Modelization of the shocks and friction in transient analysis by modal recombination

Abstract:

This document describes the physical models of contact with friction between structures and the modelization which is made by it in the algorithm of transient analysis by modal recombination of *Code_Aster* `DYNA_TRAN_MODAL` [U4.54.03]. For various linear connections not - of contact usable, one details the computation of the quantities defining the conditions of contact.

The diagrams of use used are described in [R5.06.04].

Contents

1 Introduction

the problems of shock with friction which interest EDF relate to for example the modelization of tubular structure vibrations maintained by supports with clearances, or separated by clearances weak and thus being able to make contact. The tubes of the steam generators, the pencils of the control rods, the assemblies of fuel are examples of structures which one wishes to model vibrations.

The major consequence of vibrations in the presence of clearance is to cause shocks as well as friction between structure and its bearings or structures from where risks of wear. This document describes the type of non-linearities introduced by the presence of these clearances, as well as the modelization used to take them into account in the algorithm of modal recombination.

2 Relations of contact between two structures

Two relations govern the contact between two structures:

- the relation of unilateral contact which expresses the non-interpenetrability between the solid bodies,
- the relation of friction which governs the variation of the tangential stresses in the contact. One will retain for these developments a simple relation: the friction law of Coulomb.

2.1 Relation of unilateral contact

Are two structures Ω_1 and Ω_2 . One notes $d_N^{1/2}$ the normal distance between structures, $F_N^{1/2}$ the normal reaction force of Ω 1 on Ω 2.

The model of the action and the reaction imposes:

$$F_N^{2/1}=-F_N^{1/2}$$ éq 2.1-1

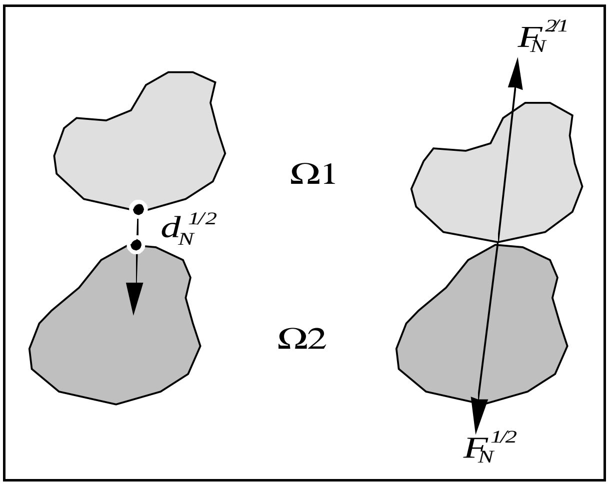


Figure 2.1-a: Outdistance normal and normal reaction

the conditions of unilateral contact, still called conditions of Signorini [bib5], are expressed in the following way:

$$d_N^{1/2} \geq 0 \;\;,\;\; F_N^{1/2} \geq 0 \;\;,\;\; d_N^{1/2} \cdot F_N^{1/2} = 0 \;\; \text{et} \;\; F_N^{1/2} = -F_N^{1/2} \qquad \text{éq 2.1-2}$$

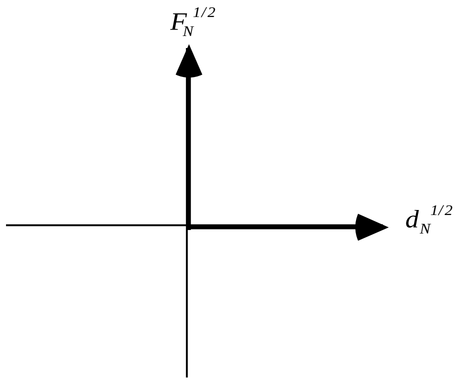


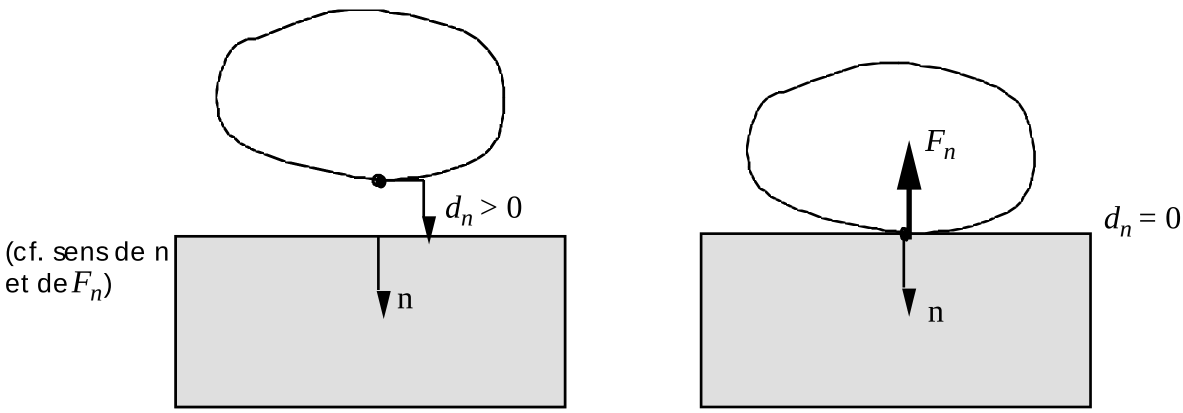
Figure 2.1-b: Graph of the relation of unilateral contact

This graph translates a relation force-displacement which is not differentiable. It is thus not usable in a simple way in an algorithm of dynamic computation.

