Gyroscopic matrixes of the straight beams and the discs

Summarized:

This document presents the formulation of the gyroscopic damping matrixes and stiffness of the elements beams and the indeformable disc.

The beams are only straight beams (Elements POU_D_T and POU_D_E). The section is constant over the length and of circular form. The material is homogeneous, isotropic.

The discs are cylinders of cross-section whose axis is confused with L `axis of the beam. The disc is supposed to be indeformable.

The assumptions selected are the following ones:

- Assumption of Timoshenko: the transverse shears and all the terms of inertia are taken into account. This assumption is to be used for weak slenderness (Elements POU_D_T).
- Assumption of Eulerian: the transverse shears are neglected. This assumption is checked for strong slenderness (Elements POU_D_E).

Clean rotational speed (along the axis of the beam) can be constant or variable.
In Code_Aster, adopted convention defines the positive meaning along the rotational axis as being the usual **trigonometrical** meaning of rotation.

Contents

1 Introduction

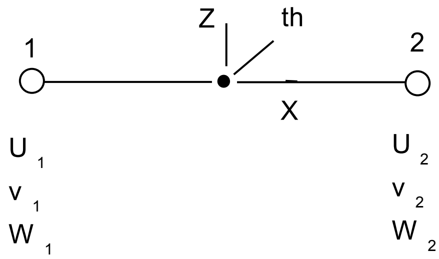
a beam are a solid generated by a surface of area S , whose geometrical center of inertia G follows a curve C called average fiber or neutral fiber.

In the frame of this modelization, only the straight beams, with constant and circular section are taken into account.

For the study of the beams in general, one formulates the following assumptions:
•The cross-section of the beam is indeformable,
•transverse displacement is uniform on the cross-section.

These assumptions make it possible to express displacements of an unspecified point of the section, according to an increase in displacement due to the rotation of the section around the transverse axes.

The discretization in "exact" elements of beam is carried out on a linear element with two nodes and six degrees of freedom by nodes. These degrees of freedom break up into three translations u v , w (displacements according to the directions x , y and z) and three rotations θ_x , θ_y and θ_z . (around the axes x , y and z).



In the case as of straight beams, the line average one is along the axis x of the local base, displacement transverse being thus carried out in the plane (y, z) .

For the storage of the quantities related to the degrees of freedom of an element in a vector or an elementary matrix (thus of dimension 12 or 12^2), one arranges initially the variables for node 1 then those of node 2. For each node, one stores initially the quantities related to the three translations, then those related to three rotations. For example, a vector displacement will be structured in the following way:

$$\underbrace{u_1, v_1, w_1, \theta_{x1}, \theta_{y1}, \theta_{z1}}_{sommet\ 1}, \underbrace{u_2, v_2, w_2, \theta_{x2}, \theta_{y2}, \theta_{z2}}_{sommet\ 2}$$

1.1 Definition of the references

One defines:
• x is the axis of neutral fiber of the beam,
• y and z are the principal axes of inertia of the section,
• R_0 is the absolute coordinate system related to a section in the initial configuration,
• R is the reference related to a section in the deformed configuration,

By not considering torsion, the transition of the reference R_0 to the reference R is carried out with the assistance 3 rotations, two following y and z , and a rotation around x , noted ϕ , such as:
$\dot{\phi}$: clean rotational speed of the shaft

2 the element beam of constant circular section

2.1 Characteristics

Each element is an isoparametric element beam of circular and constant section. One takes into account the transverse shears in the formulation of this element (straight beam of Timoshenko).

Notations:
- x is the axis of neutral fiber of line of trees,
- density: ρ
- length of the element: L
- Young modulus: E
- modulate Fish: $G=\frac{E}{2(1+\nu)}$
- section:
    - interior radius: R_i
    - external radius: R_e
    - area: $A=\pi\left(R_e^2-R_i^2\right)$
    - polar inertia: $I_x=\frac{\pi}{2}\left(R_e^4-R_i^4\right)$
    - inertia of section: $I_{yz}=I_y=I_z=\frac{\pi}{4}\left(R_e^4-R_i^4\right)$

