Algorithms of temporal integration of operator DYNA_TRAN_MODAL

Summarized

This document describes the diagrams of temporal integration which are used to solve within the space of modes of the problems of transient dynamics in linear mechanics, with, for certain diagrams, the taking into possible account of nonlocalised linearities of shocks type, frictions or fluid blade, and the possible use of the substructuring. The diagrams of NEWMARK (implicit, restricted here with linear), EULER, DEVOGELAERE, and four diagrams explicit with time step adaptive, ADAPT_ORDRE1 and ADAPT_ORDRE2 RUNGE_KUTTA_32 and RUNGE_KUTTA_54 are presented.

temporal

1 Introduction

the goal of the transitory dynamic analysis is to determine according to time the response of a structure, being given a loading external or boundary conditions functions of time, in cases where the effects of inertia cannot be neglected.

In a certain number of physical configurations, one cannot be satisfied with a modal analysis or harmonic and one must carry out a transient analysis. It is in particular the case if:

the history of the phenomenon has an importance in the study,
if the external loading is complex (seisme, excitations multi-components, etc…),
if the system is nonlinear (plasticity, shocks, frictions, etc…).

The methods of analysis transient which can be then used divide into two main categories:

the methods known as of direct integration,
the methods of Ritz, which understand inter alia the recombination of modal projections.

The integration methods direct are thus called because no transformation is carried out on the dynamic system after the discretization by finite elements. They are presented in the document [R5.05.02], algorithms of direct integration of operator DYNA_LINE_TRAN.
The methods of Ritz, on the other hand, proceed to a transformation of the initial dynamic system, by a projection on a subspace of the space of discretization departure. The resolution is done then on a modified system, which, if it is reduced, gives access only one approximation of the response of the real system.

Algorithms of temporal integration on a system in generalized coordinates are used to solve the dynamic problems in mechanics for linear structures, with taking into account possible of nonthe localised linearities the such shocks, frictions or the fluid blades. Certain algorithms allow moreover the substructuring.
These algorithms are programmed in operator DYNA_TRAN_MODAL of *Code_Aster* [U4.53.21].

2 Integration methods temporal of a dynamic problem

2.1 Introduction

One supposes that the equations governing the dynamic equilibrium of solids were discretized by finite elements. One obtains a discrete system of equations which it is a question of integrating in time. For that one chooses a discretization $\{t_i, i \in \mathbb{N}\}$ of the time interval of the study $[0,T]$ and one writes the equilibrium of structure at times t_i .

In a general way these equations take the following shape:

$$\mathbf{M}.\ddot{X}_t + \mathbf{C}.\dot{X}_t + \mathbf{K}.X_t = \mathbf{R}_{ext}(t) + \mathbf{R}_{nl}(X_t, \dot{X}_t, \ddot{X}_t) \quad \textbf{éq 3}$$

where

$\mathbf{M}$ is the mass matrix of the system,
$\mathbf{K}$ is the stiffness matrix of the system,
$\mathbf{C}$ is the damping matrix of the system,
$\mathbf{R}_{ext}(t)$ is the vector of the external forces,
$\mathbf{R}_{nl}(X_t, \dot{X}_t, \ddot{X}_t)$ is the vector of the nonlinear forces.

The damping matrix $\mathbf{C}$ is in general difficult to evaluate because damping is often function of the frequency. It is however frequent the model to simplify the catch in depreciation account by employing proportional damping, or models of Rayleigh.

The methods of reduction of Rayleigh-Ritz are presented in the document [R5.06.01], Methods of Ritz in linear and nonlinear dynamics.

If the term $\mathbf{R}_{nl}\left(X_t, \dot{X}_t, \ddot{X}_t\right)$ is not null, the technique of the pseudo-forces consists in projecting on the basis of linear system and maintaining the forces nonlinear with the second member. The technique of the pseudo-forces is always associated with a diagram of explicit integration. This fact the taking into account of nonthe linearities is available only for explicit diagrams. The addition of nonthe linearities does not modify the form of the equations.

In the method of Ritz, the field of displacement X_t is replaced by its projection on modal base such as $X_t = \Phi \eta_t$ where η_t is the vector of the generalized coordinates and Φ is modal base, generally reduced.

The dynamic system project takes the following shape, with $\eta_t \in \mathbb{R}^p$:

$$\Phi^t . \mathbf{M} . \Phi . \ddot{\eta}_t + \Phi^t . \mathbf{C} . \Phi . \dot{\eta}_t + \Phi^t . \mathbf{K} . \Phi . \eta_t = \Phi^t . \mathbf{R}_{ext}(t) + \Phi^t . \mathbf{R}_{nl}\left(\Phi . \eta_t, \Phi . \dot{\eta}_t, \Phi . \ddot{\eta}_t\right) \quad \textbf{éq 3}$$

When the assumption of Basile does not apply (damping nonproportional), the projected damping matrix is not diagonal. The integration of the coupled system is done then obligatorily with one of the three following diagrams: the implicit scheme NEWMARK, explicit diagram EULER or explicit diagrams ADAPT_ORDRE1 and ADAPT_ORDRE2.

The equation obtained η_t is same form as the equation in X_t . So in the continuation of the document, one will as well use the notation X_t for displacement in generalized coordinates as for displacement in physical space. In the case of operator DYNA_TRAN_MODAL, they are generalized coordinates.

Two classes of method can be distinguished in integration step by step from the balance equations, the integration methods clarifies integration methods and the implicit.

That is to say the linear dynamic system according to integrating in time:

$$\mathbf{M} . \ddot{X}_t + \mathbf{C} . \dot{X}_t + \mathbf{K} . X_t = \mathbf{R}_{ext}(t) \quad \textbf{éq 3}$$

This differential connection of the second order can be brought back to a first order system:

$$\mathbf{N} . \dot{U}_t = \mathbf{H} . U_t + \mathbf{F}_t \quad \textbf{éq 3}$$

where $U_t = \begin{pmatrix} X_t \\ \dot{X}_t \end{pmatrix}$ $\mathbf{N} = \begin{pmatrix} \mathbf{Id} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{pmatrix}$ $\mathbf{H} = \begin{pmatrix} \mathbf{0} & \mathbf{Id} \\ -\mathbf{K} & -\mathbf{C} \end{pmatrix}$, $\mathbf{F}_t = \begin{pmatrix} \mathbf{0} \\ \mathbf{R}_t \end{pmatrix}$

to integrate this system of equations differentials, a discretization is used $\left\{t_i, i \in \mathbb{N}\right\}$, as well as a formula of finite differences to express derivative $\dot{U}_t$.

