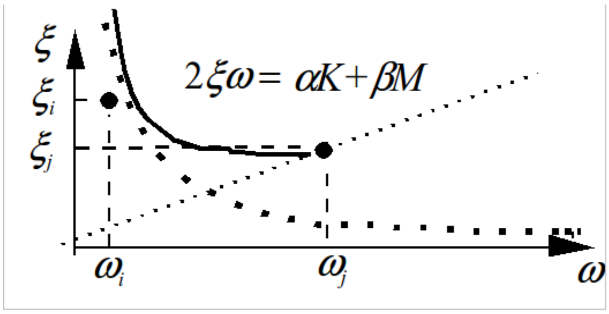
  The coefficient α can be identified, like previously from ξ_2 associated with the mode (φ_2,ω_2) from where $\alpha=2\xi_2\omega_2$. For any other pulsation one obtains a reduced modal damping $\xi=\alpha\frac{\omega}{\omega_2}$. The high modes $\omega>>\omega_2$ are very damped.



- complete proportional damping: $\alpha=\alpha,\beta=\beta$

  From an identification on two independent modes (φ_1,ω_1) and (φ_2,ω_2), one obtains for any other pulsation a reduced modal damping $\xi=\frac{1}{2}\left(\alpha\omega+\frac{\beta}{\omega}\right)=\xi_1\frac{\omega}{\omega_1}+\xi_2\frac{\omega_2}{\omega}$.

In the interval $[\omega_1, \omega_2]$, the variation of reduced damping is weak and outwards one finds the addition of the preceding disadvantages: the modes external with the interval are too damped.



In none the preceding cases, one will be able to reproduce an assumption of equal modal damping for all the modes. Methods were imagined for tending towards this purpose [bib1].

5.1.3 "Total" hysteretic damping

the generalization of the equation of the simple oscillator with hysteretic damping led to the system of equations complexes or $\boldsymbol{F}(\boldsymbol{\Omega})$ is a harmonic excitation.

$$\boldsymbol{M}\ddot{\boldsymbol{U}}+\boldsymbol{K}(1+j\eta)\boldsymbol{U}=\boldsymbol{F}(\boldsymbol{\Omega}) \quad \text{Éq 5.1.3-1}$$

Knowing the real stiffness matrix, it is possible to build a hysteretic damping matrix $\boldsymbol{K}_h=j\eta\boldsymbol{K}$, with a "total" loss ratio η.

As previously of resolution by modal recombination, from a base of real eigen modes, one obtains $\varphi^T\boldsymbol{M}\varphi\ddot{\boldsymbol{q}}+j\varphi^T\boldsymbol{K}_h\varphi\boldsymbol{q}+\varphi^T\boldsymbol{K}\varphi\boldsymbol{q}=\varphi^T F(t)$ where the generalized hysteretic damping matrix is diagonal $\varphi^T\boldsymbol{K}_h\varphi=[\mathrm{diag}\,\eta\gamma_i]$, like the generalized stiffness matrix $\varphi^T\boldsymbol{K}\varphi=[\mathrm{diag}\,\gamma_i]$.

According to the definition of reduced damping (cf [éq 1.2-2]), modal damping is constant for all the modes from where $\xi=\frac{\eta}{2}$

Advantages:

- easy to implement by means of operators `DEFI_MATERIAU (OPTION=' RIGI_MECA_HYST')` [U4.43.01 §3.1] and `ASSEMBLY` [U4.61.21]. One can also proceed using operator `COMB_MATR_ASSE` [U4.53.01], after having assembled the stiffness matrixes.;
- very useful for the validation of algorithms of resolution;
- the damping actually introduced into the model is constant for all the modes of structure, as asks it regulations of construction.

Disadvantages:

- this modelization is badly adapted for the industrial studies, because it does not make it possible to represent the heterogeneity of structure compared to damping.
- only the harmonic analysis (in complex) is possible.
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5.2 Damping located

For the analyses requiring a modelization representing the heterogeneity of structure, it is possible to affect characteristics of damping located on meshes of structure, in fact on elements of the model.