2.2 Computation of the kinetic energy of the beam of Timoshenko

One calculates the kinetic energy of the element beam of Timoshenko by considering the strains of membrane and bending. The statement of kinetic energy is obtained while integrating over the length of the element beam:

$$T=\frac{1}{2}\rho.A\int_{x=0}^{L}\left[\dot{u}^2+\dot{v}^2+\dot{w}^2\right]dx+\frac{1}{2}\rho.\int_{x=0}^{L}\vec{\Omega}_{R/R0}.[J].\vec{\Omega}_{R/R0}\,dx$$

with: $[J]=\begin{bmatrix} I_x & 0 & 0\\ 0 & I_y & 0\\ 0 & 0 & I_z\end{bmatrix}$ with I (in m^4)

That is to say a straight beam of axis $o\vec{x}$ for the undistorted configuration, it is necessary to define two intermediate bases to characterize the vector rotational speed $\vec{\Omega}_{R/R0}$.

- Transition of the base B (O $\vec{x}$ $\vec{y}$, $\vec{z}$) at the base B_1 (O $\vec{x}_1$ $\vec{y}_1$, $\vec{z}_1$) by a rotation of axis $o\vec{y}$ of amplitude $\theta_y(x,t)$ such as:
  $\vec{y}_1=\vec{y}$
- Transition of the base B (O $\vec{x}_1$ $\vec{y}_1$, $\vec{z}_1$) at the base B_2 (O $\vec{x}_2$ $\vec{y}_2$, $\vec{z}_2$) by a rotation of axis $o\vec{z}_1$ of amplitude $\theta_z(x,t)$ such as:
  $\vec{z}_2=\vec{z}_1$ and $\vec{y}_1=\cos\theta_z(x,t).\vec{y}_2+\sin\theta_z(x,t).\vec{x}_2$
- rotation at the angular velocity $\dot{\phi}(t)$ takes place along the axis $o\vec{x}_2$.

- Thus the instantaneous axis of rotation is written: $\vec{\Omega}_{R/R0}=\dot{\phi}(t).\vec{x}_2+\dot{\theta}_y(x,t).\vec{y}_1+\dot{\theta}_z(x,t).\vec{z}_1$
- Since the operator $[J]$ of the element beam is written in the base B2 which corresponds to the deformed position, it is imperative unless changing basic the operator of inertia, to write the vector rotational speed $\vec{\Omega}_{R/R0}$ in the base B2 .
  $\vec{\Omega}_{R/R0}=\dot{\phi}(t).\vec{x}_2+\dot{\theta}_y(x,t).\left(\cos\theta_z(x,t).\vec{y}_2+\sin\theta_z(x,t).\vec{x}_2\right)+\dot{\theta}_z(x,t).\vec{z}_2$
- By considering that the angles $\theta_y(x,t)$ and $\theta_z(x,t)$ are small, it is legitimate to carry out a development limited to order 1. The statement of the Flight Path Vector $\vec{\Omega}_{R/R0}$ becomes then:
  $\vec{\Omega}_{R/R0}=\left(\dot{\phi}(t)+\dot{\theta}_y(x,t).\theta_z(x,t)\right).\vec{x}_2+\dot{\theta}_y(x,t).\vec{y}_2+\dot{\theta}_z(x,t).\vec{z}_2$
- It remains to develop the following scalar product:
  $$\frac{1}{2}\rho\int_{x=0}^{L}\vec{\Omega}_{R/R0}.[J].\vec{\Omega}_{R/R0}\,dx=\frac{1}{2}\rho I_{yz}\int_{x=0}^{L}\left[\dot{\theta}_y^2+\dot{\theta}_z^2\right]dx+\frac{1}{2}\rho I_x.L.\dot{\phi}^2+\rho\dot{\phi}I_x\int_{x=0}^{L}\dot{\theta}_y.\theta_z\,dx$$

For an element beam of constant section, the statement becomes:

$$T=\frac{1}{2}\rho A\int_{x=0}^{L}\left[\dot{u}^2+\dot{v}^2+\dot{w}^2\right]dx+\frac{1}{2}\rho I_{yz}\int_{x=0}^{L}\left[\dot{\theta}_y^2+\dot{\theta}_z^2\right]dx+\frac{1}{2}\rho I_x.L.\dot{\phi}^2+\rho\dot{\phi}I_x\int_{x=0}^{L}\dot{\theta}_y.\theta_z\,dx$$

