Modelization of the rotors fissured by equivalent stiffness function of the swing angle

Summarized:

This document describes a method of taking into account of cracks in the rotors modelled in 1D.

It is based on theoretical developments described in the notes [1] and [2] and published in [3], establishing, under certain assumptions, equivalence between a model 3D nonlinear with computation of the contact and a model of beam where nonthe linearity due to crack is simulated by an equivalent stiffness concentrated between two beam elements and nonlinear tabulated function of the angle of the bending moment compared to the crack. This method used, in the computer code of lines of trees CADYRO, was extended and validated compared to the experiment on the test bench EUROPE [4]. It is taken again in operator `DYNA_VIBRA` of *Code_Aster*.

1 Introduction and purpose

For reasons of performance and implementation practice the lines of trees in dynamics are not modelled by of the finite elements 3D but by beam elements, which represent in a natural way slenderness of the rotors and which, in linear elasticity, approaches the real behavior of the system at a weak cost.

However one wishes to take into account in the computation of the dynamics of the lines of trees the impact of a crack on the rotor. If computation 3D, with fine modelization of the behavior of crack by a contact algorithm is conceptually possible, it is extremely demanding in data-processing resources. This is why one the model models the usual behavior of crack on beam of line of trees by creating two nodes on the axis of the rotor fissured with the right of crack and by introducing “a nonlinear crack constitutive law” binding average rotations of the two nodes representing the lips of crack to the bending moments applied to the beam.

The crack constitutive law is tabulated from computations 3D. It is shown that it is not necessary to traverse all the field of the couples (My, Mz) but which it is enough to a computation for each angle of the bending moment compared to the crack.

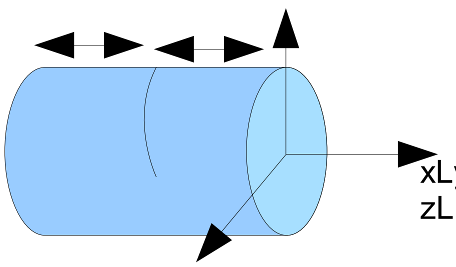
2 Assumptions

the assumptions used the model to build 1D crack are:

- the rotor has an isotropic elastic behavior;
- the crack can be of an unspecified form but it contained in a section of beam;
- the lips of crack are connected by a condition of contact without friction;
- the normal force and cutting-edges are neglected (under certain assumptions one can take into account the torsional stresses but in the frame of this element they are forgotten);
- the initial clearance of crack is null;
- the behavior of crack is quasi-static and the effects of inertia are negligible.

3 References and notations

By default the local coordinate system of the beams in Code_Aster is the axis X. This convention is preserved.



the element of fissured rotor is characterized by:

- its length $2L$;
- its radius R ;
- the position of the fissured section (in the middle of the rotor)

One considers the following mechanical characteristics:

- Young modulus: E
- Inertia of the not fissured beam: I

One follows the classical description of the kinematics of the beams. The perpendicular sections of the beam remain right and their rotation is described by:

$$\theta=\begin{pmatrix}\theta_y(x)\\ \theta_z(x)\end{pmatrix}$$

The beam is subjected to one bending moment:

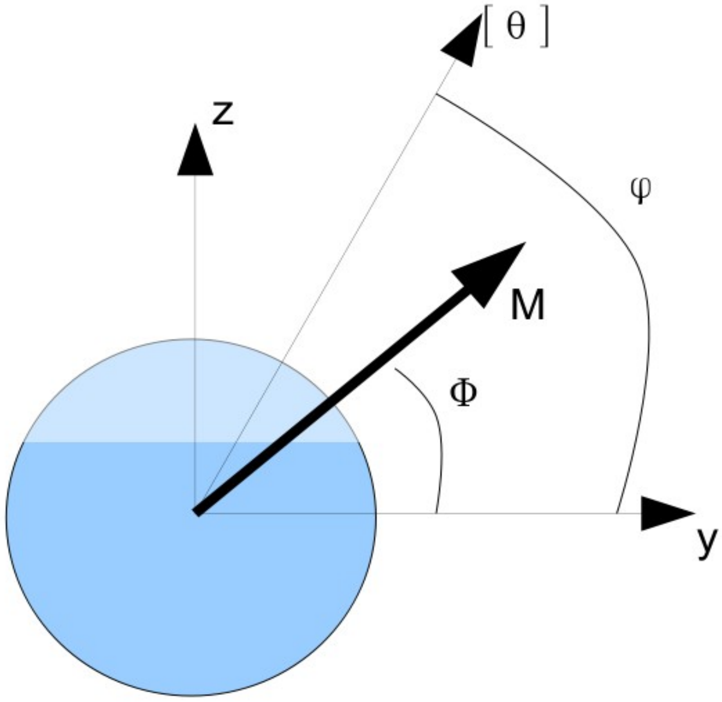
$M=\begin{pmatrix}M_y\\ M_z\end{pmatrix}$, which can open or close crack.