One will call integration methods clarifies the methods where only the derivative $\dot{U}_t$ utilizes unknowns at time t_{i+1} . From this way the determination of the quantities sought at time t_{i+1} does not result from an inversion of system utilizing the operator $\mathbf{H}$. So moreover, one carries out a "farmhouse-lumping" in order to make the matrix $\mathbf{M}$ diagonal, the determination of U_t is particularly simple. They are the main features of the integration methods there explicit.

The implicit or semi-implicit methods utilize the discretization of U_t at one posterior time with t_i , generally t_{i+1} . The determination of the variables thus passes by the resolution of a system utilizing the operator $\mathbf{H}$.

Two notions are important: consistency, or the order of the diagram of integration, and stability.

The approximations used to obtain the differential operators define consistency, or the order of the diagram of integration. One can indeed consider that the approximation with which one obtains displacement with each time step is related to the order of approximation of derivatives first and seconds compared to time.

The study of stability of a diagram consists in analyzing the propagation of the numerical disturbances in the course of time. A stable diagram preserves a finished solution, in spite of the disturbances, whereas an unstable diagram led to a numerical explosion or divergence of the solution.

To carry out a study of stability of a diagram of integration, one puts this last in the form of a linear recursive system and one determines the particular characteristics of this system. If all the eigenvalues of the operator of recursion are smaller than 1 modulates some, the diagram is stable. If not it is unstable.
The diagrams of integration clarifies are generally conditionally stable, which means that time step must be sufficiently small to ensure the stability of temporal integration.
Certain implicit algorithms have the property to be unconditionally stable, which makes their interest and makes it possible to use one time step arbitrarily large.

The diagrams retained for operator DYNA_TRAN_MODAL are an implicit scheme, explicit NEWMARK, and four diagrams, EULER, DEVOGELAERE, ADAPT_ORDRE1 and ADAPT_ORDRE2 (with time step adaptive). The choice is done by keyword METHODE : "EULER", "DEVOGE", "NEWMARK", "ADAPT_ORDRE1" or "ADAPT_ORDRE2".

2.2 Implicit integration method

2.2.1 Introduction

the implicit methods utilize the resolution of a matric system with the operator previously definite. If the solids are supposed to be elastic linear, that results in the resolution of a linear system to each time step.

The advantage of these methods is their unconditional stability, which enables them time step to integrate the equations of the dynamics with one relatively important while correctly representing the behavior of the modes low in frequency of structure.
An implicit version of the method of NEWMARK, which was programmed in DYNA_TRAN_MODAL for the linear problems.

2.2.2 The method of NEWMARK [bib1]

2.2.2.1 Presentation of diagram

NEWMARK [bib1] introduced two parameters γ and β for the computation positions and velocities to time step $t+\Delta t$:

$$\dot{\mathbf{X}}_{t+\Delta t}=\dot{\mathbf{X}}_t+\Delta t\left[(1-\gamma)\ddot{\mathbf{X}}_t+\gamma\ddot{\mathbf{X}}_{t+\Delta t}\right]$$

$$\mathbf{X}_{t+\Delta t}=\mathbf{X}_t+\Delta t\,\dot{\mathbf{X}}_t+\Delta t^2\left[(\frac{1}{2}-\beta)\ddot{\mathbf{X}}_t+\beta\ddot{\mathbf{X}}_{t+\Delta t}\right]$$

Let us consider the balance equations at time $t+\Delta t$:

$$\mathbf{M}\cdot\ddot{\mathbf{X}}_{t+\Delta t}+\mathbf{C}\dot{\mathbf{X}}_{t+\Delta t}+\mathbf{K}\,\mathbf{X}_{t+\Delta t}=\mathbf{R}_{t+\Delta t}$$

Let us defer the preceding relations while eliminating $\dot{\mathbf{X}}_{t+\Delta t}$ and $\ddot{\mathbf{X}}_{t+\Delta t}$, it comes:

$$\tilde{\mathbf{K}}\cdot\mathbf{X}_{t+\Delta t}=\tilde{\mathbf{R}}_{t+\Delta t} \text{ where } \tilde{\mathbf{K}}=\mathbf{K}+a_0\mathbf{M}+a_1\mathbf{C}$$

$$\tilde{\mathbf{R}}=\mathbf{R}_{t+\Delta t}+\mathbf{C}\cdot\left(a_1\mathbf{X}_t+a_4\dot{\mathbf{X}}_t+a_5\ddot{\mathbf{X}}_t\right)+\mathbf{M}\cdot\left(a_0\mathbf{X}_t+a_2\dot{\mathbf{X}}_t+a_3\ddot{\mathbf{X}}_t\right)$$

with:

$$a_0=\frac{1}{\left(\beta.\Delta t^2\right)} \quad a_1=\frac{\gamma}{\left(\beta.\Delta t\right)} \quad a_2=\frac{1}{\left(\beta.\Delta t\right)} \quad a_3=\frac{1}{2\beta}-1$$

$$a_4=\frac{\gamma}{\beta}-1 \quad a_5=\frac{\Delta t}{2}\left(\frac{\gamma}{\beta}-2\right) \quad a_6=\Delta t.\left(1-\gamma\right) \quad a_7=\gamma.\Delta t$$

2.2.2.2 Complete algorithm of the method of NEWMARK

a) initialization

1) initial conditions $\mathbf{X_0}$, $\dot{\mathbf{X}}_0$ and $\ddot{\mathbf{X}}_0$
2) choice of Δt γ and β computation of the coefficients $a_{1,}\ldots,a_8$ (cf above)
3) to assemble the stiffness matrixes $\mathbf{K}$, of mass $\mathbf{M}$ and damping $\mathbf{C}$
4) to form the effective stiffness matrix $\tilde{\mathbf{K}}=\mathbf{K}+a_0\mathbf{M}+a_1\mathbf{C}$
5) to factorize $\tilde{\mathbf{K}}$

b) with each time step

1) calculating the effective loading $\tilde{\mathbf{R}}$:
$$\tilde{\mathbf{R}}=\mathbf{R}_{t+\Delta t}+\mathbf{C}\cdot\left(a_1\mathbf{X}_t+a_4\dot{\mathbf{X}}_t+a_5\ddot{\mathbf{X}}_t\right)+\mathbf{M}\cdot\left(a_0\mathbf{X}_t+a_2\dot{\mathbf{X}}_t+a_3\ddot{\mathbf{X}}_t\right)$$
2) to solve $\tilde{\mathbf{K}}\cdot\mathbf{X}_{t+\Delta t}=\tilde{\mathbf{R}}_{t+\Delta t}$
3) calculate the velocities and accelerations at time $t+\Delta t$
$$\ddot{\mathbf{X}}_{t+\Delta t}=a_0\left(\mathbf{X}_{t+\Delta t}-\mathbf{X}_t\right)-a_2\dot{\mathbf{X}}_t-a_3\ddot{\mathbf{X}}_t$$
$$\dot{\mathbf{X}}_{t+\Delta t}=\dot{\mathbf{X}}_t+a_6\ddot{\mathbf{X}}_t+a_7\ddot{\mathbf{X}}_{t+\Delta t}$$
4) computation of time step according to: return out of B) 1)

2.2.2.3 Stability conditions of the diagram of NEWMARK

method of NEWMARK used in a rather widespread way in the field of the mechanics, because it makes it possible to choose the order of integration, to introduce or not numerical damping, and has a very good accuracy.