With:

$$I_x=\frac{\pi}{2}.\left[R_e^4-R_e^4\right]$$

$$I_{yz}=I_y=I_z=\frac{\pi}{4}.\left[R_e^4-R_e^4\right]$$

The various terms of kinetic energy represent:

- for the first term, kinetic energy of translation,
- for the two following terms, kinetic energy of rotation,
- for the fourth term, the gyroscopic term of effect.

2.3 Interpolation functions

For the strains of membrane (traction and compression), the field $u(x)$ is approached by a linear function of displacements of nodes 1 and 2 of the element beam:

$$u(x)=\langle N_1^L(x)\ N_2^L(x)\rangle\begin{Bmatrix}u_1\\u_2\end{Bmatrix}\quad\text{with}\quad\begin{cases}N_1^L(x)=1-\dfrac{x}{L}\\N_2^L(x)=\dfrac{x}{L}\end{cases}$$

For the strains of bending, one uses cubic functions of type modified Hermit. The degrees of freedom $v(x),\theta_y(x),w(x),\theta_z(x)$ are thus interpolated as follows:

$$v(x)=\langle \xi_1(x)\ -\xi_2(x)\ \xi_3(x)\ -\xi_4(x)\rangle \begin{Bmatrix} v_1 \\ \theta_{z1} \\ v_2 \\ \theta_{z2} \end{Bmatrix}$$

$$\theta_z(x)=\langle -\xi_5(x)\ \xi_6(x)\ -\xi_7(x)\ \xi_8(x)\rangle \begin{Bmatrix} v_1 \\ \theta_{z1} \\ v_2 \\ \theta_{z2} \end{Bmatrix}$$

$$w(x)=\langle \xi_1(x)\ \xi_2(x)\ \xi_3(x)\ \xi_4(x)\rangle \begin{Bmatrix} w_1 \\ \theta_{y1} \\ w_2 \\ \theta_{y2} \end{Bmatrix}$$

$$\theta_y(x)=\langle \xi_5(x)\ \xi_6(x)\ \xi_7(x)\ \xi_8(x)\rangle \begin{Bmatrix} w_1 \\ \theta_{y1} \\ w_2 \\ \theta_{y2} \end{Bmatrix}$$

One defines K_{yz} shear coefficient in the directions y and z.

For an element beam of constant section:

$$K_{yz}=\frac{7+20.\alpha^2}{6} \quad \text{with} \quad \alpha=\frac{R_i}{R_e.\left(1+\frac{R_i^2}{R_e^2}\right)}$$

While noting $\phi_{yz}=\frac{12.E.I_{yz}}{K_{yz}.A.G.L^2}$, the functions ξ_i are as follows defined:

$$\xi_1(x)=\frac{1}{1+\phi_{yz}}\left[2.\left(\frac{x}{L}\right)^3-3.\left(\frac{x}{L}\right)^2-\phi_{yz}.\left(\frac{x}{L}\right)+(1+\phi_{yz})\right]$$

$$\xi_5(x)=\frac{6}{L.(1+\varphi_{yz})}.\left(\frac{x}{L}\right).\left[1-\left(\frac{x}{L}\right)\right]$$

$$\xi_2(x)=\frac{L}{1+\phi_{yz}}\left[-\left(\frac{x}{L}\right)^3+\frac{4+\phi_{yz}}{2}\left(\frac{x}{L}\right)^2-\frac{2+\phi_{yz}}{2}.\left(\frac{x}{L}\right)\right]$$

$$\xi_6(x)=\frac{1}{1+\phi_{yz}}\left[3.\left(\frac{x}{L}\right)^2-(4+\phi_{yz}).\left(\frac{x}{L}\right)+(1+\phi_{yz})\right]$$

$$\xi_3(x)=\frac{1}{1+\phi_{yz}}\left[-2.\left(\frac{x}{L}\right)^3+3.\left(\frac{x}{L}\right)^2+\phi_{yz}.\left(\frac{x}{L}\right)\right]$$

