Reduction of model in linear and nonlinear dynamics: Method of RITZ

Summarized:

This document presents the principle of reduction of model by projection on reduced basis (method of Ritz). The base most usually used is modal base.

The problems of truncation due to the use of a reduced base are mentioned. Corrections of truncation are proposed.

The description and the properties of the algorithms of resolution of the system of equations differentials of the second order obtained in transient analysis are presented in the document [R5.06.04].

Contents

1 Introduction

From a description of the geometry and materials of structures, the finite element method makes it possible to create a precise and reliable but large-sized model. In the case of a problem of dynamics, one wishes to calculate the response of a system for various times (transient analysis) where for various frequencies (harmonic analysis). The size of the model finite elements obtained is often irreconcilable with the number of computations necessary to get all the desired results.
For a restricted set of dynamic stresses, there exists generally a subspace of low dimension making it possible to describe the dynamic behavior of structure under specific requests.
The projection of the model on a restricted basis is called method of Ritz or Rayleigh-Ritz.

This document comprises the following points:

- a presentation of the methods of Ritz, their use into linear,
- a detail of the possible corrections of truncation,
- the generalization in nonlinear of the methods of Ritz,
- two simple examples of illustration.

2 Methods of reduction of Ritz in linear

2.1 general Description

2.1.1 Formulation continues

the method of Ritz consists in projecting displacement on a restricted basis of functions checking the kinematical conditions of problem:

$$\tilde{u}(M,t)=\sum_{i=1}^{n}\eta_i(t)\psi_i(M) \qquad \text{éq 2.1.1-1}$$

displacement is described by a series of forms independent $\left\{\psi_i(M);i=1\ldots n\right\}$ multiplied by amplitudes functions of time $\left\{\eta_i(t);i=1\ldots n\right\}$.

The difficulty consists in defining this family of form $\left\{\psi_i(M);i=1\ldots n\right\}$ which contrary to the shape functions of the finite element method are non-zero on most of structure.
The quality of the approximation is related to the fact that displacements obtained have a good approximation in the subspace generated par. $Vect\left\{\psi_i(M),i=1\ldots n\right\}$

Projection on modal base

One knows that the eigen modes $\left\{\phi_i(M);i=1\ldots\infty\right\}$ generate the space of the kinematically admissible fields. Displacement breaks up according to:

$$u(M,t)=\sum_{i=1}^{\infty}\eta_i(t)\phi_i(M) \qquad \text{éq 2.1.1-2}$$

the option most usually used for the method of Ritz then consists in taking as projection base n the first modes:

$$\tilde{u}(M,t)=\sum_{i=1}^{n}\tilde{\eta}_i(t)\phi_i(M) \qquad \text{éq 2.1.1-3}$$

displacement obtained is an approximation of real displacement.

It can be interesting to add to n the first modes, other forms (see [§2.6.2]).

2.1.2 Approximation finite elements

In the case of an approximation of displacement by finite elements displacement is already approximate within the space of shape functions:

$$u^h(M,t)=\sum_{i=1}^{Nh} q_i(t)\,N_i(M) \qquad \text{éq 2.1.2-1}$$

One notes U the vector of the degrees of freedom of displacement: $U(t)=\left[q_1(t),q_2(t),\ldots q_{Nh}(t)\right]$;

Method of Ritz in finished dimension

If $n<Nh$, the method of Ritz applied to the field $u(M,t)$ comes then like a second approximation:

$$\tilde{U}(t)=\sum_{i=1}^{n} \eta_i(t)\,\psi_i \qquad \text{éq 2.1.2-2}$$

with $\left\{\psi_{i,i=1\ldots n}\right\}$ the base of n the independent and kinematically admissible vectors. One poses

$\Psi=\left[\psi_1,\psi_2,\psi_{3,\ldots},\psi_n\right]$. From where the matric writing: $U=\Psi\,\eta$ éq 2.1.2-3

2.2 Projection on reduced basis

Let us consider the following differential connection obtained by a method of finite elements:

$$\mathbf{M}\,\ddot{U}+\mathbf{C}\,\dot{U}+\mathbf{U}=\mathbf{F} \qquad U\in R^{Nh} \qquad \text{éq 2.2-1}$$

the solution sought in the form [éq 2.1.2-3]. While considering the same form for virtual displacement, it comes:

$$\Psi^T\mathbf{M}\,\psi\,\ddot{\eta}+\Psi^T\mathbf{C}\,\Psi\,\dot{\eta}+\Psi^T\mathbf{K}\,\Psi\,\eta=\Psi^T\mathbf{F} \qquad \eta\in R^n \qquad \text{éq 2.2-2}$$

where: η is the vector of generalized displacements, $\bar{\mathbf{K}}=\Psi^T\mathbf{K}\,\Psi$ et $\bar{\mathbf{M}}=\Psi^T\mathbf{M}\,\Psi$ are respectively called stiffness matrixes and of mass generalized.