It is unconditionally stable if: $\gamma>0.5$ and $\beta>\frac{(2\gamma+1)^2}{4}$

One introduces a positive numerical damping siet $\gamma>\frac{1}{2}$ negative if $\gamma<\frac{1}{2}$.

Lorsqueet $\gamma=\frac{1}{2}$ $\beta=0$, the formula of NEWMARK is reduced to the diagram central differences. It is thus then an explicit diagram.

A combination very often employed is $\gamma=\frac{1}{2}$ and $\beta=\frac{1}{4}$, because it leads to a diagram of a nature 2, unconditionally stable without numerical damping. In fact the choice was made in operator DYNA_TRAN_MODAL. The diagram of Newmark of this operator is thus implicit.

2.2.2.4 Employment

In DYNA_TRAN_MODAL, this diagram allows integration only linear problems. In the frame of the dynamic substructuring, it makes it possible to employ a modal base calculated by substructuring but it does not support direct computation on the basis of substructure modal bases.

2.2.2.5 Numerical damping of the implicit schemes

the numerical advantage of the direct diagrams of implicit integration lies in the fact that time step can be substantially large compared to the smaller clean period of the system without being likely to cause an instability of the results.

For modes of period clean about time step or lower than time step, the algorithms of integration introduce a strong damping which contributes to erase the contribution of high modes (cf [R5.05.02]).

There is no numerical damping in the cas particulier of the algorithm of NEWMARK with $\beta=\frac{1}{4}$ and $\gamma=\frac{1}{2}$.

On the other hand, implicit algorithms one a significant effect of lengthening of the periods of the response of structure. It is noted that to guarantee a good accuracy on the amplitude and the phase of calculated displacements, it is necessary to respect a criterion close to:

$$\Delta t<\frac{1}{(10\times F_{max})} \text{ à } \frac{1}{(100\times F_{max})}$$

where $F_{\max}$ is the high frequency of the motion which one wishes to capture.

2.3 Explicit integration methods

2.3.1 Introduction

Several integration methods clarifies are presented: a diagram of Eulerian modified of order 1, a diagram of Devogelaere-Fu of order 4 and diagrams with time step adaptive ADAPT and Runge-Kutta. These three methods are available in operator DYNA_TRAN_MODAL. The diagrams are presented by considering only linear forces. However the taking into account of the nonlinear forces from of easily deduced with the technique from the pseudo-forces.

2.3.2 Explicit diagram of Eulerian modified of order 1

2.3.2.1 Presentation

This diagram commonly is called "Eulerian modified" because it is about a very simple but conditionally stable alternative of the diagram of Eulerian of order 1, which is, him, unstable. It is thus a diagram often employed into explicit for the mechanics. In *Code_Aster*, it is quite simply called EULER.

This diagram was used in the modulus STIFF with LICE [bib3], code finite elements of beam, and in code CADYRO [bib4] for the computation of the lines of trees in rotation.

The diagram uses formulates it Eulerian of order 1 to estimate derivative in time, with a formula of front Eulerian for the velocity and a back formula of Eulerian for displacement, as follows:

$$\dot{\mathbf{X}}_{n+1}=\dot{\mathbf{X}}_n+\Delta t\,\mathbf{M}^{-1}\left(\mathbf{R}-\mathbf{K}\,\mathbf{X}_n-\mathbf{C}\,\dot{\mathbf{X}}_n\right)+o(\Delta t)$$
$$\boldsymbol{X}_{n+1}=\boldsymbol{X}_n+\Delta t\,\dot{\boldsymbol{X}}_{n+1}+o(\Delta t)$$

The algorithm is thus the following:

a) initialization: $X_0, \dot{X}_0$ given

b) to each time step:

$$\dot{\boldsymbol{X}}_{n+1}=\dot{\boldsymbol{X}}_n+\Delta t\,.\,\mathbf{M}^{-1}.\left(\mathbf{R}-\mathbf{K}.\,\boldsymbol{X}_n-\mathbf{C}.\,\dot{\boldsymbol{X}}_n\right)$$
$$\boldsymbol{X}_{n+1}=\boldsymbol{X}_n+\Delta t\,.\,\dot{\boldsymbol{X}}_{n+1}$$

2.3.2.2 Order and stability of the diagram

the approximations used in obtaining this diagram are of order 1. One can thus consider that the approximation with which one obtains displacement with each time step is of order 1. It is the consistency of the diagram.

If one puts the diagram of integration in recursive form by eliminating the terms velocity, one obtains the relation of following recurrence (without external force, nor damping):

$$\boldsymbol{X}_{n+1}+\left(\mathbf{M}^{-1}.\mathbf{K}.\Delta t^2-2\right).\boldsymbol{X}_{\boldsymbol{n}}+\boldsymbol{X}_{n-1}=\mathbf{0}$$

The eigenvalue of this diagram are for a system with a degree of freedom:

$$\lambda=\frac{2-\mathbf{M}^{-1}\boldsymbol{K}\,\Delta t^2\pm i\sqrt{\mathbf{M}^{-1}.\mathbf{K}.\Delta t^2\left(4-\mathbf{M}^{-1}.\mathbf{K}.\Delta t^2\right)}}{2} \text{ if } \Delta t<\frac{2}{\mathbf{M}^{-1}.\mathbf{K}}$$

the modulus of the eigenvalues λ is worth 1. One realizes that one is in a limiting but favorable situation. There will not be uncontrolled increase in the error. Without damping, one is right with the limit of stability. It can be an asset for the diagram: it does not introduce numerical dissipation.

If $\Delta t<\frac{2}{\mathbf{M}^{-1}.\mathbf{K}}$ one can show that one of the two eigenvalues has a modulus larger than the unit and thus that the diagram is unstable.

The criterion of stability of diagram EULER is thus: $\Delta t<\frac{2}{\mathbf{M}^{-1}.\mathbf{K}}$.

This study can be wide with a system with finished number of degrees of freedom. In this case, the criterion of stability becomes:

$$\Delta t<\frac{2}{\omega_{\max}}$$

L `analysis can be refined by considering a system with damping [bib13].