5.2.1 Elements dampers

It is possible to apply elements discrete dampers:

- on meshes POI1 : damping is related to the displacement (respectively velocity) of the node support,
- on meshes SEG2 : damping is related to the relative displacement (respectively relative velocity) of the two nodes connected.

Operator AFFE_CARA_ELEM [U4.24.01] allows to define for each discrete element:

- a damping matrix of the viscous type $\boldsymbol{a}_{discret}$ whose terms are assigned to the various degrees of freedom of the nodes concerned; several modes of description of the matrix are available.
- a hysteretic loss ratio $\boldsymbol{\eta}_{discret}$ multiplying of the stiffness matrix of the affected discrete element to the mesh support.

5.2.2 Affected damping with any type of finite element

the affected elastic material with any finite element can be defined with parameters of damping by the operator DEFI_MATERIAU [U4.23.01]:

- Viscous damping proportional with two parameters of RAYLEIGH α and β .

  AMOR_ALPHA= α

  AMOR_BETA= β

  For all element types finished (of continuums, structural or discrete), it are possible to calculate the real elementary matrixes corresponding to computation option "AMOR_MECA", after having calculated the elementary matrixes corresponding to computation options "RIGI_MECA" and "MASS_MECA".

  The elementary matrix of the affected i element of the material α_j, β_j is then of the form:

  - for a finite element

  $$\boldsymbol{c}_{\mathbf{elem}\,i}=\alpha_j \boldsymbol{k}_{\mathbf{elem}\,i}+\beta_j \boldsymbol{m}_{\mathbf{elem}\,i}$$

  - for a discrete element

  $$\boldsymbol{c}_{\mathbf{elem}\,i}=\boldsymbol{a}_{\mathbf{discret}\,i}$$

- Hysteretic damping with a coefficient of $e\,\eta$

  AMOR_HYST = coeff

  For all element types finished (of continuums, structural or discrete), it is possible to calculate the complex elementary matrixes corresponding to computation option "RIGI_MECA_HYST", after having calculated the elementary matrixes corresponding to computation options "RIGI_MECA".

  The elementary matrix of the affected i element of the material α_j, β_j is then of the form:

  - for a finite element

  $$\boldsymbol{k}^*_{\mathbf{elem}\,i}=\boldsymbol{k}_{\mathbf{elem}\,i}\left(1+j\,\eta_j\right)$$

  - for a discrete element

  $$\boldsymbol{k}^*_{\mathbf{elem}\,i}=\boldsymbol{k}_{\mathbf{elem}\,i}\left(1+j\,\eta_{\mathrm{discret}\,i}\right)$$

5.2.3 Construction of the damping matrix

the assembly of the elementary matrixes of damping is obtained with usual operator ASSE_MATRICE [U4.42.02] or by the macro command ASSEMBLY [U4.31.02]. One must use same classifications and the same mode of storage as for the stiffness matrixes and mass (operator NUME_DDL [U4.42.01]).

Note:

The damping matrix obtained is nonproportional

$\boldsymbol{C} \neq \alpha \boldsymbol{K} + \beta \boldsymbol{M}$ or $\boldsymbol{K_h} \neq j \eta \boldsymbol{K}$

6 Use of the damping matrix

6.1 Use of the viscous damping matrix

6.1.1 direct linear Dynamic analysis

the viscous damping matrix $\boldsymbol{C}$, whatever its mode of development and its character proportional or not proportional, is usable for the direct linear dynamic analysis (key word MATR_AMOR) with the operators:

- of transient analysis DYNA_LINE_TRAN [R5.05.02] and [U4.54.01]
- of harmonic analysis DYNA_LINE_HARM [R5.05.03] and [U4.54.02]

6.1.2 Dynamic analysis by modal recombination

For the analyses by modal recombination, one must project this matrix in the subspace defined by a set of φ real eigen modes, obtained on the associated undamped problem $\left(\boldsymbol{K} - \omega^2 \boldsymbol{M}\right)\varphi = 0$.