The system [éq 2.2-2] is generally a coupled differential connection, the generalized matrixes which compose it are in the general case full even if at the beginning the matrixes $\mathbf{M}$ and $\mathbf{K}$ were hollow. One thus loses structure particular to the profit of a size of problem much more reduced $n\times n$.

In the general case, the system [éq 2.2-2] provides only one approximate solution of the system [éq 2.2-1]. The mistake which one makes is called truncation error.

One has no information on the value of this error. It can be very large if the subspace of projection is badly selected. It is known only that this error decreases when the size of projection base increases.

If one has information a priori on the form of the solution, one can choose in an effective way projection base in order to minimize this error. For example, if it is known that the solution is made up only of motions of solid body, it can restrict to 6 the dimension of space.

Thereafter, one chooses the base of the eigen modes like projection base.

2.3 Projection on Eigen modes

modal base

the modes are defined like the couples $\left\{\left(\omega^{h_i},\Phi^{h_i}\right)_{i=1\ldots Nh}\right\}$ solutions of the equation:

$$\left(K-\omega^2 M\right)\Phi=0 \qquad \text{éq 2.3-1}$$

Note::

It is advisable to check that the modes calculated by approximation finite elements are sufficiently representative: $(\omega^{h_i}, \Phi^{h_i}) \approx (\omega_i, \phi_i)$. One can consider that the approximation finite elements is correct when the modal deformed shapes present a length D `higher wave in keeping with meshes of the mesh (the notion of wave length is a generalization of the notion definite on the equation of the waves, one can twice define it as the length between two nodes of the modal deformed shape).
Thereafter one omits, the index h corresponding to the approximation finite elements.

Quotient of Raleigh : energy interpretation
the pulsations and eigenforms can be defined like the solutions of the problem of minimization according to *:*

$\forall i \in [1, Nh] : \Phi_i$ minimize under space $R^{Nh} - Vect\{\Phi_j, j \in [0, i-1]\}$ the functional calculus:

$$R(X) = \frac{X^t KX}{X^t MX} \text{ on pose: } \quad \omega_i^2 = \frac{\Phi_{i'} K \Phi_i}{\Phi_{i'} M \Phi_i} = R(\Phi_i) \qquad \text{éq 2.3-2}$$

Method of reduction
a method of reduction very largely employed for the linear problems is the method of modal recombination. It consists in choosing like projection base n the first eigen modes of structure $\{\Phi_{i, i=1 \ldots n}\}$.

$$\tilde{U}(t) = \sum_{i=1}^{n} \eta_i(t) \Phi_i \qquad \text{éq 2.3-3}$$

always Let us consider the following differential connection:

$$\mathbf{M}\ddot{U} + \mathbf{C}\dot{U} + \mathbf{K}U = \mathbf{F} \qquad U \in R^{Nh} \qquad \text{éq the 2.3-4}$$

eigen modes $\{\Phi_{i, i=1 \ldots Nh}\}$ have the property to be $\mathbf{M}$ and $\mathbf{K}$ orthogonal, i.e. one has the following relations:

$$\Phi_i^T \mathbf{M} \Phi_j = m_i \delta_{ij}$$
$$\Phi_i^T \mathbf{K} \Phi_j = k_i \delta_{ij}$$

δ is the Kronecker symbol
m_i is called modal mass or generalized mass of the mode i
k_i is called modal stiffness or generalized stiffness of the mode i

the matrixes projected of $\mathbf{M}$ and $\mathbf{K}$ on the basis of eigen mode are thus diagonal; it is one of the advantages which justified the use of modal base like projection base. The system [éq 2.3 - 4] project on the basis of eigen mode first of the system is written:

$$\begin{pmatrix} \backslash & 0 & 0 \\ 0 & m_i & 0 \\ 0 & 0 & \backslash \end{pmatrix} \ddot{\boldsymbol{\eta}} + \Phi^T \mathbf{C} \Phi \dot{\boldsymbol{\eta}} + \begin{pmatrix} \backslash & 0 & 0 \\ 0 & k_i & 0 \\ 0 & 0 & \backslash \end{pmatrix} \boldsymbol{\eta} = \Phi^T \mathbf{F}_{ext} \qquad \text{éq 2.3-5}$$

the projection of the matrix $\mathbf{C}$ does not have any reason in any generality to be also diagonal. If the system is strongly damped (presence of dampers on the structure), this matrix will not be diagonal.

Note:

As opposed to what do many software, Code_Aster allows in this case to integrate the system of equations modal coupled without diagonalization of the damping matrix generalized. The integration method is in this case an implicit method of NEWMARK or clarifies EULER.

On the other hand, if only damping entering concerned is a structural damping (internal dissipation of the material for a homogeneous structure) it is then licit to make the assumption of a proportional damping, still called assumption of BASILE, in this case $\mathbf{C}$ expresses itself like linear combination of $\mathbf{M}$ and $\mathbf{K}$ (damping of RAYLEIGH), and its projection on the eigen modes is diagonal (cf Doc. [R5.05.04] on the modelization of damping).