$$\xi_7(x)=\frac{-6}{L.(1+\phi_{yz})}.\left(\frac{x}{L}\right).\left[1-\left(\frac{x}{L}\right)\right]$$

$$\xi_4(x)=\frac{L}{1+\phi_{yz}}\left[-\left(\frac{x}{L}\right)^3+\frac{2-\phi_{yz}}{2}\left(\frac{x}{L}\right)^2+\frac{\phi_{yz}}{2}.\left(\frac{x}{L}\right)\right]$$
$$\xi_8(x)=\frac{1}{1+\phi_{yz}}\left[3.\left(\frac{x}{L}\right)^2+(-2+\phi_{yz}).\left(\frac{x}{L}\right)\right]$$

Note:

In the case of elements beams of Eulerian (Elements POU_D_E) the term ϕ_{yz} is null.

The vector of the degrees of freedom of the element beam is defined by:

$$\langle q\rangle=\langle u_1\ v_1\ w_1\ \theta_{x1}\ \theta_{y1}\ \theta_{z1}\ u_2\ v_2\ w_2\ \theta_{x2}\ \theta_{y2}\ \theta_{z2}\rangle$$

One poses:

$$\langle \delta u\rangle=\langle u_1\ u_2\rangle$$
$$\langle \delta v\rangle=\langle v_1\ \theta_{z1}\ v_2\ \theta_{z2}\rangle$$
$$\langle \delta w\rangle=\langle w_1\ \theta_{y1}\ w_2\ \theta_{y2}\rangle$$

By replacing the preceding approximations in the statement of kinetic energy, one obtains:

$$T=\frac{1}{2}\langle \delta\ \dot{u}\rangle[M_1]\{\delta\ \dot{u}\}+\frac{1}{2}\langle \delta\ \dot{w}\rangle([M_2]+[M_4])\{\delta\ \dot{w}\}+\frac{1}{2}\langle \delta\ \dot{v}\rangle([M_3]+[M_5])\{\delta\ \dot{v}\}$$
$$+\dot{\phi}.\langle \delta\ \dot{v}\rangle[M_6]\{\delta\ w\}+\frac{1}{2}.\rho.I_x.\dot{\phi}^2$$

With:

$$[M_1]=\int_{x=0}^{L}\rho.A.\begin{Bmatrix}N_1^L(x\\N_2^L(x)\end{Bmatrix}.\langle N_1^L(x)\ N_2^L(x)\rangle.dx$$

$$[M_2]=\int_{x=0}^{L}\rho.A.\begin{Bmatrix}\xi_1(x)\\\xi_2(x)\\\xi_3(x)\\\xi_4(x)\end{Bmatrix}.\langle \xi_1(x)\ \ \xi_2(x)\ \ \xi_3(x)\ \ \xi_4(x)\rangle.dx$$

$$[M_3]=\int_{x=0}^{L}\rho.A.\begin{Bmatrix}\xi_1(x)\\-\xi_2(x)\\\xi_3(x)\\-\xi_4(x)\end{Bmatrix}.\langle \xi_1(x)\ \ -\xi_2(x)\ \ \xi_3(x)\ \ -\xi_4(x)\rangle.dx$$

$$[M_4]=\int_{x=0}^{L}\rho.I_{yz}.\begin{Bmatrix}\xi_5(x)\\\xi_6(x)\\\xi_7(x)\\\xi_8(x)\end{Bmatrix}.\langle \xi_5(x)\ \ \xi_6(x)\ \ \xi_7(x)\ \ \xi_8(x)\rangle.dx$$

$$[M_5]=\int_{x=0}^{L}\rho.I_{yz}.\begin{Bmatrix}-\xi_5(x)\\ \xi_6(x)\\ -\xi_7(x)\\ \xi_8(x)\end{Bmatrix}.\langle -\xi_5(x)\ \ \xi_6(x)\ \ -\xi_7(x)\ \ \xi_8(x)\rangle .dx$$

$$[M_6]=\int_{x=0}^{L}\rho.I_{x}.\begin{Bmatrix}-\xi_5(x)\\ \xi_6(x)\\ -\xi_7(x)\\ \xi_8(x)\end{Bmatrix}.\langle \xi_5(x)\ \ \xi_6(x)\ \ \xi_7(x)\ \ \xi_8(x)\rangle .dx$$