This operation is possible with macro command PROJ_BASE [U4.55.11] or with operator PROJ_MATR_BASE [U4.55.01].

For the computation of the dynamic response in force or imposed in modal space, one has following possibilities:

- use of the generalized damping matrix $\boldsymbol{\varphi}^T \boldsymbol{C} \boldsymbol{\varphi}$:
  - in transient analysis with the operator DYNA_TRAN_MODAL [R5.06.04] and [U4.54.03] and key word AMOR_GENE,
  - in seismic analysis by spectral method with the operator COMB_SISM_MODAL [R4.05.03] and [U4.54.04] and key word AMOR_GENE,
  - in harmonic analysis with the operator DYNA_LINE_HARM [R5.05.03] and [U4.54.02] and key word MATR_AMOR.

  Let us recall that in the case of heterogeneous damping (localised use of the options of damping), the matrix $\boldsymbol{\varphi}^T \boldsymbol{C} \boldsymbol{\varphi}$ is not diagonal.

- use of viscous modal damping by providing a constant reduced modal damping for all the modes ξ or a list of values ξ_i .

  Several methods of identification of these coefficients are possible but it does not exist of ordering of sytematic construction of the list of values. One can nevertheless quote the use of the assumption of BASILE $\left(2\xi_i \omega_i = \text{diag} \dfrac{\boldsymbol{\varphi}^T \boldsymbol{C} \boldsymbol{\varphi}}{\boldsymbol{\varphi}^T \boldsymbol{M} \boldsymbol{\varphi}}\right)$, the use payment RCC-G (or ETC-C) for the seismic analysis with damping of the soil, operating of results experimental,…
  - in transient analysis with the operator DYNA_TRAN_MODAL [R5.06.04] [U4.54.03] and key word AMOR_REDUIT.

- in seismic analysis by spectral method with operator COMB_SISM_MODAL [R4.05.03] [U4.54.04] and the key words AMOR or LIST_AMOR. An evolution is required to generalize key word AMOR_REDUIT.
- in harmonic analysis **a request for evolution** with operator DYNA_LINE_HARM [R5.05.03] [U4.54.02] is deposited. It is not treated in version 3.6.

For the analyses by dynamic substructuring, with the use of a modal base (RITZ bases) one will refer to [R4.06.03] and [R4.06.04].

6.2 Use of the stiffness matrix complexes

the stiffness matrix complexes $K^*=K+K_h$, where K_h is an imaginary matrix (within the meaning of the complexes!), is usable for the direct harmonic analysis with the operator DYNA_LINE_HARM [R5.05.03] and [U4.54.02] and key word MATR_RIGI.

For the analyses by modal recombination, no functionality is currently available for the use of the model of hysteretic damping.

6.3 Modal analysis complexes

the viscous damping matrix C is essential for the complex modal analysis with the operators dealing with the quadratic problem with the eigenvalues [R5.01.02]:

- by inverse iterations MODE_ITER_INV [U4.52.01]
- by simultaneous iterations MODE_ITER_SIMULT [U4.52.02]

Let us recall that the complex eigen modes allow a better adapted approach under investigation dynamic of strongly damped structures (reduced damping ξ > 20%). To date no tools of dynamic response by modal recombination using a base of complex eigen modes are available in *Code_Aster*.

7 Bibliography

1) “Damping”: Francoise GANTENBEIN and Michel LIVOLANT (CEA-DMT) in “Paraseismic Genius” Work collective - Presses of the E.N.P.C. (1985) “Damping

2) in the structural analyzes”: Forum IPSI - Φ 2 ACES Volume XVIII N°2 (June 1994) Description

8 of the versions of the document Version

Aster Author	(S) Organization (S) Description	of modifications 3 J.
3	*LEVESQUE (EDF - R&D/IMA/MMN) initial*	Text