In this case, the system [éq 2.3-4] is divided into p linear differential equations of the second order decoupled. The response of the system is then the recombination of the response of p oscillators simple associates to the eigen modes, from where the statement of "modal superposition" used usually.

Each differential equation is written m_i :

$$m_i \ddot{\eta}_i + c_i \dot{\eta}_i + k_i \eta_i = f_i \qquad \text{éq 2.3-6}$$

or while dividing by the modal mass:

$$\ddot{\eta}_i + 2\xi_i \omega_i \dot{\eta}_i + \omega_i^2 \eta_i = \frac{f_i}{m_i} \qquad \text{éq 2.3-7}$$

with:

$$\xi_i \text{ amortissement modal réduit} = \frac{c_i}{c_{\text{critique}}} = \frac{c_i}{2\ m_i . \omega_i}$$

This equation can be solved very simply in the frequential field:

$$\hat{\eta}_i = \frac{\hat{\mathbf{f}}_i(\omega)}{m_i . \left(-\omega^2 + 2 . \xi_i \omega_i \omega + \omega_i^2\right)} \qquad \text{éq 2.3-8}$$

where $\hat{}$ represents the FOURIER transform and ω the excitation frequency.

Particular numerical methods the such integral of DUHAMEL make it possible to pass this statement of the frequential field to the temporal field. (see for example Doc. [R5.05.01] on a temporal integration method).

2.4 Modal truncation error

In the case of the modal recombination with proportional damping, one can highlight the truncation error which one makes while projecting on the basis of eigen mode first of the system. Indeed, if one considers the complete base of N eigen modes of the discretized problem, there is equivalence between the initial problem and the problem project. Thus the exact solution of the problem discretized by finite elements is written:

$$U = \sum_i^{Nh} \eta_i \Phi_i$$

where the generalized coordinates are solution of:

$$\ddot{\eta}_i + 2\xi_i \omega_i \dot{\eta}_i + \omega_i^2 \eta_i = \frac{f_i}{m_i}$$

summation extending on all the **eigen modes** from the system (of finished size).

By solving the problem with a reduced number of eigen modes $n < Nh$. The solution obtained is the following one:

$$\tilde{U}=\sum_{i=1}^{n}\eta_i\Phi_i$$

The mistake made by truncating the base of representation of the solution is thus:

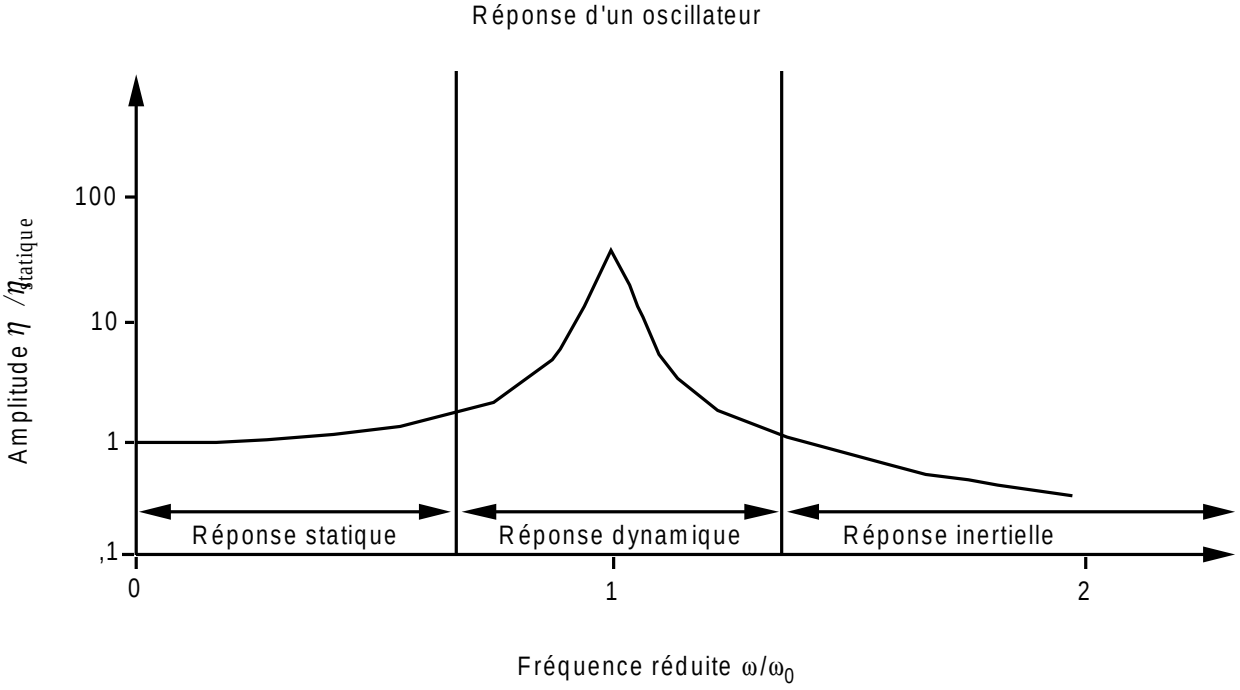
$$\mathbf{E}=U-\tilde{U}=\sum_{i=n+1}^{Nh}\eta_i\Phi_i \qquad \text{éq 2.4-1}$$

In the frequential field the statement of the error is:

$$\hat{\mathbf{E}}_{(\omega)}=\hat{U}-\hat{\tilde{U}}=\sum_{i=n+1}^{Nh}\frac{\Phi_i^t\hat{\mathbf{F}}_{(\omega)}}{m_i}.\frac{1}{\omega_i^2-\omega^2+2\mathrm{j}\,\xi_i\omega\,\omega_i}.\Phi_i \qquad \text{éq 2.4-2}$$

the summation is carried out neglected modes of the system on all the.