2.3.3 Method of Devogelaere-Fu

2.3.3.1 Presentation

to present the algorithm of Devogelaere-Fu, shortened in DEVOGE in *Code_Aster*, one puts the dynamic problem in the form:

$\mathbf{M}.\ddot{\boldsymbol{X}}_t+\mathbf{C}.\dot{\boldsymbol{X}}_t=\mathbf{G}\left(t\,,\boldsymbol{X}_t\right)$ where the matrix $\mathbf{C}$ is supposed to be diagonal.

Displacements and the velocities are calculated as follows:

a) initialization

$$\boldsymbol{X}_{-\frac{1}{2}}=\boldsymbol{X}_{\mathbf{0}}-\frac{\Delta t}{2}\dot{\boldsymbol{X}}_{\mathbf{0}}+\frac{\Delta t^2}{8}\left(4\,\mathbf{M}^{-1}.\mathbf{G}\left(t_0,\boldsymbol{X}_{\mathbf{0}}\right)-\mathbf{M}^{-1}.\mathbf{G}\left(t_0,\boldsymbol{X}_{\mathbf{0}}\right)-\mathbf{M}^{-1}.\mathbf{C}.\dot{\boldsymbol{X}}_0\right)$$

$$\dot{\boldsymbol{X}}_{-\frac{1}{2}}=4\left(4\,\boldsymbol{I}-\Delta t.\mathbf{M}^{-1}.\mathbf{C}\right)^{-1}\left(\left(4\,\boldsymbol{I}+\Delta t.\mathbf{M}^{-1}.\mathbf{C}\right)^{-1}.\dot{\boldsymbol{X}}_0-\Delta t\left(\mathbf{G}\left(t_{-\frac{1}{2}},\boldsymbol{X}_{-\frac{1}{2}}\right)+\mathbf{G}\left(t_0,\boldsymbol{X}_0\right)\right)\right)$$

b) with each time step

$$\boldsymbol{X}_{n+\frac{1}{2}}=\boldsymbol{X}_n+\frac{\Delta t}{2}\dot{\boldsymbol{X}}_n+\frac{\Delta t^2}{24}\left(4\mathbf{M}^{-1}.\mathbf{G}\left(t_n,\boldsymbol{X}_n\right)-\mathbf{M}^{-1}.\mathbf{G}\left(t_{n-\frac{1}{2}},\boldsymbol{X}_{n-\frac{1}{2}}\right)-\mathbf{M}^{-1}.\mathbf{C}\left(4\dot{\boldsymbol{X}}_n-\dot{\boldsymbol{X}}_{n-\frac{1}{2}}\right)\right)$$

$$\dot{\boldsymbol{X}}_{n+\frac{1}{2}}=4\left(4\boldsymbol{I}+\Delta t.\mathbf{M}^{-1}.\mathbf{C}\right)^{-1}\left(\dot{\boldsymbol{X}}_n+\frac{\Delta t}{4}\left(\mathbf{G}\left(t_n,\boldsymbol{X}_n\right)+\mathbf{G}\left(t_{n+\frac{1}{2}},\boldsymbol{X}_{n+\frac{1}{2}}\right)-\mathbf{M}^{-1}.\mathbf{C}.\dot{\boldsymbol{X}}_n\right)\right)$$

$$\boldsymbol{X}_{n+1}=\boldsymbol{X}_n+\frac{\Delta t}{2}\dot{\boldsymbol{X}}_n+\frac{\Delta t_2}{6}\left(4\mathbf{M}^{-1}.\mathbf{G}\left(t_n,\boldsymbol{X}_n\right)+2\mathbf{M}^{-1}.\mathbf{G}\left(t_{n-\frac{1}{2}},\boldsymbol{X}_{n-\frac{1}{2}}\right)-\mathbf{M}^{-1}.\mathbf{C}\left(\dot{\boldsymbol{X}}_n+2\dot{\boldsymbol{X}}_{n+\frac{1}{2}}\right)\right)$$

$$\dot{\boldsymbol{X}}_{n+1}=6\left(6\boldsymbol{I}+\Delta t.\mathbf{M}^{-1}.\mathbf{C}\right)^{-1}.\left(\dot{\boldsymbol{X}}_n+\frac{\Delta t}{6}\left(\mathbf{G}\left(t_{n+1},\boldsymbol{X}_{n+1}\right)+4\mathbf{G}\left(t_{n+\frac{1}{2}},\boldsymbol{X}_{n+\frac{1}{2}}\right)-\mathbf{M}^{-1}.\mathbf{C}.\left(4\dot{\boldsymbol{X}}_{n+\frac{1}{2}}+\dot{\boldsymbol{X}}_n\right)\right)\right)$$

2.3.3.2 Order and stability of the diagram

the diagram is of order 4, the approximations in the writing of temporal derivatives being in. $o\left(\Delta t^4\right)$ It thus has an excellent aptitude for the integration of regular solutions. Its interest is on the other hand less manifest if the functions to be integrated have discontinuities (shocks, friction, etc)

One can show that for an undamped linear system time step guaranteeing stability is worth: $\Delta t<\frac{2\sqrt{2}}{\omega_{\max}}$.

2.3.3.3 Employment

This method is expensive in computing times because it twice requires the evaluating of the vector of the internal forces $\mathbf{G}$, particularly heavy operation. Consequently it is used little in mechanics for direct integration. On the other hand it is employed by the French atomic energy agency [bib2] in the case of the systems projected on modal base.

This diagram allows the taking into account of nonlocalised linearities of shocks type and frictions.
In the frame of the dynamic substructuring, it makes it possible to employ a modal base calculated by substructuring but it does not support direct computation on the basis of substructure modal bases.

2.3.4 Diagrams of integration to time step adaptive

2.3.4.1 Introduction: interest of time step adaptive

To carry out the temporal integration of the transient of a structure in a nonlinear phase always poses problems as for the choice of the step time. The estimate of the error is seldom accessible during integration.

The diagrams of explicit integration oblige to respect time step maximum not to diverge. In the case of nonlinear behavior, this step cannot be a priori *given* and can change with each iteration. When the stiffness very strongly varies, time step constant and very fine to preserve the stability of the diagram led to a very large nombre of iterations and a considerable computing time.

Several algorithms of integration to time step adaptive were thus developed for DYNA_TRAN_MODAL: ADAPT_ORDRE1, ADAPT_ORDRE2, RUNGE_KUTTA_32 and RUNGE_KUTTA54. The two first lean on the diagram of central differences, order 2 and on the diagram of Eulerian, of order 1. The two last, they are diagrams of the family of Runge-Kutta with control of the error. In the continuation of this chapter one will not represent the algorithm of the adaptive diagram of order 1 because it is copied on the diagram of Eulerian. The management of the adaptation of time step is, as for it, the same one as for the adaptive diagram of order 2.

One can notice that this kind of diagram was also programmed in DYNA_LINE_TRAN (cf [R5.05.02]).