If one restricts the study with the case of a tubular structure in the presence of an indeformable support, one notes $d_n \left(d_n = d_N^{1/2} \right)$ the normal distance to the support, and F_n the reaction of this last (attention! $F_n = F_N^{2/1} = -F_N^{1/2}$ to see diagram below).

The statement of the conditions of normal contact, expressing the limitation of displacements due to the support is worth:

$$d_n \geq 0 \;\;,\;\; F_n \leq 0 \;\;,\;\; d_n \cdot F_n = 0$$



Appear 2.1-c: Outdistance normal and normal reaction between a structure and a support

2.2 Friction law of Coulomb

the model of Coulomb expresses a tangential limitation the effort of $\boldsymbol{F}_{\boldsymbol{T}}^{\mathbf{1/2}}$ tangential reaction of Ω_1 on Ω_2. Either $\dot{\boldsymbol{u}}_{\boldsymbol{T}}^{\mathbf{1/2}}$ the relative velocity from Ω_1 ratio with Ω_2 in a point of contact and or μ the coefficient of kinetic friction of Coulomb, one has [bib5]:

$$s=\|\boldsymbol{F}_{\boldsymbol{T}}^{\mathbf{1/2}}\|-\mu\cdot F_N^{1/2}\leq 0 \quad , \quad \exists\lambda\, \dot{\boldsymbol{u}}_T^{1/2}=\lambda\,\boldsymbol{F}_{\boldsymbol{T}}^{\mathbf{1/2}} \quad , \quad \lambda\leq 0 \quad , \quad \lambda . s=0 \qquad \text{éq 2.2-1}$$

and the model of the action and the reaction:

$$\boldsymbol{F}_{\boldsymbol{T}}^{\mathbf{2/1}}=-\boldsymbol{F}_{\boldsymbol{T}}^{\mathbf{1/2}} \qquad \text{éq 2.2-2}$$

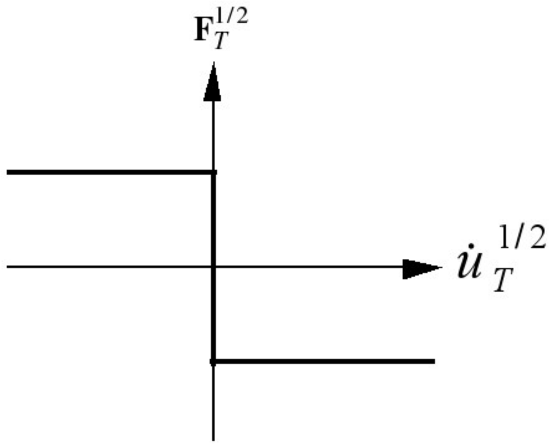


Figure 2.2-a: Graph of the friction law of Coulomb

the graph of the model of Coulomb is also nondifferentiable and is thus not simple to use in a dynamic algorithm.

If one restricts the study with the case of a tubular structure in the presence of an indeformable support, only the tangential stress $\boldsymbol{F}_{\boldsymbol{T}}^{\mathbf{2/1}}=\boldsymbol{F}_{\boldsymbol{T}}$ is used, the friction law is expressed in the following way:

$$s=\|\boldsymbol{F}_{\boldsymbol{T}}\|-\mu\cdot F_N\leq 0 \quad , \quad \exists\lambda\, \dot{\boldsymbol{u}}_T=\lambda\,\boldsymbol{F}_{\boldsymbol{T}} \quad , \quad \lambda\leq 0 \quad , \quad \lambda . s=0$$

A current extension of the model of Coulomb, resulting from the experiment, consists in having two coefficients of kinetic friction: one for the dependancy, noted μ_s , the other for the sliding, noted μ_d , with $\mu_s>\mu_d$. One has then in phase of adhérenceet $\|\boldsymbol{F}_{\boldsymbol{T}}\|\leq\mu_s\cdot F_N$ in phase of sliding $\|\boldsymbol{F}_{\boldsymbol{T}}\|=\mu_d\cdot F_N$.

3 Approximate modelization of the relations of contact between 2 structures by Model

3.1 penalization of normal force of contact

the principle of the penalization applied to the graph of the figure [Figure 2.1-b] consists in introducing a univocal relation $F_N^{1/2}=f_e\left(d_N^{1/2}\right)$ by means of a parameter ϵ . The graph of f_ϵ must tend towards the graph of Signorini when ϵ tends towards zero [bib6].

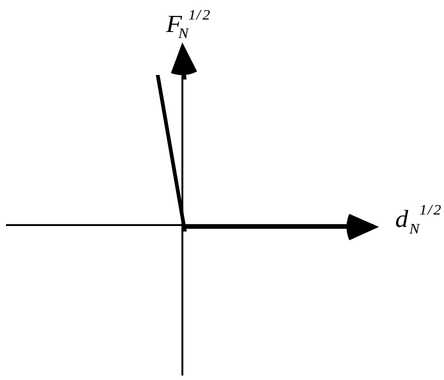
One of the possibilities consists in proposing a linear relation enters $d_N^{1/2}$ and $F_N^{1/2}$:

$$F_N^{1/2}=-\frac{1}{\epsilon}d_N^{1/2}\,\mathrm{si}\,d_N^{1/2}\leq 0 \;\;;\;\; F_N^{1/2}=0\,\mathrm{sinon} \qquad \text{éq 3.1-1}$$