2.4 Computation of the balance equations

the Lagrange's equations for the kinetic energy of the beam are written in the following form:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot q_i}\right)-\frac{\partial T}{\partial q_i}=0 \quad \text{with} \quad \langle q\rangle=\langle u\ v\ w\rangle$$

Is:

$$\begin{cases}\dfrac{d}{dt}\left(\dfrac{\partial T}{\partial(\delta\dot u)}\right)-\dfrac{\partial T}{\partial(\delta u)}=[M_1]\,\{(\delta\ddot u)\}\\ \dfrac{d}{dt}\left(\dfrac{\partial T}{\partial(\delta\dot v)}\right)-\dfrac{\partial T}{\partial(\delta v)}=([M_3]+[M_5])\,\{(\delta\ddot v)\}-\dot\varphi.[M_6]\,\{(\delta\dot w)\}-\ddot\varphi.[M_6]\,\{(\delta w)\}\\ \dfrac{d}{dt}\left(\dfrac{\partial T}{\partial(\delta\dot w)}\right)-\dfrac{\partial T}{\partial(\delta w)}=([M_2]+[M_4])\,\{(\delta\ddot w)\}+\dot\varphi.[M_6]^T\,\{(\delta\dot v)\}\end{cases}$$

These equations can be put in the form:

$$[M]\langle\ddot q\rangle+[C_{gyro}]\langle\dot q\rangle+([K]+[K_{gyro}])\langle q\rangle=\langle 0\rangle$$

The gyroscopic damping matrix $[C_{gyro}]$ of the system is made up from the matrix $[M_6]$ and of its transposed. It is skew-symmetric, and its contribution must be multiplied by the angular velocity $\dot\phi$.

While noting: $\phi=\phi_{yz}$

$$[M_6]=\frac{\rho.I_x}{30L(1+\phi)^2}\begin{bmatrix}-36 & 3\mathrm{L}(1-5\phi) & 36 & 3\mathrm{L}(1-5\phi)\\ -3\mathrm{L}(1-5\phi) & L^2(4+5\phi+10\phi^2) & 3\mathrm{L}(1-5\phi) & -L^2(1+5\phi-5\phi^2)\\ 36 & -3\mathrm{L}(1-5\phi) & -36 & 3\mathrm{L}(-1+5\phi)\\ -3\mathrm{L}(1-5\phi) & -L^2(1+5\phi-5\phi^2) & -3\mathrm{L}(-1+5\phi) & L^2(4+5\phi+10\phi^2)\end{bmatrix}$$

$$[C_{gyro}]=\frac{\rho . I_x}{30\,L(1+\phi)^2}\times$$

$$\begin{bmatrix}
- & - & - & - & - & - & - & - & - & - & - & - \\
 & 0 & 36 & - & -3L(1-5\phi) & 0 & - & 0 & -36 & - & -3L(1-5\phi) & 0 \\
 & & 0 & - & 0 & -3L(1-5\phi) & - & 36 & & - & 0 & -3L(1-5\phi) \\
 & & & - & - & - & - & - & & - & - & - \\
 & & & & 0 & L^2(4+5\phi+10\phi^2) & - & -3L(1-5\phi) & & - & 0 & -L^2(1+5\phi-5\phi^2) \\
 & & & & & 0 & - & 0 & -3L(1-5\phi) & - & L^2(1+5\phi-5\phi^2) & 0 \\
 & & & & & & - & - & & - & - & - \\
 & & & & & & & 0 & 36 & - & 3L(1-5\phi) & 0 \\
 & & & & & & & & 0 & - & 0 & 3L(1-5\phi) \\
 & & & & & & & & & - & - & - \\
 & & & & & & & & & & 0 & L^2(4+5\phi+10\phi^2) \\
 & & & & & & & & & & & 0
\end{bmatrix}$$

As the matrix $[C_{gyro}]$ is antisymmetric, only the higher triangle is represented.
(–) means that the degree of freedom is not concerned with the gyroscopic matrixes.