Let us study the response relative $\eta/\eta_{statique}$ of an oscillator to a purely sinusoidal excitation of variable frequency (diagram below), with $\eta_{statique}$ the coefficients of the static response corresponding to a static force. One can distinguish three intervals in the spectrum where the oscillator has a different behavior. In low frequency $(\omega<<\omega_0)$ the oscillator has a static answer. Around ω_0 the oscillator a dynamic response (amplification of the mode has), and high frequency the oscillator answers in an inertial way ($\frac{1}{\omega^2}$ dominating term).



Let us suppose that the excitation of the system, defined by the vector $\mathbf{F}(\omega)$, is to narrow tape, in particular that it for frequencies higher than $\omega_{\max}$ is null given.

In this case, to represent the response of the linear system correctly, it is undoubtedly necessary to take into account all the modes having a pulsation lower than $\omega_{\max}$, because the latter will answer in a dynamic way the excitation.

On the other hand, the modes such as $\omega_i>>\omega_{\max}$ nevertheless have a static contribution to the response of the system. These are often these modes that one does not take into account.

By making a development limited in ω the vicinity of 0 . One obtains the principal part of the error which is:

$$\hat{\mathbf{E}}_{(\omega)}=\hat{U}-\hat{\tilde{U}}=\sum_{i=p+1}^{n}\frac{\boldsymbol{\Phi}_i^T.\hat{\mathbf{F}}_{(\omega)}}{k_i}.\left(1-2\,j\,\xi_i\frac{\omega}{\omega_i}+0\left(\frac{\omega}{\omega_i}\right)\right).\boldsymbol{\Phi}_i \qquad \text{éq 2.4-3}$$

the error is all the more small as the generalized stiffness of the neglected modes is large. In theory thus, it will be necessary to take all the most flexible modes until the residual flexibility of an additional mode is in negligible relative value compared to the sum of the flexibilities already taken into account:

$$\frac{1}{k_{n+1}}<<\sum_{i=1}^{n}\frac{1}{k_i}$$

However, it is observed that by neglecting the modes of high frequency one makes a systematic mistake on the response of the system (even in low frequency). There exist various possibilities that we will detail now to correct the response in the beach $\left[0,\,\omega_{\max}\right]$ where the modes were chosen.

2.5 Corrections of modal truncation

to mitigate the problem of truncation due to the neglected modes, it is necessary to try to estimate their effect in the field of frequency $\left[0,\,\omega_{\max}\right]$ which interests us. We saw that the neglected modes having an own pulsation such as $\omega_i>>\omega_{\max}$ have a contribution known as static to the response of the system in the field $\left[0,\,\omega_{\max}\right]$. The techniques of correction consist in calculating this static contribution.

2.5.1 Static correction a posteriori

the truncation error, by considering only the static response of the neglected modes (transformed opposite of the principal part of the error) is:

$$\mathbf{E}_{(t)}=U-\tilde{U}\approx\sum_{i=n+1}^{Nh}\frac{\boldsymbol{\Phi}_i^T.\mathbf{F}_{(t)}}{k_i}.\boldsymbol{\Phi}_i \qquad \text{éq 2.5.1-1}$$

But a priori the neglected modes as their generalized stiffness are unknown. On the other hand, one can determine the complete static response of the system with a loading $\mathbf{F}$, the latter is worth:

$$U=\mathbf{K}^{-1}.\mathbf{F}_{(t)}\approx\sum_{i=1}^{Nh}\frac{\boldsymbol{\Phi}_i^T.\mathbf{F}_{(t)}}{k_i}.\boldsymbol{\Phi}_i$$

The correction which should be made is thus:

$$\sum_{i=n+1}^{Nh}\frac{\boldsymbol{\Phi}_i^T.\mathbf{F}_{(t)}}{k_i}.\boldsymbol{\Phi}_i\approx\mathbf{K}^{-1}.\mathbf{F}_{(t)}-\sum_{i=1}^{n}\frac{\boldsymbol{\Phi}_i^T.\mathbf{F}_{(t)}}{k_i}.\boldsymbol{\Phi}_i$$

The corrected solution of the response of the system is thus worth:

$$U=\tilde{U}+\mathbf{E}\approx\tilde{U}+\mathbf{K}^{-1}.\mathbf{F}_{(t)}-\sum_{i=1}^{n}\frac{\boldsymbol{\Phi}_i^T.\mathbf{F}_{(t)}}{k_i}.\boldsymbol{\Phi}_i \qquad \text{éq 2.5.1-2}$$

This correction is called a posteriori, because it does not intervene in the dynamic resolution of the linear system and can be calculated only well later on. If $\mathbf{F}_{(t)}$ breaks up into k products of functions of time by functions of the coordinates of space, this correction requires a factorization of $\mathbf{K}$ and k resolutions.