If one notes $K_N=\frac{1}{\epsilon}$ called commonly "**stiffness of shock**", one finds the classical relation, modelling an elastic shock:

$$F_N^{1/2}=-K_N\cdot d_N^{1/2} \qquad \text{éq 3.1-2}$$

the approximate graph of the model of contact with penalization is the following:



Appear 3.1-a: Graph of the relation of unilateral contact approached by penalization

to take account of a possible loss of energy in the shock, one introduces a "damping of shock" C_N the statement of the normal force of contact is expressed then by:

$$F_N^{1/2}=-K_N\cdot d_N^{1/2}-C_N\cdot\dot{u}_N^{1/2} \qquad \text{éq 3.1-3}$$

where $\dot{u}_N^{1/2}$ is the normal velocity relative from Ω_1 ratio to Ω_2. To respect the relation of Signorini (not blocking), one must on the other hand check a posteriori who $F_N^{1/2}$ is positive or null. One will thus take only the positive part $\langle\cdot\rangle^+$ of the statement [éq 3.1-3]:

$$\langle x\rangle^+=x\,\mathrm{si}\,x\geq 0$$
$$\langle x\rangle^+=0\,\mathrm{si}\,x<0$$

The complete relation giving the normal force of contact which is retained for the algorithm is the following one:

$$\mathrm{si}\, d_N^{1/2} \leq 0 \mathrm{F}_N^{1/2} = \langle -K_N \cdot d_N^{1/2} - C_N \cdot \dot{u}_N^{1/2} \rangle^+ \quad , \quad F_N^{1/2} = - F_N^{1/2}$$

$$\mathrm{sinon}\, F_N^{1/2} = F_N^{1/2} = 0 . \qquad \text{éq 3.1-4}$$

3.2 Models of tangential force of contact

the graph describing the tangential force with model of Coulomb is NON-differentiable for the phase of dependancy $\left(\dot{\boldsymbol{u}}_T^{1/2} = 0\right)$. One thus introduces a univocal relation binding relative tangential displacement $\boldsymbol{d}_T^{1/2}$ and the tangential force $\boldsymbol{F}_T^{1/2} = f_\xi\left(\boldsymbol{d}_T^{1/2}\right)$ by means of a parameter ξ. The graph of f_ξ must tend towards the graph of Coulomb when ξ tends towards zero [bib6].

One of the possibilities consists in writing a linear relation enters $\boldsymbol{d}_T^{1/2}$ and $\boldsymbol{F}_T^{1/2}$:

$$\boldsymbol{F}_T^{1/2} - \boldsymbol{F}_T^{1/2^0} = - \frac{1}{\xi} \cdot \left(\boldsymbol{d}_T^{1/2} - \boldsymbol{d}_T^{1/2^0} \right) \qquad \text{éq 3.2-1}$$

If one introduces a "tangential stiffness" $K_T = \frac{1}{\xi}$, one obtains the relation:

$$\boldsymbol{F}_T^{1/2} = \boldsymbol{F}_T^{1/2^0} - K_T \cdot \left(\boldsymbol{d}_T^{1/2} - \boldsymbol{d}_T^{1/2^0} \right) \qquad \text{éq 3.2-2}$$

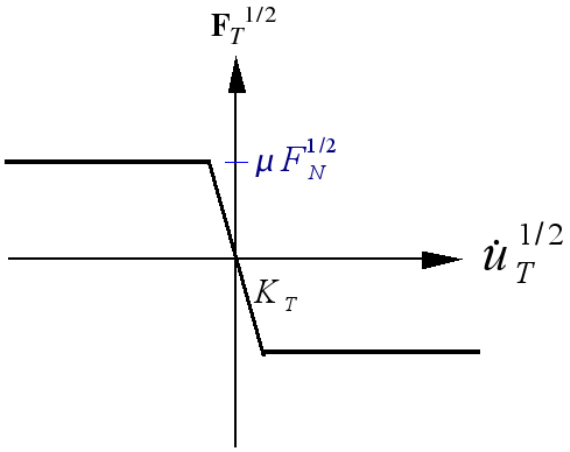
For numerical reasons, related to the dissipation of parasitic vibrations [bib7] in phase of dependancy, one is brought to add a "tangential damping" C_T in the statement of the tangential force. Its final statement is:

$$\boldsymbol{F}_T^{1/2} = \boldsymbol{F}_T^{1/2^0} - K_T \cdot \left(\boldsymbol{d}_T^{1/2} - \boldsymbol{d}_T^{1/2^0} \right) - C_T \cdot \dot{\boldsymbol{u}}_T^{1/2} , \boldsymbol{F}_T^{2/1} = - \boldsymbol{F}_T^{1/2} \qquad \text{éq 3.2-3}$$

It is necessary moreover than this force checks the criterion of Coulomb, that is to say:

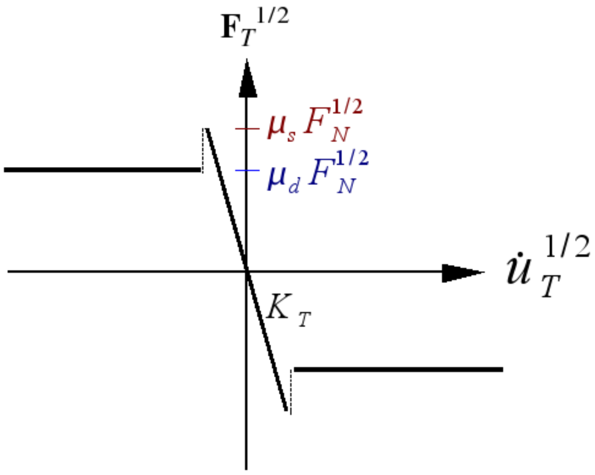
$$\| \boldsymbol{F}_T^{1/2} \| \leq \mu \cdot F_N^{1/2} \,\mathrm{sinon\ on\ applique}\, \boldsymbol{F}_T^{1/2} = - \mu \cdot F_N^{1/2} \cdot \frac{\dot{\boldsymbol{u}}_T^{1/2}}{\| \dot{\boldsymbol{u}}_{T^{1/2}} \|} \quad , \quad \boldsymbol{F}_T^{2/1} = - \boldsymbol{F}_T^{1/2} \qquad \text{éq 3.2-4}$$

the approximate graph of the friction law of Coulomb modelled by penalization is the following:



Appear 3.2-a: Graph of the friction law approached by penalization

In the case of the extension of the model of Coulomb with the distinction between the adhesion coefficient μ_s and the coefficient of friction μ_d, the approximate graph of the model becomes:



Appear 3.2-b: Graph of the alternative of the friction law approached by penalization

4 Types of modelled connections of contact

As it was specified in the paragraph [§2.2], the developments presented here relate to the implementation of nonlinear connections with unilateral contact and friction between 1 node and an obstacle or 2 nodes given.

The nodes in contact are supposed to belong to two slender structures of standard beam or to a beam and an indeformable obstacle. The nodes on which will carry the condition of contact are supposed to be carried by the line average one of the beams.

4.1 Connections between a node and an indeformable obstacle

4.1.1 Connections of contact node on plane obstacle

One considers a slender structure represented by elements of type beam. Its displacement is limited in a point by the presence of an obstacle made up of two infinite half-planes in the direction Y (see [Figure 4.1.1-a]).

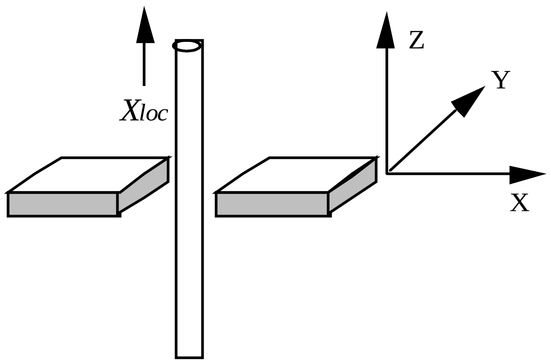


Figure 4.1.1-a : Slender structures with contact node on plane

to analyze the conditions of contact, one places oneself in the reference perpendicular to the axis $Xloc$, direction of neutral fiber or a generator of the beam. Maybe $NO1$, the node of the connection considered on the beam, the geometry of connection contact node on plane (called `PLAN_Y` in *the Code_Aster* [bib3]) is described on the figure below.

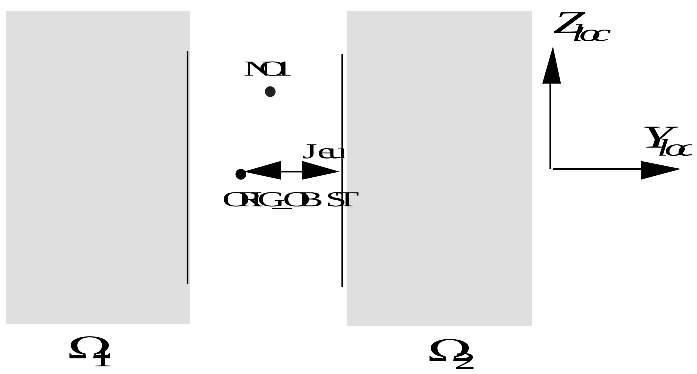


Figure 4.1.1-b : Geometry of connection node on plane obstacle

Are $\begin{pmatrix} Y_{loc} \\ Z_{loc} \end{pmatrix}$ the coordinates of the node $NO1$ in the reference (Y_{loc}, Z_{loc}), the origin of this reference is point `ORIG_OBST`.

The normal distance d_N in this case, by neglecting rotations of the sections is expressed then by:

$$d_N = -|Y_{loc}| + jeu$$ éq 4.1.1-1

the contact in this connection is judicious to take place whatever the shift of Z_{loc} between two structures.

The normal vector $\boldsymbol{n}$ in the reference (Y_{loc}, Z_{loc}) has as components:

$$\boldsymbol{n} = \begin{pmatrix} signe(Y_{loc}) \\ 0 \end{pmatrix}$$ éq the 4.1.1-2

other quantities $\dot{u}_N$ F_N $\dot{\boldsymbol{u}}_{\boldsymbol{T}}$, $\boldsymbol{F}_{\boldsymbol{T}}$ are calculated in a general way as specified with [§3].

4.1.2 Connections of contact node on concave circular obstacle

One considers a hurled structure, represented by elements of type beam. Its displacement is limited in a point by the presence of an obstacle made up of a bored infinite plane of a circular hole (see figure below).