The strain energy of the part of rotor fissured under the loading of bending imposed is noted $W^f(M)$.

The strain energy brought by the discrete element modelling crack for a discontinuity of rotation $[\theta]$ is noted $W^d([\theta])$.

The description of the fissured section, though realised in the modelization of beam, requires however a certain attention, because the directional sense of the response to the level of crack is not systematically the same one as that of the force. It depends on the fissure shape.
Two angles are thus defined in the reference of the fissured section:

- Φ , directional sense of the force imposed in the revolving reference;
- φ , directional sense of the response in the revolving reference.

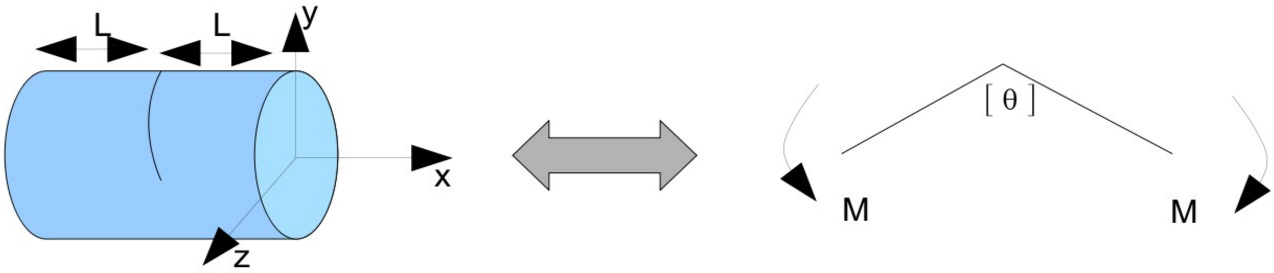


Lastly, one notes $s(M)$ the local flexibility of the element are equivalent 1D representing crack and $k([\theta])$ his local stiffness.

4 Determination of the equivalent stiffness of crack

4.1 Models equivalent of crack

One seeks in this chapter to determine the constitutive law $M=f([\theta])$ 1D connecting for the crack element the bending stresses M to the strains expressed by the jump of rotation $[\theta]=\begin{pmatrix}[\theta_y(x)]\\ [\theta_z(x)]\end{pmatrix}$ of the lips of crack represented by the two nodes borders of crack.



4.2 Strain energy

4.2.1 Formulates strain energy with force imposed

In [1] one establishes that the energy of the model 3D into quasi-static is given by a formula of "Clapeyron":

$$W^f(M_y, M_z)=\frac{1}{2}\left(M_y\theta_y(2\mathrm{L})+M_z\theta_z(2\mathrm{L})\right)$$

On the model finite element 3D, one can thanks to this formula compute the energy of the system for all the possible couples (M_y, M_z) and 1D to determine the equivalent $k(M_y, M_z)$ stiffness for the element of beam fissured.

However to explore a space equivalent to $\mathbb{R}^2$ would be too expensive for a practical application. The properties of the functional calculus of energy of the elastic problem with contact make it possible to reduce the problem to only one variable.

It is shown that W^f is convex and positively homogeneous of degree 2 compared to the applied moments:

$$\forall\lambda\geq 0,\quad W^f(\lambda M_y,\lambda M_z)=\lambda^2 W^f(M_y, M_z)$$

This property is interesting because she wants to say, in practice, that the contact zone on the lips of crack is independent of the amplitude M of bending. Energy is quadratic in M, as in the linear case. Its form depends only on the direction on (M_y, M_z). One can thus rewrite the function of energy with, for variables, the amplitude of the bending stress $\|M\|$ and its angle ϕ.

In the cylindrical coordinate system the moment is written:

$M_y=\|M\|\cos\phi$
$M_z=\|M\|\sin\phi$ and strain energy: $W_f(M)=\frac{1}{2}\|M\|^2 S(\phi)$

The problem is reduced to the identification of $S(\phi)$ on the interval $[0,2\pi]$.

Energy is rewritten in a form which distinguishes the flexibility $S(\phi)$, is brought by the beams (left healthy and fissured) and the flexibility $s(\phi)$, brought by crack:

$$W^f(M)=\frac{L}{EI}\|M\|^2(1+s(\phi))$$

By calculating strain energy on a model finite elements 3D of the fissured bar (figure 1), one can determine $s(\phi)$.