This method has the advantage of not increasing the number of vectors taken into account in the base. This method is applicable in the case of a narrow tape excitation, or at least having a known cut-off frequency. The correction is exact in the low frequency field but can distort the response of the system in high frequency [§Annexe1].

2.5.2 Addition of static modes to the base

Let us suppose that the loading $\mathbf{F}(t)$ is written:

$$\mathbf{F}(t)=\sum_j \alpha_j(t).\mathbf{F}_j$$

The second way of correcting the truncation error consists in adding to the base of the initial eigen modes of **the definite static** Ψ_j modes like the deformed shape with each force $\mathbf{F}_j$ given:

$$\Psi_j=\mathbf{K}^{-1}.\mathbf{F}_j \qquad \text{éq 2.5.2-1}$$

new projection base to be considered is the following one:

$$\hat{\Phi}=[\Phi_1,\ \Phi_2,\ \ldots,\Phi_p,\ \Psi_1,\ \Psi_2,\ \ldots,\Psi_m]=[\Phi\,\Psi] \qquad \text{éq the 2.5.2-2}$$

components generalized to use are the following ones:

$$\hat{\eta}=[\eta_1,\ \eta_2,\ \ldots,\eta_p,\ \mu_1,\ \mu_2,\ \ldots,\mu_m]=[\eta,\mu] \qquad \text{éq 2.5.2-3}$$

the problem project on the supplemented basis is:

$$\begin{bmatrix} \begin{pmatrix} \backslash & 0 & 0 \\ 0 & m_i & 0 \\ 0 & 0 & \backslash \end{pmatrix} & \Phi^T.\mathbf{M}.\Psi \\ \Psi^T.\mathbf{M}.\Phi & \Psi^T.\mathbf{M}.\Psi \end{bmatrix} \begin{Bmatrix} \ddot{\eta} \\ \ddot{\mu} \end{Bmatrix} + \begin{bmatrix} \begin{pmatrix} \backslash & 0 & 0 \\ 0 & k_i & 0 \\ 0 & 0 & \backslash \end{pmatrix} & \Phi^T.\mathbf{K}.\Psi \\ \Psi^T.\mathbf{K}.\Phi & \Psi^T.\mathbf{K}.\Psi \end{bmatrix} \begin{Bmatrix} \eta \\ \mu \end{Bmatrix} = \begin{Bmatrix} \Phi^T.\mathbf{F}.\alpha \\ \Psi^T.\mathbf{F}.\alpha \end{Bmatrix} \qquad \text{éq 2.5.2-4}$$

One notes that one lost the diagonal character of the generalized matrixes, but the advantage obtained is that the base supplemented with static modes makes it possible to correctly represent the low frequency behavior of the initial system.

For example it is simple to show that with frequency null the solution of this system is:
$\eta=0$ and $\mu=\alpha$ which is the exact solution of the initial static problem.

One presents in appendix 1, the comparison on a discrete system with 3 degrees of freedom between the exact solution, the solution projected on 1 mode, that projected on a mode with a static correction and the solution consisted 1 eigen mode and 1 static mode.

One realizes that the addition of static modes makes it possible to extend beyond the interval $\left[0,\ \omega_{\max}=\max(\omega_j)\right]$ the good dynamic representation of the system. This technique thus seems very interesting, it has the virtue to carry out the correction immediately what will be interesting for the nonlinear methods where one needs the knowledge of physical displacements to each time step.

Implementation the practical of this technique is specified in [U2.06.04].

3 Extension of the methods of reduction of Ritz in non- linear

3.1 general Problem

the nonlinear problem of dynamics discretized without damping can be generally put in the following form:

$$\mathbf{M}.\ddot{\mathbf{X}}+\mathbf{G}(\mathbf{X})=\mathbf{F}(t) \qquad \mathbf{X}\in R^n \qquad \text{éq 3.1-1}$$

$\mathbf{G}(\mathbf{X})$ is a nonlinear function of $\mathbf{X}$ which represents the internal forces of the system as all the other forces which are dependant on displacement, $\mathbf{F}$ the vector of the external forces and $\mathbf{M}$ the mass matrix of the system.
The tangent stiffness matrix of the system is by definition:

$$\mathbf{K}^{tg}_{(\mathbf{x})}=\frac{\partial \mathbf{G}}{\partial \mathbf{X}}(\mathbf{x}) \qquad \text{éq 3.1-2}$$

It makes it possible to define a modal base at every moment by:

$$\left(-\omega^{tg^2}_{i(\mathbf{X})}\mathbf{M}+\mathbf{K}^{tg}\right).\Phi^{tg}_{i(\mathbf{X})}=0 \qquad \text{éq the 3.1-3}$$

modes thus defined depend on $\mathbf{X}$, therefore time t.
Knowing that the computation of modal base is very expensive in computing times, the idea to want to project with each time step the model on a modal base, then to solve, is irrelevant compared to a direct resolution.