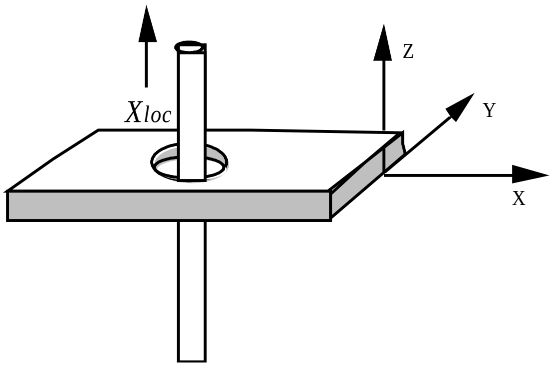


Figure 4.1.2-a : Slender structures with contact node on circular obstacle

to analyze the conditions of contact, one places oneself in the reference perpendicular to the axis X_{loc}, direction of neutral fiber or a generator of the beam. Are $NO1$, the node of the connection considered, the geometry of the connection of contact node on circle (called CERCLE in *Code_Aster* [bib3]) is described on the figure below.

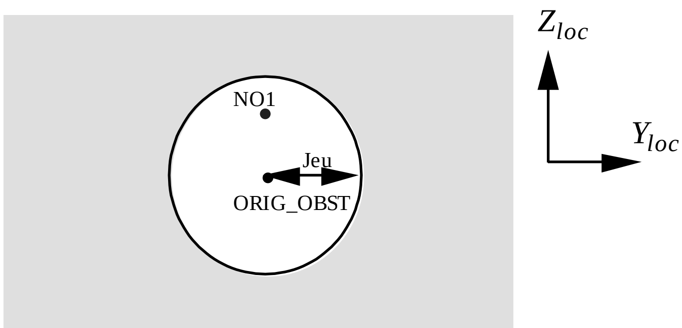


Figure 4.1.2-b : Geometry of connection circular node obstacle

Are $\begin{Bmatrix} Y_{loc} \\ Z_{loc} \end{Bmatrix}$ the coordinates of the node $NO1$ in the reference (Y_{loc}, Z_{loc}), of origin `ORIG_OBST`.

The normal distance d_N, by neglecting rotations of the sections is expressed then by:

$$d_N = -\sqrt{(Y_{loc} - Y\,ORIG_{obst})^2 + (Z_{loc} - Z\,ORIG_{obst})^2} + jeu$$

One poses like normal vector n the vector:

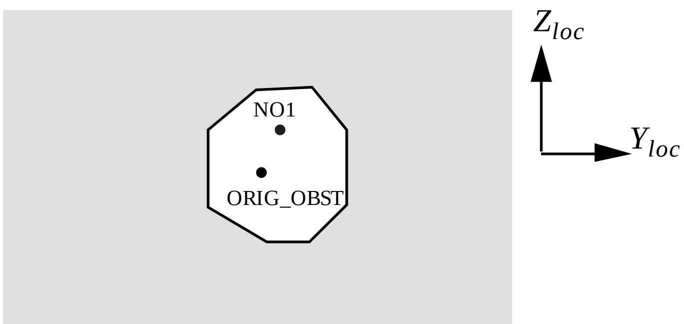
$$n = \frac{ORIG_{obst} - NOEUD1}{\|ORIG_{obst} - NOEUD1\|}$$

jeu is a strictly positive distance.

The other quantities $\dot{u}_N$ F_N $\dot{\boldsymbol{u}}_{\boldsymbol{T}}$, $\boldsymbol{F}_{\boldsymbol{T}}$ are calculated in a general way as specified with [§3].

4.1.3 Connections of contact node on concave obstacle discretized by segments

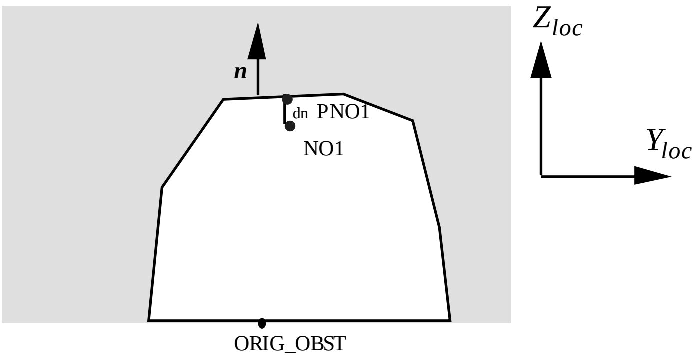
One considers a hurled structure, represented by elements of type beam. Its displacement is limited in a point by the presence of an obstacle made up of a bored infinite plane of a hole of concave form unspecified whom can be discretized in polar coordinates by segments (see figure Ci - below).



Appear 4.1.3-a: Geometry of connection node on discretized concave obstacle

Are $\begin{Bmatrix} Y_{loc} \\ Z_{loc} \end{Bmatrix}$ the coordinates of the node $NO1$ in the reference (Y_{loc}, Z_{loc}), of origin `ORIG_OBST`.

One searches the facet of contact nearest to the node $NO1$, the normal vector n is defined like the direct orthogonal vector with the facet:



Either PNO1 the projection of node NO1 on the facet, the normal distance d_N in this case is worth:

$$d_N = (NO1 - PNO1) . \boldsymbol{n}$$

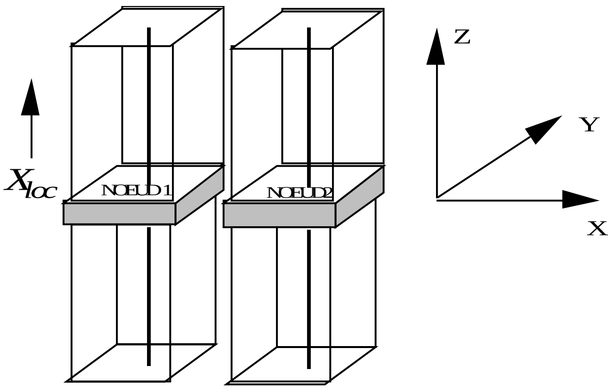
The other quantities $\dot{u}_N$ F_N $\dot{\boldsymbol{u}}_{\boldsymbol{T}}$, $\boldsymbol{F}_{\boldsymbol{T}}$ are calculated in a general way as specified with [§3].