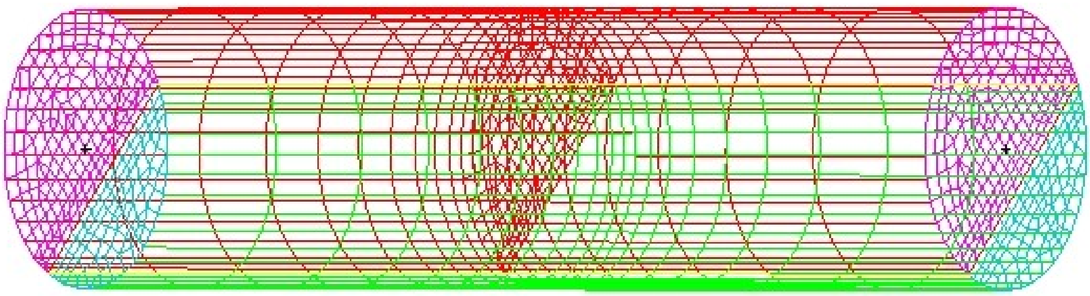


Illustration 1: Mesh 3D of the bar fissured

4.2.2 Energy with imposed displacement

It now remains to establish the relation between energy and $[\theta]$. One uses for that energy with imposed displacement w^d. It has the same properties of convexity and homogeneity of degree 2 that energy with imposed force.

While posing:

$[\theta_x]=\|[\theta]\|\cos(\varphi)$

$[\theta_y]=\|[\theta]\|\sin(\varphi)$

one obtains the following statement for strain energy:

$$w^d([\theta])=\frac{EI}{4\mathrm{L}}\|[\theta]\|^2 k(\varphi)$$

4.2.3 constitutive law of the fissured element

One obtains M according to $[\theta]$ by derivative of energy:

$$M_y=\frac{EI}{4L}\frac{\partial(k(\varphi)([\theta_y]^2+[\theta_z]^2))}{\partial[\theta]_y}$$
$$M_z=\frac{EI}{4L}\frac{\partial(k(\varphi)([\theta_y]^2+[\theta_z]^2))}{\partial[\theta]_z}$$
with $[\theta]=\sqrt{[\theta_y]^2+[\theta_z]^2}\begin{pmatrix}\cos(\varphi)\\ \sin(\varphi)\end{pmatrix}$

Some intermediate computations lead to the following relations:

$$\begin{pmatrix}M_y\\ M_z\end{pmatrix}=\frac{EI}{2L}\begin{pmatrix}k(\varphi) & -\frac{1}{2}k'(\varphi)\\ \frac{1}{2}k'(\varphi) & k(\varphi)\end{pmatrix}\begin{pmatrix}[\theta_y(x)]\\ [\theta_z(x)]\end{pmatrix}$$ where $\varphi=\arctan(\frac{[\theta_z(x)]}{[\theta_y(x)]})$

4.2.4 Relation between the stiffness of crack and its aperture

the direction φ of the average angle of opening of crack $[\theta]$ is not *a priori* the same one as ϕ, that of the forces of the moments M.

It is thus a question now of establishing a direct relationship between the stiffness (or its reverse, flexibility) and the opening of crack. It is established in [1] and [2].

By operating of the principle of complementary energy and thanks to the convexity of strain energy, one shows that:

$$w^d([\theta])=\underset{M}{Sup}\left(M.[\theta]-\frac{L}{EI}\|M\|^2 s(\phi)\right)$$

The function $s(\phi)$ was established in the preceding paragraphs. A complementary mathematical processing consists in applying to $s(\phi)$ a quadratic interpolation apart from the interval of nullity of $s(\phi)$ and an interpolation power at the boundaries of nullity of $s(\phi)$.

In addition, in the same way that energy at imposed time, energy with imposed strain is convex and homogeneous of order 2. That implies that it does not depend on the norm on $[\theta]$, but only of its direction φ. One can thus limit oneself to identify $w^d([\theta])$ for $\|[\theta]\|=1$.
Energy is written then:

$$w^d([\theta])=\frac{EI}{4L}k(\varphi).$$

If λ is the amplitude of the applied moment and ϕ its direction, by identification one finds the formula giving the equivalent crack stiffness:

$$k(\varphi)=\frac{4L}{EI}\underset{M=1}{Sup}\,\underset{\lambda\geq 0}{Sup}\left(\lambda M.[\theta]-\frac{L}{EI}\lambda^2 s(\phi)\right)$$

In practice that amounts finding the field $[\varphi_{1,}\varphi_2]$ which the direction of crack can have then to maximize for all the possible crack angles φ the following statement:

$$k(\varphi)=\underset{\phi\in[\varphi-\frac{\pi}{2,}\varphi+\frac{\pi}{2}]}{Sup}\frac{\cos^2(\phi-\varphi)}{s(\phi)}$$

The derivative $k'(\varphi)=\frac{d\,k(\varphi)}{d\,\varphi}$ is then obtained using a derivative by Finite differences of $k(\varphi)$.

5 Bibliography

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3.Stephane Andrieux, Christophe Varé “A 3D cracked beam model with unilateral contact. Application to rotors”; European Newspaper of Mechanics A/Solids 21 (2002) 793-810

4.Carlo Maria Stoisser, Sylvie Audebert “ A understanding theoretical, numerical and experimental approach for ace detection in power seedling rotating machinery”; Mechanical Systems and Processing Signal 22 (2008) 818-844

6 History of the versions of the document

Version Aster	Author (S) or contributor (S), organization	Description of the modifications
11.1	Mr. TORKHANI	initial Version.