The method most usually used consists in defining a projection base while adding to the modes calculated on an initial configuration of the forms making it possible to project to it not linearity.
Example: if it not linearity comes from a specific shock, one proposes to enrich modal base with static modes allowing to project the force undergone by structure lasting the shock [R5.06.04].

The method of Ritz remains always relevant in nonlinear computations, if the selected base makes it possible to project correctly displacements and the forces.

The nonlinear problem project on an unspecified Ψ basis is written:

$$\Psi^T\mathbf{M}\Psi.\ddot{\eta}+\Psi^T\mathbf{G}(\mathbf{X})=\Psi^T\mathbf{F} \quad \eta\in R^n \qquad \text{éq 3.1-4}$$

Two possibilities are then possible:

- nonthe linearities are located and one can evaluate to it not linearity on projection base: $\mathbf{G}(\mathbf{X})=\mathbf{G}(\Psi\,\eta)$.
- The problem to be solved is a nonlinear differential connection into cubes η smaller size. Various strategies are possible to solve this problem, depending primarily on the technique of integration which one wishes to use.
- nonthe linearities are total, and it is necessary to compute: to pass by again within the space of physical degrees of freedom the internal forces: $\mathbf{G}(\mathbf{X})$.

This second method is more expensive it is much less current.

One presents in appendix 2, the comparison on a system with 3 degrees of freedom with nona linearity of x^3 between the exact solution and the solution obtained by the method above with 1 then 2 modes. One realizes that it is necessary to take more modes counts some than for the linear problem. On the other hand, on this example 2 modes are enough very well to describe the system.

3.2 Indication of the error of projection

For the nonlinear problems the physical meaning amongst modes to take into account is completely lost, and if the methods of reduction always give a solution it is necessary to know the degree of confidence which one can grant to them. A way of proceeding, which is a little expensive but essential is to calculate the residue of the initial system to each time step. It is defined by:

$$\mathbf{R}=\mathbf{M}.\ddot{\mathbf{X}}+\mathbf{G}(\mathbf{X})-\mathbf{F}(t)$$

This vector residue is unfortunately not null, it is only its projection on the basis used which is it.

A norm can then be calculated for this residue; more the norm of the residue will be small more one will be able to grant confidence to the solution.

To use a relative value, one may find it beneficial to calculate the following fraction:

$$r=\frac{\|\mathbf{R}\|}{\max\left(\|\mathbf{F}(t)\|,\|\mathbf{G}(\mathbf{X})\|,\|\mathbf{M}.\ddot{\mathbf{X}}\|\right)}$$ éq 3.2-1

Note::

This indicator is not currently established in Code_Aster.

4 Use in *Code_Aster*

In *Code_Aster,* the methods of Ritz are usable out of transient primarily by the operator DYNA_TRAN_MODAL [U4.53.21].

A phase of projection of the stiffness matrixes and mass on a basis of vectors is carried out by operators PROJ_MATR_BASE [U4.63.12] and PROJ_VECT_BASE [U4.63.13].

The generalized dynamic problem is then solved in operator DYNA_TRAN_MODAL by a diagram of explicit integration (EULER or DEVOGELEARE) or implicit (NEWMARK). The characteristics and properties of the diagrams of integration are presented in the note [R5.06.04]. For the structures for which the assumption of BASILE does not apply (damping nonproportional) one will project also the damping matrix which does not become diagonal. The integration of the coupled system is done then obligatorily with the implicit scheme (NEWMARK) or clarifies (EULER).

Localised non-linearities are specified directly in operator DYNA_TRAN_MODAL. One can introduce localised non-linearities of the standard shock and friction (see [R5.06.03] Modelization of the shocks and frictions), modal forces function of displacement or velocity (see [R5.06.05] on the modelization of a force of fluid blade).
Static corrections of truncation a posteriori are available in the case of a single excitation (see [R4.05.01] seismic Response).

The addition of static modes can by means of be made as a preliminary operators MODE_STATIQUE [U4.52.14] and DEFI_BASE_MODALE [U4.64.02]. When the problem comprises non-linearities only the explicit diagrams can be used.

For total non-linearities [éq 3.1-4], it is possible to use command DYNA_NON_LINE [U4.53.01] with the operand PROJ_MODAL, which makes it possible to calculate with each time step internal forces according to the physical degrees of freedom, then to project the problem on a modal base.

An operation of return to physical base is then necessary to obtain the physical quantities such as displacement, velocity or acceleration on the structure. This operation is carried out by the operator REST_GENE_PHYS [U4.63.31] for the group of structure or RECU_FONCTION, (key key factor RESU_GENE) [U4.32.01] for the follow-up of a quantity in a node. In the case of total non-linearities (computation with DYNA_NON_LINE) operator REST_COND_TRAN [U4.63.33] is used.