4.2 Connections between two nodes of two deformable structures

4.2.1 Connections of plane contact on plane

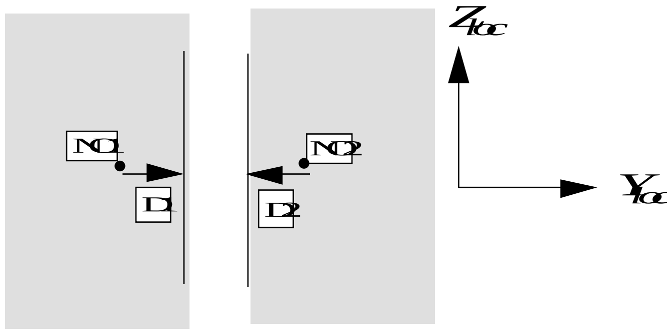
the contacts between assemblies fuel, on the level as of grids of mixture, constitute an example of plane contact on plane (see [Figure 4.2.1-a]).

One thus considers two hurled structures, being able to be modelled by beams of rectangular section on the level as of contact zones.



Appear 4.2.1-a: Slender structures with plane contact on plane

to analyze the conditions of contact, one places oneself in the reference perpendicular to the axis X_{loc}, direction of neutral fiber of the beams. Are $NO1$ and $NO2$, the two nodes of the connection considered, the geometry of connection plane contact on plane (called `BI_PLAN_Y` in *Code_Aster* [bib3]) is described on the figure below.



Appear 4.2.1-b: Geometry of connection plane on plane

Are $\begin{Bmatrix} Y^i_{loc} \\ Z^i_{loc} \end{Bmatrix}$ the coordinates of $NŒUDi$ in the reference (Y_{loc}, Z_{loc}), of origin `ORIG_OBST` (`ORIG_OBST` can be provided by the user, by default `ORIG_OBST` is selected like the medium of the nodes $NO1 \quad NO2$.

The normal distance $d_N^{1/2}$ in this case, by neglecting rotations of the sections is expressed then by:

$$d_N^{1/2} = |Y^1_{loc} - Y^2_{loc}| - D_1 - D_2$$ éq 4.2.1-1

D_1 and D_2 is strictly positive distances.

The contact in this connection is judicious to take place whatever the shift of Z_{loc} between two structures.

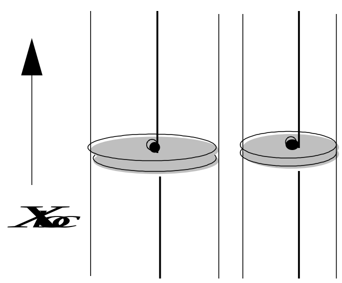
The normal vector $n^{1/2}$ in the reference (Y_{loc}, Z_{loc}) has as components:

$$n^{1/2} = \begin{Bmatrix} signe(Y^2_{loc} - Y^1_{loc}) \\ 0 \end{Bmatrix}$$ éq the 4.2.1-2

other quantities $\dot{u}_N^{1/2} \quad F_N^{1/2} \quad \dot{\boldsymbol{u}}_{\boldsymbol{T}}^{\boldsymbol{1/2}}$, $\boldsymbol{F}_{\boldsymbol{T}}^{\boldsymbol{1/2}}$ are calculated in a general way [§ 2.4].

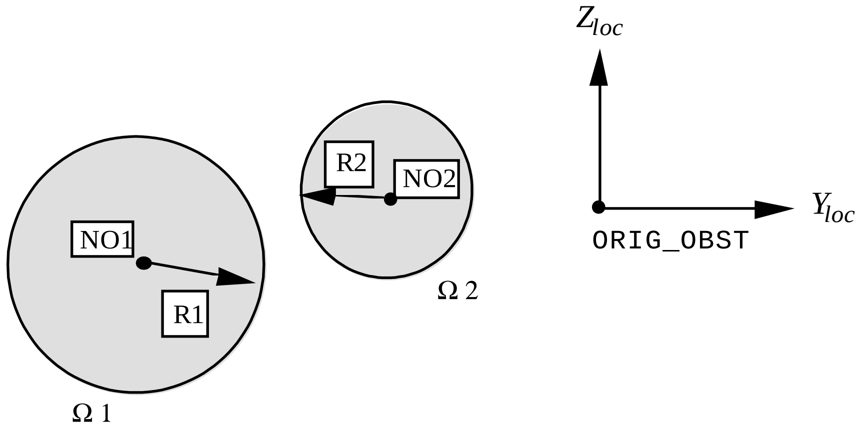
4.2.2 Connections of contact rings on circle

If one considers now two cylinders of circular section, modelled by beam elements. The connection of contact between two nodes of the average lines is supposed to take place between two circles as shown in the figure following:



Appear 4.2.2-a: Slender structures with contact rings on circle

One is placed in the reference perpendicular to the axis X_{loc} parallel with a generator of the cylinders. Are $NŒUD1$ and $NŒUD2$, the two nodes of connection considered, the geometry of connection contact rings on circle (called `BI_CERCLE` in *Code_Aster* [bib3]) is described on geometry Ci - below:



Appear 4.2.2-b: Geometry of connection rings on circle

the normal distance $d_N^{1/2}$ has as a statement:

$$d_N^{1/2}=\sqrt{\left(Y_{loc}^1-Y_{loc}^2\right)^2+\left(Z_{loc}^1-Z_{loc}^2\right)^2}-R_1-R_2$$

One poses like normal vector of Ω_1 worms Ω_2 the vector:

$$n^{1/2}=\frac{\boldsymbol{NOEUD2}-\boldsymbol{NOEUD1}}{\|\boldsymbol{NOEUD2}-\boldsymbol{NOEUD1}\|}$$

5 Use of the localised nonlinear forces of shock and friction in modal recombination

the nonlinear forces expressed above are explicit functions of the position and velocity of the nodes to which the conditions of contact relate.