2.3.4.2 The diagrams with adaptive steps ADAPT_ORDRE1 and ADAPT_ORDRE2

2.3.4.2.1 Diagram of the central differences with constant step

One presents D" access the diagram of the central differences to constant step on which diagram ADAPT_ORDRE2 is based. He is written as follows:

$$\dot{X}_{n+\frac{1}{2}} = \dot{X}_{n-\frac{1}{2}} + \Delta t . \ddot{X}_n\left(t_n, X_n, \dot{X}_n\right) + o\left(\Delta t^2\right)$$
$$X_{n+1} = X_n + \Delta t \dot{X}_{n+\frac{1}{2}} + o\left(\Delta t^2\right)$$

with the following notations:

It is noted that the velocity is expressed with indices half integers of the discretization in time whereas displacements and accelerations are expressed with the whole indices. Written this way the diagram is of order 2. However acceleration is not immediately computable because the velocity is known only with the half not preceding. To circumvent this difficulty, one can time step use several approximations velocity to whole.

method 1: to suppose that $\ddot{X}_n\left(X_n, \dot{X}_n, t_n\right) \equiv \ddot{X}_x\left(X_n, \dot{X}_{n-\frac{1}{2}}, t_n\right)$ what constitutes a valid approximation if damping is sufficiently weak $\dot{X}_n = \dot{X}_{n-\frac{1}{2}} + o(1)$. If damping is important, the diagram loses then its accuracy of order 2.

method 2: to use an approximation of order 1 for the velocity: what makes it possible $\dot{X}_n = \dot{X}_{n-\frac{1}{2}} + \frac{\Delta t}{2}\ddot{X}_{n-1} + o(\Delta t)$ to preserve order 2 of the diagram.

method 3: to use a diagram of type correct predictor/:

predictor: $\begin{cases} \dot{X}_{n^p} = \dot{X}_{n-\frac{1}{2}} + \gamma \Delta t \ddot{X}_{n-1} \\ \ddot{X}_{n^p} = \ddot{X}\left(t_n, X_n, \dot{X}_{n^p}\right) \end{cases}$

corrector: $\begin{cases} \dot{X}_n = \dot{X}_{n-\frac{1}{2}} + \frac{\Delta t}{2}\left(\beta \ddot{X}_{n^p} + (1-\beta)\ddot{X}_{n-1}\right) \\ \ddot{X}_n = \ddot{X}\left(t_n, X_n, \dot{X}_n\right) \end{cases}$

where α and β are two parameter to be chosen. Park and Underwood [bib6] report that to carry out additional iterations does not improve in a significant way the stability of the diagram.

2.3.4.2.2 Adaptation of the diagram with time step variable

When time step varies, the statements of the preceding paragraph are not valid any more, acceleration $\ddot{X}_n$ being necessarily expressed in the center of the interval $\left[\dot{X}_{n-\frac{1}{2}}, \dot{X}_{n+\frac{1}{2}}\right]$, as one sees it on the diagram below:

To take account of this, the velocity is calculated as follows:

$$\dot{X}_{n+\frac{1}{2}}=\dot{X}_{n-\frac{1}{2}}+\frac{\Delta t_{n-1}+\Delta t_n}{2}\ddot{X}_n$$

Complete diagram ADAPT_ORDRE2 is written then as follows:

1. estimate of $\dot{X}_x$ according to methods 1,2 or 3
2. $\dot{X}_{n+\frac{1}{2}}=\dot{X}_{n-\frac{1}{2}}+\frac{\Delta t_{n-1}+\Delta t_n}{2}\ddot{X}_n\left(t_n, X_n, \dot{X}_n\right)+o\left(\Delta t^2\right)$
3. $X_{n+1}=X_n+\Delta t\,\dot{X}_{n+\frac{1}{2}}+o\left(\Delta t^2\right)$

The order of the diagram is not rigorously any more equal to 2, the diagram having lost its centered character.
More Δt_n and Δt_{n+1} are different, more the order of the diagram tends towards 1. Strong variations of time step thus lead to a loss of accuracy.

It is possible to find statements more complex, which use the velocity or acceleration with the preceding iteration [bib7]. However the formula presented here gives satisfactory results when time step decreases but it cause a drop in the limit of stability when time step increases. The remedy is to control the step so that it increases only slowly.

2.3.4.2.3 Stability and accuracy of the diagram

to study the diagram, one was satisfied with the analysis of a system to only one degree of freedom, free and linear, of own pulsation ω and reduced damping ξ :

$$\ddot{\xi}+2\omega\xi\dot{\xi}+\omega^2\xi=0$$

The approached solution, by means of the diagram with step of constant, is obtained by the relation of following recurrence:

$$\mathbf{A}.Y_n+\mathbf{B}.Y_{n-1}=\mathbf{0}$$

with $Y_n=\begin{bmatrix}\ddot{x}_n\\ \dot{x}_{n+\frac{1}{2}}\\ x_{n+1}\end{bmatrix}$

$\mathbf{A}$ and $\mathbf{B}$ are two matrixes which depend on the method chosen to compute: the contribution of the term of damping. A solution of the form is sought: $Y_n=\lambda Y_{n-1}$.

λ is an eigenvalue of $\mathbf{A}^{-1}.\mathbf{B}$ and can be written in the following form:
$\lambda_c=\exp\left(\omega_c\Delta t\left(-\xi_c\pm i\sqrt{1-\xi_{c^2}}\right)\right)$ where ω_c and ξ_c are the pulsation and reduced damping calculated by the algorithm.

One can compare them with the exact solution $\lambda_e=\exp\left(\omega\Delta t\left(-\xi\pm i\sqrt{1-\xi^2}\right)\right)$, which makes it possible to evaluate the error on the pulsation and the error on damping: $\frac{|\omega_c-\omega|}{\omega}$ and $\frac{|\xi_c-\xi|}{\xi}$.

One studied [feeding-bottle 8] and [bib10] the properties of the diagram according to the method employed to estimate the velocity with the whole steps. It was empirically found that method 3 is at the same time more precise and more stable than methods 1 and 2. The method, without overcost of computation, makes it possible to increase the order of the diagram and gives in most case a better accuracy, except in the event of weak damping. It is however less stable than method 1. It is the method 2 which was finally adopted in diagram ADAPT. These studies made it possible moreover to estimate the number of points per period necessary to guarantee a stable integration. 20 is a value which gives a good safety margin. It is the selected value by default.

2.3.4.2.4 Criteria of adaptation of time step

the preceding developments make it possible to quantify the errors introduced during the computation of a free and linear system. These criteria do not make it possible however to adapt time step. They are indeed delicate to implement in the nonlinear cases and do not take account of the variations of the excitation.

Another approach consists in studying the site error introduced by the diagram using restricted developments.