More generally, the approach of Ritz can be used in harmonic computation by the command DYNA_LINE_HARM [U4.53.22] and of power spectral density by the command DYNA_ALEA_MODAL [U4.53.23].

Finally the dynamic substructuring can be regarded as a method of Ritz specific [R4.06.02].

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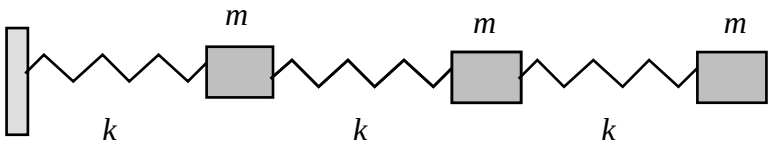
6	*(EDF R & D/ AMV) initial Text*	7,4 L.RATIER
7.4	*G. JACQUART (EDF R & D/ AMA, EDF/CNPE of Tricastin) Let us consider*	

Annexe 1

the discrete system with three masses according to: The stiffness matrixes



and of mass are: That is to say:

$$\mathbf{M}=\begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix} \qquad \mathbf{K}=\begin{pmatrix} k & -k & 0 \\ -k & 2\mathrm{k} & -k \\ 0 & -k & 2\mathrm{k} \end{pmatrix}$$

The eigen modes $\omega_0^2=\frac{k}{m}$

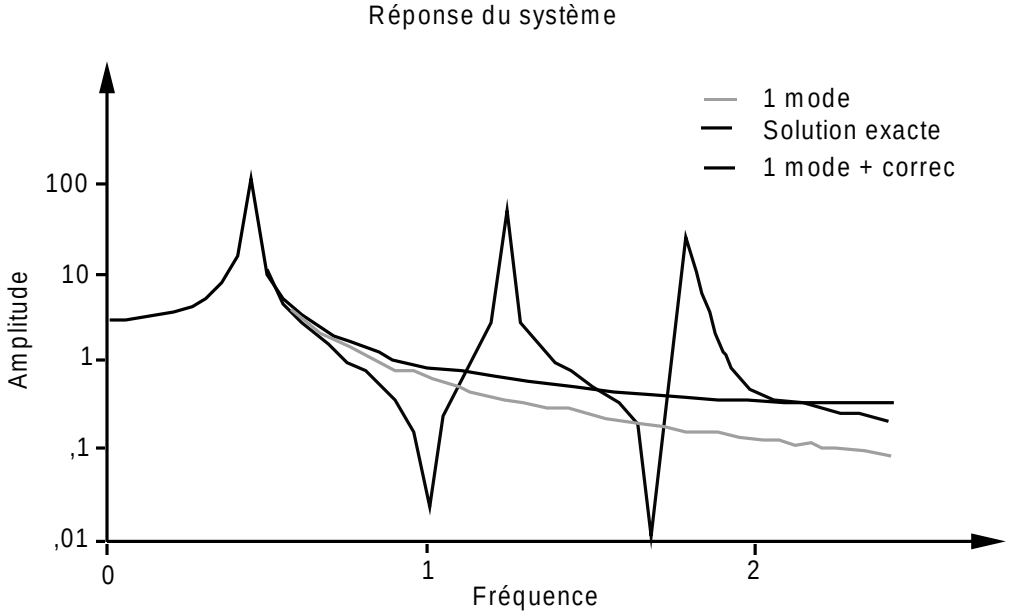
and their pulsation are worth: Let us compare

$$\omega_1^2=0,198\,\omega_0^2,\ m_1=1,841\quad m,\ \boldsymbol{\Phi_1}=\begin{pmatrix} 1 \\ 0,802 \\ 0,445 \end{pmatrix}$$

$$\omega_2^2=1,555\,\omega_0^2,\ m_2=2,863\quad m,\ \boldsymbol{\Phi_2}=\begin{pmatrix} 1 \\ -0,555 \\ -1,247 \end{pmatrix}$$

$$\omega_3^2=3,247\,\omega_0^2,\ m_3=9,296\quad m,\ \boldsymbol{\Phi_3}=\begin{pmatrix} 1 \\ -2,247 \\ 1,802 \end{pmatrix}$$

the responses of the system modelled by only one eigen mode with or without static correction: One notes



that static correction makes it possible to correct the low frequency response, the model with 1 mode plus correction sticks perfectly to the exact solution in low frequency. On the other hand, in high frequency (beyond the first mode), this correction results in over-estimating the response enormously. The use of static correction will have to thus be used with prudence and in the frame of a narrow tape excitation. Let us look at