One chooses to use the technique of pseudo-forces to solve the dynamic problem project. If the direct dynamic system is written:

$$M\ddot{\boldsymbol{X}}_t+C\dot{\boldsymbol{X}}_t+K\boldsymbol{X}_t=\boldsymbol{F}_{ext}(t)+\boldsymbol{F}_{choc}(\boldsymbol{X}_t,\dot{\boldsymbol{X}}_t)$$

The technique of pseudo-forces consists in projecting on the basis of linear system and maintaining the forces nonlinear with the second member.

The dynamic system project takes the shape:

$$\boldsymbol{\Phi}^t M\boldsymbol{\Phi}\ddot{\boldsymbol{\eta}}_t+\boldsymbol{\Phi}^t C\boldsymbol{\Phi}\dot{\boldsymbol{\eta}}_t+\boldsymbol{\Phi}^t K\boldsymbol{\Phi}\boldsymbol{\eta}_t=\boldsymbol{\Phi}^t\boldsymbol{\Phi}_{ext}(t)+\boldsymbol{\Phi}^t\boldsymbol{\Phi}_{choc}(\boldsymbol{\Phi}\boldsymbol{\eta}_t,\boldsymbol{\Phi}\dot{\boldsymbol{\eta}}_t)$$

The problem project is integrated numerically by an explicit diagram.

Recommendations are given in [U2.06.04] for the choice of this base.

6 Accuracy on the use of the non- linearities of shock with friction

non-linearities of shock between a structure and an obstacle or two structures were introduced into the algorithms of modal recombination of *Code_Aster* : an algorithm of Eulerian of order 1 and Devogelaere of order 4 [bib4] [R5.06.04].

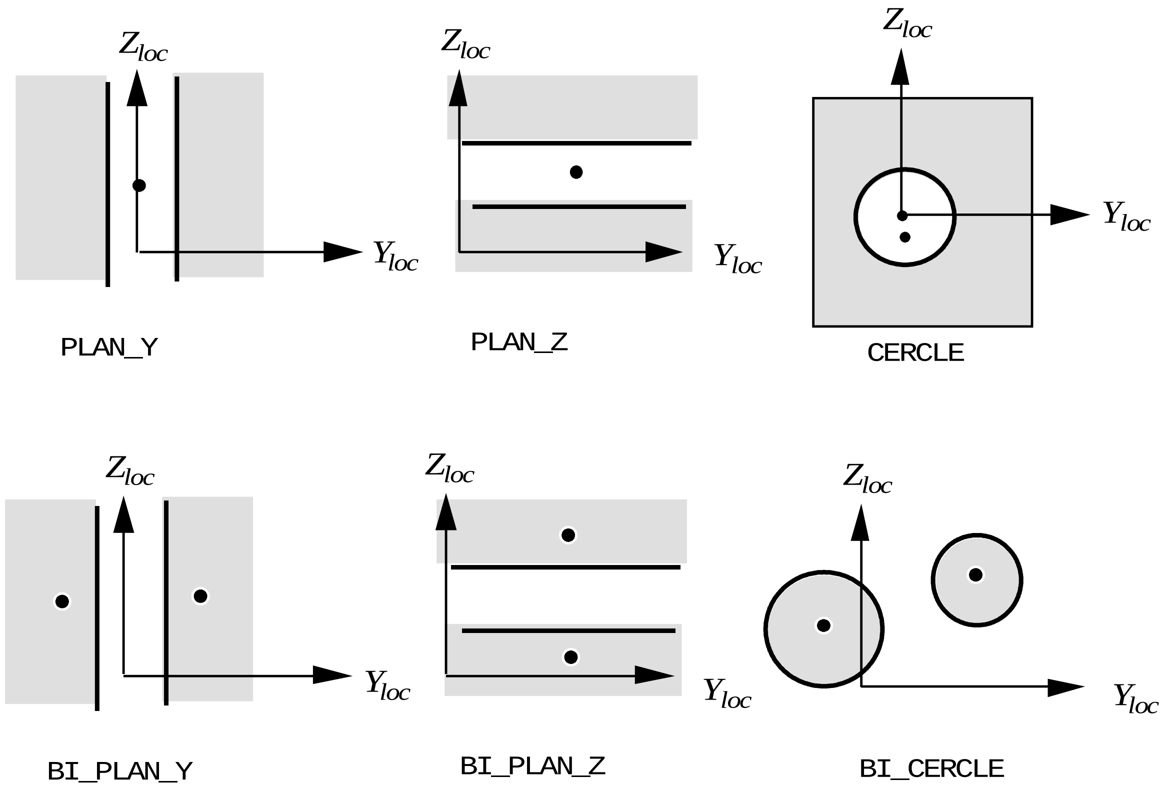
These algorithms are used by the operator `DYNA_TRAN_MODAL` [bib1], [U4.54.03]. The type of connection of shock between the two nodes is specified by a specific command: `DEFI_OBSTACLE` [U4.21.07].