The exact solution of a system to a degree of freedom checks:

$$\begin{cases} X\left(t+\frac{\Delta t}{2}\right)=X(t)+\frac{\Delta t}{2}\dot{X}(t)+\frac{\Delta t^2}{8}\ddot{X}(t)+\frac{\Delta t^3}{48}\dddot{X}(t)+o(\Delta t^3) \\ X\left(t-\frac{\Delta t}{2}\right)=X(t)-\frac{\Delta t}{2}\dot{X}(t)+\frac{\Delta t^2}{8}\ddot{X}(t)-\frac{\Delta t^3}{48}\dddot{X}(t)+o(\Delta t^3) \end{cases}$$

$$\Rightarrow X\left(t+\frac{\Delta t}{2}\right)=X\left(t-\frac{\Delta t}{2}\right)+\Delta t\,\dot{X}(t)+\frac{\Delta t^3}{24}\dddot{X}(t)+o(\Delta t^3)$$

The formula of integration of the differences thus leads to a truncation error being worth:

$$E_n=\frac{\Delta t^3}{24}\dddot{X}(t_n)\equiv\frac{\Delta t^2}{12}\left(\ddot{X}_n-\ddot{X}_{n-1}\right)$$

One can normalize this error to obtain a relative error:

$$e_n=\frac{\Delta t^2}{12}\frac{|\ddot{X}_n-\ddot{X}_{n-1}|}{X_n}\quad,\quad X_n\neq 0$$

Park and Underwood [bib7] interpreted this error by defining a "apparent pulsation":

$$\omega_{A_n^2}=\frac{\ddot{X}_n}{X_n}$$

Applied to the diagram with central difference, this definition makes it possible to interpret the relative error e_n like a variation of the apparent pulsation:

$$e_n\equiv\frac{\Delta t^2}{12}|\omega_{A_n^2}-\omega_{A_{n-1}^2}|$$

Many algorithms time step use a criterion of adaptation of founded on the truncation error ([bib9], [bib11]). However in the case of a conditionally stable diagram, this method neither to make sure of the stability of integration, nor to guarantee an accuracy for the computation of the transients.

Other methods use an approximation of the instantaneous own pulsation of the system [bib12], using the mass matrixes and of stiffness. They have the default not to adapt to the external forces and their fluctuations in frequency.

It is thus useful to find a criterion which takes account of the two approaches. This is why Park and Underwood introduced the notion of "frequency connect disturbed":

$$f_{AP_n}=\frac{1}{2\mathrm{p}}\sqrt{|\frac{\ddot{X}_n-\ddot{X}_{n-1}}{X_n-X_{n-1}}|}$$

This quantity is interpreted like the "instantaneous" frequency of the system.

In the case of a system with several degrees of freedom, it is necessary to calculate an apparent frequency for each degree of freedom and to take the maximum. Time step can be then selected to respect a minimum of points per apparent period.

If the denominator of the statement of the apparent frequency tends towards zero, this one can become very large and not to have meaning more. This leads to an unjustified refinement when the velocity is cancelled. To cure it a criterion of the type is added:

$$\frac{|X_n-X_{n-1}|}{Dt}<\dot{X}_{\min}\Rightarrow f_{AP_n}=\frac{1}{2\mathrm{p}}\sqrt{|\frac{\ddot{X}_n-\ddot{X}_{n-1}}{\dot{X}_{\min}Dt}|}$$

It is an intermediary between the disturbed apparent frequency and the truncation error. The adequate value of $f_{AP_n}=\frac{1}{2\pi}\sqrt{|\frac{\ddot{X}_n-\ddot{X}_{n-1}}{X_n-X_{n-1}}|}$ is difficult to choose *a priori* and an unsuited value involves an artificial reduction in the apparent frequency. In the case of a system with several degrees of freedom one circumvents this difficulty by employing the "close" degrees of freedom:

$$f_{AP_n}=\max_{1\le i\le \mathrm{nb}\,ddl}\left(\frac{1}{2\pi}\sqrt{|\frac{\ddot{X}^i_n-\ddot{X}^i_{n-1}}{b^i_n}|}\right)$$
$$b^i_n=\max\left(\max_{|i-j|\le lb}|X^j_n-X^j_{\mathrm{n-1}}|,\dot{X}_{\min}\Delta t\right)$$

where lb for example the bandwidth of the stiffness matrix indicates and where can be $\dot{X}_{\min}$ selected very small.

This method appears very effective if indicate X^i_n them of the components physical (displacements). In the case of a projection on modal base it is not relevant to use the close components to compute: the apparent frequency. In this case it is to better return to the first criterion and to use one of the two following methods, specified by key key VITE_MIN :

if VITE_MIN: "NORM" then it is a variable parameter equal to $\frac{\|\dot{X}_n\|}{100}$ ($\|\dot{X}_n\|=\sqrt{\sum_{1\le i\le \mathrm{nb}\,ddl}(\dot{X}^i_n)^2}$). This method gives good performances when the number of degrees of freedom is large and is inapplicable with the case with only one degree of freedom. It is not indicated any more if the order of magnitude velocity is very different from a degree of freedom to another.

if VITE_MIN: "MAXI" then it is a variable and different parameter for each degree of freedom $\dot{X}^j_{\min}=\max_{1\le m\le n}\frac{|\dot{X}^j_m|}{1001}$. This method has the advantage of functioning whatever the number of degree of freedom of the system but it cannot be used if the order of magnitude

velocity varies too much during computation because, in this case, one would obtain systematically: $\frac{|X^j_n - X^j_{n-1}|}{\Delta t} \leq \dot{X}^j_{\min}$.

2.3.4.2.5 Algorithm of the diagram of the central differences with adaptive step

the rules mentioned above make it possible to fix a number of time step desired per period of the response according to the wanted accuracy N . It is adjustable by key key NB_POINT_PERIODE. Time step Δt_n must then be lower than $\frac{1}{Nf_{AP_n}}$. The key key PAS gives the time step initial one Δt_{ini} , and key key PAS_MAXI the time step maximum one not to exceed $\Delta t_{\max}$. In the old versions of the code (before version 10.1.20) the key key PAS defined at the same time the maximum step, key key PAS_MAXI not existing.