The gyroscopic stiffness matrix $[K_{gyro}]$ system is made up from the matrix $[M_6]$. Its contribution must be multiplied by acceleration $\ddot{\phi}$.

$$[K_{gyro}]=\frac{\rho . I_x}{30\,L(1+\phi)^2}\times$$

$$\begin{bmatrix}
- & - & - & - & - & - & - & - & - & - & - & - \\
- & 0 & 36 & - & -3L(1-5\phi) & 0 & - & 0 & -36 & - & -3L(1-5\phi) & 0 \\
- & 0 & 0 & - & 0 & 0 & - & 0 & & - & 0 & 0 \\
- & - & - & - & - & - & - & - & & - & - & - \\
- & 0 & 0 & - & 0 & 0 & - & 0 & & - & 0 & 0 \\
- & 0 & 3L(1-5\phi) & - & -L^2(4+5\phi+10\phi^2) & 0 & - & 0 & -3L(1-5\phi) & - & L^2(1+5\phi-5\phi^2) & 0 \\
- & - & - & - & - & - & - & - & - & - & - & - \\
- & 0 & -36 & - & 3L(1-5\phi) & 0 & - & 0 & 36 & - & 3L(1-5\phi) & 0 \\
- & 0 & 0 & - & 0 & 0 & - & 0 & 0 & - & 0 & 0 \\
- & - & - & - & - & - & - & - & - & - & - & - \\
- & 0 & 0 & - & 0 & 0 & - & 0 & 0 & - & 0 & 0 \\
- & 0 & 3L(1-5\phi) & - & L^2(4+5\phi+10\phi^2) & 0 & - & 0 & -3L(1-5\phi) & - & -L^2(4+5\phi+10\phi^2) & 0
\end{bmatrix}$$

the matrix full $[K_{gyro}]$ is filled in integer (triangles higher and inferior).

Recall:

- with $\langle q\rangle=\langle u_1\ v_1\ w_1\ \theta_{x1}\ \theta_{y1}\ \theta_{z1}\ u_2\ v_2\ w_2\ \theta_{x2}\ \theta_{y2}\ \theta_{z2}\rangle$
- in the case of elements beams of Eulerian (Elements POU_D_E) the term ϕ_{yz} is null.

3 The circular disc

the purpose of this chapter is to characterize the gyroscopic matrixes of an infinitely rigid circular disc, subjected at a constant or variable rotational speed.

The characteristics of the disc are the following ones:

- center disc confused with the axis of neutral fiber of the beam (axis $\vec{x}$)
- center of gravity of the disc: C
- interior radius: R_i

- external radius: R_e
- thickness: h
- presumedly uniform density: ρ

Deduced values:

- mass disc: $M=\pi\,\rho\,h\,(R_e^2-R_i^2)$
- main moment of inertia mass/axes y or z calculated at the center of gravity C :

  $I_{yz}=\frac{M}{12}(3.R_e^2+3.R_i^2+h^2)$
- mass main moment of inertia compared to the axis x calculated at the center of gravity C :

  $I_x=\frac{M}{2}(R_e^2+R_i^2)$

Note:

- The axes $C\vec{x}$, $C\vec{y}$ and $C\vec{z}$ being principal axes of inertia of the disc, the products of inertia I_{xy} , I_{yz} and I_{xz} are null.
- The symmetry of the disc compared to the axes $C\vec{y}$ and $C\vec{z}$ imposes: $I_{yz}=I_y=I_z$

The displacement of the center of the disc is given by: $u.\vec{x}+v.\vec{y}+w.\vec{z}$

One notes:

- $\vec{\Omega}_{R/R0}$: the vector rotational speed of the disc
- $\vec{x}.\vec{\Omega}_{R/R0}=\dot{\phi}(t)$: clean rotational speed