what the method of addition of a static mode gives. If one applies

a unit force to item 1, the static deformed shape is worth: The projected

$$\Psi_s = \frac{1}{k}\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

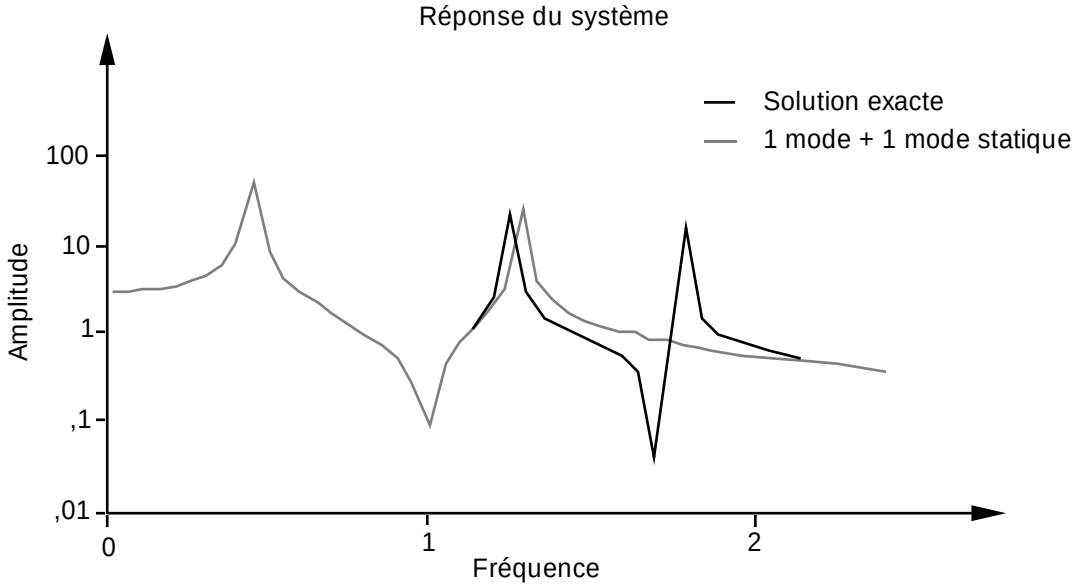
matrixes of mass and stiffness which one obtains are the following ones: this system

$$\hat{\mathbf{M}} = \begin{pmatrix} 1,841\, m & \frac{5,049}{\omega_0^2} \\ \frac{5,049}{\omega_0^2} 14 & \frac{m}{k^2} \end{pmatrix} \quad \text{et} \quad \hat{\mathbf{K}} = \begin{pmatrix} 0,365\, k & 1 \\ 1 & \frac{3}{k} \end{pmatrix}$$

has as eigenfrequencies: and formulates

$$\omega_1^2 = 0,198\,\omega_0^2 \quad \omega_1^2 = 1,667\,\omega_0^2$$

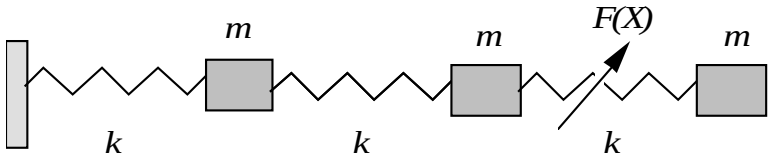
of the system modelled with an eigen mode and a static mode is the following one: One realizes



that one corrects very well in low frequency, (effect of static correction), one models well the dynamics of the system beyond the first mode taken into account. On the other hand the effect of the second mode is badly represented (shift on the frequency). Let us consider

Annexe 2

the discrete system with three masses according to: The stiffness matrixes



and mass are: Let us make this

$$\mathbf{M}=\begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix} \qquad \mathbf{K}=\begin{pmatrix} k & -k & 0 \\ -k & 2\mathrm{k} & -k \\ 0 & -k & 2\mathrm{k} \end{pmatrix}$$

system nonlinear by adding a term of internal force enters and cubic $x1 \quad x2$: Let us seek

$$F=k.\left(x1-x2\right)^3$$

to evaluate the response of this system with a forced excitation of frequency close to the first eigenfrequency of the linear system (one chose), with $\omega=0,18\omega_0$ an important amplitude. In this $F_m=3.k$

configuration, the response of the system cannot be evaluated by the response of the linear system (the cubic term is well too important), it is necessary to implement a nonlinear computation with pseudo-forces as one showed in [§3.2]. One can see

in the ratio [bib8] the curves of the transitory results of this method, in taking into account one or 2 eigen modes of the initial linear system. With only one

eigen mode, one realizes that one makes a relatively important mistake (reaching 50% sometimes), on the other hand it is satisfactory to note that the extrema of vibrations are rather well envisaged. One could have hoped that while exciting in on this side first eigenfrequency it would have is enough to only one eigen mode to model the response of the system, one sees here that it is not the case. As one often notes it for nonlinear systems, the system also answers with the surharmoniques ones of the excitation frequency. On the other hand

, by taking 2 eigen modes to model the response of this structure with 3 d.o.f., one obtains one result very satisfactory (a few % of error on the amplitude), with the eye one has difficulty distinguishing the difference. This shows that by choosing a sufficiently rich projection base one can thanks to a method of pseudo - forces model a dynamic system very well complexes with non-linearities.