The algorithm is described schematically below:

1) initialization: X_0 and $\dot{X}_0$ given

$\Delta t_{-1}=0$, $\Delta t_0=\Delta t_{ini}$ and $\ddot{X}_0=\ddot{X}(t_0, X_0, \dot{X}_0)$

2) initialization of $\dot{X}_{\min}$

3) A each step of tempsinitialisation

1) of the search of time step: $N_{iter}=0$

2) computation of $\dot{X}_{n+\frac{1}{2}}$ then of X_{n+1} :

estimate the velocity (method 2): $\dot{X}_n=\dot{X}_{n-\frac{1}{2}}+\frac{\Delta t}{2}\ddot{X}_{n-1}$

velocity with the semi step: $\dot{X}_{n+\frac{1}{2}}=\dot{X}_{n-\frac{1}{2}}+\frac{\Delta t_{n-1}+\Delta t_n}{2}\ddot{X}_n(t_n, X_n, \dot{X}_n)$

displacement: $X_{n+1}=X_n+\Delta t\,\dot{X}_{n+\frac{1}{2}}$

- computation of acceleration $\ddot{X}_{n+1}$
- computation of the apparent frequency:

$$\frac{|X_n-X_{n-1}|}{\Delta t}\geq \dot{X}_{\min} \Rightarrow f_{AP_n}=\frac{1}{2\pi}\sqrt{|\frac{\ddot{X}_n-\ddot{X}_{n-1}}{X_n-X_{n-1}}|}$$

$$\frac{|X_n-X_{n-1}|}{\Delta t}< \dot{X}_{\min} \Rightarrow f_{AP_n}=\frac{1}{2\pi}\sqrt{|\frac{\ddot{X}_n-\ddot{X}_{n-1}}{\dot{X}_{\min}\Delta t}|}$$

1) checking of the adequacy enters time step and the apparent frequency:

computation of the indicator $err=\Delta t_n Nf_{AP_n}$

if $err\geq 1$ and $N_{itez}<N_{iter\ \max}$ then reduction of time step and new iteration of search of the step: $\Delta t_n \leftarrow 0,75\,\Delta t_n$ $N_{iter}\leftarrow N_{iter}+1$, return in 2)

so $err\leq 0,75$ since more than 5 time step consecutive, then increase in time step $N_{iter}\leftarrow N_{iter}+1$

•the archivage in the solution, possible computation of $\dot{X}_{\min}$ and return in 1) for the following iteration.

2.3.4.2.6 Comments on the parameters of the algorithm

makes It fix a limit higher $N_{iter\ \max}$ by key key NMAX_ITER_PAS than the number of reductions of time step makes it possible to make sure of the convergence of the algorithm in the difficult cases (for example in the event of discontinuity in the external forces).

When the indicator err is higher than 1, time step is multiplied by a fixed factor (0,75 per default but it can be modified by the user thanks to operand COEF_DIVI_PAS). It would have been possible to write directly: $\Delta t_n \rightarrow \frac{1}{err}\Delta t_n = Nf_{AP_n}$, which more intuitive. But this strategy leads to an excessive refinement, the calculated apparent frequency being often largely higher than the real frequency, when the error is large. However, in only one time step, Δt_n can be considerably reduced (factor $0,75^{N_{iter}}$).

On the other hand the increase in time step is always much slower (ratio of intensification per default of 1,1 definable by COEF_MULT_PAS) and takes place only if the indicator is lower than 1 during five time step consecutive. These restrictions are justified by the risks of loss of stability or accuracy of the diagram when time step varies too quickly. A coefficient of 1,2 or 1,3 can allow a faster computation but exposes sometimes at the risks of error.

In short the values by default were validated by many tests and in general give satisfaction in terms of accuracy and stability [bib8].

2.3.4.2.7 Performance of the algorithm

A equal accuracy, the nombre of iterations carried out by an adaptive diagram is at least five times weaker than with a constant step in the phenomena which justify the use of a variable step by the irregular aspect of their evolution (shocks, fluid blade, excitations discontinuous, etc).

The empirical studies showed that the time step adaptive one makes it possible in the successful outcomes to gain a factor two or three in computing times. This diagram makes it possible moreover to control the accuracy of integration by the method of the control amongst points per period of the response.
In the case of the very damped systems, the gains can be even more important (computations five to ten times faster).

On the other hand when time step the "ideal one" is about constant, the use of the adaptive diagram appears useless.

It of course allows the taking into account of nonthe localised linearities of shocks type or frictions, as well as the fluid blades.

In dynamic substructuring, it is compatible as well with the transient analysis on the modal base restored on the whole system or transient computation on the bases distinct from substructures.

2.3.4.3 Diagrams with time step adaptive of the family of Runge-Kutta

the explicit integration methods to a step of the Runge-Kutta type seek to determine an approximate solution of the problem of following Cauchy:

$$\begin{cases} \dot{y}(t) = f(t, y(t)) \quad t \in [t_0, T] \\ y(t_0) = y_0 \quad y_0 \in \mathbb{R} \end{cases}$$

For that, one subdivides the interval $[t_0 : T] : t_0 < t_1 < t_2 < \cdots < t_N$ and one poses $\Delta t = t_{n+1} - t_n$. At every moment of the interval the solution of the problem of Cauchy is given by the integral statement:

$$y(t_{n+1}) = y(t_n) + \int_{t_n}^{t_n + \Delta t} f(t, y(t))$$

the approximate solution $y_n \approx y(t_n)$ proposed by the recursive diagrams of Runge-Kutta take the shape:

$$y_{n+1} = y_n + \Delta t \, \Psi(t_n, y_n, \Delta t)$$

with:

$$\Psi : [t_0, T] \times \mathbb{R}^d \times \mathbb{R} \to \mathbb{R}^d$$

where one identifies the term easily $\Delta t \Psi(t_n, y_n, \Delta t)$ like the integral approximation of the term $\int_{t_n}^{t_n+\Delta t} f(t, y(t))$.

Thus, a method of Runge-Kutta with s stages making it possible to approach the solution of the problem of Cauchy is given by the method of following squaring:

$$\begin{cases} k_1 = f(t_n, y_n) \\ k_2 = f(t_n + c_2 \cdot \Delta t, y_n + a_{21} \cdot k_1) \\ \vdots \\ k_s = f(t_n + c_s \cdot \Delta t, y_n + a_{s1} \cdot k_1 + a_{s2} \cdot k_2 + \cdots + a_{s\ s-1} \cdot k_{s-1}) \\ y_{n+1} = y_n + \Delta t \cdot (b_1 \cdot k_1 + b_2 \cdot k_2 + \cdots + b_s \cdot k_s) \end{cases}$$

a diagram of integration of Runge-Kutta is thus entirely defined by the coefficients b_i, c_i and a_{ij} with, in addition, $c_i = \sum_j a_{ij}$. In the case of an explicit diagram of Runge-Kutta, the coefficients must also check the condition $a_{ij} = 0 \ \forall\, j \geq i$.