6.1 Definition of the type of connection of shock

the type of connection of shock is a generic notion, which does not comprise any physical information like a distance or unspecified dimension. The type of connection specifies simply the geometrical form of connection considered.

The types of connection with shock with two nodes accepted by the command `DEFI_OBSTACLE` are described by the following key words:

`PLAN_Y`, `PLAN_Z` or `CERCLE`
`BI_PLAN_Y`, `BI_PLAN_Z` or `BI_CERCLE` (see figure below).



Appear 6.1-a: Geometries of connections of shock

prefix BI_ specifies that it is about a connection with two nodes.

6.2 Definition of the local coordinate system for the conditions of contact

the treated structures, being regarded as cylindrical slender (circular or rectangular section), are modelled by beam elements. The contact is treated, as one saw with [§3.1] and [§3.2] in a plane perpendicular to the direction X_{loc} of the generator of the cylinders.

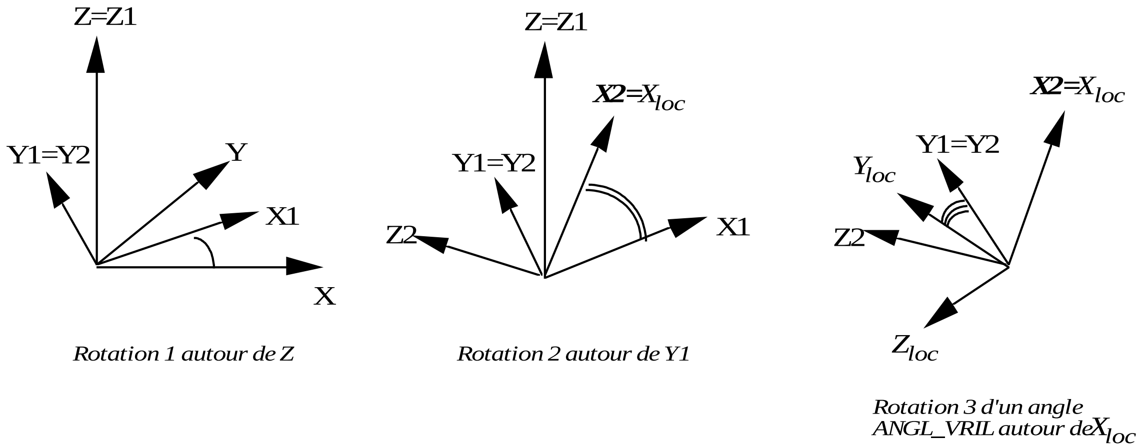
To define this change of reference completely, a local coordinate system is introduced $(X_{loc}, Y_{loc}, Z_{loc})$.

The vector X_{loc} is the vector with 3 components provided behind the key word NORM_OBST.

Using the first two nautical angles, one passes in a single way of the total reference (X, Y, Z) to a reference having X_{loc} like first basic vector (see [Figure 6.2-a] Ci - afterwards). The third rotation whose angle is provided behind key word ANGL_VRIL gives a single correspondence between the principal reference and the local coordinate system.

Note :

the directional sense of this local coordinate system is important because it is in this reference that the conditions of contact are analyzed, and are provided the local positions of the nodes of shock.



Appear 6.2-a: Rotations defining the local coordinate system

operand ORIG_OBST makes it possible to define the origin of the local coordinate system $(Orig, X_{loc}, Y_{loc}, Z_{loc})$. This operand is optional and in theory will not be used in the case as of shocks between two nodes. The code considers whereas the origin is located in the middle of the segment connecting the two nodes.

6.3 Definition of the nodes of connections

One specifies, behind key word NOEU_1 and NOEU_2, the names of the two nodes of the structures on which will carry the conditions of shock. If it is about a connection between a node and an obstacle, only NOEU_1 is indicated.

6.4 Definition of dimensions characteristic of the sections

operand JEU is used for the conditions of contact between a node and an obstacle.

Operands DIST_1 and DIST_2 make it possible to specify dimensions characteristic of the sections of structures surrounding the nodes of shock. In the plane case as of connections on plane, they are the thickness of matter surrounding the node of shock in the direction considered.

In the case of connections rings on circle, it acts of radius of the sections surrounding the nodes of shock.

6.5 Definition of the parameters of contact

the parameters stiffness and damping of shock were introduced with the §3.1 and §3.2, one specifies the key words here making it possible to define them for a given connection.

Operand RIGI_NOR is compulsory, it makes it possible to give the value of normal stiffness of shock K_N.

The other operands are optional.

Operand AMOR_NOR makes it possible to give the value of normal damping of shock C_N.

Operand RIGI_TAN makes it possible to give the value of tangential stiffness K_T.

Operand AMOR_TAN makes it possible to give the tangential value of damping of shock C_T.

Operand COULOMB makes it possible to give the value of the coefficient of Coulomb.

Note :

If a stiffness K_T is defined and that key word AMOR_TAN is absent, the code calculates a damping optimized in order to minimize the residual oscillations in dependancy [bib7]:

$$C_T=2\cdot\sqrt{(k_i+K_T).m_i}-2\cdot x_i\cdot\sqrt{k_i.m_i} \quad ,$$

where I am the index of the dominating mode in the response of the structure (the modal mass most important).

7 Bibliography

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8 of the versions of the document Version

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3	*(EDF/EP/AMV) initial*	Text