3.1 Computation of the kinetic energy of the disc

One calculates the kinetic energy of the disc by applying the formula of Huygens:

$$T=\frac{1}{2}M\ .\left(\vec{V}_{C,D/R0}\right)^2+\frac{1}{2}\vec{\Omega}_{R/R0}.[J].\vec{\Omega}_{R/R0}$$

$$T=\frac{1}{2}M\ .\left(\dot{u}^2+\dot{v}^2+\dot{w}^2\right)^2+\frac{1}{2}\vec{\Omega}_{R/R0}.[J].\vec{\Omega}_{R/R0}$$

with: $[J]=\begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}$ with $I_{yz}=I_y=I_z$

By developing the preceding statement, one obtains:

$$T=\frac{1}{2}\rho.\left(\dot{u}^2+\dot{v}^2+\dot{w}^2\right)^2+\frac{1}{2}I_{y\,z}.(\dot{\theta}_y^2+\dot{\theta}_z^2)+\frac{1}{2}\ I_x.(\dot{\phi}^2+2\dot{\theta}_y.\dot{\phi}.\theta_z)$$

The various terms of kinetic energy represent:

- for the first term, kinetic energy of translation,
- for the second term, kinetic energy of rotation,
- for the term $\frac{1}{2}I_x.\dot{\phi}^2$, "clean" energy of rotation,
- and for the term $I_x.(\dot{\theta}_y.\dot{\phi}.\theta_z)$, the gyroscopic effect.

3.2 Computation of the balance equations

the Lagrange's equations are used to formulate the dynamic equilibrium of the disc. In this case, typical case strain energy is null (disc infinitely rigid) and no external force is considered, one thus has:

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}=0$ with $\langle q\rangle=\langle u\ v\ w\ \theta_y\ \theta_z\rangle$: vector of the degrees of freedom of the element disc.

Account of the degree of freedom is not taken ϕ because it is considered that clean rotational speed is imposed and thus known. The following equations then are obtained:

$$\begin{cases}\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{u}}\right)-\frac{\partial T}{\delta u}=M.\ddot{u}\\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{v}}\right)-\frac{\partial T}{\delta v}=M.\ddot{v}\\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{w}}\right)-\frac{\partial T}{\delta w}=M.\ddot{w}\\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_y}\right)-\frac{\partial T}{\partial \theta_y}=I_{yz}.\ddot{\theta}_y+I_x.\dot{\phi}.\dot{\theta}_z+I_x.\ddot{\phi}.\theta_z\\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_z}\right)-\frac{\partial T}{\partial \theta_z}=I_{yz}.\ddot{\theta}_z-I_x.\dot{\phi}.\dot{\theta}_y\end{cases}$$

These equations can be put in the form:

$$[M]\langle\ddot{q}\rangle+[C_{gyro}]\langle\dot{q}\rangle+([K]+[K_{gyro}])\langle q\rangle=\langle 0\rangle$$

the gyroscopic damping matrix of the disc is obtained as from the main moment of inertia I_x. It is skew-symmetric, and its contribution must be multiplied by the clean angular velocity $\dot{\phi}$.

$$[C_{gyro}]=\dot{\phi}.\begin{bmatrix}0&0&0&-&0&0\\0&0&0&-&0&0\\0&0&0&-&0&0\\-&-&-&-&-&-\\0&0&0&-&0&I_x\\0&0&0&-&-I_x&0\end{bmatrix}$$

with $\langle u\ v\ w\ \theta_x\ \theta_y\ \theta_z\rangle$ vector of the degrees of freedom of the element disc and such as: $\dot{\theta}_x=\dot{\phi}$
The indent corresponds to the degree of freedom of rotation along the axis of the beam and leads obviously to null terms.

The gyroscopic stiffness matrix of the disc is also obtained as from the main moment of inertia I_x. Its contribution must be multiplied by clean acceleration $\ddot{\phi}$.

$$[K_{gyro}]=\ddot{\phi}.\begin{bmatrix} 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & - & 0 & 0 \\ - & - & - & - & - & - \\ 0 & 0 & 0 & - & 0 & I_x \\ 0 & 0 & 0 & - & 0 & 0 \end{bmatrix}$$

4 Description of the versions

Version Aster	Author (S) Organization (S)	Description of the modifications
9.4	E. BOYERE, X. RAUD EDF/R & D AMA	initial Text
9.8	Mr. Torkhani EDF/R & D AMA	Correction of shells