In practice, all the coefficients of a diagram of Runge-Kutta are presented in the shape of a table of Butcher like illustrates it below the figure:

$$\begin{array}{c|cccc} c_1 & & & & \\ c_2 & a_{21} & & & \\ c_3 & a_{31} & a_{32} & & \\ \vdots & \cdots & \cdots & \ddots & \\ c_s & a_{s1} & a_{s2} & \cdots & a_{s\ s-1} \\ \hline y_{n+1} & b_1 & b_2 & \cdots & b_s \end{array}$$

Moreover, one will say that a method is of order p if $\forall\, n, 0 \leq n \leq N, e_n = O(\Delta t^{p+1}), \Delta t \to 0$

2.3.4.3.1 Diagrams of Runge-Kutta encased for the control of time step adaptive

During the resolution numerical of the problem of Cauchy describes higher, the choice of time step Δt is determining on the order of magnitude of the total error. Thus, the control of the made mistake e_n during the computation of the solution $y_n \approx y(t_n)$ makes it possible to determine a choice of time step known as "optimal" so as to guaranteeing an increase of the error by a tolerance provided by the user.

With this intention, a classical procedure consists in employing two methods of Runge-Kutta known as encased. The first method of order p with s stages is used to calculate the approximate solution y_{n+1}, whereas the second method of order $\hat{p} < p$ is used to estimate the error $e_n = \| y_{n+1} - \hat{y}_{n+1} \|$ for the control of time step. In general, there is $\hat{p} = p - 1$ and one notes the method $RK\ p(\hat{p})$.
The advantage of this approach is that the approximation of order the weakest $\hat{p}$ uses the same evaluatings of f and thus the same coefficients a_{ij}.

In the frame of operator DYNA_TRAN_MODAL, two explicit diagrams of integration of the Runge-Kutta family are available:

1.Diagram "RUNGE_KUTTA_32" : It is the diagram of Bogacki-Shampine here 3(2). This diagram is of order 3 with 4 stages. It also integrates an approximation of a nature 2 allowing the control of the error. Although there are 4 stages, it uses really 3 stages because he has property FSAL (First Same Ace Last). The table of associated Butcher is presented below:

0				
1/2	1/2			
3/4	0	3/4		
1	2/9	1/3	4/9	
y_{n+1}	2/9	1/3	4/9	0
$\hat{y}_{n+1}$	7/24	1/4	1/3	1/8

2.diagram "RUNGE_KUTTA_54" : It is here the diagram of Dormand-Prince also known under the name of DOPRI 5(4). This diagram is of order 5 with 7 stages. With an approximation of a nature 4 allowing the control of the error. As the preceding diagram it has property FSAL and it uses really only 6 stages. The table of associated Butcher is presented below:

0							
1/5	1/5						
3/10	3/40	9/40					
4/5	44/45	−56/15	32/9				
8/9	19372/6561	−25360/2187	64448/6561	−212/729			
1	9017/3168	−355/33	46732/5247	49/176	−5103/18656		
1	35/384	0	500/1113	125/192	−2187/6784	11/84	
y_{n+1}	35/384	0	500/1113	125/192	−2187/6784	11/84	0
$\hat{y}_{n+1}$	5179/57600	0	7571/16695	393/640	−92097/339200	187/2100	1/40

These two diagrams programmed in operator DYNA_TRAN_MODAL consider a vector of state y_n as being the concatenation of the displacement vector and velocities, namely:

$$y_n = \begin{Bmatrix} X_n \\ \dot{X}_n \end{Bmatrix}$$

In addition, the norm used for the control of the relative error is given by [bib14]:

$$err = \frac{1}{d} \sum_{k=1}^{d} \sqrt{\left(\frac{y_{n+1}^k - \hat{y}_{n+1}^k}{sc^k} \right)^2}$$

where d is the dimension of the vector of state y, y_{n+1}^k formula $\hat{y}_{n+1}^k$ formula k components of the vecteursformule y_{n+1} and $\hat{y}_{n+1}$ respectively. Lastly, sc^k is given by:

$$sc^k = tol \cdot MAX(|y_n^k|, |y_{n+1}^k|)$$

where tol is a relative tolerance given by the user. Thus, the algorithm controls the increase of the relative error by the statement $err \leq 1$.

Lastly, the statement of time step optimal the function of the made mistake is:

$$\Delta t_{opt} = 0{,}9 \cdot \Delta t_n \cdot (\frac{1}{err})^{\frac{1}{p+1}}$$

with $p=5$ for diagram "RUNGE_KUTTA_54" and $p=3$ for diagram "RUNGE_KUTTA_32"

For a better comprehension, the two algorithms are presented in the table below.

1) Initialization: X_0 , $\dot{X}_0$, t_0 and Δt_0 given:
$n=0$, $\Delta t=\Delta t_0$ and $\ddot{X}_0=\ddot{X}(t_0, X_0, \dot{X}_0)$

2) As long as the condition $t_n<T$ is satisfaiteCalcul

1) following state y_{n+1} and of the relative error err by a method of Runge_KuttaSi
2) the condition $err \leq 1$ is satisfied (time step is accepted):

- Archivage of X_0 , $\dot{X}_0$ and $\ddot{X}_0$
- in a following state:
  - $y_n := y_{n+1}$
  - $t_n := t_n + \Delta t$
  - $n := n+1$

1) Computation of Δt_{opt} under the stress $0{,}2 \cdot \Delta t_n \leq \Delta t_{opt} \leq 5 \cdot \Delta t_n$ (in order to avoid brutal changes)
2) Selection of time step $\Delta t := min(\Delta t_{opt}, t_n - T)$

3 Conclusion

As a conclusion, summarized here the various possibilities of temporal integration which offer the operator:

Eulerian ("EULER") clarifies modified to ensure a conditional stability,
Diagram of Newmark ("NEWMARK") parameterized in order to be implicit,
Diagram of Devogelaere-Fu ("DEVOGE") of order 4,
Four explicit adaptive diagrams:

- "ADAPT_ORDRE1" leaning on the diagram of Euleur,
- "ADAPT_ORDRE2" based on the diagram of central differences,
- two diagrams of the Runge-Kutta family: "RUNGE_KUTTA_32" and RUNGE_KUTTA_54'.

The diagram by default is EULER but it is not systematically adapted more. The diagram of NEWMARK available in *the Code_Aster* is implicit and guarantees an unconditional stability but is usable only for purely linear problems.

Diagram DEVOGE is of order 4 and thus is more precise but is it is expensive in computing times.

The adaptive diagrams are more particularly indicated for the problems with located non-linearities, where time step the "ideal one" is not constant during the transient. It is thus the experiment of the

modelization which makes it possible to choose the diagram best adapted to the problem according to the ratio (computing time) /précision.

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5 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
5	*G. JACQUART EDF-R&D/AMV*	initial Text
6.4	*E. BOYERE,* R & D LIGHT */AMA A.C.,* EDF-R&D/TESE, *G. JACQUART DER/AMV*	
10.4	N.GREFFET, F.VOLDOIRE R & D /	Addition of syntax PAS_MAXI for diagram

	AMA	ADAPT (file 14906). Addition of a diagram ADAPT based on the diagram of Eulerian (file 16222).